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Biographical Sketches of Authors

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Ran Wei, Ph.D., Director, specializes in the areas of finance and accounting. She consults on securities litigation, securities market efficiency, investments, and corporate valuation. Her academic research involved estimation of the securities markets effects of regulation and firm disclosures, and modeling of corporate risk exposures. Her research has been published in several academic journals in economics and finance. She has a B.A. in Economics and Mathematics from Ohio Wesleyan University. She earned her Ph.D. in Applied Economics and a M.A. in Statistics from the Wharton School at the University of Pennsylvania.

Mark Zmijewski, Ph.D., Principal, is a founding partner of Chicago Partners, L.L.C. He is also a deputy dean and chaired professor at The University of Chicago Booth School of Business. Professor Zmijewski has also been the Executive Director of The University of Chicago's Center for Research in Security Prices. He has been a member of The University of Chicago Graduate School of Business faculty since 1984. Professor Zmijewski earned his Ph.D. at the State University of New York at Buffalo, where he also earned his B.S. and M.B.A degrees. Professor Zmijewski has taught courses and consulted in various matters related to accounting, financial analysis, valuation, mergers and acquisitions, securities litigation, damage awards, bankruptcy and solvency. He was awarded the Emory Williams Award for Excellence in Teaching (University of Chicago, 1988) and the Einhorn Award for Excellence in Teaching in the Executive M.B.A. Program (University of Chicago, 1999). Professor Zmijewski has authored several articles and papers for academic research journals related to his academic fields of study in journals such as the Journal of Accounting Research and the Journal of Accounting and Economics, and he has also served on the editorial boards of various academic journals.
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Abstract

Aggregate damages in securities class action lawsuits have two distinct elements: inflation in the prices of the subject company’s securities and the trading behavior of the subject company’s investors. Experts typically have not been able to use statistical and empirically-based methods to measure the trading behavior of all investors in the class. In this paper we outline a methodology, the actual-trader model, to quantify aggregate damages for non-institutional investors. This methodology provides appropriate and sufficient empirical foundation for the aggregate damage calculations in securities class action lawsuits. When implemented appropriately, we believe this methodology meets Daubert and other standards of expert testimony.
I. Introduction

Aggregate damages in securities class action lawsuits have two distinct elements: inflation in the prices of the subject company’s securities and the trading behavior of the subject company’s investors. Experts typically use various statistical and empirically-based valuation methods to measure inflation in the prices during the class period. Appropriate implementation of these methods has been accepted by the courts. Because of a lack of trading data, however, experts typically have not been able to use statistical and empirically-based methods to measure the trading behavior of all investors in the class, which they typically partition into two groups, institutional investors and non-institutional investors.

For institutional investors, experts typically have sufficient information to use actual investment positions and empirically-based methods to measure the trading behavior of each investor. For non-institutional investors, however, experts typically do not have data for specific investors. Instead, experts often use non-empirical, broad-based trading behavior assumptions for the group of non-institutional investors. Using such broad assumptions has various limitations that include a lack of foundation for the

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1 An institutional investor is an investment company that files its security positions as of the end of each calendar quarter with the U.S. Securities and Exchange Commission (Form 13-F filing).

2 Non-institutional investors refer to individual investors and institutions that do not file Form 13-F. In addition to institutional and individual investors, other types of investors include insiders, dealers, and certificate holders. Insiders include directors and officers of a company and stockholders who own more than 10% of equity. Dealers include individuals or firms in the securities industry who buy and sell stocks and bonds as principals rather than as agents. Certificate holders include investors who actually have the physical piece of paper representing ownership in a company.
assumptions underlying the analysis and a lack of individual investor level information. The lack of foundation has resulted in certain courts rejecting the models based on non-empirically broad-based trading behavior assumptions.  

Not observing the trading behavior of individual investors also limits the expert’s ability to measure net inflationary losses. An investor’s net inflationary loss is equal to the investor’s inflationary losses minus the investor’s inflationary gains, with a minimum value of zero. Knowing each investor’s inflationary gains and losses allows the expert to measure the investor’s net inflationary loss. Since the courts have not yet settled the issue of offsetting inflationary gains against losses, defendants typically interpret the law to include offsetting and plaintiffs interpret the law to exclude offsetting.

In this paper, we outline a methodology, the actual-trader model, to quantify aggregate damages for non-institutional investors. The actual-trader model uses a sample of the actual trading history of investors of the subject company. Thus, it provides appropriate and sufficient empirical foundation for the aggregate damage calculations in securities class action lawsuits. When implemented appropriately, we believe this

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3 For example, a judicial ruling in Kaufman v. Motorola, Inc. states, “The proportional trading model does not meet any of the Daubert standards.” Kaufman v. Motorola, Inc., 2000 U.S. Dist. LEXIS 14627 (N.D. Ill. Sept. 21, 2000) at *2-3. Also, in In re Broadcom Corp. Securities Litigation, the Court excluded the presentation of aggregate damages, stating, “The Court does not need to finally decide whether the trading model technique passes the Daubert test. All that need[s] to be decided is that, because of its Daubert shortcomings, and because it is of questionable accuracy, the trading model technique is of significantly questionable reliability.” In re Broadcom Corp. Sec. Litig., 2005 U.S. Dist. LEXIS 12118 (C.D. Cal. June 3, 2005) at *8. In this litigation, the Court directed the jury to determine a daily inflation per share that the Court would use in the claims administration process to calculate total damages.
methodology meets Daubert and other standards of expert testimony. In addition, since this methodology uses actual individual non-institutional investor trading data, experts can use it as a basis for offsetting individual investors’ inflation-based trading gains against losses as they do for institutions.

For ease of exposition, we focus the remainder of this paper on using the actual-trader model to measure damages for common equity shares. However, this methodology is equally applicable to aggregate damages for any security for which sufficient data are available. We organized the remainder of this paper as follows. We provide background and a review of the literature in Section II. In Section III, we describe the actual-trader model and illustrate the effect of using the actual-trader model vis-à-vis other trading models.

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4 In 1993, the Supreme Court used the case of Daubert v. Merrell Dow Pharmaceuticals, Inc. to set forth standards on which the District Courts were to evaluate the admissibility of expert testimony offered under Federal Rule of Evidence 702; these standards include:

1. whether the theory or technique can be and has been tested,
2. whether the technique or theory has been subjected to peer review and publication,
3. the known or potential rate of error, and
4. the “general acceptance” of the theory.


behavior models. In Section IV, we discuss how to use the actual-trader model to offset inflation-based trading gains against losses for non-institutional investors. In Section V, we compare the damages from different trading behavior models using data from actual companies. We present a summary and conclusions in Section VI.

II. Background and Literature Review

Most inflationary losses result from an investor purchasing the security during the class period and holding through the end of the class period (buy and hold securities). The inflationary loss for these investors is equal to the inflation on the date the investor purchased the security. Inflationary losses also result if an investor purchases the security during the class period and later sells it during the class period (in-and-out securities). The loss for these investors is equal to the decrease in inflation between the date the investor purchased the security and the date the investor sold the security. If, however, inflation increased instead of decreased during the in and out period, the investor benefited from the inflation; in other words, the investor had an inflationary gain.

Investors who owned the security before the beginning of the class period and sold the security during the class period (when the security price was inflated) also have an inflationary gain.

More generally, inflation on a share of stock at a date during the class period is equal to the difference between the actual price and the correct (inflation-free) price of the stock on that day.\footnote{While we typically think of inflation as positive (actual price is more than the corrected price), it is also possible for inflation to be negative (actual price is less than corrected price) at different dates during the class period.} Experts generally assume inflation is equal to zero before and
after the dates included in the class period. Damage on a share of stock during the class period is equal to the difference between the inflation on the date the stock was purchased and the inflation on the date it was sold or was assumed to be sold (for stock held as of the end of the class period). Thus, the damage is positive (i.e., the investor suffers a loss) if inflation decreased during the investor’s holding period and negative (i.e., the investor enjoys a gain) if inflation increased during the investor’s holding period.

For example, assume an investor purchased 200 shares of stock on a date in the class period when the stock price was inflated by $3 per share. The investor sold 100 shares later in the class period when the inflation in the stock price was $1 per share. Since inflation decreased during the investor’s holding period, the investor’s damages are equal to $200, or 100 shares sold multiplied by the difference between $3 of inflation per share on the purchase date and $1 of inflation per share on the sale date. If the investor sold the remaining 100 shares when inflation in the stock price increased to $4 per share, the investor benefited from the inflation. The benefit is equal to $100 or 100 shares multiplied by the $1 increase in the inflation from $3 to $4 in the stock price during the investor’s holding period.

This simple example illustrates the importance of both elements in aggregate damage calculations – inflation per share and trading behavior (estimating the dates when each share of stock trades). In addition, experts who offset investors’ inflation-based gains against their inflation-based losses must also estimate all buy and sell trades of each investor in the class. As we discuss below, all of the variants of the proportional trading model make certain assumptions about the number of different types of traders and the

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Note that for purposes of this discussion we are ignoring any potential cap on damages.
relative propensity to trade for each type of trader. Without empirical foundation, these assumptions are simply arbitrary methods for allocating volume.

Experts can measure the trading behavior of institutional investors because institutional investment positions are available on a quarterly basis. More specifically, experts use the change in an institution’s position from the last quarter to measure its net trading during the quarter. Experts then convert quarterly net trading to daily net trading by assuming the net trading occurs in proportion to the subject company’s total trading volume in that security.\(^8\) Since this approach uses trading data for institutional investors, experts measure both an investor’s inflationary gains and losses.

Non-institutional investors include all investors who are part of the class but are not included as institutional investors. Experts do not typically use actual trading history for these investors because such information is not publicly available.\(^9\) Instead, experts make an assumption about the number of different types of investors and the propensity of each type of investor to trade a security once the investor purchases it. Since these

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\(^8\) We can measure institutional volume as the sum of all net daily trades, which represents the minimum volume of institutional trading. Alternatively, we can use the New York Stock Exchange Trade and Quote (TAQ) data, when available, to classify each trade as institutional or non-institutional based on the size of the trade. The TAQ database contains intra-day transactions data (trades and quotes) for all securities listed on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX), as well as NASDAQ National Market System (NMS) and SmallCap issues. In the market microstructure literature, institutional trading behaviors are identified using a cutoff rule. Trades above a cutoff size are classified as institution; trades below the cutoff size are classified as individual. Since the TAQ data provide the size of the trade for each transaction, the database affords us an efficient way to classify each trade.

\(^9\) The experts at Chicago Partners, who use the approach based on this research, are an exception.
assumptions have no empirical foundation for the subject company, certain courts have rejected their use. We review some of these models below.

A. The Proportional Trading Model

The assumption underlying the proportional trading model is that, on any particular day during the class period, each share is equally likely to trade, regardless of when it was purchased. For example, assume the subject company has 1000 shares available for trading (the float) during the class period. Also assume that 100 shares traded on the first day of the class or damage period, 200 shares traded on the second day, 50 shares on the third day, and 300 shares on the fourth and final day of the class period.

As shares trade on a day during the class period, they enter the class on that day; thus, on the first day during the class period, 100 shares traded and all entered the class on that day. In order to measure damages for these shares, we must identify the dates on which these shares are sold. This model assumes that all of the shares that entered the class on Day 1 were retained (not sold) at the end of Day 1. On Day 2 of the class period, 200 of the 1000 shares (or 20%) trade. This model assumes that the probability of a share being sold on Day 2 is 20%. Therefore, 20% of the 100 shares that entered the class on

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11 As noted by McCann and Hsu, the float may need to be adjusted for holdings by insiders and institutions, and for short interest. McCann & Hsu, supra note 10.
Day 1 (20 shares) are assumed to be sold on Day 2, and 80 shares are assumed to be retained as of the end of Day 2.

On Day 3, 50 shares (5%) are traded, which results in the assumption that 5% of the 80 retained shares on Day 2, or 4 shares, are sold on Day 3, and 76 shares are retained as of the end of Day 3. Finally, on Day 4, 300 shares (30%) of the shares trade, which results in the assumption that 30% of the 76 retained shares, or 23 shares, are traded, and 53 shares are retained at the end of the class period. These 53 shares are the shares purchased on Day 1 of the class period and held through the end of the class period, which we call *buy and hold class shares*. We call shares that were purchased and sold during the class period *in-and-out class shares*. In Panel A of Table 1, we show the results of these and similar calculations for the other days in example. Since the maximum loss for a share typically occurs if the share is held through the end of the class period, the key summary statistic is the percentage of the buy and hold shares, which is 521 of the 650 shares (80%) purchased during the class period.

In an effort to test the models, Cone and Laurence used claims data to test the assumptions of the proportional trading model and found that it significantly overestimates claimed damages on the order of one to four times. The weakness of the Cone and Laurence paper is, of course, that their analyses rely on claims submitted by class members, which are a subset of all potential claims. Barclay and Torchio compare

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12 Naturally, we illustrate this model using a simple example; many potential nuances exist in these models such as adjusting for newly issued shares, addressing short sales, and so forth, but the above example describes the basic working of the proportional trading model.

different variations of the proportional trading model to demonstrate that results from the proportional trading model can be consistent with the results of multi-trader models when certain assumptions and parameters are used.\footnote{Michael Barclay & Frank C. Torchio, \textit{A Comparison of Trading Models Used for Calculating Aggregate Damages in Securities Litigation}, 64 n.2-3 LAW & CONTEMP. PROBS. 105-36 (2001).}

\subsection*{B. Accelerated Trading Model}

Experts have proposed several variants on the proportional trading model. For example, Koslow and Simmons and Hoyt described accelerated trading models, and the models were later improved by McCann and Hsu.\footnote{Koslow, \textit{supra} note 5; Richard W. Simmons & Richard C. Hoyt, \textit{Economic Damage Analysis in Rule 10b-5 Securities Litigations}, 3 n.1 J. LEGAL ECON. 71-87 (1993); McCann & Hsu, \textit{supra} note 10.} The accelerated trading model assumes that shares already in the class trade at a greater rate than those not yet traded. One variation of this model assumes that on any given day, shares that have already traded in the class period are more likely to trade than non-class shares by a constant multiple. To understand how this works, return to our previous example and assume that class shares trade at twice the rate of non-class shares.

For Day 2, the model assumes that the 100 shares already in the class are two times more likely to trade than the shares not in the class. The choice of this ratio (twice as likely to trade in this example) has no empirical foundation but is chosen by the expert. This assumption results in a probability of the 100 shares in the class trading on Day 2 of
.364; thus, the model assumes that 36 of the 100 shares traded on Day 2 are in-and-out class shares. At the end of Day 2, the number of shares remaining in the class is equal to the shares traded on Day 1, plus the shares traded on Day 2, minus the Day 1 class shares that are included in the Day 2 volume (264 = 100 + 200 – 36). On Day 3, 50 shares traded and the probability of the shares in the class trading on Day 3 is .079; thus, the model assumes 7.9% of the Day 1 shares remaining in the class at the end of Day 2 (= 100 – 36) traded in Day 3 (5 = .079 x 64).

This assumption results in the propensity to trade for class shares (turnover likelihood) to be higher for the accelerated trading model relative to the proportional trading model, which we observe by comparing the turnover likelihood columns in Panels A and B. The effect of the accelerated trading model is to reduce the number of shares that are purchased and held to the end of the class period relative to the proportional trading model. In our example, the proportional trading model has 521 shares (80%) held to the end of the class period, while the accelerated trading model has

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16 This model calculates the probability on a trading day \( p_t \) of a class share trading as follows, where \( M \) is the adjustment multiple (which is 2 in our example) and \( \text{Shares in Class} \) is the number of shares that are in the class as of the end of the previous trading day:

\[
\begin{align*}
p_t &= \frac{M \times \text{Volume}_t}{\text{Float}_t + (M-1) \times \text{Shares in Class}_t} \\
p_2 &= \frac{2 \times 200}{1000 + (2-1) \times 100} = .364 \\
p_3 &= \frac{2 \times 50}{1000 + (2-1) \times (100 - 36 + 200)} = .079
\end{align*}
\]

17 The calculation of the probability is
457 shares (70%). Recall, however, that the multiple (two in our accelerated trading model example) generally has no empirical foundation.

C. The Two-Trader Model

Beaver, Malernee, and Keeley proposed partitioning traders into two types: active and inactive traders, where the active traders have a much higher propensity to trade than the inactive ones.18 While the two-trader model is more general and more intuitively appealing than the one-trader model, as the authors claim,19 it also requires specific assumptions regarding the propensity of different groups to trade. Implementation of the two-trader model requires an assumption regarding the proportion of the float held by each type of trader and the ratio of the propensity to trade between the two groups. Using the depository records for a particular company, they show that by selecting certain parameters (proportion of shares held and relative propensity to trade) for the two-trader model, they are better able to mimic the observed trading behavior than when they use the proportional trading model.

In Panel C of Table 1, we continue our example but now assume that 20% of the shares are held by active traders and that they are 8 times more likely to trade than inactive traders. Since for both groups a buy and a sell are equally likely, the percentage of shares held by each group remains constant. On Day 1, active traders trade 67 of their 200 available shares while inactive ones trade 33 of their 800 available shares; on Day 2, active traders trade 133 shares and inactive ones trade 67 shares, and so on. Comparing


19 Id. at 6.
the turnover likelihood columns for active and inactive traders, on each day during the class period, active traders are 8 times more likely to trade. The two-trader model reduces the number of buy and hold shares to 398 (61%; 46% for the active traders and 91% for the inactive traders), which is lower than both of the other two models. Again, however, both the number of traders and their propensity to trade are key assumptions for which most experts have little, if any, foundation.

Marcia Kramer Mayer developed an empirically based model that considers a multi-sector, multi-trader environment, and is a more sophisticated version of the two-trader model. She introduced a third type of traders, intraday traders, in addition to the active and inactive traders in the two-trader model. She estimated parameters for this model using brokerage firm data that were obtained in 12 shareholder class actions. However, there is no reason to believe that these parameters from the pooled sample would be reliable in any particular case.

III. The Actual-Trader Model

Since the focus of this paper is to develop an appropriate analysis of trading behavior and not measure inflation in the stock price, we assume we know the inflation per share on each day (\(I_t\)) during the class period. We state damages on the sale of a share of stock, purchased during class period, in terms of the change in inflation during an investor’s holding period. In other words, damage is equal to the inflation on the date

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purchased, \( I_p \), minus inflation on the date the share is sold, \( I_s \). If an investor that purchased a share during the class period and held it until the end of the class period, the damage per share is simply equal to \( I_p \) (in effect, we assume the share is sold with zero inflation).

We measure damages for shares purchased before the beginning of the class period in the same way as for shares purchased during the class period. Damage is equal to the inflation on the date a share is purchased, which is equal to zero for these shares, minus inflation on the date the share is sold. An investor can benefit from selling stock that the investor purchased before the beginning of class period (with zero inflation), if the investor sold that stock during the class period when the stock price was too high (with positive inflation). It is also possible for an investor to be damaged on shares purchased before the beginning of the class period if the shares were sold during the class period when the stock price was too low (in other words, with negative inflation).

To identify non-institutional shares in the class, we exclude investors who are not in the class – such as investors with inside information (insiders) as well as institutional investors, who are in the class, but for whom we estimate aggregate damages separately using the quarterly information about each institutional investor’s security positions. We then collect the trading history for a representative sample of non-institutional investors. We call this sample the observed trading sample. We then apply the distribution of trading behavior of the observed trading sample to the remaining non-institutional investors. We discuss each step in this process below.
A. Aggregate Damages for the Observed Trading Sample

We directly observe the share purchases and sales on each day in the class period for each investor in the observed trading sample. Since we calculate damage on the sale of a share of stock during the class period as the change in inflation during the investor’s holding period, we must identify the specific day a share is purchased (p) and sold (s). For notation purposes, $X_{p,s}$ refers to the number of shares purchased on day $p$ and sold on day $s$. We assume we know both of these dates for the observed trading sample.\(^{21}\) The

\(^{21}\) While the trading records allow direct observation of purchases and sales on each day, they do not necessarily contain information that matches purchases and sales. In other words, we know how many shares were purchased and sold on a day, but we typically do not know the purchase date of the shares sold. Without certain constraints on aggregate damages either imposed by the law or interpretations of the law, an investor’s aggregate economic loss does not change using alternative assumptions about the purchase date of a share of stock sold; thus, inventory flow assumptions are irrelevant to the calculations of aggregate economic loss. However, the Private Securities Reform Act of 1995 specifies limits (90-Day Look Back Period) on recoverable damages that make it necessary to track the damages on individual shares owned by an investor. Private Securities Litigation Reform Act of 1995, 15 USCA §§77z-1 - 78u-5 (Supp 1995).

The choice of an inventory flow assumption is relevant if potential constraints on calculating an investor’s aggregate damages affect damages. The alternative inventory flow assumptions include: (1) specific identification of the share sold (matching each sale with a specific purchase); (2) sell the share purchased the earliest (first-in, first-out assumption, FIFO); and (3) sell the share purchased the latest (last-in, first-out assumption, LIFO). Generally, we cannot implement the specific identification flow assumption because such detailed information does not exist. The U.S. Internal Revenue Service (IRS) requires taxpayers to use the FIFO inventory flow assumption unless the taxpayer specifically identifies shares sold. See, e.g., INTERNAL REVENUE SERVICE, U.S. DEP’T OF THE TREASURY, PUB. NO. 550,
calculation of aggregate damages to individual investor i equals the sum of the product of
the number of shares and the change in the inflation between the purchase date and the
sale date. Using this notation, we calculate an investor’s aggregate inflation-based
trading losses as the sum of the inflation-based trading losses (I_p – I_s > 0):

\[ D_{i,L} = \sum_{p=1}^{P} \sum_{s=1}^{S} X_{p,s} \max(0, (I_p - I_s)) \]

The above formula excludes inflation-based trading gains. We calculate an
investor’s aggregate inflation-based trading gains as the sum of the inflation-based
trading gain (I_p – I_s < 0):

\[ D_{i,G} = \sum_{p=1}^{P} \sum_{s=1}^{S} X_{p,s} \min(0, (I_p - I_s)) \]

If the expert offsets inflation-based trading gains against inflation-based trading
losses, we calculate an investor’s aggregate damages as the sum of all inflation-based
trading losses and inflation-based trading gains:

\[ D_i = \sum_{p=1}^{P} \sum_{s=1}^{S} X_{p,s} (I_p - I_s) = D_{i,L} + D_{i,G} \]

If the expert does not offset inflation-based trading gains against inflation-based
trading losses, aggregate total damages for the observed trading sample accounts (a total

\textit{Investment Income and Expenses} (2008). For measuring damages, however, the LIFO method is
implementable with the available information while FIFO may not implementable.
of I number of accounts) is equal to the sum of all individual investor damages based on
the no offsetting assumption:

\[ D_{\text{Observed}, \text{L}} = \sum_{i=1}^{I} D_{i, \text{L}} \]

If the expert offsets inflation-based trading gains against inflation-based trading
losses, aggregate total damages for the observed trading sample accounts is equal to the
sum of all positive individual investor damages. We only include the positive individual
investor damages because we do not offset an investor’s aggregate gains against other
investor’s aggregate losses:

\[ D_{\text{Observed}, \text{Net}} = \sum_{i=1}^{I} D_{i} \quad \forall \quad D_{i} > 0 \]

We have not discussed the calculation of damages for insiders because they are
not typically included in the class. If insiders were members of the class, we would
calculate damages for insiders in the same manner described above. We have also not
discussed the calculation of damages for institutional investors. Once we measure daily
purchases and sales, we use the above method to measure damages for institutional
investors as well.

**B. Propensity to Trade**

Using the observed trading sample, we measure the propensity of investors to
hold the subject security for certain periods of time. Based on the observed sample data
for the subject security, we calculate a trading probability, \( TP_{h} \) (h is the number of days
after the purchase of the security). For example, \( TP_{0} \) is the proportion of shares sold on
the same day as purchased (day trades); \( TP_{1} \) is the proportion of shares sold one day after
purchase, and so forth. Consider a pure day trader who sells every share purchased on the same day. Then, TP\(_0\) for this trader is 1 and all other TP\(_h\)'s are 0. Consider another example investor that purchases 100 shares and sells 10 shares each day after the purchase date. TP\(_0\) for this trader is 0 because the trader does not sell any shares on the same day the shares were purchased; TP\(_1\) through TP\(_{10}\) is 0.1 because the trader sells 10 out of 100 shares every day. After 10 days, the trading probability for this trader is 0 because as of day 10 the trader sold all shares purchased. We refer to the series of TP\(_h\)'s as the *trading probability vector*. Using the observed sample data, we pool together the trading experience of all observed investors and calculate the average probability of selling a share of stock h days from the date of purchase.

To gain an insight into the trading probability vector, we contrast it with the assumption of the simple proportional trading model. The single trader proportional trading model assumes that the propensity to trade is simply a function of the proportion of the shares that trade on a particular day (for a detailed example, please see Panel A of Table 1). It is indifferent to the holding period. This implies that someone who has held a share for one day would have the same propensity to trade as someone who has held the share for 30 days.

The proportional trading model assumption is inconsistent with the evidence in the literature\(^{22}\) and the data we have reviewed. The trading data we review and discuss later in this paper indicates that investors who purchased a share yesterday can be, on average, 20 times more likely to trade than investors who purchased a share 10 days ago. This is the issue that experts have attempted to address using the accelerated trading

\(^{22}\) *See* Cone & Laurence, *supra* note 13.
model. However, the accelerated trading models are not based on any empirical data, therefore lack foundation. Versions of the accelerated trading model generally only adjust the propensity to trade based on whether a share was purchased before or during the class period.

To measure the trading probability vector, we first organize the observed trading sample data into two groups: sold shares and held shares. We group the sold shares by the number of days in the holding period – that is, the number of days from the purchase date to the sales date. We group the held shares on the number of days from the purchase date to the end of the class period. If the observed trading sample does not contain information past the end of the class period, we must adjust the elements of the trading probability vector for the missing data. For example, we might observe 100 shares purchased one day before the end of the class period but not observe when those shares were sold after the end of the class period; thus, we would not observe how many of these shares were sold 2 days after the purchase date, three days after the purchase date, and so forth. We can adjust our trading probability vector for the missing data by using statistical methods that deal with censored data. We use the Kaplan-Meier estimation method for this adjustment. This is a non-parametric method designed to estimate survival probability with censored data.\(^{23}\)

\(^{23}\) E. L. Kaplan & Paul Meier, *Nonparametric Estimation from Incomplete Observations*, 53 n.282 J. AMER. STATISTICAL ASSOC. 457-81 (1958). The Kaplan and Meier estimation method is commonly used in a wide variety of fields, such as the medical literature and the insurance literature. We can think of the propensity to hold a share of stock as the survival probability of that stock.

More specifically, we let \(t_0 < t_1 < \ldots < t_\delta\) represent the distinct number of days from the date of purchase until the date of sale (uncensored observations) or until the end of the observed period (censored...
C. Unobserved Trading Sample

The next step in our methodology is to measure the level of non-institutional investor trading that is not in the observed trading sample (unobserved trading sample).

We know that for every share purchased there is a share sold and define total trading volume (adjusted for market maker activity) on day \( t \), \( V_t \). Thus, the shares purchased and sold on that day equal total trading volume, \( V_t \). We deduct the volume of purchases by observations). For each \( h = 0, 1, \ldots, \delta \), let \( n_h \) be the number of shares available for sale at \( t_h \). In the absence of censoring, \( n_h \) is the number of shares still not sold just prior to time \( t_h \), the survivors. When censoring is present, \( n_h \) is the number of survivors less the number of shares censored at \( t_h \). Let \( s_h \) be the number of units sold at \( t_h \). The Kaplan-Meier estimator of element \( h \) of the TPV is calculated as follows:

Let \( CTP \) denote the cumulative probability for TPV,

\[
CTP_n = \left[ 1 - \prod_{j=0}^{\delta} \left( 1 - \frac{s_j}{n_j} \right) \right]
\]

Then, \( TP_0 = CTP_n \), and \( TP_h = CTP_h - CTP_{h-1} \) for \( h = 1, 2, \ldots, \delta \).

If \( y \) corresponds to the largest number of days between purchase and sale in the observed data, then \( TP_t = 0 \) if \( t > y \). \( TP_t \) corresponds to the trading probability of the censored data. The Kaplan-Meier estimation method will yield an unchanging estimate of \( TP_t \) for all \( t \) greater than the observed largest number of days between purchase and sales. In order to correct for the under-estimation of \( TP_t \) we apply a method described by Brown, Hollander, and Korwar. For all \( t > y \), the estimate of \( CTP_t = 1 - (1 - CTP_y)^{(t/y)} \).


24 Due to market maker activities, the reported volume for a Nasdaq stock needs to be reduced by at least 50% for damage calculation. See John F. Gould & Allan W. Kleidon, *Market Maker Activity on Nasdaq: Implications for Trading Volume*, 1 n. 11 STAN. J.L. BUS. & FIN. 11-27 (1994). The reported volume for a
insiders, institutions, and the observed trading sample on each day during the class period from total purchases to calculate the remaining purchases, called unobserved purchases. We conduct a similar calculation for sales to calculate unobserved sales. Using statistical and other analytical methods, we use the information in the observed trading sample and the trading probability vector to measure the damages for the unobserved trading sample.

We propose a simulation model that uses the trading probability vector and satisfies the constraint that predicted sales on a particular day cannot exceed the unobserved sale volume on that day. We begin by dividing the unobserved purchases into smaller lot sizes. For example, for 10,000 unobserved purchases on a particular day, create round lots such as 10 lots of 1,000 shares or 100 lots of 100 shares. We round the unobserved purchases and unobserved sales to the nearest lot size so we have no odd lot sizes. Next, we randomly select a lot from the unobserved purchases during the class period and choose a sale date. To select the sale date, we create a cumulative probability distribution from the trading probability vector and randomly select a point along the distribution.

For example, assume a simple case where there are two days in the class period. Also assume a trading probability vector that shows 50% of shares are sold on the same day, 30% trade one day later, 10% trade two days later and the remaining 10% hold for NYSE stock needs to be reduced by about 20%. See Anne-Marie Anderson & Edward A. Dyl, Market Structure and Trading Volume, 28 n. 1 J. FIN. RES. 115-31 (2005).

25 A lot size of one share is, of course, the smallest lot size. The smaller the lot size, results in the more accurate damage calculation, but it also takes the most computation time. We recommend testing alternative lot sizes on a case-by-case basis and decreasing the lot size until the most recent reduction does not materially affect damages.
more than two days. This would imply a cumulative trading probability distribution equal to 50% for a holding period of 0 day, 80% for a holding period of no more than 1 day, and 90% for a holding period of no more than 2 days. We then choose a uniform random number between zero and one and use this cumulative probability distribution to select the sale date. For example, if the random number was .30, it would indicate the share was sold on the same day. Likewise, if the random number was .87, it would imply the share was sold two days after purchase.

Once we select the sales date, we look to see if there are any unobserved sales on that date available. If there are, we assume it was sold according to the prediction and calculate damages accordingly. We then reduce the number of unobserved sales on the sales date by that amount. We next select another purchase and repeat the same procedure as above. We may reach a point in the process when the randomly selected sales date is a date with no remaining unobserved sales. At this point, we select a new random number along the probability distribution and follow the process again until we select a day with unobserved sales.26

IV. Offsetting Inflation-Based Trading Gains Against Inflation-Based Trading Losses for the Unobserved Trading Sample

As we stated in the introduction, without actual trading history, experts cannot offset inflation-based trading gains against inflation-based trading losses at the individual level. We can easily offset such gains against losses when we have observed trading for

26 We assume the trading probability vector is time-invariant. In principle, if sufficient data are available, one could estimate a separate trading probability vector for each purchase date and apply those vectors in the procedure described above.
individual investors using a process as we described above for the observed trading sample.\textsuperscript{27} Below, we show how we can use the observed trading sample to offset such gains against losses for the unobserved trading sample.

Since the observed trading sample is, by assumption, representative of the unobserved trading sample, the proportion of offsetting in the observed trading sample can be reasonably applied to the unobserved trading sample. The proportion of offsetting inflation-based trading gains against inflation-based trading losses in the observed trading sample is equal to the ratio of aggregate damages with offsetting, $D_{\text{Observed, Net}}$, to aggregate damages without offsetting, $D_{\text{Observed, L}}$:

$$
\text{Offsetting Ratio} = \frac{D_{\text{Observed, Net}}}{D_{\text{Observed, L}}} = \frac{\sum_{i=1}^{I} D_i}{\sum_{i=1}^{I} D_{i,L}} \quad \forall \ D_i > 0.
$$

The expert can reasonably apply the above ratio to the aggregate damages for the unobserved trading sample to measure aggregate damages with offsetting inflation-based trading gains against inflation-based trading losses.

V. An Example of the Effect of Using the Actual-Trader Model

In this section, we compare the actual-trader model to several other trading behavior models using a sample of actual trading data for a common stock that was at issue in a securities class action lawsuit.\textsuperscript{28} The relevant portion of the data included the

\textsuperscript{27} The expert can use this same process for institutional investors as well, once the expert converts the quarterly security holding information to daily net trading information.

\textsuperscript{28} Protective order prohibits us from disclosing the companies and securities involved, as well as the names of the brokerage houses from which we received data.
following fields; account number, purchase/sale indicator, purchase/sale date, number of shares traded, and transaction price. We refer to this common stock as the common stock of Company X. These data provide the observed trading sample for Company X and allow us to calculate the propensity of investors to trade given the holding period. There is a large variation in daily purchase volume represented by the observed trading sample. It ranges from 0 to more than 350,000 shares. On average, about 33% of the daily trading volume for individual investors is represented by the observed trading sample.

In Figure 1, we present the chart showing the trading probability vector and the cumulative trading probability for the observed trading sample of Company X. The probability of intraday trading is over 40%.

In Figure 2, we compare the trading probability vector for the observed trading sample to the average trading probability vector that results from our simulation methodology. This chart suggests that the trading probability vector of the simulation samples is quite similar to the observed trading sample. We test whether the trading probability vectors for the two samples are statistically different from each other. We do not reject the null hypothesis that the trading probability vector of the simulation samples is the same as the observed trading sample at any generally accepted level of statistical significance.

In Panel A of Table 2, we show the aggregate damage calculations for non-institutional investors for Company X using the trading behavior models that we discuss.

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29 Marcia Kramer Mayer clearly identified this as important. Table 5 of her study shows that, on average, the observed intraday trading rate was 33.7%. Mayer, supra note 20 at 13.
in this paper. We index aggregate non-institutional damages to $100 under the proportional trading model. By assuming class shares are two times more likely to trade than non-class share, the accelerated trading model reduces the damage to $91. The actual-trader model further reduces damages by 32% to $61.5. This reduction in damage reflects that Company X’s shares have high turnover likelihood. The maximum loss for a share typically occurs if the share is held through the end of the class period. By incorporating the high turnover likelihood estimated from a representative sample of Company X’s shares, the actual-trader model reduces the number of buy and hold shares at the end of the class period. The reduction in damage by the actual-trader model also reflects that 40% of Company X’s shares are purchased and sold within the same day. Recall that damage on a share of stock during the class period is equal to the difference between the inflation on the date the stock was purchased and the inflation on the date it was sold. Thus, shares traded intraday are not damaged. Both the proportional and the accelerated trading model overstate damages because they fail to take into account the Company X’s actual trading propensity.

As we discussed in earlier sections of the paper, since the Actual Trading Model uses trading data for individual investors, experts can use this approach to offset non-institutional investors’ inflation-based trading gains against losses. Offsetting inflation-based trading gains against trading losses is an issue that is not specifically addressed in the law or settled by the courts; thus, plaintiffs typically choose to not offset these gains against these losses while defendants choose to offset. Recall that the only trading behavior model that has sufficient information to offset inflation-based trading gains against inflation-based trading losses is the actual-trader model.
In Panel A of Table 2, we show the aggregate non-institutional damages when we
do not offset inflation-based trading gains against inflation-based trading losses. In Panel
B of Table 2, we show the effect of using the actual-trader model to offset. By offsetting
gains against losses, the observed sample’s damage is reduced to $14.9 from $21.7. This
implies an Offsetting Ratio of 69%. We apply the same ratio to damages for the
unobserved individual investors, and it reduces the damage to $27.4 from $39.8. The
total non-institutional damages using actual-trader model is $42.3, which is 57.7% less
than damages from the proportional trading model.

VI. **Summary and Conclusions**

In this paper, we describe a new methodology to calculate damages for non-
institutional investors, the actual-trader model. This model uses a sample of actual
trading history of investors of the subject company. We believe that by using the
sampling and simulation technique, this model meets the Daubert standard. It also offers
the experts a basis for offsetting individual investors’ inflation-based trading gains
against losses as they do for institutions. Such an approach to class action securities
litigation is long overdue.
## Table 1
Illustration of Assumptions Underlying Alternative Models for Trading Behavior

<table>
<thead>
<tr>
<th>Day in Class Period</th>
<th>Share Available (Float)</th>
<th>Shares Traded (Volume)</th>
<th>Turnover Likelihood for Class Shares</th>
<th>Day Purchased</th>
<th>Assumed Day Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Day 1</td>
<td>Day 2</td>
</tr>
<tr>
<td>Panel A: Share Holding and Sale Matrix for Proportional Trading Model¹</td>
<td>1 1000</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2 1000</td>
<td>200</td>
<td>200</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>3 1000</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>4 1000</td>
<td>300</td>
<td>300</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>300</td>
<td>350</td>
<td>650</td>
<td>521</td>
</tr>
<tr>
<td>Percentage of Retained Shares</td>
<td>80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Share Holding and Sale Matrix for Accelerated Trading Model (Ratio = 2)²</td>
<td>1 1000</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>2 1000</td>
<td>200</td>
<td>200</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>3 1000</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>4 1000</td>
<td>300</td>
<td>300</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>300</td>
<td>350</td>
<td>650</td>
<td>457</td>
</tr>
<tr>
<td>Percentage of Retained Shares</td>
<td>70%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel C: Share Holding and Sale Matrix for Two-Trader Model (20% active traders, Ratio = 8)³</td>
<td>1</td>
<td>200</td>
<td>67</td>
<td>67</td>
<td>0</td>
</tr>
<tr>
<td>Active Traders</td>
<td>2</td>
<td>200</td>
<td>133</td>
<td>133</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>200</td>
<td>33</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>67</td>
<td>200</td>
<td>233</td>
<td>433</td>
<td>200</td>
</tr>
<tr>
<td>Percentage of Retained Shares</td>
<td>46%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactive Traders</td>
<td>1</td>
<td>800</td>
<td>33</td>
<td>33</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>800</td>
<td>67</td>
<td>67</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>800</td>
<td>17</td>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>800</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>100</td>
<td>117</td>
<td>217</td>
<td>198</td>
</tr>
<tr>
<td>Percentage of Retained Shares</td>
<td>91%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Traders</td>
<td>1</td>
<td>1000</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1000</td>
<td>200</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1000</td>
<td>50</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1000</td>
<td>300</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>300</td>
<td>350</td>
<td>650</td>
<td>398</td>
</tr>
<tr>
<td>Percentage of Retained Shares</td>
<td>61%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

¹ This model assumes that each share is equally likely to trade, regardless of when it was purchased.

² This model assumes that class shares are two times more likely to trade than non-class shares.

³ This model assumes that there are two types of trader: active and inactive traders. Active traders hold 20% of shares and are eight times more likely to trade than inactive traders.
<table>
<thead>
<tr>
<th>Trading Behavior Model</th>
<th>Damages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Without Offsetting Gains Against Losses</strong></td>
<td></td>
</tr>
<tr>
<td>Proportional Trading Model</td>
<td>$ 100.0</td>
</tr>
<tr>
<td>Accelerated Trading Model (Ratio of 2)</td>
<td>$ 91.0</td>
</tr>
<tr>
<td>Actual-Trader Model</td>
<td></td>
</tr>
<tr>
<td>Observed Trading Sample</td>
<td>$ 21.7</td>
</tr>
<tr>
<td>Unobserved Trading Sample</td>
<td>$ 39.8</td>
</tr>
<tr>
<td>Total Damages</td>
<td>$ 61.5</td>
</tr>
<tr>
<td><strong>Panel B: With Offsetting Gains Against Losses</strong></td>
<td></td>
</tr>
<tr>
<td>Actual-Trader Model</td>
<td></td>
</tr>
<tr>
<td>Observed Trading Sample (Offsetting Ratio = 69%)</td>
<td>$ 14.9</td>
</tr>
<tr>
<td>Unobserved Trading Sample</td>
<td>$ 27.4</td>
</tr>
<tr>
<td>Total Damages</td>
<td>$ 42.3</td>
</tr>
</tbody>
</table>

1. This model assumes that each share is equally likely to trade, regardless of when it was purchased.
2. This model assumes that class shares are two times more likely to trade than non-class shares.
3. We index aggregate non-institutional damages equal to $100 under the Proportional Trader Model.
4. The offsetting ratio is equal to the ratio of aggregate damages with offsetting to aggregate damages without offsetting.
Figure 1: Company X's Trading Probability Vector and Cumulative Trading Probability for the Observed Trading Sample (For Holding Period from 0 Days to 150 Days)
Figure 2: Company X's Trading Probability Vector of the Observed Trading Sample and Simulated Trading Probability Vector of the Unobserved Trading Sample (For Holding Period from 0 Days to 150 Days)
References


