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Torsional Waveguide Density Sensor

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TORSIONAL WAVEGUIDE DENSITY SENSOR

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ABSTRACT

As a part of National Science Foundation-Research for Undergraduates program, a torsional wave guide density sensor has been investigated in the Laboratory of Surface Science and Technology at the University Of Maine. This simple design uses torsional piezo-electric transducers to generate torsional mode in a long circular rod.
The non-circular portion of the wave guide allows for the measurement of different properties of fluids. Theory is presented from energy balance perspective and from the acoustic wave propagation approach. A series of experiments have been conducted with variety of liquids and gases. Several relationships have been verified and also new have been developed.

ACKNOWLEDGEMENT

I would like to express my gratitude to my advisors: Mr. Lawrence Lynnworth, Professor John Vetelino and William Spratt for their patience and priceless assistance during the span of the project.

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INTRODUCTION

A sensor that its functioning is based on propagation of torsional waves through solids and liquids have been researched during the ten-week NSF-REU program at the University Of Maine. The main purpose of this project was to develop a tensional wave density sensor that could be used in variety of applications. Theoretical research leads me to the work of Kim, Bau and Lynnworth that basically laid foundation in the torsional wave-guide sensors in the USA. A short explanation of the torque-torsion and the properties strength of material are essential to understand fully the mechanism of the torsional movement through the solids. In addition, the density and viscosity measurements are closely related to the fluid mechanics properties that are dependent on the shapes of wave-guides and the types of fluids involved. The propagation of the acoustic wave in the circular rod can be analyzed not only with the energy balance approach and mechanical engineering, but also trough the standard electrical engineering approach. An experimental set-up that included an adjustable stand/holder and a gas chamber was manufactured during the first six weeks of the program. Once the theoretical part was analyzed, a series of experiments have been conducted using rods with different shapes of the wave-guides and a variety of fluids. Experimental results were analyzed and it is shown that they follow either the theory or experimental data from the past. Possible applications of the torsional wave-guide sensor are shown in different industrial and technological settings.

OBJECTIVES OF THE PROJECT

The main purpose of this project was not only the investigation of the theory behind the torsional wave propagation through the circular and noncircular wave guides, but also the design and manufacturing of the sensor and its set-up. Evidently, an experimental testing of the sensor in order to obtain different densities of several fluids was the next step. By the in depth analysis of the available literature and experiments conducted our task was accomplished.

THEORY

Torsional wave propagation through a solid circular rod has to be looked first from the mechanics point of view. First, let us consider a

circular rod that is being exposed to a mechanical torque applied at its ends. As the torque is applied, a shear stress is being formed inside the rod and "twisting" of the rod occurs.

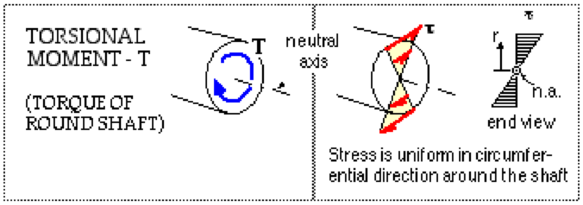


Fig.1 Shows torque, torsional stress in circular bar[1]

The stress present in the rod can be described by equation (1):

$$\tau = \frac{P}{A} \tag{1}$$

where τ is shear stress, P is the force applied and A is the area exposed to the force. In order to quantify a geometrical deformation let us take a look at differential part of the rod that is depictured below.

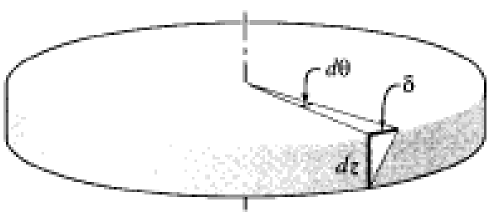


Fig.2 Differential element of the circular rod[2]

Therefore, the relative tangential displacement of the top portion of the cylinder is defined by equation (2)

$$\delta = r d\theta \tag{2}$$

Consequently, shear strain present in this differential cylinder is defined by equation (3):

$$\gamma_{z\theta} = \frac{\delta}{dz} = r\frac{d\theta}{dz} \tag{3}$$

For the material in the elastic regime Hooke's Law defines shear stress in (4):

$$\tau_{\theta z} = G \cdot \gamma_{\theta z} = G \cdot r \cdot \frac{d\theta}{dz} \tag{4}$$

where G is the shear modulus of materials and r is the radius of the bar. Once the torque T is applied to the bar, we can obtain a momentum-shear balance on the rod depicted below

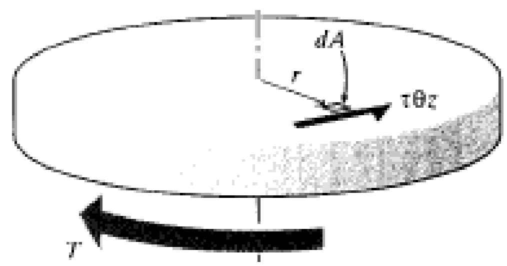


Fig.3 Torque applied on the differential element[3]

to find that rod is in the rotational equilibrium and shown in equation (5):

$$T = \int_A \tau_{\theta z} r dA = \int_A Gr\frac{d\theta}{dz} r dA = G\frac{d\theta}{dz}\int_A r^2 dA$$

where the last integral on the right side is the polar moment of inertia of the disk. The quantity in equation (6):

$$\frac{d\theta}{dz} = \frac{T}{GJ} \rightarrow \theta = \int_z \frac{T}{GJ} dz \tag{6}$$

[1] http://www.mech.uwa.edu.au/DANotes/SSS/loads/blocks.gif
[2] http://web.mit.edu/course/3/3.11/www/modules/torsion.pdf
[3] Ibid

Moreover, because values of T, J and G are constant within circular rod along the z axis therefore the angle of twist can be written in equation (7) as:

$$\frac{d\theta}{dz} = constant = \frac{\theta}{L} \quad \cdots \quad \theta = \frac{TL}{GJ} \qquad (7)$$

Referring back to Hooke's Law, we can establish the relationship for shear stress in equation (8) in terms of angle of twist:

$$\tau_{\theta z} = Gr\frac{d\theta}{dz} = Gr\frac{\theta}{L} = \frac{Gr}{L}\frac{TL}{GJ} = \frac{Tr}{J} \qquad (8)$$

As we can see the shear stress is directly proportional to the Torque, radius of the point of interest and inversely proportional to the polar moment of inertia. Also, it is important to illustrate that the distribution of shear stress is not symmetrical in waveguides with non-circular cross sections. It is shown on the figures below. We will see that this analogy appears again when analyzing propagation of torsional waves through a rod as an acoustic wave.

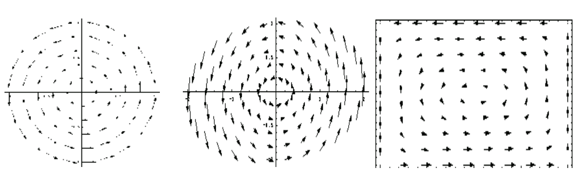


Fig.4 Shear stress distribution for waveguides with different cross-sections.

ENERGY BALANCE IN A NON-CIRCULAR ROD

Torsional wave propagation can be analyzed from an energy balance approach. This allows obtaining the expression for the speed of the torsional wave in terms of densities of the fluid of interest. H. Bau proposed the following method in one of his early papers[4]. While analyzing an effect of surrounding fluid on the transmission of stress waves in the rod we consider an elastic rod with a non-circular cross-section submerged in the fluid. The theory states that the speed of propagation of torsional waves is decreasing with increased density of the fluid. A torsional pulse is induced and is propagating along z-axis. If the angle of rotation per unit length is $\frac{\partial\varphi}{\partial z}$ then the elastic energy in the rod can be defined by equation (9):

$$E_1 = \frac{1}{2}\int_o^L GD\left(\frac{\partial\varphi}{\partial z}\right)^2 dz \qquad (9)$$

Where D is the torsional rigidity G is the shear modulus of the material and the derivative represents the rate of deformation. Furthermore, the corresponding kinetic energy in the rod is expressed by equation (10):

[4] Torsional Wave Sensor – A Theory, Journal of Applied Mechanics 1986, Vol. 108, H.H. Bau.

$$E_2 = \frac{1}{2}\int_o^L \rho_s I_s \left(\frac{\partial\varphi}{\partial t}\right)^2 dz \qquad (10)$$

with the density ρ_s of the material that the rod is made of and polar inertia of the cross sectional area of the rod I_s. While torsional wave moves through the rod along *z*-axis, it causes acceleration and deceleration of the surrounding fluid. Consequently, the kinetic energy of the fluid can be defined by equation (11) as:

$$E_3 = \frac{1}{2}\int_o^L \rho_f I_f \left(\frac{\partial\varphi}{\partial t}\right)^2 dz \qquad (11)$$

with the density and the apparent polar inertia of the fluid respectively. We assume that the fluid is inviscid and the thickness of the viscous boundary layer is significantly smaller that the wave guide area $\sqrt{\nu T}$ is the approximation of the boundary layer with kinematic viscosity of the fluid and wave's period. We can see that both inertias and D are not functions of z. Setting up a differential equation (12):

$$\frac{\partial^2\varphi}{\partial t^2} = c^2 \frac{\partial^2\varphi}{\partial z^2} \qquad (12)$$

and solving it to obtain torsional value c in equation (13):

$$c = K\sqrt{\frac{G}{\rho_s}}\left(1 + \frac{I_f\rho_f}{I_s\rho_s}\right)^{-\frac{1}{2}} \qquad (13)$$

with $K = \sqrt{\frac{D}{I_s}}$. The relationship between the densities and the torsional speed is dependent on the geometry of the wave guide and specified by K and I values. Most shapes that are used in wave guides have tabulated expressions for the torsional rigidity and polar inertia. For the rectangular shape of the wave guide the expression for K according to Sokolnikoff [5] is given in equation (14):

$$K^2 = \frac{4}{1 + \left(\frac{a}{b}\right)^2}\left[1 - \frac{192}{\pi^5}\left(\frac{b}{a}\right)\sum_{n=0}^{\infty}\frac{tanh\frac{(2n+1)\pi}{2}\frac{a}{b}}{(2n+1)^5}\right]$$

and for practical purposes, the first term of the series brings the largest contribution to the result. For the elliptical cross-sectional wave guide the value of torsional rigidity is given by equation (15):

$$K = \frac{2ab}{a^2+b^2} \qquad (15)$$

where a and b are the larger and smaller distances in the ellipse. Analytically torsional rigidity can be obtained in the following manner. Let a stress function H (x,y) be defined in such a way that the stresses caused by the torsional wave are expressed by equation (16):

[5] Sokolnikoff Mathematical Theory of Elasticity

$$\tau_{zx} = G\frac{d\varphi}{dz}\frac{\partial H}{\partial y} \quad \text{and} \quad \tau_{zy} = -G\frac{d\varphi}{dz}\frac{\partial H}{\partial x} \qquad (16)$$

the equilibrium allows for the stress function to satisfy Poisson's equation (17):

$$\nabla^2 H = -2 \qquad (17)$$

In addition, the lateral surface of the rod is stress free and H is constant. Solving the above equation analytically we obtain H(x,y) and then using the following area integral we obtain the torsional rigidity D expressed by equation (18)

$$D = 2\int\int H(x,y)dxdy \qquad (18)$$

Usually the torsional rigidity is expressed as $K = \sqrt{\frac{D}{I_s}}$ and available for simple shapes in the strength or mechanics of materials publications.

SOLID-FLUID INTERACTION

As the torsional waves proceeds along the wave guide it accelerates and decelerates the fluid. The effect of the deformation of wave guide on the surrounding fluid can be observed in streamlines around the sensing portion of the wave guide. The following are samples of a graphical representation of stream lines for rectangular, elliptical and diamond shaped wave guides.

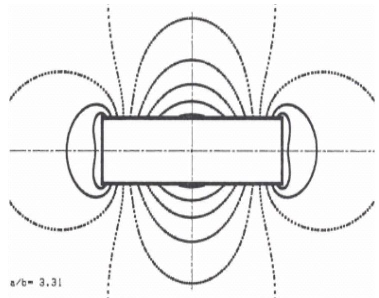


Fig.5 Streamlines around rectangular cross-section

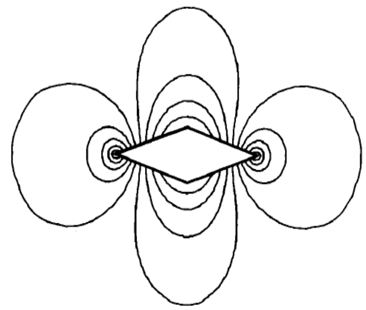


Fig.6 Streamlines around diamond cross-section

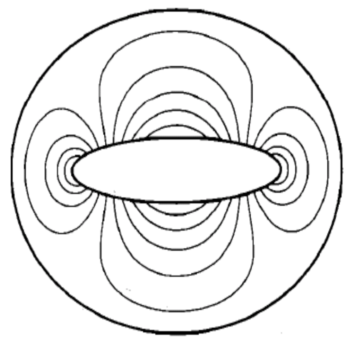
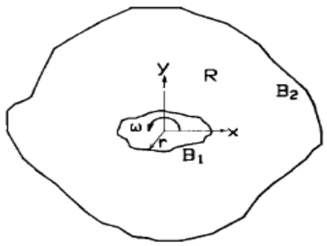


Fig.7 Streamlines around elliptical cross-section

Another aspect associated with solid-fluid interaction is the apparent inertia of the fluid. It is necessary to obtain a numerical value for it in order to calculate the density of the fluid of interest. There are different methods available including computer programs such as finite elements methods or other numerical methods. Kim and Bau[6] proposed the following approach to obtain its value.

[6] Kim and Bau, "On-line real time densimeter,



[7]

Fig.8 Integration domain for the tip of the waveguide

Consider the wave guide of interest with contact surfaces B_1, B_2 and with the domain occupied by the fluid R. Stream function can be obtained from the set of equations (19):

$$\nabla^2\psi = 0 \; in \; R$$
$$\psi = \frac{1}{2}\,\omega r^2 \;\; on \;\; B_1 \qquad (19)$$
$$\psi = constant \;\; on \;\; B_2$$

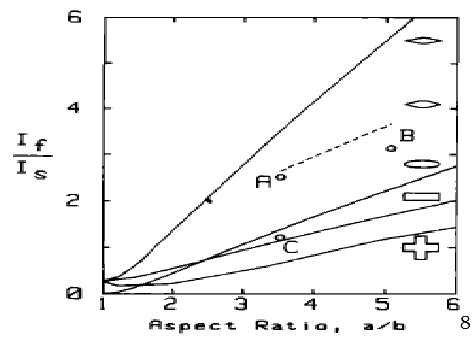
For complex shapes a finite elements method can be used and for simple shapes, computational methods are used. Once the stream function is calculated, the kinetic energy of the fluid per unit length is obtained from the integration in equation (20):

$$[KE]_f = \frac{1}{2}\rho_f \int\int_R \nabla\psi \cdot \nabla\psi dA = \frac{1}{2}\rho_f \int_{B_s} \psi \frac{\partial\psi}{\partial n} ds$$

Finally, kinetic energy is used to calculate the apparent inertia of the fluid using equation (21):

$$I_f = \frac{[KE]_f}{\frac{1}{2}\rho_f\omega^2} \qquad (21)$$

where omega is the frequency of operation. Another method of obtaining apparent inertia of the fluid is to use tabulated or graphical data from previously collected experimental data for know shapes of wave guides. Graph below shows the ratio of inertias in terms of geometrical ratio of the/b that is known from the dimensions of the sensor.



[8]

Fig.9 Sensitivity as a function of shape-ratio a/b.

ANALYSIS OF THE TORSIONAL WAVE PROPAGATION THROUGH A CIRCULAR ROD

One of the methods of obtaining the speed of the torsional wave in a circular rod is to analyze it from a harmonic approach. While working in a cylindrical coordinate system as shown below we can assume that the wave guide is linear, homogenous and made out of elastic medium.

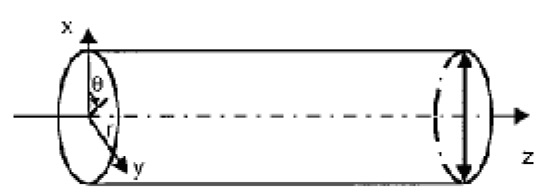


Fig.10 Circular rod in rectangular coordinate system

[7] Ibid

[8] Ibid

The torsional wave relationship for a cylindrical rod of radius r is defined by equation (22):

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{C_t^2}\frac{\partial^2 u}{\partial t^2} \qquad (22)$$

where u is the displacement induced in radial direction and C_t is the shear wave velocity[9]. For the time-harmonic wave propagation, the solution to above equation is in the form given in (23):

$$u(r,z,t) = \Phi(r)e^{i(\omega t - kz)} \qquad (23)$$

where ω = 2Πf and k is the wave number. The corresponding amplitude equation is given in (24):

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} - \frac{\Phi}{r^2} + q^2\Phi = 0 \qquad (24)$$

where the value of q is defined as (25):

$$q^2 = \frac{\omega^2}{C_t^2} - k^2 \qquad (25)$$

A conventional analytical approach gives the general solution to the amplitude equation in equation (26):

$$\Phi(r) = AJ_1(qr) + CK_1(qr) \qquad (26)$$

where J_1 and K_1 are the equivalents of Bessel function of the first type of order 1 and also a modified second type Bessel function of order 1. In the case of r = 0 we obtain C = 0 and the general solution is given by equation (27):

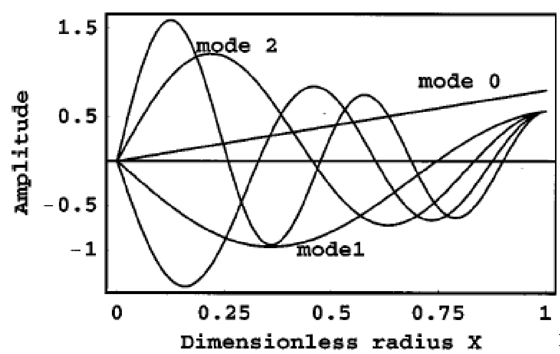
$$u(r,z,t) = AJ_1(qr)e^{i(kz-\omega t)} \qquad (27)$$

and the second boundary condition states that r = a and gives the equation (28) and simplifies to equation (29):

$$(qa)J_0(qa) - 2J_1(qa) = 0 \qquad (28)$$

$$J_2(qa) = 0 \qquad (29)$$

The solutions of the above equation gives in the set of the values $\{\beta n = qna\}$ that are the roots of the first type Bessel function of order 2 and therefore $\{\beta n\}=\{0.0, 5.13, 8.41, 11.62, 14.80, 17.96, 21.11\}$. For these values we can compute the values of $\{kn\}$. The following graph shows the amplitude of the first five torsional modes.



[10]

Fig.11 Amplitude vs. radius-first five torsional modes.

It is interesting to mention tat the first torsional mode does not have attenuation. The remaining modes contribute to the attenuation in the rod.

[9] J.D.Achenbach, Wave Propagation in Elastic Solids (North-Holland/Amaerican, Elsevies, 1973), Chapter 3

[10] J.C. Aime"Spatial analysis of torsional wave propagation in a cylindrical wave guide". J.Acoust. Soc. Am., 2001

EQUIPMENT-INSTRUMENTATION

The purpose of the series of experiments conducted during the last two weeks of the program was to generate torsional wave in the rod and to obtain a variety of experimental data. The following equipment and instrumentation was used:

- Pair of 2.25 MHz - 0.50 V 154-RB Shear Transducers - Panametrics
- Ritec Advanced Measuring System RAM-5000
- LeCroy WavePro 7100 1 Ghz Oscilloscope
- Standard thermocouple digital thermometer
- Thermocouple type -T from Omega
- 0.4 kWh vacuum pump
- Circular and rectangular wave guides
- Gas chamber and a waveguide holder.

During the experiment, the torsional wave travels on the same route until it reaches the medium of interest. First case is when air is used as a medium. Second case is when it is in the liquid in the beaker. Third case is in the gas chamber. Diagram below shows those cases in different color.

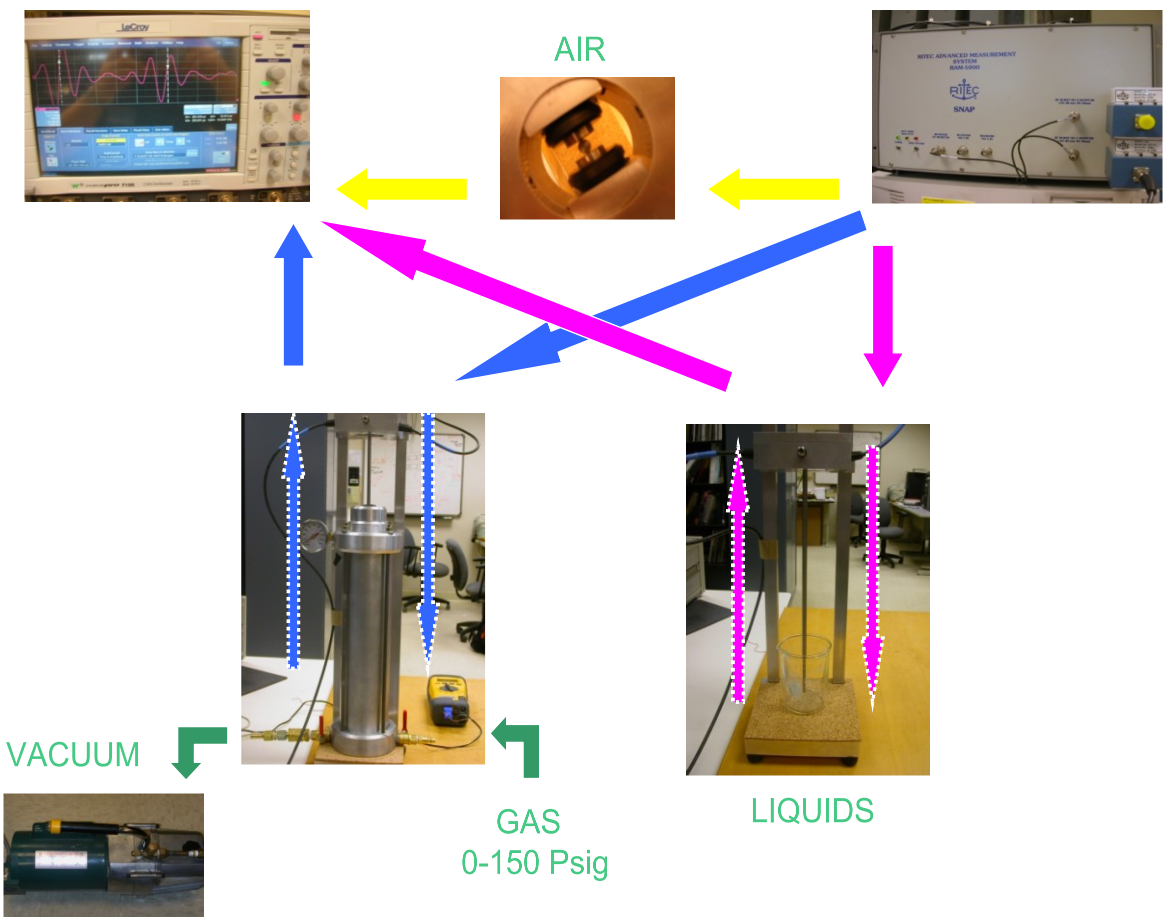


Fig. 12 Experimental Set-up

1. Torsional wave propagation through air (yellow path)

2. Torsional wave propagation through fluids (purple path)

3. Torsional wave propagation through gases (blue path)

As we can see, the paths vary by the medium in which the torsional waves travel. The description of experimental data that follows in the next section will not only explain fully the nature of the experiments, but also provide with numerical and graphical examples associated with each experiment.

EXPERIMENTAL RESULTS

First experiment focused on generating a torsional wave in a SS-306-12” circular waveguide and comparing the travel time in air and water. The picture below shows waveforms obtained during this experiment. The time flight is the same for both cases. Therefore, it confirms the theory that a circular waveguide cannot be used for density measurements.

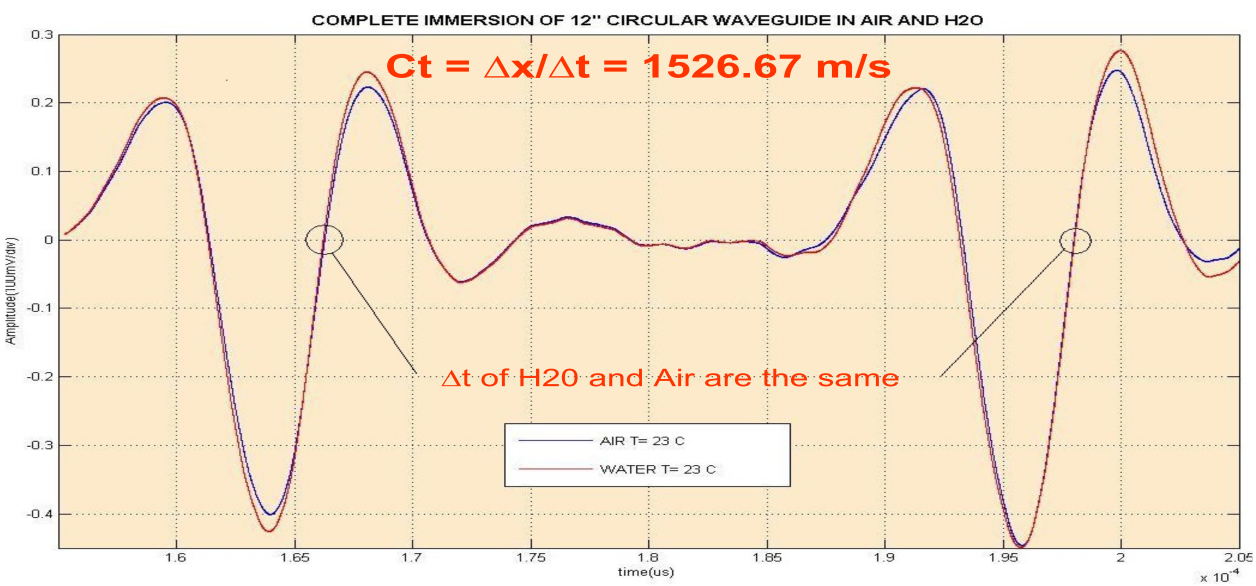


Fig. 13 12” circular waveguide in air and in water.

In second experiment we used SS-316-14” rectangular waveguide that was immersed in air and in the water. There was a difference in the flight time observed between air and water. Consequently, non-circular waveguide is a useful tool in density measurement

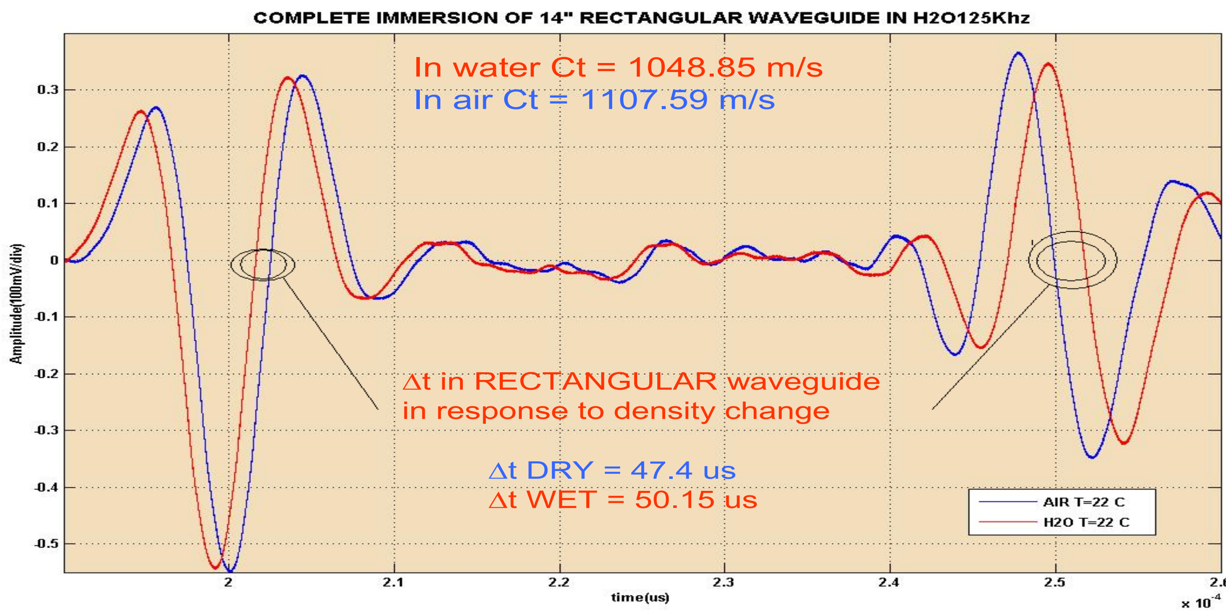


Fig. 14 14" rectangular waveguide in air and in water.

Third experiment focused on determining the difference in the flight time dependent on the fluid level. Again SS-314-14" rectangular waveguide was used and gradually immersed in water. As we can see from the graph below the flight time increases as the fluid level increases ant this also confirms the theory.

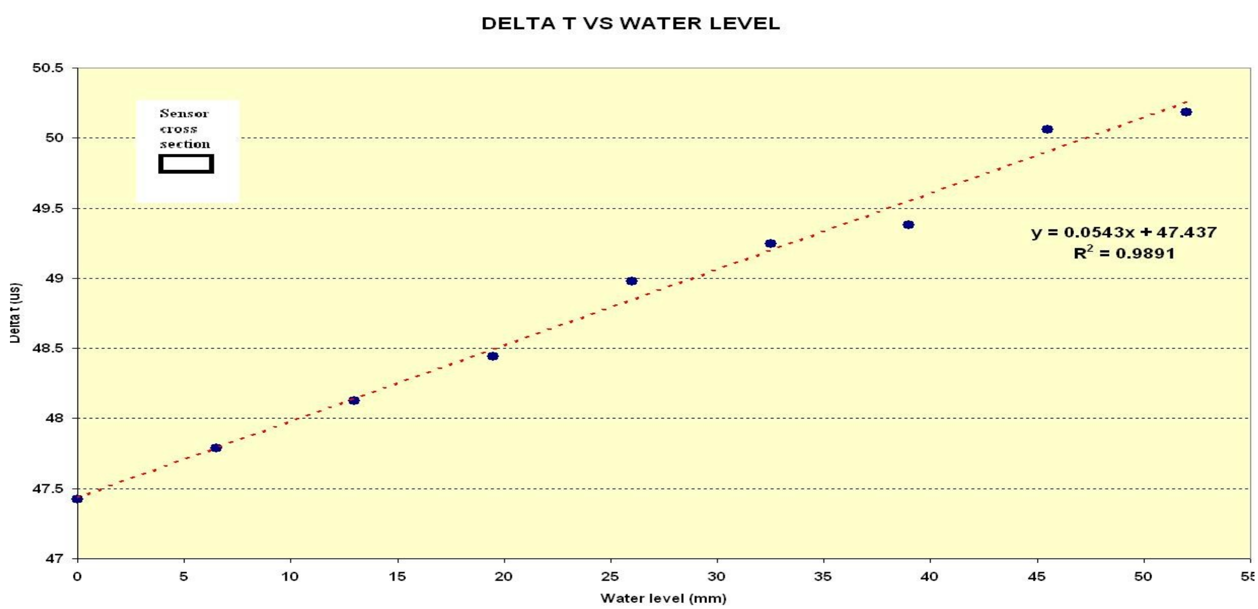


Fig. 15 14" rectangular waveguide in water.

Next experiment examined the influence of the temperature of the waveguide on the flight time. Again, a SS-316-14" rectangular waveguide was used to obtain the results. A standard thermocouple was mounted inside a small axial hole on at the bottom of the rectangular waveguide and the temperature of the waveguide was increased gradually. The relationship between the temperature and the time of flight is shown below.

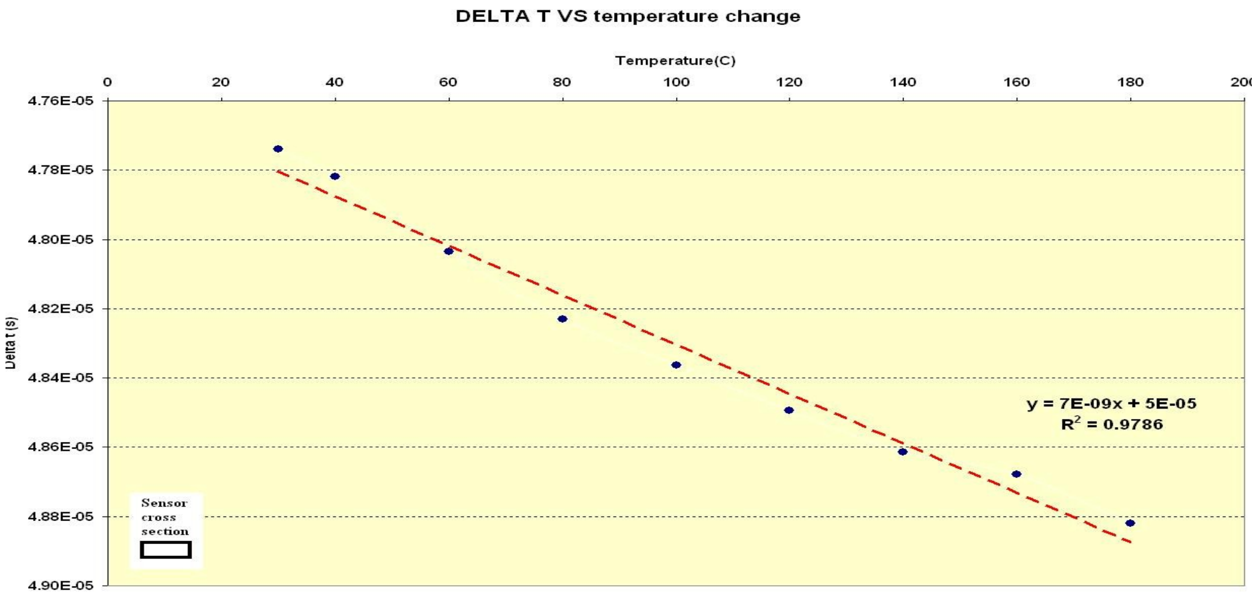


Fig. 16 14” rectangular waveguide in air.

Fifth procedure leads us to fluid density determination. For this experiment, we used four different fluids: water, acetone, isopropyl alcohol 93%, hydrogen peroxide 3 % and air. Again a SS-316-14” rectangular waveguide was used. The first graph below shows the waveform from the first step (‘ marked with circle’),to the end-tip of the 2” rectangular part of the wave-guide(‘ marked with rectangle’).The second graph below show the close-up with the detailed end-tip. The difference in the flight time is quite visible in the rectangular window on the right side.

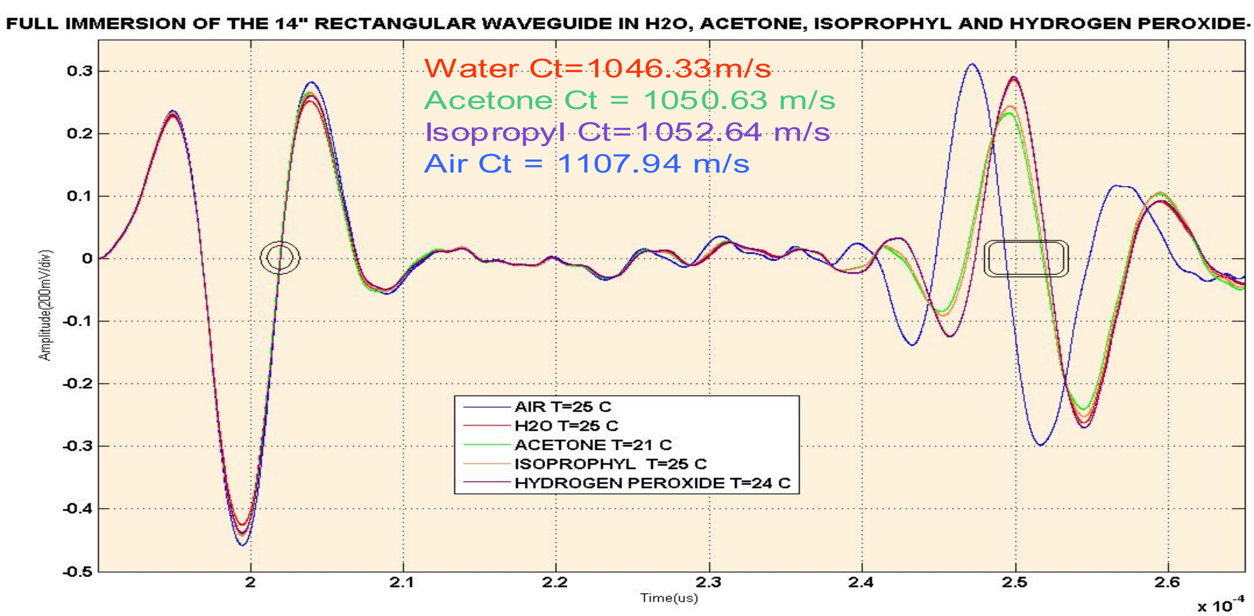


Fig. 17 14” rectangular waveguide in different fluids and air.

As we can see in the large rectangle, there is a difference in the flight time between T0 at the “circle” and T1 at the “rectangle”. This difference can be used in the calculation of the density of unknown fluids that in our case was in the range of 0.25%.

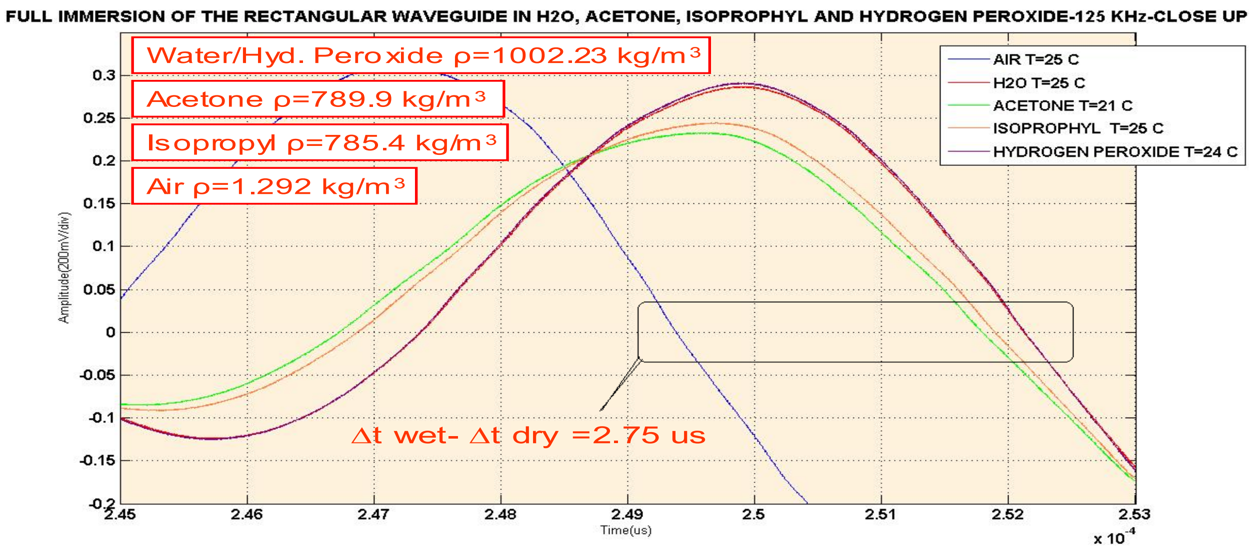


Fig. 18 14" rectangular waveguide in different fluids and air – close-up.

Another interesting experiment dealt with the simulation of gradual "bio-build up" on the tip of the waveguide. This time we used SS-316-14" circular waveguide. Single layers of silicone sealant were applied on the end tip of the waveguide. On the graph we can clearly see a gradual almost linear increase in both amplitudes (blue-A, red-B) and the natural logarithm of the ratio (a/b) against the numbers of layers.

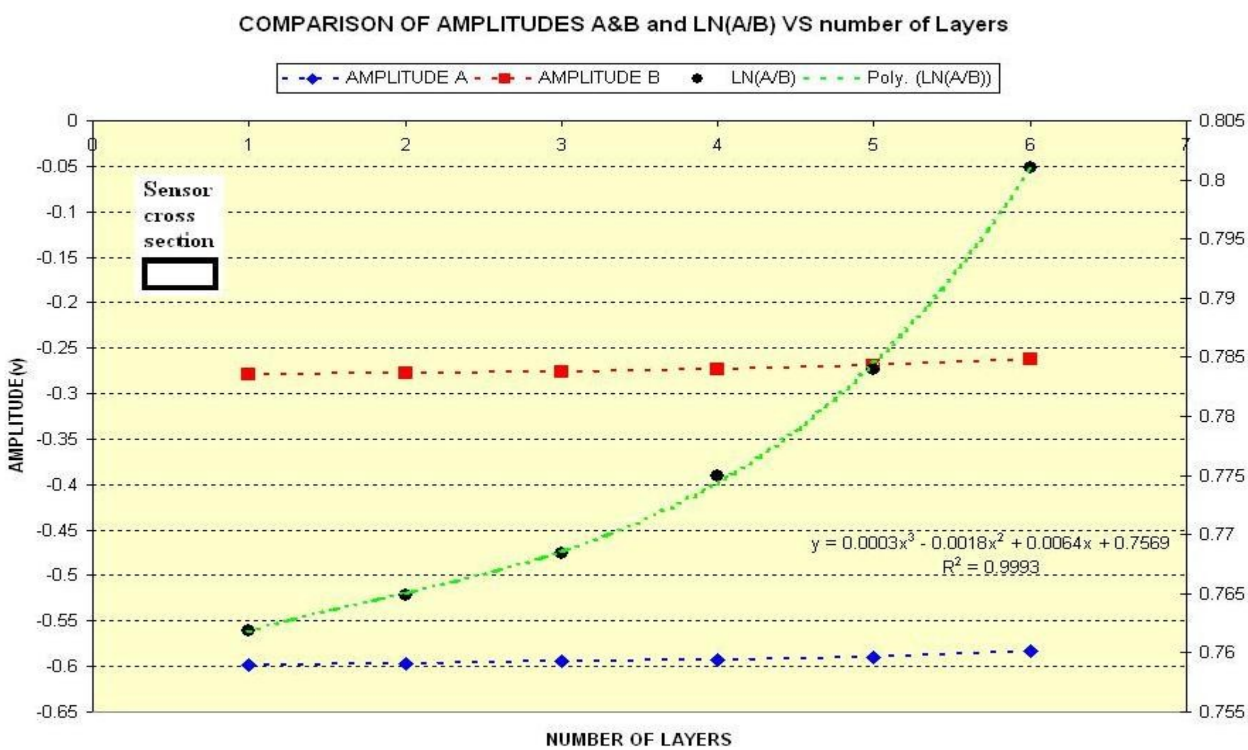


Fig. 19 14" circular waveguide with several coatings of liquid silicone.

Our last experiment involved Nitrogen, Helium and Argon gas testing. Again we used SS-316-14" waveguide, but this time it was placed in small gas chamber. The pressure changed gradually from vacuum to 150 psig and several readings were taken for each gas. Waveforms below show the flight times of the torsional waves in the rectangular waveguide for each gas.

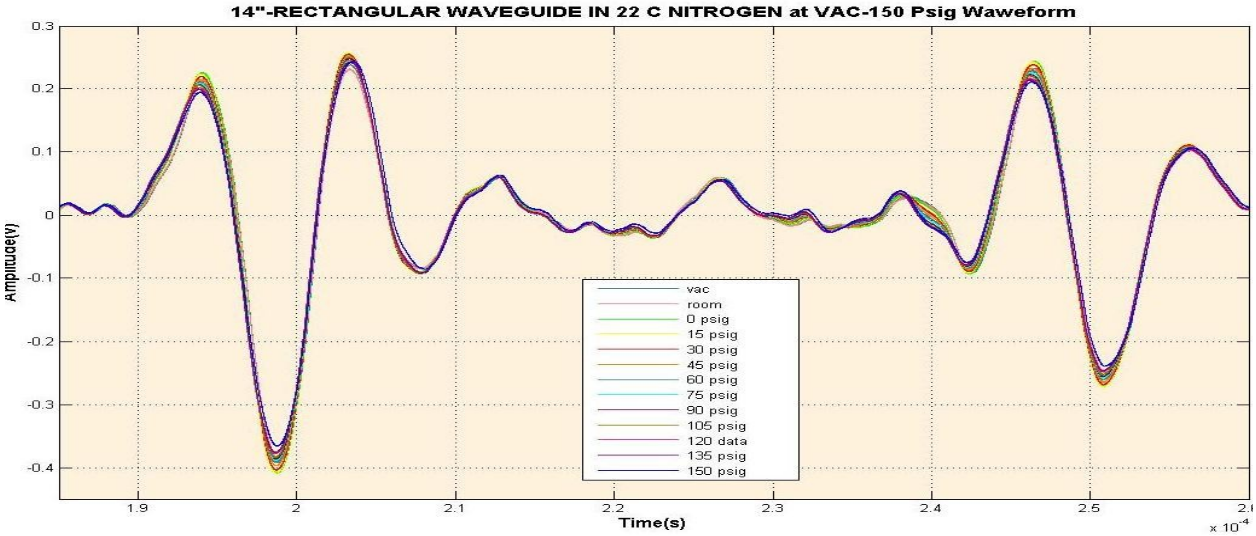


Fig. 20 14” rectangular waveguide in Nitrogen.

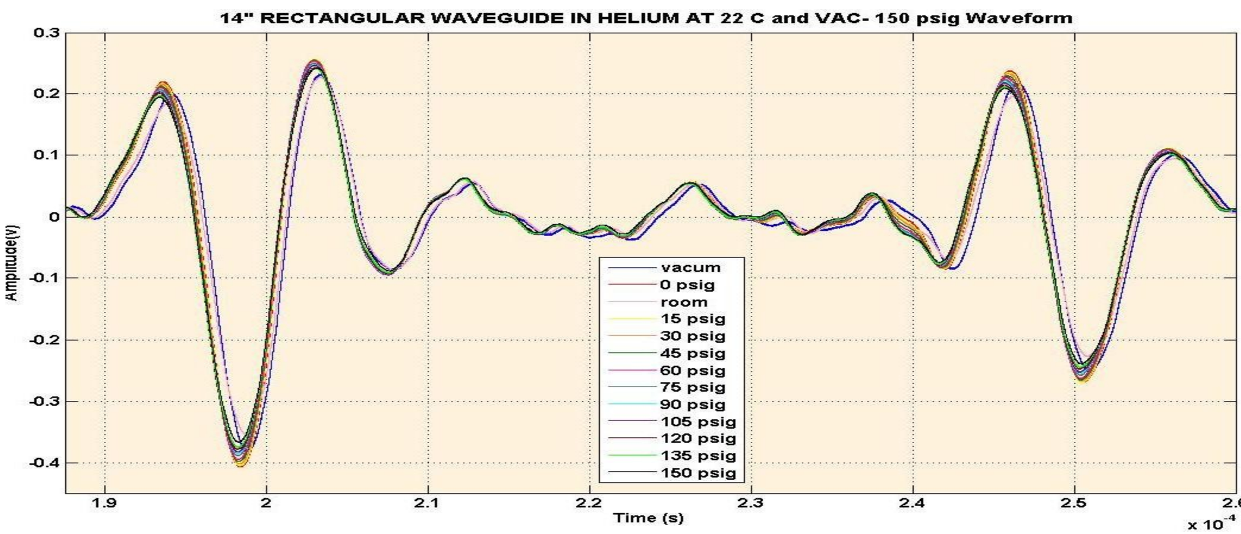


Fig. 21 14” rectangular waveguide in Helium.

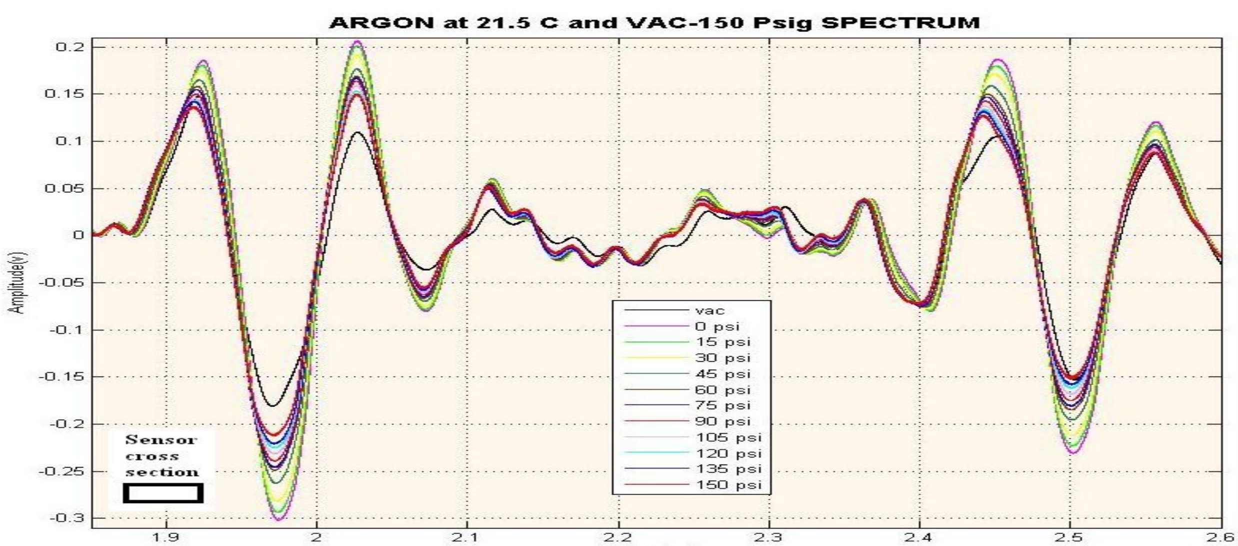


Fig. 22 14” rectangular waveguide in Argon.

Gas testing was the most challenging procedure. The key in obtaining good linear results is to control pressure and temperature in equilibrium steps and then obtain the correct readings. Although we were able to take the readings, the final result was not accurate enough and the expected linearity was missing. Therefore, I would repeat these experiments with improved pressure-temperature control.

APPLICATIONS

There are many potential applications of devices that use the torsional wave propagation phenomena. Currently on the market, there are torsional wave sensors that measure the following:

1. density
2. viscosity
3. fluid level
4. flow rate
5. steam quality
6. temperature
7. G value.
8. Corrosion and crack detection

Undoubtedly, there are many more possible applications of sensors based on the torsional wave propagation through circular and non-circular waveguides. Future research will investigate such possibilities.

SUMMARY-CONCLUSIONS

During the ten-week program, we were able to accomplish the following:

- generated the torsional mode using shear transducers
- obtained shear velocities for circular and rectangular sensors
- established correlation between the temperature change and flight time
- correlation between the level of liquid and the change of flight time
- simulated "bio build up" and obtained relationship between the thickness of the build up and the amplitude
- verified the density of several liquids within 0.5- 0.75 %
- obtained experimental data for Nitrogen, Helium and Argon

Overall, the project was very successful. Improvement is required in the gas density measurement part.

FUTURE WORK

Undoubtedly, there is a lot to be done in the continuation phase of the project. The following suggestions were give by our mentor Larry Lynnworth:

- Improve accuracy of time and delta time, from ns to sub-ns
- Avoid interferences
- Use materials with smaller thermoelastic coefficients e.g. fused silica, impervious graphite,Ni-Span C902 Use material of lower density than 316SS as the sensor, to improve sensitivity, e.g. Ti, Al, fused silica, impervious graphite
- Utilize attenuation to complement sound speed, derive more information about fluid

Potential applications

- Measure multiparameters e.g. density and also temperature, viscosity, flow velocity, Reynolds number
- Select target opportunities, e.g. steam quality, steam flow velocity, mass flow rate
- Measure remotely from transducers, measure at multiple sites along one wave guide, n wave guides
- Biological, industrial and environmental applications. Fused silica may be acceptable in laboratory environment; may need robustness for harsh environments

LITERATURE

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