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The Observed Choice Problem in Estimating the Cost of Policies

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Abstract

A policy will be used more heavily when its marginal cost is lower. In a regression setting, this can mean that the equation to be estimated is actually $y_i = \beta_i x(\beta_i)$. The analyst who treats times and places as identical will underestimate the policy’s average cost. OLS is biased towards small coefficients, and instrumental variables should be used.

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It is common to estimate policy effects by looking at data from various locations. Suppose Impact = \( \beta \cdot \text{Policy} \), or

\[
y_i = \beta x_i,
\]

and that the impact is undesirable. In this setting, \( x_i = x(\beta) \) because policies are chosen in recognition of their marginal impacts in particular locations, and \( \beta \) varies across locations. This causes a predictable bias in OLS estimation which I call “the observed choice problem”. This problem has not been directly discussed in the econometrics literature. The closest I have found is Garen (1984). In my own Rasmussen (1996) I develop the problem more fully and apply it to the slightly more complicated case where the policy impact is desirable.

The following three-equation model illustrates the bias.

\[
y_i = \beta_i x_i + \epsilon_i
\]

(2)

\[
\beta_i = \bar{\beta} + v_i
\]

(3)

\[x_i = \gamma_1 + \gamma_2 \beta_i + \gamma_3 z_i + u_i\]

(4)

Assume that: (i) \( \gamma_1 + \gamma_2 \bar{\beta} + \frac{\gamma_3}{N} \sum z_i > 0 \), (ii) \( \bar{\beta} > 0 \), (iii) \( z \) and \( \bar{\beta} \) are nonstochastic, (iv) \( \epsilon, u \) and \( v \) are independent stochastic disturbances with mean zero and finite variance, (v) \( v \) has a symmetric distribution, (vi) \( \gamma_2 < 0 \). Assumptions (i) and (ii) are just normalizations, but (vi) represents that \( y \) is an undesirable impact of \( x \), so \( x \) is used less when \( \beta_i \) is greater.

The OLS estimate of \( \bar{\beta} \) is

\[
\hat{\beta}_{OLS} = \frac{\sum x_i y_i}{\sum x_i^2},
\]

(5)

which has the expectation

\[
E \left( \frac{\sum x_i (\beta x_i + v_i x_i + \epsilon_i)}{\sum x_i^2} \right) = E \left( \frac{\sum x_i^2}{\sum x_i^2} \right) + E \left( \frac{\sum x_i^2 v_i}{\sum x_i^2} \right) + E \left( \frac{\sum x_i \epsilon_i}{\sum x_i^2} \right).
\]

(6)

The first and last terms of (6) equal \( \bar{\beta} \) and 0, and the middle term equals 0 if \( E(x_i^2 v_i) = 0 \). If \( x_i \) and \( v_i \) are independent, OLS is unbiased.
This model, however, violates the OLS assumptions in two ways, each harmless by itself, but bad in combination: random parameters and stochastic regressors. The simpler system of just (2) and (3) has random parameters, and the simpler system of just (2) and (4) (so \( \beta_i = \bar{\beta} \)) has stochastic regressors, but in each of those two simple systems, OLS would be unbiased.

To see that the OLS estimate of \( \bar{\beta} \) is biased in the full system, combine equations (3) and (4) to get

\[
x_i = \gamma_1 + \gamma_2 \bar{\beta} + \gamma_2 v_i + \gamma_3 z_i + u_i.
\]

The critical middle term in equation (6), which for unbiasedness must equal zero, can be written using (7) as

\[
\frac{\sum (\gamma_1 + \gamma_2 \bar{\beta} + \gamma_2 v_i + \gamma_3 z_i + u_i)^2 v_i}{\sum x_i^2}.
\]

The summed quantity in the numerator has the expectation

\[
2 \gamma_2 [\gamma_1 + \gamma_2 \bar{\beta} + \gamma_3 z_i] \sigma_v^2,
\]

since \( E(v^3) = 0 \) by assumption (v), and \( u \) and \( v \) are independent.

Expression (9) has the same sign as \( \gamma_2 [\gamma_1 + \gamma_2 \bar{\beta} + \gamma_3 z_i] \). Summed across the \( n \) observations, this takes the same sign as \( \gamma_2 \), since the term in square brackets is positive by assumption (i). Since \( \gamma_2 < 0 \), \( \beta \) is underestimated.

This is similar to the folk wisdom that estimation problems lead to coefficients being too small. Instrumental variables can be used to solve the observed-choice problem, as I show in Rasmussen (1996), if the analyst can observe \( z \).

Figure 1 illustrates the problem. It shows two localities with their own relationships between policy \( x \) and impact \( y \) depicted as rays through the origin. Localities 1 and 2 have slopes \( \beta_1 \) and \( \beta_2 \), an average slope of \( \bar{\beta} = \frac{(\beta_1 + \beta_2)}{2} \). Policymakers 1 and 2 choose points on their respective rays. If they choose \( x \) ignoring local conditions, \( x_1 \) and \( x_2 \) have the same expected value, and the expected average of the two observations is on the middle ray. This
corresponds to OLS being unbiased.

If, however, $y$ is a cost of $x$, and a steeper slope makes a policymaker choose a lower level of $x$, then Locality 1, with a greater marginal cost, chooses a lower $x$ than Locality 2: $x_1 < x_2$. If the econometrician draws a line through the origin to lie between the two observations and minimize the squared deviations, that line will have a slope of less than $\beta$. OLS underestimates the marginal cost.

![Diagram of Cost vs Policy for Iowa and Wisconsin](image)

**FIGURE 2: ESTIMATING THE MARGINAL COST OF A POLICY**

**REFERENCES**
