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A Method for Thermodynamic Work Potential Analysis of Aircraft Engines

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A Method for Thermodynamic Work Potential Analysis of Aircraft Engines

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Abstract

The objective of this paper is to provide a tool to facilitate the application of thermodynamic work potential methods to aircraft and engine analysis. This starts with a discussion of the theoretical background underlying these methods, which is then used to derive various equations useful for thermodynamic analysis of aircraft engines. The work potential analysis method is implemented in the form of a set of working charts and tables than can be used to graphically evaluate work potential stored in high-enthalpy gas. The range of validity for these charts is 300 to 36,000 ^oR, pressures between 0.01 and 100 atm, and fuel-air ratios from zero to stoichiometric. The derivations and charts assume mixtures of Jet-A and air as the working fluid. The thermodynamic properties presented in these charts were calculated based upon standard thermodynamic curve fits.

Symbols and Notation

Note: thermodynamic properties expressed in lower case represent mass-specific quantities.

b = availability
c_P = constant pressure specific heat
ex = exergy
g_c = gravitational constant
ghp = gas horsepower
h = enthalpy
J = work equivalent of heat
P = pressure
R = mass-specific gas constant
s = entropy
sa = stream thrust
T = temperature
u = flight velocity
w = work output
wp = thrust work potential
γ = ratio of specific heats, c_P/c_v

Subscripts

ref = reference conditions
in = inlet conditions
out = outlet conditions
loss = lost work potential
i = conditions at station 'i'

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Introduction

The concept of thermodynamic work potential holds considerable promise as a general analysis tool for thermodynamic cycles. Specifically, work potential methods are a convenient and intuitive means of evaluating thermodynamic performance and loss in engine cycles. Although the basic thermodynamic concepts underlying work potential methods have been known for decades, they have yet to receive mainstream application to the analysis of engine performance. This is in part because there is little practical reference material available to the propulsion community.

The objective of this paper is to develop the basic theory of work potential methods in a clear, concise form. These principles are then used to develop and present work potential data for Jet-A-air mixtures in the form of working charts and graphs quantified in terms of standard units and measures. These charts provide a great deal of insight relating temperature, pressure, and fuel-air ratio to work potential. The data and charts presented in this report should be regarded as a ready-reference for analysis of cycles employing mixtures of Jet-A fuel and air as the working fluid.

Fundamental Concepts and Relations

The fundamental concept on which this research is based is the notion that all substances have a quantifiable and calculable thermodynamic property called work potential. Work potential can take a variety of forms: potential energy of a rock at the top of a hill, kinetic energy of a body in motion, heat energy, chemical energy stored in the molecular bonds of substances, nuclear energy stored in the subatomic bonds of atoms, etc. This section describes in simple terms an analytical framework that formalizes the intuitive concept of work potential. This can in turn be used to gain insight regarding the thermodynamic performance of prime movers.

Work potential is defined as that portion of the energy contained within a substance that can be converted into useful work. It is used herein as a generic term for one of several figures of merit used to measure the amount of work stored in a system. The maximum possible fraction of the total energy that can be converted into useful work is governed by the laws of thermodynamics, particularly the second law. The second law states that the entropy of the universe can never decrease. *Entropy* is essentially a measure of the disorder of a system; the lower the entropy, the more heat energy is available to be converted into useful work. One could therefore view work as energy with zero entropy, or in other words, work is the transfer of energy in a perfectly ordered fashion. The essence of work potential analysis methods is calculation of total work potential initially available in an energy source (usually Jet-A fuel) and tracking of how that work potential is used or lost in the engine. This in turn leads to loss management methods[1] designed to target the largest losses and minimize their impact if possible.

The Relationship Between Equilibrium and Work Potential

Work potential is intimately related to the concept of *equilibrium*. When the entropy of a system is maximized, it is said to be in equilibrium with its environment.[2] In the equilibrium state, the system has no tendency to depart from the equilibrium condition. It therefore has no capacity to do work. On the other hand, a system that is not at equilibrium has a natural predisposition to move towards equilibrium with its environment. It also has potential to do work in going from non-equilibrium to the equilibrium state. The further a system is from equilibrium with its environment, the more stored work potential is contained within it.

To understand this, consider again a rock at the top of a hill. This rock is in a state of non-equilibrium with its environment. If it is perturbed, it will tend to roll down the hill until it reaches the bottom, at which point it is in equilibrium with its environment. In rolling down the hill, the potential energy initially stored in the rock is dispersed into the surrounding environment, thereby increasing the total entropy of the rock plus environment system. In its equilibrium state at the bottom of the hill, the rock has no potential to do work. However, instead of allowing the rock to roll uncontrolled down the hill, one could construct an elevator mechanism that utilizes the potential energy in the rock to do work as the rock is lowered to the bottom of the hill. In this case, the rock produces work while being brought into equilibrium with its environment. Furthermore, the higher the hill, the further the rock is from equilibrium with its environment, and the more work can be extracted from it in taking it to the bottom of the hill. This simple example is directly analogous to work potential analysis of an engine, the chief difference being that in the latter case, work potential is stored and extracted from the chemical bonds of a fuel instead of a gravitational potential field.

The Concept of Reference State in Relation to Work Potential

A second important concept relating to work potential analysis is that of the *equilibrium state,* also referred to as the *reference state* or *dead state*. The work potential present in any substance is always measured relative to a datum representing the equilibrium condition. In the rock example discussed previously, it was always necessary to define a reference state as the "bottom of the hill," with the height of the hill measured from this datum. It should be noted that the choice of datum is entirely arbitrary and could be chosen to be anything. For example, the zero potential energy datum could have been chosen to be at the top of the hill, in which case the rock would have no work potential relative to that datum. This is a perfectly valid choice of reference state, though it is not particularly convenient when the objective is to calculate *usable work potential* relative to the bottom of the hill.

The reference state is usually chosen to be representative of the ambient environment in which the system is immersed because the selection of this datum yields a realistic estimate of the true work potential available in a system. When the system of interest is immersed in an ambient environment that changes significantly with time (such as an aircraft engine), the reference state is often allowed to float to match the instantaneous ambient conditions surrounding the system.[3] Finally, note that a reference state must be defined for each form of work potential of interest. For example, if heat transfer work is significant, one must define a reference temperature; if adiabatic expansion is of interest, one must define a reference pressure or area ratio; if electric power is significant, a reference (ground) voltage must be defined, and so on.

The Concept of Usable Work Potential

The previous section mentioned the concept of usable work potential. Substances can contain work potential in a variety of forms ranging from heat energy, chemical energy, electric energy, nuclear energy, etc. Not all of these forms are readily accessible or useable in a given situation. Typically, only one or two work potential mechanisms are of interest or are readily accessible. Accessible in the sense used here means that the machine or component being analyzed can readily tap into and utilize the source of work potential. For instance, the work potential contained in the nuclear bonds of the molecules in jet fuel is many orders of magnitude greater than the work potential stored in the chemical bonds. However, nuclear energy is not readily accessible when a gas turbine engine is being used. It is therefore common to ignore this source of work potential when analyzing a gas turbine.

There are also situations where it is useful to discount a specific portion of work potential that might otherwise be readily accessible. For instance, for a Brayton cycle of a given design turbine inlet temperature and pressure ratio, a portion of the work potential is inherently inaccessible and will appear in the exhaust stream as heat, even if all components in the cycle have perfect performance. For example, presume that one desires to compare the performance of a real Brayton cycle against that of an ideal Brayton cycle having the same pressure ratio and turbine inlet temperature. Given these bounding assumptions, it is of little value to bookkeep that portion of work potential which is inherently inaccessible (i.e. non-equilibrium combustion and associated exhaust heat). It would instead be more revealing to choose a work potential FoM that discounts those portions of work potential not readily accessible using a machine based on the Brayton cycle (such as gas horsepower). Otherwise, the work potential contained in the exhaust heat (most of which is in fact not readily accessible using the Brayton cycle) will obscure the true component loss relative to the ideal Brayton cycle machine. This is directly analogous to the concept of a "sunk cost" in economics—the inherent losses due to the Brayton cycle should play no role in the analysis process once the bounding assumptions are set.

This leads directly to the concept of usable work potential. *Usable work potential* is essentially that portion of work potential that is theoretically accessible using a given machine. The definition of what is usable is somewhat subjective in that it depends largely on the intended scope of the analysis. For instance, if one is starting with a "clean sheet of paper", it may be useful to understand usage and loss of work potential relative to the absolute bounds of the laws of thermodynamics (in which case exergy would be the tool of choice). On the other hand, if the objective is to estimate loss relative to an ideal machine of a given configuration and ideal cycle, a more limited work potential figure of merit may be more appropriate. This concept is discussed in further detail in Ref. [4].

Common Work Potential Figures of Merit and Their Interrelations

Several distinct figures of merit (FoM) have been proposed for use in propulsion system analysis.[5] This paper presents three such figures of merit: exergy, gas horsepower (GHP), and thrust work potential. Each is a successively more specialized case of the previous, and each is useful for a specific type of analysis. The fundamental differences between the FoM are summarized in Ref. 4:

> Exergy can be thought of as a Carnot FoM in that a Carnot cycle will appear to have no losses when analyzed using exergy methods, whereas any departure from a Carnot cycle will appear as a loss in exergy. It is the most comprehensive and consistent FoM [of the three] in that it can be shown to capture the effect of all losses relevant to contemporary propulsive cycles, including non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy. [It is measured relative to a reference temperature and pressure.]
>
> GHP can be thought of as a Brayton FoM because a Brayton cycle will appear to have no loss of gas horsepower, whilst any departure from the ideal Brayton cycle will appear as a loss in gas horsepower. It appears to be most useful for analysis of gas-turbine power generation units and turboshaft engines, and is measured relative to a [reference] pressure but not [a reference] temperature. However, gas horsepower counts exhaust residual kinetic energy as a loss even though this portion of the exhaust gas horsepower is *inherently unavailable* to jet propulsion applications if the cycle is taken as given. Gas horsepower [is] a special case of exergy wherein only mechanical (pressure) equilibrium with the environment is enforced.
>
> Thrust work potential produces results suggesting that it is a pure jet propulsion figure of merit because it is a direct index on the ability to produce thrust work. In effect, thrust work potential is a measure of ability to project thrust work into the Earth-fixed reference frame and is related to gas horsepower through propulsive efficiency. Thus, thrust work potential is a special case of gas horsepower, and by extension, a special case of exergy. [It is measured relative to a reference pressure and a prescribed inertial coordinate system.]

The application of these three FoM to describe engine component performance is discussed extensively in Ref. 6. The following sections define each FoM and develop useful relations that can be used for common engine analysis tasks.

Exergy

Exergy is a thermodynamic property describing the maximum theoretical (Carnot) work that can be obtained in taking a substance from a given chemical composition, temperature, and pressure to a state of thermal, mechanical, and chemical equilibrium with its environment. It is defined as:

$$ex \equiv h - h_{ref} - T_{ref}\left(s - s_{ref}\right) \tag{1}$$

Note that while energy is a conserved quantity, exergy is not—it is always destroyed when entropy is produced. The theoretical underpinnings of exergy analysis are discussed in detail in Refs. 7, 8, and 9.

Some exergy relations are useful for engine analysis and are worth noting. The exergy of a calorically perfect gas (neglecting kinetic and potential energy as well as chemical potential) is derived using the definition of constant pressure specific heat:

$h \equiv c_p T$ (2)

and the second Gibbs relation:

$$s - s_{ref} = c_p \ln\left(T / T_{ref} \right) - R \ln\left(P / P_{ref} \right). \quad (3)$$

Substituting into Eq. (1) and collecting terms:

$$ex = c_p \left(T - T_{ref}\right) - c_p T_{ref} \ln\left(T / T_{ref} \right) + R T_{ref} \ln\left(P / P_{ref} \right). \quad (4)$$

The exergy loss inside any arbitrary system can be calculated by summing the exergy fluxes into and out of the system. The difference between the exergy fluxes in and out is equal to the sum of the power output and the exergy loss rate:

$$\dot{e}x_{in} - \dot{e}x_{out} = \dot{w}_{out} + \dot{e}x_{\text{Loss}}. \quad (5)$$

Note that the term *availability* is often used interchangeably with the term exergy and the differences between the two are subtle. Availability is a thermodynamic property defined by Keenan[10] as:

$$b = h - T_{ref} s. \quad (6)$$

It therefore follows that exergy can be expressed in terms of a change in availability between two distinct states (Ref. 7, P. 127):

$$ex = b - b_{ref} = \left(h - T_{ref} s\right) - \left(h_{ref} - T_{ref} s_{ref}\right). \quad (7)$$

One could also view availability as being a change in Gibbs free energy ($g \equiv h - Ts$) relative to a prescribed reference state.

Gas Horsepower

Gas horsepower is defined as the work that would be obtained by isentropic expansion of a gas from a prescribed temperature and pressure to some reference pressure. Expressed mathematically:

$$ghp \equiv h\left(T_i, P_i\right) - h\left(P = P_{ref}, s = s_i\right) \quad (8)$$

where subscript 'i' denotes the thermodynamic state of the gas at the initial condition. The reference pressure is usually taken to be atmospheric pressure, and the temperature at the end of the process is a fall-out of the analysis. If the gas is calorically perfect, the temperature at the expanded condition can be found using standard isentropic flow relations:

$$T\left(P_{ref}, s_i\right) = T\left(P_{ref} / P_i \right)^{\frac{\gamma - 1}{\gamma}}. \quad (9)$$

An expression for GHP of a calorically perfect gas can be obtained by substitution of Eqs. 2 & 9 into Eq. 8:

$$ghp = c_p T_i \left[1 - \left(P_{ref} / P_i \right)^{\frac{\gamma - 1}{\gamma}} \right] \quad (10)$$

The GHP loss inside any steady system can be calculated by summing the GHP fluxes into and out of the system. The net difference between fluxes is equal to the sum of the power output and the GHP loss rate:

$$g\dot{h}p_{in} - g\dot{h}p_{out} = \dot{w}_{out} + g\dot{h}p_{loss}. \quad (11)$$

Gas horsepower is also referred to by various authors as *available energy*[11] or *barergy*.[12] It is very easy to confuse the term 'available energy' with 'availability,' and care is required in order to avoid this. The term 'gas horsepower' is used herein because it is the least ambiguous and best known term.

Thrust Work Potential

Thrust work potential is defined as the thrust work obtained via expansion of a gas at a given temperature and pressure to a prescribed reference pressure.[13] It is similar to GHP in this regard but instead of expanding the gas in an imaginary turbine to produce shaft work, the gas is expanded in an imaginary thrust nozzle to produce thrust work. The definition of thrust work potential is dependent on the existence of an inertial reference frame relative to the system because thrust work is equal to thrust produced (which is independent of reference frames) and velocity of the system relative to a prescribed reference frame. This dependence upon definition of reference frame makes it difficult to present thrust work potential data in a compact set of tables or charts. However, the mass-specific impulse function (also known as stream thrust) can be used as a close surrogate because it can be expressed as a function of temperature, pressure, and fuel/air ratio using only a few charts. Stream thrust can be expressed in terms of Mach number:[14]

$$sa = \sqrt{\frac{R T_0}{\gamma}} \frac{1 + \gamma M^2}{M \sqrt{1 + \left(\gamma - 1\right) M^2 / 2}} \quad (12)$$

and is also related to GHP via the relation:

$$sa = \sqrt{\frac{2\left(ghp\right) J}{g_c}}. \quad (13)$$

Thrust work potential is obtained by multiplying the stream thrust by the velocity of the system center of mass relative to the reference frame of interest:

$$wp \equiv sa\left(u\right) / J. \quad (14)$$

Earth-fixed reference frames are typically used for calculation of thrust work potential in flight vehicle propulsion systems.

Charts for Work Potential Analysis

The definitions given in the previous section can be used to calculate work potential properties for Jet-A-air mixtures. Figures 1-4 contain a number of such charts that are useful for aircraft engine analysis.[15] These charts are valid for mixtures of Jet-A fuel and air from temperatures of 300 ^{o}R to 36,000 ^{o}R, pressures from 0.01 to 100 atmospheres, and fuel/air ratios of 0 and 0.066 (equivalence ratios of 0 and 1.0). Each figure consists of four charts, with the top pair corresponding to a stoichiometric Jet-A-air mixture and the bottom corresponding to dry air. The left pair of charts in each figure is a log-log plot of the entire temperature and pressure range. The right pair in each figure is plotted over a range of pressure and temperature typically encountered in modern aircraft engines.

Brief perusal of the exergy and GHP charts (Figs 1 and 2) reveals that the spacing of the exergy and GHP contours is somewhat irregular. Specifically, the contour spacing is much closer in some regions, forming three distinct bands. These bands correspond to the vibrational excitation temperature of N_2 and O_2 in the lowest band, the dissociation of O_2 and N_2 in the middle band, and ionization of N and O in the upper left band. It is noteworthy that these chemical effects have a marked impact on the total work potential of the fluid. Though these effects are insignificant in the operating range of modern gas turbines, it nevertheless serves to help in understanding the broader thermodynamic picture of how work potential is related to temperature and pressure. Note also that the concavity of the exergy curves changes from concave up to concave down at roughly 4,500 ^{o}R. This is also due to changes in chemical composition in those temperature ranges.

Comparison of the plots for pure air and stoichiometric fuel-air mixtures shows that the latter has discontinuities and break-points in the 400-600 ^{o}R temperature range. This is due to phase change of the products of combustion in this region. Specifically, the upper break in the curves is due to condensation of water vapor. Therefore, the locus of breakpoints in the contours is the dew line for the combustion products. The lower break in the contours is caused by freezing of water into ice at 492 ^{o}R (32 ^{o}F). Note that although the line is shown with a slight slope, it is actually horizontal across the freezing line.

Methods Used to Estimate Thermodynamic Properties

The work potential plots in Figs. 1-4 were created based on thermodynamic curve fits for the properties of fuel-air mixtures. The calculations were carried out using Gordon and McBride's well-known Chemical Equilibrium and Applications (CEA) code.[16,17] The calculations assume equilibrium mixtures of dry air and Jet-A fuel. The thermodynamic properties of all species are based on CEA's default thermodynamic curve fits. Perfect gas effects such as vibrational excitation, dissociation, ionization, and chemical reactions are accounted for in the equilibrium calculations. Exergy is calculated by: 1) finding the enthalpy and entropy of the fuel/air mixture at the temperature and pressure of interest, 2) finding enthalpy and entropy at atmospheric conditions, and 3) calculating exergy via Eq. 1. Gas horsepower is calculated by: 1) finding the enthalpy and entropy of the fuel/air mixture at the temperature and pressure of interest, 2) finding enthalpy of the same fuel/air mixture at the same entropy as in step one but at atmospheric pressure, and 3) calculating GHP via Eq. 8. Stream thrust is calculated via Eq. 11. All work potential plots assume standard atmospheric temperature and pressure as the reference state against which work potential is measured.

Transformation of Reference Conditions

The plots presented in herein are calculated assuming the reference state is sea level standard conditions. However, it frequently occurs that one must calculate work potential relative to a reference that is not at sea level standard conditions. It is very simple to correct exergy calculations for non-standard reference conditions by using the exergy plots in Fig. 1. The procedure for correcting to non-SLS reference is:

1) Look up exergy at the temperature and pressure of interest relative to SLS reference.
2) Look up exergy of the fluid at the new reference conditions relative to SLS reference.
3) Subtract the result of step 2 from that of step 1.

It should be noted that if the new reference conditions are at a lower temperature and pressure than SLS, then the result from step 2 will be a negative number, implying that the exergy relative to the new reference condition is more than it is for SLS conditions.

The procedure for calculating GHP relative to a non-standard reference condition is similar to that for exergy. However, since GHP is independent of reference temperature, it must be calculated presuming isentropic expansion from the conditions of interest:

1) Look up GHP at the temperature and pressure of interest relative to SLS reference (Fig. 2).
2) Look up the entropy at the temperature and pressure of interest relative to SLS reference (Fig. 4).
3) Follow the entropy contour found in step 2 down to the point where it intersects the new reference pressure (x-axis)—read off the new reference temperature (y-axis).
4) Look up the GHP (Fig. 2) for the new reference pressure and temperature (from step 3).
5) Subtract the result from step 4 from step 1.

Once again, if the new reference conditions are at a lower pressure than SLS, then the result from step 2 will be a negative number, implying that the GHP relative to the new reference condition is higher than it is for SLS reference conditions.

Transformation of stream thrust to alternate reference conditions is somewhat complicated by the fact that is does not possess a "conservation" property analogous to Eqs. 5 and 11. The simplest approach for calculating stream thrust relative to a reference other than SLS is to calculate GHP first and then transform to thrust work potential via Eq. 13.

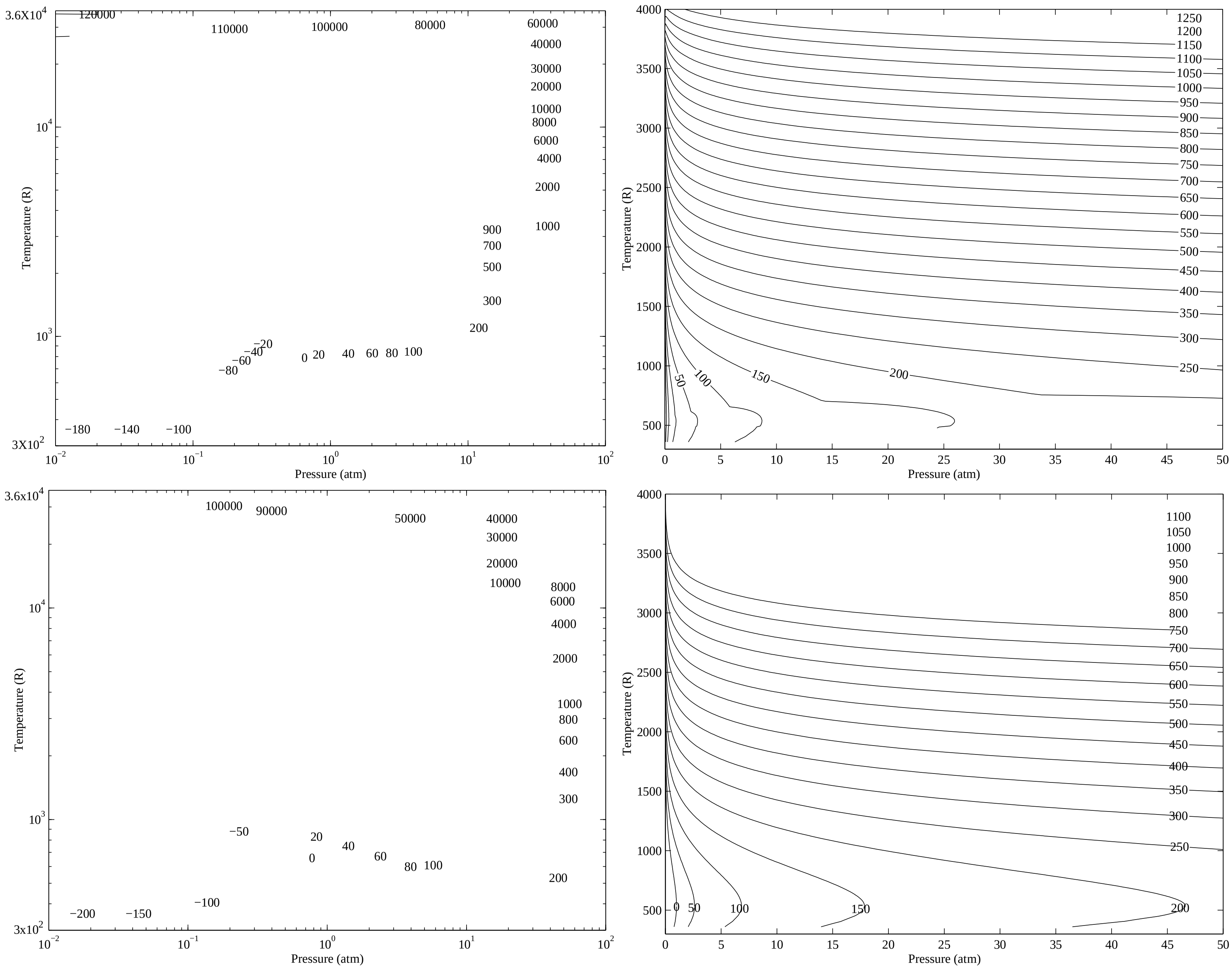

Fig. 1: Contours of Exergy for Equilibrium Stoichiometric Mixtures of Jet-A and Air (Top) and Pure Air (Bottom, HP/pps).

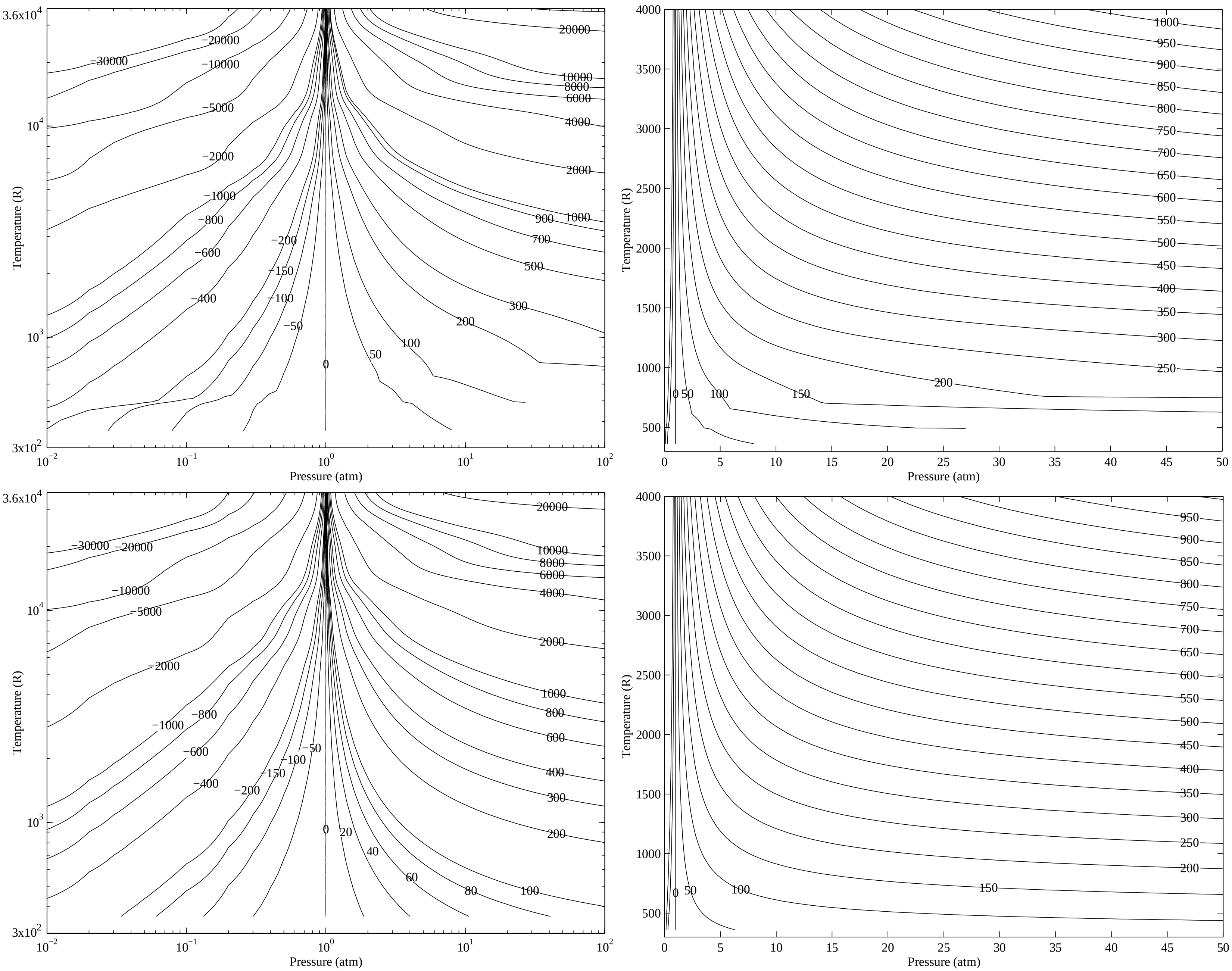

Fig. 2: Contours of Gas Horsepower for Equilibrium Stoichiometric Mixtures of Jet-A and Air (Top) and Pure Air (Bottom, HP/pps).

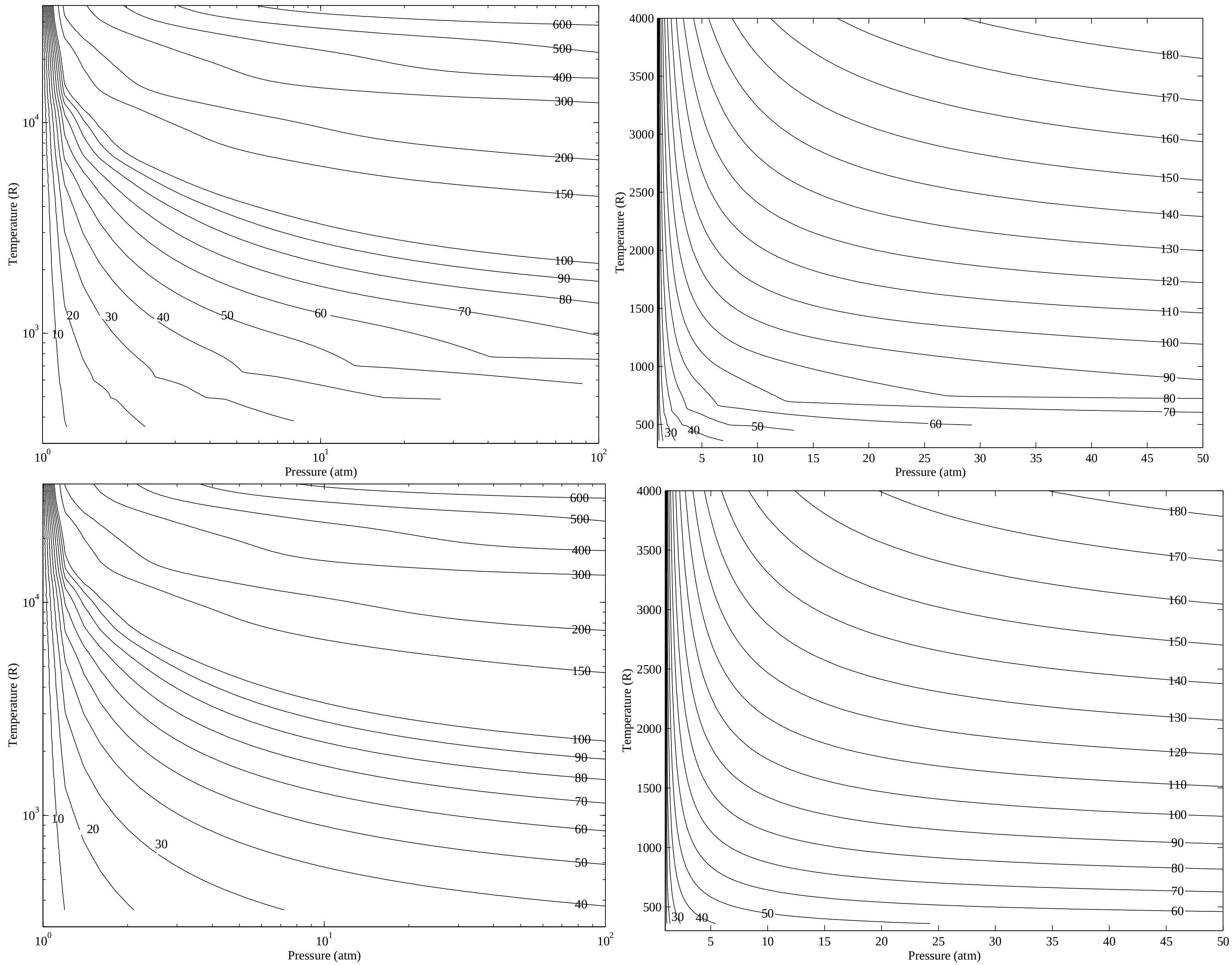

Fig. 3: Contours of Stream Thrust (Specific Thrust) for Equilibrium Stoichiometric Mixtures of Jet-A and Air (Top) and Pure Air (Bottom, lbf/pps).

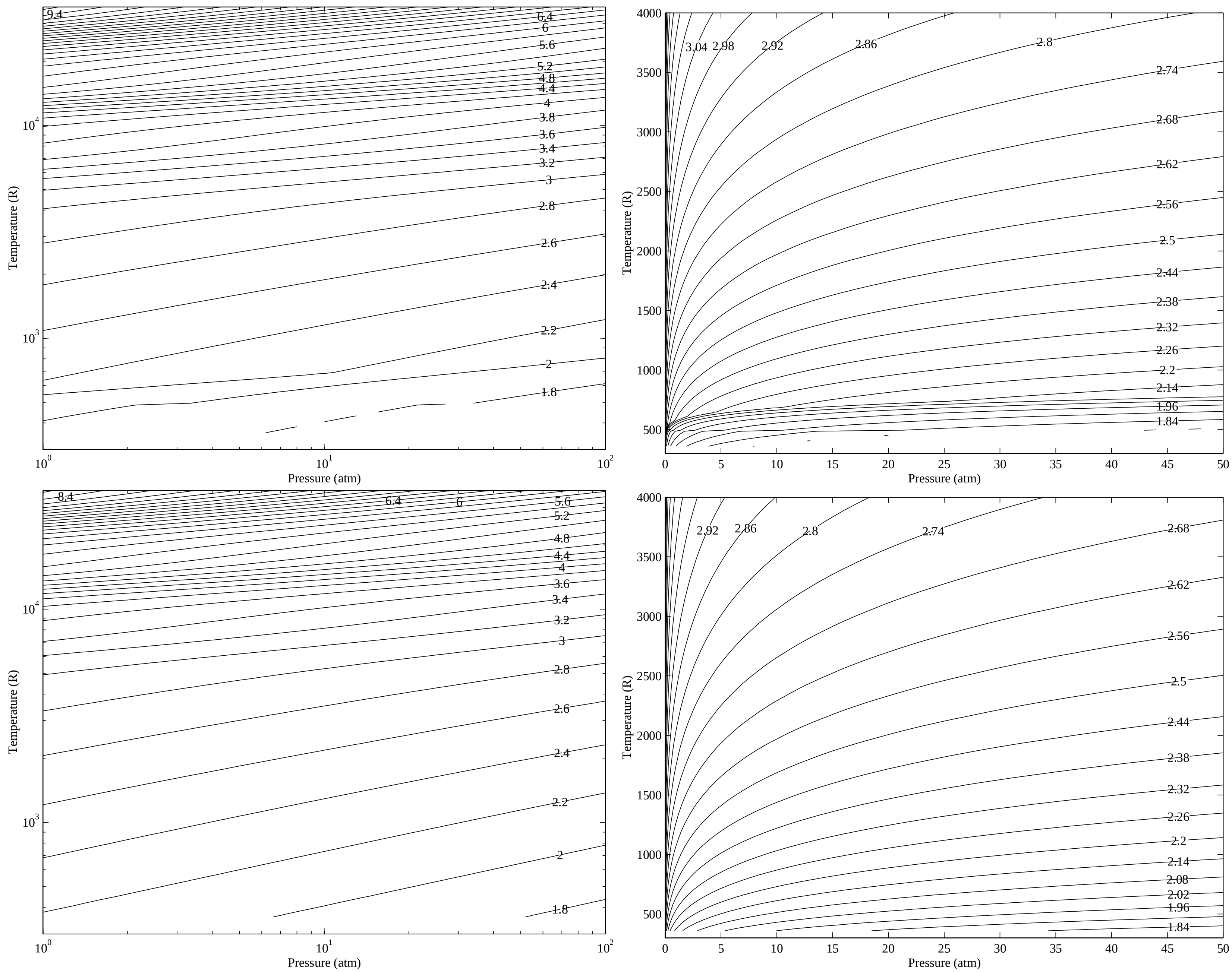

Fig. 4: Contours of Entropy for Equilibrium Stoichiometric Mixtures of Jet-A and Air (Top) and Pure Air (Bottom, HP/pps-R).

Illustrative Examples

Example 1: Calculation of Exergy for a Non-Standard Reference Condition

Modern turbofan engines typically operate with very high turbine inlet temperatures and pressures. The amount of work that could be theoretically extracted from this flow is tremendous. As an example, calculate the exergy contained in a turbine inlet flow at a fuel-air ratio of 0.033, a temperature of 3,300 R and a pressure of 40 atmospheres relative to a reference condition of 5,000 ft altitude (0.832 atm) and a temperature of 86 F.

SOLUTION:

Step 1: calculate the flow exergy at f/a=0.033, 3,300R, 40 atm using STP reference condition.

a. Use Fig. 1 to find exergy of equilibrium air at above conditions: ex=905 HP/pps
b. Use Fig. 1 to find exergy of air-fuel mixture for f/a=0.066, above conditions: ex=970 HP/pps
c. Interpolate on f/a to get exergy at 3,300R, 40 atm, f/a=0.033: ex=938 HP/pps

Step 2: calculate exergy of new reference condition relative to the standard reference condition.

a. Use Fig. 1 to estimate specific exergy at 0.832 atm, 546 R, pure air: ex=-10 HP/pps

Step 3: correct the flow exergy found in step 1 by subtracting the flow exergy at the new reference condition found in step 2: 938 HP/pps – (-10 HP/pps) = 948 HP/pps.

This is the amount of work that could theoretically be extracted per pound-mass of turbine inlet flow through a modern turbofan engine operating at 5,000 ft altitude on a hot day if that flow could be used in a Carnot engine.

Example 2: Calculation of Gas Horsepower for a Non-Standard Reference Condition

The total gas horsepower typically present in the turbine inlet flow of a modern turbofan engine is much less than the exergy. As an illustration, calculate GHP in the turbine inlet flow for the previous example.

SOLUTION:

Step 1: calculate the GHP at f/a=0.033, 3,300R, 40 atm using STP reference condition.

a. Use Fig. 2 to find gas horsepower of equilibrium air at above conditions: ghp=785 HP/pps
b. Use Fig. 2 to find gas horsepower of air-fuel mixture for f/a=0.066, above conditions: ghp=815 HP/pps
c. Interpolate on f/a to get gas horsepower at 3,300R, 40 atm, f/a=0.033: ghp=800 HP/pps

Step 2&3: look up entropy at f/a=0.033, 3,300R, 40 atm and follow isentrope down to new reference pressure:

a. Use Fig. 4 to estimate entropy for stoichiometric mixture: 2.72 HP/pps-R; this corresponds to an isentropic turbine discharge temp. of 1,395 R @ 0.8 atm.
b. Use Fig. 4 to estimate entropy for pure air: 2.64 HP/pps-R; this corresponds to an isentropic discharge temperature of 1,285R at 0.8 atm reference pressure.
c. Interpolate on temperature: 1,340R

Step 4: look up gas horsepower of new reference condition relative to the standard reference condition:

a. Use Fig. 2 to estimate specific gas horsepower at 0.832 atm, 1,340 R, f/a=0.0: ghp= -30 HP/pps
b. Use Fig. 2 to estimate specific gas horsepower at 0.832 atm, 1,340 R, f/a=0.066: ghp= -28 HP/pps
c. Interpolate: ghp= -29 HP/pps

Step 3: correct the flow gas horsepower found in step 1 by subtracting the flow gas horsepower at the new reference condition found in step 2: 800 HP/pps – (-29 HP/pps) = 829 HP/pps.

Note that the theoretical gas horsepower of the turbine flow is substantially less than the exergy found in example 1. The various loss mechanisms present in the engine components tend to further magnify the differences between gas horsepower and exergy, as explained in Ref. 6.

Example 3: Calculation of Thrust Work Potential for a Non-Standard Reference Condition

Calculate the thrust work potential at the turbine inlet for the conditions used in the first two examples presuming that the aircraft is moving at 300 ft/s.

SOLUTION:

If standard day conditions were present, one could use Fig. 3 to estimate thrust work potential by finding stream thrust at the turbine inlet and then multiplying by flight velocity. However, Fig. 3 cannot be corrected to non-sea level standard conditions because stream thrust is not conserved in the same way as GHP and exergy. However, Eqs. 12 and 13 provide a convenient means of determining thrust work potential for non-standard conditions.

Step 1: calculate stream thrust at turbine inlet conditions via Eqn. 13:
sa=sqrt[2(829)550/32.17]=168.4 lbf/pps.

Step 2: calculate thrust work potential via Eqn. 14:
wp=168.4(300)/550=91.8 HP/pps.

Note that this is far less work potential than previously calculated for exergy or gas horsepower. Of the 803 HP/pps gas horsepower available in the stream, only 91.8 HP/pps would materialize as thrust work if the turbine inlet flow were to be expanded in a thrust nozzle. The remaining gas horsepower would be converted into residual kinetic energy, which is a loss if the objective is to produce thrust work. This is the Achilles heel of turbojet engines and is the reason that turbofan engines are dominant today. The turbofan engine allows the cycle to be tailored for maximum effectiveness in transferring gas horsepower of the core stream into thrust work potential of the fan stream.

Example 4: Calculation of Gas Horsepower Loss

Presume that the HP turbine in example 2 delivers 150 shaft HP per lbm core flow in order to drive the HP compressor. Further presume that the conditions at the exit of the HP turbine are f/a=0.033, 2,750 R and 25 atm. Find the loss in gas horsepower inside the turbine relative to SLS reference conditions.

SOLUTION:

Step 1: calculate the gas horsepower flowing into the turbine (found in example 2): ghp=800 HP/pps

Step 2: calculate gas horsepower of the flow leaving the turbine.

a. Use Fig. 2 to find gas horsepower of equilibrium air at exit conditions: ghp=597 HP/pps
b. Use Fig. 2 to find gas horsepower of air-fuel mixture for f/a=0.066, exit conditions: ghp=618 HP/pps
c. Interpolate on f/a to get GHP at 3,300R, 40 atm, f/a=0.033: ghp=608 HP/pps

Step 3: use Eq. 11 to calculate loss of gas horsepower inside the HP turbine: Loss=800-608-150=42 HP/pps

Thus, the turbine loses 42 HP of flow work potential per pound-mass flow through the machine. See Refs. 4, 18, and 19 for a detailed example applying these concepts to a full propulsion system.

Conclusions

Thermodynamic work potential-based analysis of aircraft engines is greatly facilitated by the availability of practical working charts and tables from which the propulsion system engineer can draw useful analysis data. This paper presented a very abbreviated set of charts that can be used for this purpose, with a more extensive set being available in Ref. 15. These charts are particularly useful in the cycle analysis process for determining absolute magnitude of loss in each component of the engine. The charts are also quite revealing relative to how work potential of a high-enthalpy gas mixture varies with temperature, pressure, and fuel-air ratio.

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Note: many of the references cited herein are available for download at: www.asdl.gatech.edu

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