Mission Performance Considered as Point Performance in Aircraft Design

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The cruise or loiter performance of an aircraft is intimately tied to its wing loading and its thrust-to-weight ratio. Paradoxically, mission performance is often not considered when these fundamental aircraft parameters are determined in conceptual design. In this paper, the traditional constraint diagram is extended to include contours of range or endurance parameter. These performance metrics represent the mission-performance capability of the aircraft without sizing the aircraft to a particular mission. This gives the designer an immediate and intuitive understanding of the tradeoff between the point and mission performance of the aircraft. The potential freedom for the designer to choose the operating condition of the aircraft is also considered. This improved constraint diagram presents the designer with a more complete basis for understanding and weighing the consequences of decisions. Through improved insight, the secondary impact of technologies can be better understood early in the design process.

Nomenclature

\[ \begin{align*}
\alpha &= \text{speed of sound} \\
C_D &= \text{drag coefficient} \\
C_L &= \text{lift coefficient} \\
D &= \text{drag} \\
E &= \text{endurance} \\
EP &= \text{endurance parameter} \\
h &= \text{altitude} \\
L &= \text{lift} \\
M &= \text{Mach number} \\
PC &= \text{engine power code} \\
P_s &= \text{specific excess power} \\
q &= \text{dynamic pressure} \\
R &= \text{range} \\
RP &= \text{range parameter} \\
S &= \text{wing reference area} \\
SR &= \text{specific range} \\
T &= \text{atmospheric temperature, thrust} \\
TSFC &= \text{thrust specific fuel consumption} \\
T/W &= \text{thrust to weight ratio} \\
T_{sl,max} &= \text{sea level static rated thrust} \\
V &= \text{velocity} \\
V_{app} &= \text{approach speed} \\
V_{SL} &= \text{stall speed in landing configuration} \\
W &= \text{weight} \\
W/S &= \text{wing loading} \\
W_e &= \text{empty weight} \\
W_f &= \text{fuel weight} \\
W_f &= \text{fuel flow} \\
W_p &= \text{payload weight} \\
W_{TO} &= \text{maximum takeoff gross weight} \\
\alpha &= \text{thrust ratio, } T/T_{sl,max} \\
\alpha_{req} &= \text{equilibrium required thrust fraction} \\
\beta &= \text{mission segment weight fraction, } W/W_{TO} \\
\theta &= \text{atmospheric temperature ratio, } T/T_d \\
\rho &= \text{atmospheric density} \\
\sigma &= \text{atmospheric density ratio, } \rho/\rho_0 \\
\end{align*} \]

I. Introduction

The relationships between thrust level, wing size, point performance requirements, cruise condition, and mission efficiency are fundamental to the design of an aircraft. These relationships are subject to a great deal of subtle interplay, which can manifest counterintuitive behavior. Incomplete understanding of these relationships early in the design process can lead to costly design changes or to a compromised aircraft that fails to meet its objectives or potential.

The early stages of conceptual aircraft design may be considered in two phases: unsized and sized. In the unsized phase of aircraft design, the aircraft is described and analyzed in terms of normalized (often dimensionless) parameters. Most notably, the gross weight and fuel required for the aircraft are as yet undefined; the aircraft has not yet been sized to fly a particular mission. In the sized phase of design, estimates of the aircraft gross weight are available and all of the aircraft dimensions and weights can be calculated.

To transition from the unsized to sized phase of aircraft design, a number of additional assumptions and models are required; these include a mission model with a complete set of mission rules and a set of weight estimating relationships. These additional assumptions and models make achieving insight into the fundamental trades of aircraft design even more difficult.

The decisions made during the unsized phase of conceptual design can have great impact on the mission performance of an aircraft, yet mission performance is not normally considered until the sized phase of conceptual design. This paper presents a technique to consider mission performance in the unsized phase of design by considering range or endurance parameter; these are metrics of mission performance that are independent of the size of the vehicle and the length of the mission. This approach provides greater insight during the earliest stages of design. In addition, an optimization of aircraft operating condition is performed that allows the designer to consider the role of the mission requirements in these fundamental trades.

II. Current Practice

When the conceptual designer is faced with the choice of how much thrust and how large a wing an aircraft should have, the constraint diagram with axes of thrust to weight ratio and wing loading \((T/W \text{ vs } W/S)\), like the one shown in Fig. 1, is often the tool of choice. Sometimes called the matching plot, the constraint
diagram concisely represents an aircraft’s ability to meet the specified point performance requirements [1, 2]. The requirements typically represented include takeoff distance and/or balanced field length, landing distance, approach and/or stall speed, rate of climb, climb gradient, ceiling, dash, sustained and/or instantaneous turn, etc.

The traditional constraint diagram depicts only point performance metrics for an aircraft that has not been sized to a particular mission. Unfortunately, when used in this way, the constraint diagram gives no guidance as to the effect of thrust and wing loading on the mission performance of the aircraft. However, these parameters are crucial in determining the cruise or loiter efficiency of the aircraft. The fundamental tradeoffs between point and mission performances are overlooked by the traditional constraint diagram.

To investigate the effect of thrust and wing loading on mission performance, the designer usually must delve deeper into the design process, sizing the vehicle to a design mission. Because it does not conventionally consider mission performance, some practitioners skip the constraint diagram and the unsized phase of design entirely. Skipping a clear trade process involving thrust level and wing size risks nonideal choice of these fundamental parameters and a compromised design; practitioners who conduct this trade during the sized phase of design are faced with a more complex task.

The impact of $T/W$ and $W/S$, including their influence on mission performance, can be considered in the sized phase of design through an explicit trade process involving sized aircraft. These studies are sometimes presented as a carpet plot of $W_{TO}$ for a range of $T/W$ and $W/S$ [4–7]. Convention of presentation differs, with some practitioners preferring dimensional thrust and wing area in place of $T/W$ and $W/S$ [8]. In addition to plotting $W_{TO}$, carpet plots can be constructed depicting other figures of merit such as unit production cost [9] or measures of operating cost [10] as functions of $T/W$ and $W/S$. When contours of $W_{TO}$ or range are plotted on axes of engine size and wing area, the results are called a sizing matrix [11], thumbprint plot, or knothole plots [12,13]. In all of these cases, generation of the information that is presented requires a trade study of closed vehicle designs sized to a particular mission with full mission rules and weights models.

The complexity of this kind of study typically requires use of a comprehensive sizing and synthesis code. This class of program has been developed in many forms by government and industry alike since the mid 1960s [14]. These codes are usually specialized to a certain vehicle type such as helicopter [15], vertical or short takeoff and landing [16], or hypersonic vehicles [17,18]. Programs intended for fixed wing aircraft are often further specialized to fighter [19,20], general aviation [21], or transport aircraft [22–24]. The Flight Optimization System (FLOPS) [25,26] program is a comprehensive sizing and synthesis code for fixed wing aircraft that is in broad use and is actively developed. This class of program requires significant

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**Fig. 1** Traditional point-performance constraint diagram.

**Fig. 2** DC-10 cruise aerodynamics [8].
modification to handle unconventional and advanced concepts; even
the versatile FLOPS program required modification to model
blended wing body aircraft [27].

Although sizing and synthesis codes can enable complex system
level multidisciplinary trade studies for a class of vehicles, they are
often treated as black box tools by their users. The sophistication of
these tools hide many of the assumptions and models required to
solve a particular problem. This lack of transparency can make it
difficult or impossible to gain insight into the driving factors of
fundamental trades. Addressing the most fundamental aspects of
mission performance in the unsized phase of conceptual design
provides maximum insight for decision making before proceeding to
the more complex sized phase of design.

III. Example Aircraft

An example aircraft will be employed to illustrate the concepts
discussed in this paper. The example aircraft is defined in terms of a
drag polar and an engine deck. Rather than use simplified equations
for the drag, thrust, and fuel consumption, tabulated data corre-
sponding to a real world aircraft were used. All of the equations and
procedures developed in this work are applicable to an arbitrary
aircraft model. The drag of a DC 10 in cruise configura-
tion was used for the example aircraft. Figures 2a and 2b depict the drag polar and
drag rise behavior for a variety of lift coef-
ficients for this aircraft. These drag data were digitized from data presented in the
McDonnell Douglas performance course [8].

The thrust and fuel consumption of a CF6 50 were used for the
element aircraft. Figure 3a depicts the full throttle thrust lapse of the
engine throughout the operational envelope; near cruise, the engine is
only able to produce about 30% of the rated thrust. Figure 3b depicts
the thrust specific fuel consumption (TSFC) of the engine at 80%
thrust throughout the operational envelope. As expected, TSFC
improves with altitude approximately proportional to the square root
of the temperature ratio and degrades approximately linearly with
Mach. The part power performance of the engine from full power to
flight idle is exemplified by the thrust hook for operation at $M = 0.8$
and $h = 36,089$ ft given as Fig. 3c. Two throttle settings in terms of
percentage of thrust are specifically identified (29.3 and 86%); these
points correspond to behavioral changes later in the analysis. These
engine data were digitized from data presented in the CF6 Engine
Installation Manual [28].

In the unsized phase of conceptual design, construction of a
constraint diagram relies on a rubber engine assumption; this as-
sumption represents the engine as a certain cycle that may be scaled
up or down with $T = W$ as desired. This neglects any effects of scale on
the engine cycle and presumes an engine company willing to build
any engine to order. To rubberize a given engine, the thrust is scaled
by the sea level rated thrust. This thrust ratio, $/\text{W} = 0.8$
represents both the
thrust lapse due to speed and altitude, but also any reduction in thrust
due to throttling.

The unsized phase of conceptual design also relies on a rubber
airframe assumption to construct a constraint diagram; this assump-
tion represents the airframe as a drag polar held
fixed throughout the
study. For all wing and some other advanced concepts, this assump-
tion is quite good, only neglecting any Reynolds number scale effects
on the aircraft drag. However, for transport aircraft with
fixed
payload requirements, conceptual design changes in wing loading
are most often realized through changes in wing area; these changes
can have significant effect on the drag polar for a con-
figuration.

IV. Efficient Mission Performance

The Breguet range equation provides insight into the integrated
mission performance of an aircraft without requiring the complexity
of a complete mission model. When written as Eq. (1), the crucial
elements for long range flight are clearly evident. An aircraft designed for efficient cruise should have high aerodynamic efficiency \( VL/D \) and low TSFC, and it requires a large fuel load \( W_f \) and a low empty weight \( W_e \) for a given payload weight \( W_p \):

\[
R = \frac{V}{\text{TSFC} D} \left( 1 + \frac{W_f}{W_p + W_e} \right)
\]  

(1)

The coefficients in the front of this equation can be grouped into quantities that are useful for further simplifying the range behavior of an aircraft. The specific range \( SR \), defined in Eq. (2), is typically expressed in units of nautical miles per thousand pounds of fuel. The specific range is directly analogous to fuel efficiency expressed as miles per gallon for a family car. These metrics give an indication of a vehicle’s efficiency, but do not indicate how much fuel is required for a particular mission (distance and payload). They also leave out the effects of fuel fraction of the vehicle, but include some effect of the vehicle size:

\[
SR = \frac{V}{\text{TSFC} D}
\]  

(2)

Similarly, the range parameter \( RP \), sometimes called range factor, defined in Eq. (3), is typically expressed in units of nautical miles. The range parameter abstracts the specific range by removing the effect of the vehicle scale. This makes it suitable for consideration during the unsized stage of design when the constraint diagram is used to guide decisions. However, the range parameter does not enjoy the intuitive interpretation of specific range:

\[
RP = \frac{V}{\text{TSFC} D}
\]  

(3)

In the same manner, integrated and point measures of endurance performance exist that provide insight without undue complexity. An integrated endurance equation analogous to the Breguet range equation can be written as Eq. (4). From this, we see that an aircraft designed for long duration flight requires high aerodynamic efficiency as measured by \( L/D \) vs \( VL/D \) for long range flight:

\[
E = \frac{1}{\text{TSFC} D} \left( 1 + \frac{W_f}{W_p + W_e} \right)
\]  

(4)

The intuitive and scale dependent measure of endurance efficiency for an aircraft is the fuel flow, \( W_f \), presented in Eq. (5), which is usually expressed in units of pounds of fuel per hour:

\[
W_f = \text{TSFC} D
\]  

(5)

Finally, the endurance parameter \( EP \), defined in Eq. (6), is typically expressed in units of hours. Like range parameter, endurance parameter abstracts the fuel flow by removing the effect of the vehicle scale, making it suitable for use before the size of the vehicle has been determined. However, without consideration of the fuel fraction or size of the vehicle, the endurance parameter does not give a direct measure of the endurance capability of an aircraft:

\[
EP = \frac{1}{\text{TSFC} D}
\]  

(6)

In this study, we aim to treat the range and endurance parameters as point performance quantities that can be considered alongside traditional point performance constraints. The quantities that appear in each of these equations are sometimes grouped in order to simplify discussion and to provide intuition to the designer for how to affect changes in a vehicle’s performance; at this stage, this is predomi-

nantly accomplished through consideration of the thrust to weight ratio \( T/W \) and wing loading \( W/S \) of the aircraft. Although the abbreviated symbol \( T/W \) is often used, the quantity being considered is the ratio of the sea level static rated thrust to the maximum takeoff gross weight of the aircraft, \( T_{\text{sl,ma}}/W_{\text{TO}} \). Similarly, the abbreviated symbol \( W/S \) refers to the ratio of the maximum takeoff gross weight to the reference area \( W_{\text{TO}}/S \).

V. Aerodynamic Efficiency

To understand the impact of \( T/W \) and \( W/S \) on mission efficiency, we first break down the range and endurance parameters into the relevant aerodynamic and propulsive components. Then, we investigate how the designer’s choice of \( T/W \) or \( W/S \) affects mission efficiency and how those choices interact with the choice of operating condition. For aerodynamic efficiency, we are primarily interested in \( ML/D \) or \( L/D \) and how they are impacted by choice of \( W/S \) and operating condition.

The effect of aerodynamic efficiency on aircraft range can be understood by considering the term \( VL/D \) or often \( ML/D \). Similarly, the effect of aerodynamic efficiency on aircraft endurance is understood through the term \( L/D \). A drag polar can be viewed as providing the drag coefficient as a function of the aircraft operating condition, say \( C_D = f(C_L, M) \). This function can be used to likewise provide \( ML/D \) or \( L/D \) as a function \( ML/D = f(C_L, M) \). This perspective on \( ML/D \) is depicted in Fig. 2c. This function can be maximized with respect to \( C_L \) allowing \( ML/D_{\text{max}} \) to be plotted as a function of the remaining parameter, \( M \), as in Fig. 2d.

Figures 2c and 2d are reflections of the drag polar alone; they do not represent the aerodynamic efficiency of a particular aircraft at a particular operating condition. Figure 2d provides the limit of the best achievable aerodynamic efficiency for this drag polar at any Mach number; if the cruise altitude and wing loading are properly selected, this performance may be achieved at a single Mach number. It is not possible for a single aircraft to attain this level of performance at any arbitrary operating condition.

The aerodynamic efficiency metrics \( ML/D \) and \( L/D \) can be calculated and plotted for a particular aircraft as contours across the operating envelope as in Figs. 4 and 5.
The wing loading and flight condition (through dynamic pressure) determine the lift coefficient and thereby the operating point on the drag polar. Through the wing loading, the designer can effectively influence the aerodynamic efficiency at a specified flight condition.

VI. Propulsive Efficiency

As with aerodynamic efficiency, we seek to understand the impact of $T/W$ and $W/S$ on mission efficiency through propulsive efficiency. For propulsive efficiency, we are primarily interested in TSFC and how it is impacted by choice of $T/W$ and operating condition.

The effect of propulsive efficiency on aircraft range and endurance can be understood directly through TSFC. As with the drag polar, the engine deck may be viewed as providing the propulsive performance as a function of the aircraft operating condition, say $(\alpha, \text{TSFC}) = f(M, h, \text{PC})$. Figure 3b depicts typical behavior of TSFC throughout the flight envelope for a fixed throttle setting. TSFC improves with increasing altitude until the tropopause and is constant thereafter; it generally degrades with increasing Mach number.

As depicted in Fig. 3c, TSFC is also dependent on the engine throttle setting; note the clear TSFC penalty for operating at flight idle or maximum throttle. TSFC is minimum at the tropopause (and constant above) and can be minimized with respect to throttle setting allowing $\text{TSFC}_{\text{min}}$ to be plotted as a function of Mach number as in Fig. 3d. Similar to Fig. 2d, Fig. 3d represents the engine performance alone; it does not represent the propulsive efficiency of a particular aircraft at a particular operating condition. Figure 3d provides the limit of the best achievable propulsive efficiency for this engine at any Mach number; if the cruise altitude and thrust to weight ratio are properly selected, this performance may be achieved at a single Mach number. It is not possible for a single aircraft to attain this level of performance at any arbitrary operating condition.

The engine designer, through selection of a thermodynamic cycle and its parameters, has the greatest influence over this bulk behavior of TSFC. The aircraft designer typically chooses between existing engines or works with an engine manufacturer to develop an engine for an application. Consistent with the rubber engine assumption, $T/W$ is used by the conceptual aircraft designer to scale a given engine cycle to the desired aircraft application.

The effect of $T/W$ on the cruise efficiency of an aircraft is best understood by comparing the thrust hooks depicted in Fig. 6. These hooks represent the same engine cycle scaled to three $T/W$ ratios each capable of sustaining flight $(T \geq D)$ at the specified flight condition. The first case, labeled 83%, represents an undersized engine where the engine must operate at nearly full power during cruise; this incurs a TSFC penalty. The second case, labeled 100%, represents an engine sized such that cruise operation occurs at minimum TSFC. The final case, labeled 150%, represents an oversized engine where the engine must operate at greatly reduced power during cruise; operation near flight idle incurs a TSFC penalty as well as an implied weight and cost penalty.
VII. Best Attainable Mission Performance

The best case aerodynamic and propulsive performance represented by Figs. 2d and 3d can be combined to provide a limit for the best attainable cruise or loiter performance for a drag polar and engine deck combination. These calculations have been performed for a variety of altitudes up to the tropopause and have been plotted in Fig. 7.

As with Figs. 2d and 3d, Fig. 7 represents the limit of possible performance and does not represent a level of performance that can be achieved in general. In order for an aircraft to meet this level of performance at a particular Mach number and altitude, it must have the correct \( T/W \) and \( W/S \). Although the performance levels presented in Fig. 7 are theoretically achievable, there is no guarantee that doing so is practical. For example, maximizing endurance performance according to Fig. 7b requires flight at the tropopause at very low speed, which would imply an aircraft with impractically low wing loading.

Figure 7 also begins to indicate how strongly opposed the goals of extreme range and endurance can be. Although they both appear to favor high altitudes, they are strongly opposed in preference for speed. These preferences will lead to conflicting trends for \( W/S \) and \( T/W \).

VIII. Mission Performance for an Unsized Aircraft

The range and endurance parameters have been identified as point performance metrics indicative of the efficiency of an aircraft. Furthermore, these parameters are sensitive to changes in \( T/W \) and \( W/S \). As such, it is appropriate to overlay a traditional constraint diagram with contours of range or endurance parameter. Calculation of the mission performance metrics for a particular \( T/W \) and \( W/S \) requires determination of the throttle setting for cruise flight at the specified flight conditions.

First, recall that the thrust ratio for a rubberized engine is the thrust at a particular operating condition divided by the rated thrust of the engine: \( \alpha = \frac{T}{T_{\text{sls, max}}} \). This ratio includes thrust changes due to lapse and throttling. In equilibrium flight typical of cruise or loiter, thrust must equal drag. This condition is used to identify the equilibrium required thrust fraction \( \alpha_{\text{req}} \), as specified in Eq. (8):

\[
\alpha_{\text{req}} = \frac{D}{T_{\text{sls, max}}}
\]

The definition of the thrust coefficient is substituted for the drag in this equation. Then the numerator and denominator are each multiplied by the takeoff gross weight \( W_{\text{TO}} \). This expression can be rearranged to be in terms of \( W/S \) and \( T/W \). Equation (9) gives the equilibrium required thrust fraction for operation at a given flight condition. An arbitrary engine deck will give engine performance as a function of flight condition and power code (PC): i.e., \( \alpha \cdot \text{TSFC} = f(M, h, \text{PC}) \). In this case, the power code required to match \( \alpha = \alpha_{\text{req}} \) can be solved for numerically:

\[
\alpha_{\text{req}} = C_{\theta}(\frac{S}{W_{\text{TO}}})\left(\frac{W_{\text{TO}}}{T_{\text{sls, max}}}ight)
\]

Table 1 details the computational procedure followed to calculate range and endurance parameters for a particular \( T/W \), \( W/S \), and flight condition. Section IX discusses the effect of choice in flight condition on mission performance. The arbitrary drag polar used in step 4 must give drag coefficient as a function of flight condition and lift coefficient. Once the equilibrium required thrust fraction is calculated in step 5, a numerical solution procedure is used to solve for the cruise power code; this calculation will also yield the cruise fuel consumption, TSFC. The arbitrary engine deck used in step 6 must give engine performance as a function of flight condition and throttle setting.

IX. Role of Mission Operating Point

The aircraft designer often has limited ability to choose the operating conditions of the aircraft. Cruise speed and altitude may be stated as requirements from the prospective customer. Furthermore, a commercial aircraft will need to work within the existing air traffic control infrastructure, flying at appropriate flight levels and at speeds set by air traffic controllers to maintain safe separation limits. In the situation where the operating conditions are fixed, the designer’s job is to tailor the aircraft to maximize performance at that condition. On the other hand, the pilot and performance engineer’s job is to tailor the existing aircraft’s operation to maximize performance within regulatory and operational limits. In other circumstances, the designer may have some capacity to choose the operating conditions of the aircraft. For example, the required airspeed for a long endurance surveillance platform may not be specified by the customer.
In addition, the customer is not always right and a poorly balanced set of requirements can greatly compromise the performance of an aircraft under consideration. Consequently, the ability to compre hensively and quantitatively explore the impact of requirements trades for requirement pushback is critical. It is also best to do this as early in the design process as possible. Presenting these fundamental trades in as clear and concise a manner as possible is critical to making the right decisions for properly balanced requirements.

In this study, four classes of problems are considered for each range and endurance. The most simple problem presents the designer with choice of $T/W$ and $W/S$ when operating Mach number and altitude are specified. Then the more complex problems where the designer has limited choice in operating condition are addressed. First, choice in Mach number is considered, then choice in altitude. Finally, the most complex problem is addressed, where the designer has complete freedom to choose the aircraft operating condition.

The fundamentally different nature of these problems is often overlooked or understated. In terms of aerodynamic efficiency, the problem of maximizing $ML/D$ reduces to maximizing $L/D$ when Mach number is fixed. The simplistic derivation of the lift coefficient for maximum range presented in many texts will give misleading results for the common case where cruise Mach number is specified. Care must be taken when considering variation of cruise Mach number during the design process. An aircraft wing is designed to a particular drag divergence Mach number; this plays a strong role in determining wing sweep, thickness, and airfoil. Choice of cruise Mach number significantly beyond drag divergence will have a strong (negative) impact on performance. The designer must be fully aware of the parameters varied during any trade study.

In this study, when one or more operational variables are free to be chosen, their value is selected through numerical optimization, which takes into account the full complexity of the problem as represented by the drag polar and engine deck. This includes the transonic drag rise and the nonconstant TSFC variation along the thrust hook. The optimization problem constructs a feedback loop between steps 8 and 2 of the computational procedure depicted in Table 1.

Because of the tabulated and interpolated nature of the aircraft data, these optimization problems present many challenges. Non gradient based optimization algorithms were employed including a golden section search for the one variable case and a Nelder Mead simplex search for the two variable case. In these problems, the need for algorithm robustness far outstrips the need for efficiency.

The optimization of the endurance cases proved to be especially problematic. One reason for this can be seen in Fig. 2d. The nearly constant value of $L/D_{\max}$ for a range of Mach number is perturbed by small fluctuations that are most likely artifacts of the data introduced by the fitting, fairing, and presentation of the original data as well as the digitization, smoothing, and interpolation of those data for their current use. These fluctuations create numerous local optima in endurance performance that can trap the optimizer. Multiplication of this function by Mach number does much to weaken the impact of these oscillations and prevent them from causing false optima in range performance.

Significant effort was required in order to obtain satisfactory results for the more complex cases considered. This primarily consisted of careful selection of the starting point and bounds for the optimizer. In the case of endurance performance with full freedom of the operating condition, it was necessary to perform some smoothing of the drag polar and also on the resulting optimal Mach number and altitude. These calculations would prove challenging to automate in a robust manner unless there can be assurance of smooth behavior of the input drag polar and engine deck across all operating variables.

### X. Point-Performance Constraint Diagram

The representative point performance constraint diagram pre sented in Fig. 1 depicts the unsized design space in terms of $T/W$ and $W/S$, which will allow the example aircraft to meet the specified point performance requirements. This constraint diagram was based on the requirements listed in Table 2. These requirements were selected to represent the variety of point performance requirements that may be imposed; they are not intended to represent the critical sizing requirements for this or any other particular design.

The traditional constraint diagram exemplified by Fig. 1 depicts the role of $T/W$ and $W/S$ in meeting a set of point performance requirements. This diagram does not depict the influence $T/W$ and $W/S$ have in the aircraft achieving efficient mission performance; the aim of this paper is to extend the constraint diagram to depict that influence without sizing the aircraft.

The four mission performance problems from Sec. IX are considered in Sec. XI in terms of $T/W$ and $W/S$, resulting in Figs. 8 and 14. For clarity, the point performance constraints of Fig. 1 are omitted from Figs. 8 and 14; however, these requirements can and should be considered simultaneously. In practice, superimposing the point performance constraints and mission performance objective on the same figure provides the best means for the designer to consider these design elements concurrently.

### XI. Results

#### A. Mach and Altitude Fixed

Contours of range and endurance parameter for fixed Mach number and altitude on axes of $T/W$ vs $W/S$ for the example trans port aircraft were generated and plotted as Fig. 8. A hatched constraint line depicting zero specific excess power for that flight condition is also included. When $V$ is specified, as is the case when both $M$ and $h$ are specified, the range and endurance parameters are directly proportional. Consequently, for the same operating conditions, contours of range and endurance performance would have identical shape, with differing values and units. In this case, different operating Mach numbers have been chosen to illustrate typical differences between range and endurance missions.

In terms of range parameter, the optimum $T/W$ is approximately 0.21 and the optimum $W/S$ is approximately 131 lbf/ft$^2$. Changing $W/S$ results in moving along the drag polar at the specified Mach number to different values of $L/D$ at cruise. The optimum cruise point is located directly above the minimum $T/W$ point in the $P_e = 0$ constraint curve. This minimum occurs at minimum drag and

### Table 1 Computational procedure for calculations

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\beta$, $W_{TO}/S$, $T_{\text{flaps max}}/W_{TO}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\text{Cruise flight condition specified or free to optimize.}$</td>
</tr>
<tr>
<td>3.</td>
<td>Calculate atmospheric conditions, velocity, and dynamic pressure.</td>
</tr>
<tr>
<td>4.</td>
<td>Evaluate lift and drag coefficient.</td>
</tr>
<tr>
<td>5.</td>
<td>Evaluate required thrust fraction.</td>
</tr>
<tr>
<td>6.</td>
<td>Evaluate thrust fraction at throttle setting.</td>
</tr>
<tr>
<td>7.</td>
<td>Iterate on PC to match thrust fractions.</td>
</tr>
<tr>
<td>8.</td>
<td>Evaluate range and endurance parameters.</td>
</tr>
</tbody>
</table>

### Table 2 Example point-performance requirements

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$P_e = 100$ ft/min at the tropopause, $M = 0.8$, and $\beta = 0.95$.</td>
</tr>
<tr>
<td>2.</td>
<td>Approach: $140$ KEAS $V_{\text{app}} = 1.23 V_{\text{cr}}$ at $\beta = 0.75$.</td>
</tr>
<tr>
<td>3.</td>
<td>Go around: $3.2%$ climb gradient at approach conditions [29].</td>
</tr>
<tr>
<td>4.</td>
<td>Takeoff: $9000$ ft ground roll distance.</td>
</tr>
</tbody>
</table>
therefore best $L/D$. This heuristic can be used to identify the optimum cruise point in lieu of the complete range parameter calculation.

Changing $T/W$ results in moving along the thrust hook for the engine with accompanying changes in TSFC as depicted in Fig. 6. If $T/W$ is increased greatly, the aircraft cruises near flight idle with a significant penalty in TSFC. If $T/W$ is decreased to the $P_{s0}$ contour, the aircraft cruises at maximum throttle. In this case, full throttle operation comes with a slight TSFC penalty.

The odd shape of the endurance parameter contours in Fig. 8b is a direct result in the bubble in $ML/D$ for $M = 0.6$ observed in Fig. 2c around $C_L = 0.6$. The cutoff of the contours for high $W/S$ is due to $C_L$ required to sustain flight at the specified condition exceeding $C_{L_{\text{max}}}$. 

**B. Altitude Fixed and Optimal Mach**

For the second class of problem, contours of range and endurance parameter for fixed altitude, but optimal Mach number, on axes of $T/W$ vs $W/S$ were plotted as Figs. 9a and 10a. The corresponding optimal Mach number for each case was plotted in Figs. 9b and 10b, respectively. A hatched constraint line depicting zero specific excess power for flight at the tropopause at any Mach number was also

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**Fig. 8 Mission performance with both Mach and altitude specified.**

**Fig. 9 Best range with Mach free, $h = 36,089$ ft.**

**Fig. 10 Best endurance with Mach free, $h = 36,089$ ft.**
included; the freedom to choose Mach allows this constraint to be lower and flatter than in the Mach fixed case.

Comparison of Figs. 9 and 10 for range and endurance performance highlights the conflicting nature of these objectives. Extreme range performance demands a specific, fairly high, wing loading, whereas extreme endurance performance demands as low a wing loading as possible. Any aircraft required to have good performance in both regards will face this fundamental tradeoff.

C. Mach Fixed and Optimal Altitude

For the third class of problem, contours of range and endurance parameter for fixed Mach number, but optimal altitude, on axes of $T/W$ vs $W/S$ were plotted as Figs. 11a and 12a. The corresponding optimal altitude for each case was plotted in Figs. 11b and 12b, respectively. A hatched constraint line depicting zero specific excess power for flight at the specified Mach number at any altitude was also included.

Comparison of Figs. 11 and 12 for range and endurance performance reveal the similarity of these problems when Mach number is considered fixed; if the specified Mach number were the same, these problems would only differ due to the change in the speed of sound below the tropopause. These figures reveal a clear valley in range or endurance performance where high performance can be achieved across a range of wing loadings. The lowest $T/W$ required for high performance occurs at high $W/S$. Reducing $W/S$ requires more thrust and higher operating altitude to maintain mission performance.

Figure 12b includes a dotted line corresponding to a 30% throttle setting. Above this line, $T/W$ is so great that the aircraft must operate at flight idle conditions with a severe TSFC penalty, as seen in Fig. 3c.

D. Optimal Mach and Altitude

The final class of problem allows the optimal selection of both Mach number and altitude for best mission performance. Contours of best range and endurance parameter on axes of $T/W$ vs $W/S$ as well as corresponding contours of optimal operating conditions were plotted as Figs. 13 and 14. A hatched constraint line depicting zero specific excess power for flight at sea level at any Mach number was also included; this line represents an absolute lower limit for thrust required for this aircraft to fly.

Comparison Figs. 13 and 14 reveals the fundamentally different nature of designing an aircraft for extreme range or endurance performance. An aircraft designed for long range performance requires fairly high $W/S$ and moderate to high $T/W$; it operates very near the Mach number for $ML/D_{\text{max}}$ and at altitudes near or above the tropopause. Alternatively, an aircraft designed for long endurance performance requires very low $W/S$ and $T/W$; it operates near the minimum Mach number and altitude possible.

The design space for aircraft designed for long range performance can be divided into two clear regions of distinctly different behavior. These regions are divided by the dotted line in Fig. 13, which corresponds to an 86% required throttle setting. Below that line, the aircraft are generally under powered and the required throttle setting approaches 100%; above that line, the aircraft generally have sufficient thrust for efficient cruise. The 86% throttle setting corresponds to a point where increased throttle is accompanied by increased penalty in TSFC, as seen in Fig. 3c.

The region of the design space with sufficient thrust corresponds to an area of nearly constant optimal Mach number; that optimal Mach number is close to that for $ML/D_{\text{max}}$. The Mach number selected for the earlier Mach fixed problem ($M = 0.8$) was also close to this optimal value; consequently, the corresponding regions of Fig. 11 are substantially similar to those of Fig. 13.

![Figure 11](image1.png)  
**Fig. 11** Best range with altitude free, $M = 0.8$.

![Figure 12](image2.png)  
**Fig. 12** Best endurance with altitude free, $M = 0.6$. Dotted line corresponds to 30% throttle setting.
Fig. 13  Best range with Mach and altitude free. Dotted line corresponds to 86% throttle setting.

Fig. 14  Best endurance with Mach and altitude free. Dotted lines correspond to throttle settings.
Aircraft corresponding to the under powered region of the design space pay multiple distinct penalties. These aircraft have insufficient thrust to cruise at the Mach number for $ML/D_{max}$. Consequently, they must fly at reduced Mach number and altitude. This results in sacrificed range performance as well as reduced cruise speed resulting in longer block times for a given mission. These aircraft require more fuel and more time to complete the same mission as an otherwise identical aircraft with higher $T/W$. Finally, these aircraft have minimal excess power margins at cruise required for climb, maneuver, and operational flexibility.

Aircraft in service that cruise near the maximum continuous power setting and have low operational ceilings likely do not have sufficient thrust to achieve the Mach number for $ML/D_{max}$. Increasing the thrust of these aircraft would potentially benefit their range, speed, ceiling, and excess power.

Designers intending to minimize $T/W$ while remaining in the upper region of the design space must take special care to ensure that thrust targets for the program are met. Any reduction in thrust will place the design in the under powered region of the design space. Conversely, the reduction in $T/W$ caused by feature weight targets will be somewhat offset by the corresponding increase in $W/S$ and may not place the aircraft in the under powered region.

The design space for aircraft designed for long endurance performance has regions of distinct behavior, but they are not as distinct as those for long range performance. In contrast to long range performance, the behavior changes for optimal long endurance performance correspond to behavior changes at low power settings (30–40%). The extremes of long endurance performance correspond to very low $T/W$, $W/S$, Mach number, and altitude. It is difficult to imagine a need for an aircraft in which all of those conditions would be compatible. One possibility would be an aircraft designed solely for low level scientific data collection over friendly waters. On the other hand, most surveillance missions that would use an aircraft designed for long endurance performance would not place the design in the under powered region of the design space.

Consequently, it is expected that the problem of fixed altitude endurance performance illustrated in Sec. XII.B will be more appropriate for typical use. An aircraft that requires both long range and endurance should first be constrained to the region of the design space corresponding to sufficient power to reach Mach for $ML/D_{max}$. This is most readily done by constructing a $P_s = 0$ contour with altitude free at the appropriate Mach number (say $M = 0.8$) and the appropriate throttle setting (86% for this case). This line should correspond to the 86% throttle dotted line in Fig. 13, but should be far more straightforward to construct. This constraint will ensure good range performance while allowing the balance and tradeoff of any remaining point performance requirements and endurance performance desires.

The conflicting requirements for required thrust for an aircraft that must have both long range and long endurance suggest an aircraft with $T/W$ that is variable in flight. This would be most readily achieved by shutting down a centerline engine during loiter and deploying fairings to minimize the drag of the inoperative engine. The 3X concept is one such proposal that uses two dissimilar engines and has been proposed for other claimed performance benefits [30]. Although the derivation is not presented in this paper, a handful of reasonable assumptions for flight above the tropopause lead to the result that the constant throttle line and the valley of optimum performance in Fig. 13 follow a trend where $T/W$ is proportional to $(1/\sigma)(W/S)$, where $\sigma$ is the atmospheric density ratio at the operating altitude. This result is reflected in the character of the optimum region in both Figs. 13a and 11a.

### E. Results with Limited Information

Early in the design process, detailed information such as that presented in Figs. 2 and 3 may not be available and the designer must make due with limited information about the aerodynamic and propulsive performance of the aircraft. In those situations, a number of simplifications to the method presented are possible. In addition, heuristics based on results from aircraft with complete data can provide guidance for the limited case.

A likely scenario with limited aerodynamic estimates is that the designer has a basic drag buildup yielding a parabolic drag polar, but no detailed information about the Reynolds number and drag rise behavior of the aircraft. Instead of a drag polar in the form $C_D = f(C, M, h)$, the designer is limited to $C_D = f(C)$. In such a situation, Mach number must be held constant at a reasonable estimate; for best range, a cruise Mach number at or near the drag divergence Mach number is appropriate. The previously noted similarities between Figs. 11 and 13 indicate that this restriction is not too great.

Two scenarios of limited propulsion estimates are likely in the early phases of design. In the first, the designer does not have information about the partial power performance of the engine; i.e., a complete thrust hook is unavailable. Instead of an engine deck in the form $(\sigma, TSFC) = f(M, h, PC)$, the designer is limited to $(\sigma, TSFC) = f(M, h)$. This limitation has the benefit of greatly simplifying the computational procedure presented in Table 1 by eliminating the iteration associated with steps 6 and 7.

The problem of design for range or endurance fundamentally involves the selection of an appropriate Mach number (say $M_{cr}$) where $W=S$. This would be most readily done by constructing a $P_s = 0$ constraint curve. Although this shortcut accurately identifies the location of the optimum, it does not give any indication of the penalty associated with a nonoptimal design point.

In the second likely scenario of limited propulsion estimates, the designer does not have detailed information about the behavior of TSFC with Mach and altitude. Instead of a drag polar in the form $(\sigma, TSFC) = f(M, h, PC)$, the designer frequently only has one or two values of TSFC; sea level static or cruise conditions are most common. Fortunately, as exemplified by Fig. 3b, TSFC is relatively well behaved with Mach and altitude for most turbine engines. The variation of TSFC with altitude can be approximated as proportional to $h$, where $\theta$ is the atmospheric temperature ratio [1]. The variation of TSFC with Mach number is typically linear; the slope of the variation typically increases with bypass ratio. These general trends and data for similar engines can be used to create an estimate for the variation of TSFC with Mach and altitude sufficient for the early stages of design.

### XII. Conclusions

The traditional $T/W$ vs $W/S$ constraint diagram used in the unsized phase of design has been extended to include a measure of mission efficiency: not as a constraint, but as contours of merit. These range and endurance parameter contours are developed using rubber engine and rubber airplane assumptions consistent with those used in the generation of a traditional constraint diagram. Generation of these contours has been demonstrated for an example aircraft based on complex, real world aerodynamic and engine performance models. Design problems where the operating conditions of the aircraft were both fixed and free were considered. The additional insight provided by this extension provides the designer with an immediate means to better understand the fundamental tradeoff between an aircraft’s point performance and mission performance.

The range and endurance problems are dominated by the subtle details of the drag polar and the engine deck. This is particularly true when one or more operational conditions are free to be chosen through optimization. The TSFC behavior of the thrust hook from maximum power to flight idle is critically important to this problem, yet it is often overlooked.

The problem of design for range or endurance fundamentally changes when one considers whether the operational speed (or Mach number) is fixed or free to be determined. Any design problem must
start with identifying and justifying whether the designer has freedom to choose the operational conditions.

References