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Revisiting Galvanomagnetic Effects in Conducting Ferromagnets

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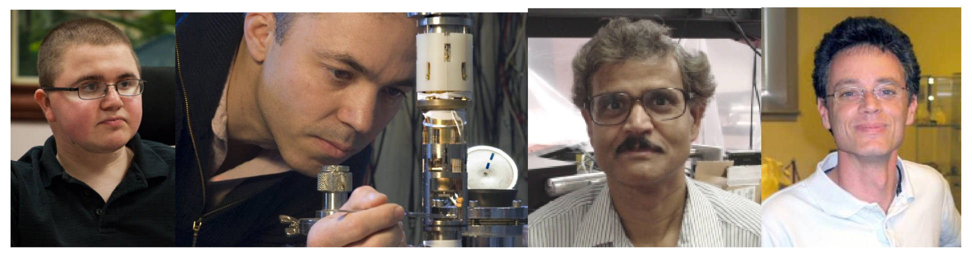

Abstract

The recently proposed coupling between the angular momentum density and magnetic moments is shown to provide a straightforward alternative explanation for galvanomagnetic effects, i.e., for both anisotropic magnetoresistance (AMR) and planar Hall effect (PHE). Such coupling naturally reproduces the general formula associated with AMR and PHE and allows for the occurrence of so-called 'negative AMR'. This coupling also provides a unifying link between AMR, PHE and the anomalous Hall effect (AHE) since this same coupling was previously found to give rise to AHE (Bellaiche et al 2013 Phys. Rev. B **88** 161102).

Fundamental Quantities

The (classical) electromagnetic field angular momentum density is

$$\mathcal{J} = \frac{1}{c^2}\mathbf{r}' \times (\mathbf{E} \times \mathbf{B})$$

for position vector $\mathbf{r}'$, electric field $\mathbf{E}$, and magnetic field $\mathbf{B}$. Consider the magnetization of a conducting electron $\mathcal{M}(\mathbf{r}') = \boldsymbol{\mu}\delta(\mathbf{r}' - \mathbf{r})$ for $\mathbf{r}$ the center of the volume V' around the electron and $\boldsymbol{\mu}$ the electron's magnetic moment with V' sufficiently small so $\mathbf{E}$ and $\mathbf{B}$ are homogeneous. We examine the physical energy resulting from coupling $\mathcal{J}$ with $\boldsymbol{\mu}$:

$$\begin{aligned}\mathcal{E} &= -\frac{a}{2}\int_{V'} [\mathbf{r}' \times (\mathbf{E} \times \mathbf{B})] \cdot \mathcal{M}(\mathbf{r}') d^3r' \\ &= -\frac{a}{2}\mathbf{r} \times (\mathbf{E} \times \mathbf{B}) \cdot \boldsymbol{\mu}\end{aligned}.$$

Research Program

Since its recent proposal [2], this coupling has been used to explain a wide variety of phenomena:

- demonstrates existence of physical energy of magnetic vortices via such coupling and sense of vortex rotation switched by reversing sign of $\mathbf{E} \times \mathbf{B}$ [2]
- characterizes spin current model and existence of magnetic cycloids in multiferroics [2, 3, 4]
- predicts antiferroelectricity-driven magnetic anisotropy [2]
- provides an alternative explanation for the anomalous Hall effect (AHE) that links to more standard semiclassical electron dynamics and complicated Berry phase curvature theories of AHE [5]
- predicts a novel Hall effect that can conceivably be tested by experiment
- predicts novel spintronic effects near interfaces between two different materials [6]
- provides an explanation for the topological Hall effect in magnetic skyrmions (see poster by Charles Paillard)

References

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Contact + Acknowledgement

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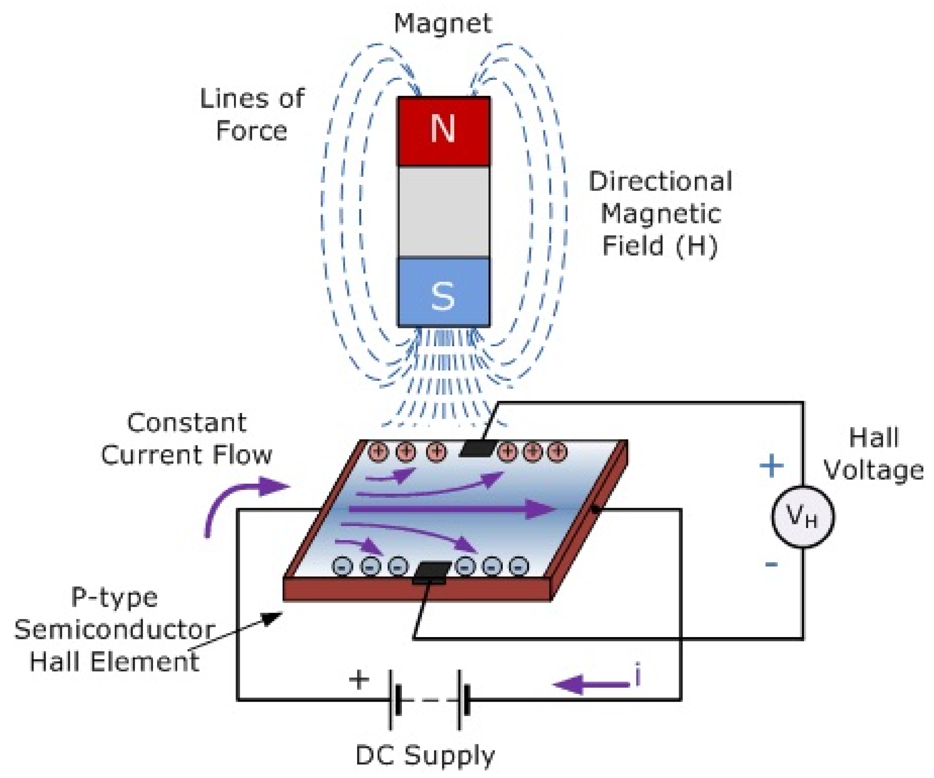
Ordinary Hall Effect+AHE

The Hall effect is the production of a voltage difference across an electrical conductor, *transverse* to an electric current $\mathbf{j} = \sigma\mathbf{E}$ in the conductor (say, in the x-direction) and a magnetic field perpendicular to the current (say, in the z-direction). This effect is frequently used in automative sensors, keyboards, and timing devices. It was discovered by Edwin H. Hall in 1879, and in 1880 he discovered the anomalous Hall effect (AHE) in ferromagnets. In AHE, the conductivity σ_{xy} has a component directly proportional to the magnetization of the material. Modern theories of intrinsic AHE invoke sophisticated ideas of Berry phase curvature, but the proposed coupling of electromagnetic angular momentum density and magnetic moments provides a simpler model readily related to Berry phase curvature.



AMR + PHE

Anisotropic magnetoresistance (AMR) essentially consists of dependence of *longitudinal* resistivity in a ferromagnet on the orientation of magnetization relative to applied electric field. The planar Hall effect (PHE) (also called *transverse* AMR) consists of the formation of a *transverse* electric field for magnetization with *longitudinal* and *transverse* components (if *longitudinal* direction is x and *transverse* direction is z, then this *transverse* electric field is along z-axis) with an applied *longitudinal* electric field. This is expressed mathematically as

$$\begin{aligned}\rho_{xx} &= \rho_{xx,\perp} + (\rho_{xx,\parallel} - \rho_{xx,\perp})\cos^2(\Theta) \qquad (1)\\ \rho_{zx} &= \frac{1}{2}(\rho_{xx,\parallel} - \rho_{xx,\perp})\sin(2\Theta)\end{aligned}$$

where $\rho_{xx,\perp}$ and $\rho_{xx,\parallel}$ are the *longitudinal* resistivities when the magnetization is along the z-axis and x-axis, respectively, while Θ is the angle between this magnetization and the direction of the applied electric field. Prevailing theories of AMR involve spin-orbit interaction. Use of second-order perturbation theory suggests our coupling constant a originates from spin-orbit interaction [2] and is a material dependent constant. In particular, if the sign of a depends on the material, then for some materials a may be negative and hence 'negative AMR' is natural in our coupling-based theory.

Derivations+Extensions

In a ferromagnet consider homogeneous $\mathbf{E}$ and $\mathbf{B}$, the latter restricted to the xz-plane and proportional to magnetization: $\mathbf{E} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y + \hat{\mathbf{z}}E_z$ and $\mathbf{B} = \alpha\mathcal{M} = \alpha(\mathcal{M}_z\hat{\mathbf{z}} + \mathcal{M}_x\hat{\mathbf{x}})$. Embedding force $\mathbf{F} = -\nabla\mathcal{E}$ in a Drude model with the Lorentz force gives the equation of motion

$$\frac{d\mathbf{p}}{dt} = -e\mathbf{E} - \frac{e}{m}\mathbf{p} \times \mathbf{B} + \mathbf{F} - \frac{\mathbf{p}}{\tau},$$

for $\mathbf{p}$ momentum of the electron, e magnitude of electron charge, m mass of the electron, and τ mean time between two successive electronic collisions. Going componentwise in steady-state ($\frac{d\mathbf{p}}{dt} = 0$), multiplying through by $\frac{-e\tau}{Vm}$, summing over all electrons in that volume, noting $M_y = 0$, identifying current density $\mathbf{j} = \frac{-ne}{m}\mathbf{p}$, and considering $j_y = 0 = j_z$ in steady-state, we obtain

$$\begin{aligned}j_x &= \left(\frac{ne^2\tau}{m} + \frac{a\alpha e\tau}{2m}\mathcal{M}_z^2\right)E_x - \frac{a\alpha e\tau}{2m}\mathcal{M}_x\mathcal{M}_zE_z,\\ \frac{e\tau}{m}B_zj_x &= -\frac{ne^2\tau}{m}E_y - \frac{a\alpha e\tau}{2m}E_y\mathcal{M}^2,\\ \frac{a\alpha e\tau}{2m}\mathcal{M}_x\mathcal{M}_zE_x &= \left(\frac{ne^2\tau}{m} + \frac{a\alpha e\tau}{2m}\mathcal{M}_x^2\right)E_z.\end{aligned}$$

Dividing the second equation by $\frac{e\tau}{mE_y}B_z$ derives ordinary and anomalous Hall effect contributions to conductivity σ_{xy}. Resistivities for AMR and PHE are given by $\rho_{xx} = E_x/j_x$ and $\rho_{zx} = E_z/j_x$, obtained from the first and third equations. Consider three cases.
Case (1): $\mathcal{M} = \mathcal{M}_x\hat{\mathbf{x}} \implies \rho_{xx} = E_x/j_x = \frac{m}{ne^2\tau} =: \rho_{xx,\parallel}$
Case (2): $\mathcal{M} = \mathcal{M}_z\hat{\mathbf{z}} \implies$

$$\rho_{xx} = \frac{m}{ne^2\tau}\left(1 + \frac{a\alpha}{2en}\mathcal{M}^2\right)^{-1} =: \rho_{xx,\perp}$$

Case (3): $\mathcal{M} = \mathcal{M}cos\Theta\hat{\mathbf{x}} + \mathcal{M}sin\Theta\hat{\mathbf{z}}$, for Θ angle between the magnetization vector and the x-axis. Let $\xi = \frac{a\alpha}{2en}$ and solve the first and third equations simultaneously:

$$\rho_{xx} = \rho_{\parallel}\frac{1 + \xi\mathcal{M}^2\cos^2\Theta}{1 + \xi\mathcal{M}^2} = \rho_{xx\perp} + (\rho_{xx\parallel} - \rho_{xx\perp})\cos^2\Theta$$

$$\rho_{zx} = \frac{1}{2}\rho_{\parallel}\frac{\xi\mathcal{M}^2\sin 2\Theta}{1 + \xi\mathcal{M}^2} = \frac{1}{2}(\rho_{xx\parallel} - \rho_{xx\perp})\sin 2\Theta.$$

Evidently we have reproduced Eq. (1). Thus coupling between EM angular momentum density and magnetic moments in a Drude model provides a unified explanation for AHE, AMR, and PHE consistent with their respective dependencies on spin-orbit interaction and existing theories of these effects. It is an open question whether a Berry phase curvature theory of AMR is possible, as was the case for AHE; our theory suggests this possibility.