
Rachit Bhatia, *Utah State University*
David Geller

Available at: https://works.bepress.com/rachit-bhatia/7/
Autonomous Navigation using Gravity Gradient Measurements

Rachit Bhatia(1), David K. Geller(2)

(1) Graduate Student, 4130 Old Main Hill, Utah State University, Logan-84322, Utah (USA), Email: rachitbhatia@aggiemail.usu.edu
(2) Associate Professor, 4130 Old Main Hill, Utah State University, Logan-84322, Utah (USA), Email: david.geller@usu.edu

ABSTRACT

The future of deep space exploration depends upon technological advancement towards improving spacecraft’s autonomy and versatility. This study examines the feasibility of orbit determination using gravity gradient measurements, and investigate its potential as a prospective solution for futuristic autonomous orbital navigation.

The long term objective of this research is to ascertain specific sensor requirements to meet pre-defined mission navigation error budgets. For this paper, a Monte Carlo simulation based on a simple three degree of freedom environment model and an extended Kalman filter are developed. While the results are low fidelity, they can be used as a guide for more detailed and complete analysis.

Traditional inertial navigation (dead reckoning and external aiding) is not considered. Instead, measurements from pairs of advanced, highly sensitive accelerometers (e.g., cold atom accelerometers), on three mutually perpendicular baselines, are used to determine gravity field gradients which are then correlated to onboard gravity field maps and used to determine orbital information. This study is repeated for a variety of orbital regimes and initial conditions.

1. INTRODUCTION

The objective of this research is to determine the feasibility and requirements for an autonomous navigation that can potentially apply to all flight regimes. For any space mission, navigation relies primarily on external aids such as the Global Positioning Systems (GPS), the Tracking and Data Relay Satellite (TDRSS), or the Deep Space Network (DSN). This limits the range of our exploration capability, and requires specialized communication and ground-based navigation systems to achieve acceptable level of spaceflight safety. These additional systems not only require precious onboard resources, but are also subject to failures that can result in the Loss of Crew/Loss of Vehicle condition. For next generation space navigation, there is a need to relieve the traditional navigation techniques by implementing autonomous navigation system onboard and thus reduce the risk level of Loss of Crew/Loss of Vehicle condition. Autonomous navigation with reduced dependence on ground-based infrastructure can provide the capability to better optimize and control mission trajectories. Enhanced automation and versatility can be beneficial for both crewed and robotic space missions, and may result in increased scientific productivity over mission timeline.[1]

This study evaluates the role of advanced accelerometers, used in recent gravity-mapping missions like GRACE-2 and GOCE, in developing and executing autonomous orbital navigation for different mission requirements. Future autonomous orbital navigation architectures need to be versatile and reliable for varying space environments. The navigation approach addressed in this study has the potential to satisfy these requirements and, considering the universal nature of gravity, this approach provides a generic solution for autonomous navigation in almost all types of space environment, thus giving it an edge over other potential autonomous orbital navigation techniques.

Gravity gradiometry has been in use since mid 20th century and has find applications in wide ranging fields like mineral exploration[2], field survey, submarine navigation[3] and gravitational mapping[4]. The technology has been used for many airborne and terrestrial surveys[4], predominantly to image subsurface geology to aid hydrocarbon and mineral exploration. Over 2.5-million-line km has been surveyed using the technique.[2] During the Cold War, US Navy submarines used gravity gradiometry for covert navigation.[3] In recent years, the technology has matured and requisite instruments have evolved and been upgraded. Because of this, there is renewed interest in space applications for this technique. Recently, engineers and scientists have used various measurement principles based on electrostatics, superconductivity, and cold atom interferometry, to considerably advance the measurement sensitivity and precision of accelerometers.[3]

This research explores the viability of using pairs of advanced accelerometers and onboard gravity field maps to autonomously determine orbital position and velocity for LEO, GEO, and other orbital regimes. The results presented in this paper have been generated for nearly-ideal conditions (not possible for real mission scenario), nevertheless the analysis of these results provide an important glimpse into the technological requirements for autonomous orbital navigation, and help contemplate a
plan for future work required to develop the technology. Development of autonomous orbital navigation can help break new ground in improving safety for human exploration, reducing complexity and cost of spacecraft navigation systems, and reducing costly efforts of traditional ground-based navigation. This study presents a step in this direction.

Section 2 provides an overview and history of the gravity gradiometry, Section 3 shows the derivation of the gravity gradient measurement model, Section 4 outlines the basic simulation, and Section 5 provide an overview of the extended Kalman filter. Simulation results and analysis are presented in Section 6. Conclusions and required future work are listed in Section 7 and Section 8.

Table 1: Gravity Gradiometer Instruments (GGI) \(^{[3]}\)

<table>
<thead>
<tr>
<th>Gradiometer</th>
<th>Developer</th>
<th>Noise, (1 - \sigma) Eötvös</th>
<th>Data Rate, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotating Accelerometer GGI</td>
<td>Bell Aerospace/Textron</td>
<td>2 (Lab.), 10 (Air)</td>
<td>10</td>
</tr>
<tr>
<td>Rotating Torque GGI</td>
<td>Hughes Research Lab</td>
<td>0.5 (Goal)</td>
<td>10</td>
</tr>
<tr>
<td>Floated GGI</td>
<td>Draper Lab</td>
<td>1 (Lab.)</td>
<td>10</td>
</tr>
<tr>
<td>Falcon AGG</td>
<td>Lockheed Martin (LM)/BHP Billiton</td>
<td>3</td>
<td>Post Survey</td>
</tr>
<tr>
<td>ACVGG</td>
<td>Lockheed Martin (LM)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3D FTG</td>
<td>LM/Bell Geospace</td>
<td>5</td>
<td>Post Survey</td>
</tr>
<tr>
<td>FTGeX</td>
<td>LM/ARKeX</td>
<td>10 (Goal)</td>
<td>1</td>
</tr>
<tr>
<td>UMD SGG (Space)</td>
<td>University of Maryland (UMD)</td>
<td>0.02 (Lab.)</td>
<td>1</td>
</tr>
<tr>
<td>UMD SAA (Air)</td>
<td>University of Maryland (UMD)</td>
<td>0.3 (Lab.)</td>
<td>1</td>
</tr>
<tr>
<td>UWA OQR</td>
<td>University of Western Australia (UWA)</td>
<td>1 (Lab.)</td>
<td>1</td>
</tr>
<tr>
<td>Exploration GGI</td>
<td>ARKeX</td>
<td>1 (Goal)</td>
<td>1</td>
</tr>
<tr>
<td>HD-AGG</td>
<td>Gedex/UMD/UWA</td>
<td>1 (Goal)</td>
<td>1</td>
</tr>
<tr>
<td>Electrostatic GGI</td>
<td>European Space Agency (ESA)</td>
<td>0.001 (Goal)</td>
<td>10</td>
</tr>
<tr>
<td>Cold Atom Interferometer</td>
<td>Stanford University/Jet Propulsion Laboratory (JPL)</td>
<td>30 (Lab.)</td>
<td>1</td>
</tr>
</tbody>
</table>

2. OVERVIEW OF GRAVITY GRADIOMETRY

The study and measurement of the changes in the gravitational acceleration, at a given position, requires the measurement of the gravity gradient tensor at that position. Gravity Gradiometer Instrument (GGI) is used to measure the gravity gradient tensor.

Hungarian physicist Loránd (Roland von) Eötvös is credited for inventing the first gravity gradiometer instrument, in the late 1880s.\(^{[5]}\) While working on series of experiments on the proportionality of inertial and gravitational masses, Eötvös specialized torsion balance to measure gravitational gradient.\(^{[5]}\) To recognize this ingenious invention, the unit of the gravitational gradient has been named after him.\(^{[5]}\) One Eötvös is equal to \(10^{-9} \text{s}^{-2}\).\(^{[6]}\)

The gravity gradient tensor (GGT) is the 3x3 matrix, consisting of 9 components of the derivative of the gravitational vector with respect to position vector.

\[
\nabla g = \begin{bmatrix}
\n\nabla g_{XX} & \nabla g_{XY} & \nabla g_{XZ} \\
\n\nabla g_{YX} & \nabla g_{YY} & \nabla g_{YZ} \\
\n\nabla g_{ZX} & \nabla g_{ZY} & \nabla g_{ZZ}
\end{bmatrix}
\]

(1)

\[
\nabla g_{ij} = \frac{\partial^2 U}{\partial r_i \partial r_j}, \ i, j = X, Y, Z
\]

(2)

where \(U\) is the gravitational potential at the given position vector \(r\).

The gravity gradient matrix is symmetric (\(\nabla g_{ij} = \nabla g_{ji}\)) with zero trace (\(\sum_i \nabla g_{ii} = 0\)). Thus, only five out of nine components are independent.\(^{[4]}\)

Table 1 summarizes the chronological evolution of the gravity gradiometer instruments and parallel improvement in their sensitivity, respectively.\(^{[3]}\) This study is based upon the measurement model for electrostatic gravity gradiometer instrument, used onboard European Space Agency’s GOCE (Gravity field and steady-state Ocean Circulation Explorer) mission.\(^{[4]}\)

3. MEASUREMENT MODEL

Each gradiometer instrument comprises a configuration of 3 pairs of accelerometer, arranged on three mutually perpendicular baselines. And each accelerometer is designed to detect the difference between the acceleration of the spacecraft/spacecraft-frame and that of the proof-mass, by measuring the electrostatic force required to keep the proof-mass in the center of the accelerometer-frame.

An analysis of the forces acting on the proof-mass helps to model the accelerometer measurement as a function of forces acting on the spacecraft. Acceleration of the proof-mass (\(\bar{a}_{proof}\)) as observed in inertial frame, can be given as
\[
a_I^{\text{proof}} = a_I^{\text{veh}} + \omega_I \times (\omega_I \times \mathbf{R}_{\text{com}}^{\text{veh}}) + \omega_I \times \mathbf{R}_{\text{com}}^{\text{rel}} + 2 \omega_I \times \bar{\mathbf{R}}_{\text{rel}}^I + \bar{\mathbf{R}}_{\text{rel}}^I
\]

where \( a_I^{\text{proof}} \) is the acceleration of the COM of the proof-mass, \( a_I^{\text{veh}} \) is the acceleration of the COM of the spacecraft, \( \mathbf{R}_{\text{com}}^{\text{veh}} \) is the position of the COM of the proof-mass of the accelerometer \( A_I \) relative to the COM of the spacecraft, \( \mathbf{R}_{\text{com}}^{\text{rel}} \) is the velocity of the COM of the proof-mass as viewed relative to the rotating spacecraft-fixed frame, and \( \bar{\mathbf{R}}_{\text{rel}}^I \) is the acceleration of the COM of the proof-mass as viewed relative to the rotating spacecraft-fixed frame. All vectors with superscript 'I' are coordinatized in an inertial frame.

Using Newton’s second law

\[
F_I^{\text{veh}} = m_{\text{veh}} a_I^{\text{veh}} \tag{4}
\]

\[
F_I^{\text{proof}} = m_{\text{proof}} a_I^{\text{proof}} \tag{5}
\]

where \( F_I^{\text{veh}} \) is the total force acting on the spacecraft, \( m_{\text{veh}} \) is mass of the spacecraft, \( F_I^{\text{proof}} \) is the total force acting on the proof-mass, \( m_{\text{proof}} \) is mass of the proof-mass within the cage.

Force analysis of the vehicle, yields

\[
F_{\text{veh}} = F_{\text{gravity}_{\text{veh}}} + N \tag{6}
\]

where \( F_{\text{gravity}_{\text{veh}}} \) is the force acting on the spacecraft due to gravitation, and \( N \) is the force acting on the spacecraft due to non-gravitational forces, like drag.

And, force analysis of the proof-mass, yields

\[
F_{\text{proof}} = F_{\text{gravity}_{\text{proof}}} + F_{\text{emf}} \tag{7}
\]

where \( F_{\text{gravity}_{\text{proof}}} \) is the force acting on the proof-mass due to gravitation, and \( F_{\text{emf}} \) is the electro-motive force acting on the proof-mass, to keep the proof-mass at the center of the cage.

From the free body analysis of the vehicle and the proof-mass, the forces acting on the vehicle and the proof-mass can be expressed as below

\[
a_I^{\text{veh}} = g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} \right) + \frac{N^I \left( \mathbf{R}_{\text{com}}^{\text{veh}}, V_{\text{veh}}^I \right)}{m_{\text{veh}}} \tag{8}
\]

\[
a_I^{\text{proof}} = g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} + \mathbf{R}_{\text{com}}^{\text{rel}} \right) + \left( \frac{F_{\text{emf}}^I}{m_{\text{proof}}} \right) \tag{9}
\]

where \( \mathbf{R}_{\text{com}}^{\text{veh}} \) is the position of the COM of the spacecraft, \( V_{\text{veh}}^I \) is the velocity of the COM of the spacecraft, \( g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} \right) \) is the gravitational acceleration at the position \( \mathbf{R}_{\text{com}}^{\text{veh}} \), and \( g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} + \mathbf{R}_{\text{com}}^{\text{rel}} \right) \) is the gravitational acceleration at the position \( \left( \mathbf{R}_{\text{com}}^{\text{veh}} + \mathbf{R}_{\text{com}}^{\text{rel}} \right) \).

Since, the detected acceleration is the difference between the acceleration of the spacecraft/spacecraft-frame and the acceleration of the proof-mass. This means that by taking the difference between Eq. 8 and Eq. 9, and noting that the accelerometer measurement is proportional to \( \left( \frac{F_{\text{emf}}^I}{m_{\text{proof}}} \right) \), the detected acceleration is given as

\[
a_I^{\text{detected}} = \frac{F_{\text{emf}}^I}{m_{\text{proof}}} = g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} \right) - g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} + \mathbf{R}_{\text{rel}}^I \right) + \frac{N^I \left( \mathbf{R}_{\text{com}}^{\text{veh}}, V_{\text{veh}}^I \right)}{m_{\text{veh}}} + \omega_I \times \left( \omega_I \times \mathbf{R}_{\text{com}}^{\text{veh}} \right) + \omega_I \times \left( \mathbf{R}_{\text{com}}^{\text{rel}} + \omega_I \times \bar{\mathbf{R}}_{\text{rel}}^I \right) + 2 \omega_I \times \bar{\mathbf{R}}_{\text{rel}}^I \tag{10}
\]

The difference of the terms \( g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} \right) \) and \( g^I \left( \mathbf{R}_{\text{com}}^{\text{veh}} + \mathbf{R}_{\text{rel}}^I \right) \) is approximately (to first order) equal to \( - \left( \nabla g^I \right) \mathbf{R}_{\text{com}}^{\text{rel}} \), where \( \nabla g^I \) is the gravity gradient at the position \( \mathbf{R}_{\text{com}}^{\text{rel}} \). Thus further simplification yields

\[
a_I^{\text{detected}} = - \left( \left[ \left( \nabla g^I \right) - \left( \Omega^I \times \Omega^I \right) \right] \right) \mathbf{R}_{\text{com}}^{\text{rel}}, + \frac{N^I \left( \mathbf{R}_{\text{com}}^{\text{veh}}, V_{\text{veh}}^I \right)}{m_{\text{veh}}} + 2 \omega_I \left( \mathbf{R}_{\text{rel}}^I \right) + \bar{\mathbf{R}}_{\text{rel}}^I \tag{11}
\]

where \( \nabla g^I \) is a matrix consisting of the components of the GGT, and \( \Omega^I \), \( \Omega^I \), and \( \left( \Omega^I \right)^2 \) are matrices, defined as

\[
\Omega^I = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}, \quad \Omega^I = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix},
\]

\[
\left( \Omega^I \right)^2 = \begin{bmatrix} -\omega_z^2 & -\omega_y^2 & \omega_x \omega_y \\ \omega_x \omega_z & -\omega_x^2 & \omega_y \omega_z \\ \omega_z \omega_y & \omega_z \omega_x & -\omega_z^2 \end{bmatrix}
\]

where \( \omega_x, \omega_y, \omega_z \) are the components of the angular velocity of the spacecraft, relative to inertial frame, and \( \dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z \) are their time derivatives (angular accelerations).

For this study, several important assumptions have been made. These assumptions have been used to create an
hypothesized situation, so as to help generate a simplistic result and thereby, determine the feasibility of this concept. It is understood that such a hypothetical situation may never exist in reality, and thus for any practical use, further evaluation (with more realistic models) is required.

Figure 1: Schematic Model of a Gravity Gradiometer\(^4\)

This study assumes a conceptual scenario with a non-rotating spacecraft. An ideal gradiometer is assumed, except for the measurement noise. The measurement noise has been assumed to be zero mean Gaussian, and a constant and uncorrelated noise has been applied to all the components of gravitational gradient matrix \((\nabla g)\). However, based on the analysis of the problem, it is suspected that in reality, the noise on different components of gravitational gradient matrix \((\nabla g)\) is correlated and should be taken into account, for more accurate modeling.

For this study, there exist no accelerometer bias, scale factor, and misalignment. It is assumed there is no center of mass uncertainty, and uncertainty in the accelerometer location. Lastly, the environmental model has no drag, solar radiation pressure or any other non-gravitational acceleration, like acceleration due to external magnetic field or self-gravity. Using the aforementioned assumptions and Eq. 11, to isolate the components of \(\mathbf{N}\) and \((\nabla g)\), common-mode acceleration and differential-mode acceleration, for each pair of accelerometer, are computed as\(^4\)

\[
a_{i,j}^{c} = \frac{1}{2} (a_{i}^{c} + a_{j}^{c})
\]

\[
a_{i,j}^{d} = \frac{1}{2} (a_{i}^{d} - a_{j}^{d})
\]

where \(a_{i,j}^{c}\) is common-mode acceleration measured by the accelerometer \(A_{i}\) and \(A_{j}\), and \(a_{i,j}^{d}\) is differential-mode acceleration measured by the accelerometer \(A_{i}\) and \(A_{j}\), such that accelerometer \(A_{i}\) and \(A_{j}\) are the accelerometers on the same axis of the gradiometer, as shown in Fig. 1

On expanding and simplifying expressions for common-mode acceleration and differential-mode acceleration, important equations defining the measurement model are computed, and listed in Table 3

In Table 3, equations for the elements of gravitational gradient matrix \((\nabla g)\) and components of angular acceleration have been approximated to first order.

<table>
<thead>
<tr>
<th>Table 3: Measurement Model Equations(^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Components of Non-gravitational forces</strong></td>
</tr>
<tr>
<td>(a_{c,14}, X = a_{c,25}, X = a_{c,36}, X = N_{X}) (14)</td>
</tr>
<tr>
<td>(a_{c,14}, Y = a_{c,25}, Y = a_{c,36}, Y = N_{Y}) (15)</td>
</tr>
<tr>
<td>(a_{c,14}, Z = a_{c,25}, Z = a_{c,36}, Z = N_{Z}) (16)</td>
</tr>
<tr>
<td><strong>Elements of Gravitational Gradient Matrix ((\nabla g))</strong></td>
</tr>
<tr>
<td>(\nabla g_{XX} = -\frac{a_{d,14}, X}{A_{1}, X} - \omega_{y}^{2} \omega_{z}^{2}) (17)</td>
</tr>
<tr>
<td>(\nabla g_{YY} = -\frac{a_{d,25}, Y}{A_{2}, Y} - \omega_{x}^{2} \omega_{z}^{2}) (18)</td>
</tr>
<tr>
<td>(\nabla g_{ZZ} = -\frac{a_{d,36}, Z}{A_{3}, Z} - \omega_{x}^{2} \omega_{y}^{2}) (19)</td>
</tr>
<tr>
<td>(\nabla g_{XY} = \frac{1}{2} \left( -\frac{a_{d,14}, Y}{A_{1}, X} - \frac{a_{d,25}, X}{A_{2}, Y} \right) + \omega_{x} \omega_{y}) (20)</td>
</tr>
<tr>
<td>(\nabla g_{XZ} = \frac{1}{2} \left( -\frac{a_{d,14}, Z}{A_{1}, X} - \frac{a_{d,36}, X}{A_{3}, Z} \right) + \omega_{x} \omega_{z}) (21)</td>
</tr>
<tr>
<td>(\nabla g_{YZ} = \frac{1}{2} \left( -\frac{a_{d,25}, Z}{A_{2}, Y} - \frac{a_{d,36}, Y}{A_{3}, Z} \right) + \omega_{y} \omega_{z}) (22)</td>
</tr>
<tr>
<td><strong>Components of Angular Acceleration</strong></td>
</tr>
<tr>
<td>(\omega_{x} = \frac{1}{2} \left( -\frac{a_{d,25}, Z}{A_{2}, Y} - \frac{a_{d,36}, Y}{A_{3}, Z} \right)) (23)</td>
</tr>
<tr>
<td>(\omega_{y} = \frac{1}{2} \left( -\frac{a_{d,25}, X}{A_{2}, Y} + \frac{a_{d,36}, X}{A_{3}, Z} \right)) (24)</td>
</tr>
<tr>
<td>(\omega_{z} = \frac{1}{2} \left( -\frac{a_{d,25}, X}{A_{2}, Y} + \frac{a_{d,14}, Y}{A_{1}, X} \right)) (25)</td>
</tr>
</tbody>
</table>

4. SIMULATION MODELING

In this section, the simulation dynamics and measurement models are presented.
4.1 Dynamics

The true state vector is defined as

\[ x^t = \begin{bmatrix} r^t \\ v^t \end{bmatrix} \]  
(26)

where \( r^t \) and \( v^t \) are the true position and velocity of the spacecraft, respectively. Correspondingly, the dynamics for the truth model are given as

\[ \dot{x}^t = f(x^t, t) = \begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} (r^t) \]  
(27)

where \( g(r^t) \) is 18x18 spherical harmonics gravity model, and is computed analytically by evaluating the second derivative of gravitational potential \( U \) with respect to the position vector \( r \), expressed as an infinite series of spherical harmonics \([7, 8]\).

\[ U = \frac{\mu}{r} \left\{ 1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{r_E}{r} \right)^n P_{n,m}(\phi, \lambda) (C_{n,m} \cos(m\phi) + S_{n,m} \sin(m\phi)) \right\} \]  
(28)

where \( \mu \) is the universal gravitational parameter, \( r \) is the position vector from a point \( O \) fixed in body \( E \) (say Earth) to a generic point \( Q \), \( r_E \) denotes the magnitude of \( r \), \( r_E \) is a scaling radius for body \( E \), \( P_{n,m} \) is the associated Legendre function of the first kind, of degree \( n \) and order \( m \), and has as its argument \( \phi, \lambda \), the latitude and longitude of \( Q \). \([7, 8]\) The longitude of \( Q \) is denoted by \( \phi \). \( C_{n,m} \) and \( S_{n,m} \) are unnormalized gravitational coefficients of degree \( n \) and order \( m \). \([7, 8]\) If point \( O \) is coincident with the mass center of \( E \), then \( C_{1,0}, C_{1,1} \), and \( S_{1,1} \) all become zero. \([7-9]\)

4.2 Measurements

For simulation modeling, the measurement model is the same as presented in Section 3, in this paper. Although, for easier computation, the measurement vector is re-arranged using the elements of gravity gradient matrix, given as

\[ \bar{z} = \nabla g_{\text{measured}} + \eta \]  
(29)

\[ \bar{z} = \begin{bmatrix} \nabla g_{XXX} \\ \nabla g_{XY} \\ \nabla g_{XZ} \\ \nabla g_{YY} \\ \nabla g_{YZ} \\ \nabla g_{ZZ} \end{bmatrix}^T + \eta \]  
(30)

where \( \bar{z} \) is the true measurement vector, \( \eta \) is the white gaussian noise (with standard deviation as \( \sigma_l = 1E6 \)) added to the measurements, and \( \bar{R} \) denotes the corresponding measurement covariance, respectively.

5. EXTENDED KALMAN FILTER FORMULATION

The Extended Kalman Filter for orbital navigation using gravity gradient measurements, is outlined below

5.1 Defining State Vector

The state vector is defined as

\[ \hat{x}^t = \begin{bmatrix} \hat{r}^t \\ \hat{v}^t \end{bmatrix} \]  
(31)

where \( \hat{x}^t \) and \( \hat{v}^t \) are the position and velocity of the spacecraft, respectively.

5.2 Initial State and State Covariance

The initial state is defined as the position and velocity of the spacecraft at the given epoch. The initial velocity error is computed using the following thumb rule

\[ \sigma_v = \frac{2\pi}{T} * \sigma_r \]

where \( T \) is the orbital period of the spacecraft’s orbit, \( \sigma_r \) and \( \sigma_v \) denote initial standard deviation of the position and velocity, respectively. Thus, the initial filter state vector \( (\hat{x}_0) \) and initial state covariance \( P_0 \) are defined as

\[ \hat{x}_0 = \begin{bmatrix} \hat{r}_0^T \\ \hat{v}_0^T \end{bmatrix}, P_0 = \begin{bmatrix} (P_0)_r \\ (P_0)_v \end{bmatrix} \]  
(32)

where \( (P_0)_r = \sigma_r^2 (I_{3x3}) \) is initial position covariance matrix, and similarly, \((P_0)_v = \sigma_v^2 (I_{3x3}) \) is initial velocity covariance matrix, while \( I_{3x3} \) is a square identity matrix of size \( 3x3 \), and \( Z_{3x3} \) is a square matrix with all elements as zero, and has size of \( 3x3 \)

5.3 Dynamics, State and Covariance Propagation Equations

The dynamics are non-linear and continuous, given as

\[ \begin{bmatrix} \dot{\hat{r}}_{DM}^t \\ \dot{\hat{v}}_{DM}^t \end{bmatrix} = \begin{bmatrix} v_{DM}^t \\ g(r_{DM}^t) + \begin{bmatrix} 0_{3x3} \\ w \end{bmatrix} \end{bmatrix} \]  
(33)

where \( g(r_{DM}^t) \) is 8x8 spherical harmonics gravity model, and the subscript \( DM \) denotes the design model.

Now, linearizing the dynamics as

\[ \delta \dot{x}_{DM}^t = F \delta x_{DM}^t + Gw \]  
(34)

\[ G = \begin{bmatrix} 0_{3x3} \\ I_{3x3} \end{bmatrix} \]  
(35)

\[ F = \frac{\partial f}{\partial \hat{x}} |_{\hat{x}_0} = \begin{bmatrix} 0_{3x3} \\ (\frac{\partial f}{\partial \hat{x}})_{3x3} \end{bmatrix} \]  
(36)

where \( \delta \dot{x}^t \) is the defined filter state vector, \( w \) is the process noise, and \( \nabla g_{3x3} \) is the gravity gradient at the given position.

The process noise is considered as a zero-mean stationary white noise, constituting the unmodeled perturbation accelerations.\([10]\)
Once the state and state covariance has been initialized, the recursive propagation steps of the filter are defined and executed at each time step, by first computing the state estimation vector \( (\hat{x}_k) \), using the above defined dynamical equations, and then the state transition matrix is computed at the midpoint by

\[
\phi = I_{6 \times 6} + F dt + F^2 (dt)^2 / 2
\]

(37)

such that

\[
F = \frac{\partial f}{\partial x} |_{x_k} = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}
\]

(38)

Next, the state covariance matrix is propagated using the Discrete Lyapunov Equation as

\[
P_{k+1} = \phi_k P_k \phi_k^T + Q_{d,k}
\]

(39)

where \( k \) is the time index in the recursive estimation process by the filter, \( Q_{d,k} \) is the strength of the discrete white noise.

### 5.4 Measurement and update equations

The estimated measurement vector \( (\tilde{z}) \) is defined and linearized as

\[
\tilde{z}_k = h(\hat{x}_k, t_k) + \eta_k
\]

(40)

\[
H_k = \frac{\partial h}{\partial x} |_{\hat{x}_k} = \begin{bmatrix} \frac{\partial h}{\partial x} |_{6 \times 3} \end{bmatrix} = \begin{bmatrix} \frac{\partial h}{\partial x} |_{6 \times 3} \\ 0_{6 \times 3} \end{bmatrix}
\]

(41)

where \( H_k \) is the measurement partial.

The measurement partials are computed analytically, using Eq. 42 and the measurement model (as presented in Section 3, in this paper). The complete derivation of the measurement partial is given in the Appendix. The measurement partial has been derived as

\[
H_k = \begin{bmatrix}
-2 L_x \frac{\partial (A_{D,21,k})}{\partial \sigma} & 0_{1 \times 3} \\
-2 L_y \frac{\partial (A_{D,22,k})}{\partial \sigma} & 0_{1 \times 3} \\
-2 L_z \frac{\partial (A_{D,23,k})}{\partial \sigma} & 0_{1 \times 3} \\
-L_x \frac{\partial (A_{D,11,k})}{\partial \sigma} & 0_{1 \times 3} \\
-L_y \frac{\partial (A_{D,12,k})}{\partial \sigma} & 0_{1 \times 3} \\
-L_z \frac{\partial (A_{D,13,k})}{\partial \sigma} & 0_{1 \times 3}
\end{bmatrix}
\]

(42)

The measurement residual is defined as

\[
\text{residual} = \tilde{z}_k - \hat{z}_k
\]

(43)

Next, residual covariance matrix \( V_k \) is computed using the measurement partial, the measurement covariance and the state covariance, as \([11]\)

\[
V_k = H_k P_k H_k^T + R_k
\]

(44)

where \( R_k \) is the measurement covariance (as defined in Section 4.2)

Further, the Kalman gain \( K_k \) is given as \([11]\)

\[
K_k = P_k H_k^T (V_k)^{-1}
\]

(45)

Using the Kalman gain and measurement residual, the state vector is updated as \([11]\)

\[
\dot{x}_k^+ = \dot{x}_k^- + K_k (\tilde{z}_k - \hat{z}_k)^T
\]

(46)

And the state covariance matrix \( (P_k) \) is computed using Joseph formulation, as \([11]\)

\[
P_k^+ = (I_{6 \times 6} - K_k H_k) P_k^- (I_{6 \times 6} - K_k H_k)^T + K_k R_k K_k^T
\]

(47)

To ensure that the state covariance matrix remains symmetric

\[
P_k^- = \frac{P_k^+ + P_k^T}{2}
\]

(48)

Finally, the new measurement residual and residual covariance matrix are computed and the procedure is repeated for the next time step. \([11]\)

### 6. SIMULATION RESULTS AND ANALYSIS

Simulation results have been generated for varying important modeling parameters like filter gravity model, measurement noise, measurement frequency, initial position error, gradiometer baseline length, and orbital regime. This is to achieve the objective of determining the role and importance of each modeling parameter in designing an autonomous orbital navigation system. This analysis will help to decide modeling parameters and sensitivity requirements for achieving stipulated mission objectives. Nominal parameters have been listed in Table 5

#### 6.1 Effect of varying Filter Gravity Model

In this section, the effect of filter gravity model, varied from a simple two body model to 18x18 spherical harmonics gravity model, is investigated. The results have been generated using nominal modeling parameters (as given in Table 5), while varying the degree and order of the spherical harmonics gravity model used in the filter.

It was determined that there is negligible change in 3σ error with respect to the change in the filter gravity model, except for the case when the filter gravity model is set to two body. And, the filter performance is nearly the same for any filter gravity model, above two body model.

In Fig. 2, the filter seems to be tuned for filter gravity model set at two-body plus J2, with 18x18 true gravity model, and noise level of 1Eö 1 - σ. This result can
help conserve onboard resources, because this means that under specific conditions, a low resolution filter gravity model can be used onboard while ensuring reasonable estimation of filter states. These results have been verified using the Monte Carlo Analysis, and the $3\sigma$ errors have been found to be within 95% confidence interval.

<table>
<thead>
<tr>
<th>Table 5: Nominal Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Altitude</td>
</tr>
<tr>
<td>Eccentricity</td>
</tr>
<tr>
<td>Inclination</td>
</tr>
<tr>
<td>Right Ascension of the Ascending Node</td>
</tr>
<tr>
<td>Argument of Perigee</td>
</tr>
<tr>
<td>True Anomaly</td>
</tr>
<tr>
<td>Integration step-size</td>
</tr>
<tr>
<td>Number of Orbits</td>
</tr>
<tr>
<td>Measurement Frequency</td>
</tr>
<tr>
<td>Truth Gravity Model</td>
</tr>
<tr>
<td>Filter Gravity Model</td>
</tr>
<tr>
<td>$1\sigma$ Initial Position uncertainty</td>
</tr>
<tr>
<td>$1\sigma$ Initial Velocity uncertainty</td>
</tr>
<tr>
<td>$1\sigma$ Measurement Noise</td>
</tr>
<tr>
<td>Gradiometer Baseline Length</td>
</tr>
<tr>
<td>Process Noise</td>
</tr>
</tbody>
</table>

6.2 Effect of varying Measurement Noise

The noise levels were varied from 0.0001Eö $1\sigma$ to 100Eö $1\sigma$, at a factor 10, to record the effects of varying noise levels on the filter performance. The results have been generated using nominal modeling parameters (as given in Table 5), while measurement noise level is varied. Results have been generated over five orbital periods. It was observed that the $3\sigma$ error is directly proportional to the measurement noise levels.

After 5 orbital periods, $3\sigma$ position error for 0.0001Eö $1\sigma$ was observed to be approximately 4 cm, while the corresponding $3\sigma$ velocity error being 0.004 cm/s. This contrasts to $3\sigma$ position error of 36.7 km and $3\sigma$ velocity error of 38.9 m/s for one measurement every 5 minutes, while for one measurement every 2 seconds (time step), $3\sigma$ position error of 162.3 m and $3\sigma$ velocity error of 0.179 m/s is recorded.

6.3 Effect of varying Measurement Frequency

In this case, the effect of measurement frequency, varying from one measurement every 2s to one measurement every 300s, are studied. The results have been generated using nominal modeling parameters (as given in Table 5), while measurement frequency is varied. For all cases, a fixed integration time-step equal to 2 sec is used.

After 2 orbital periods, $3\sigma$ position error for no measurements has been noted to be about 1550.9 km, while corresponding $3\sigma$ velocity error being 1.77 km/s. This contrasts to $3\sigma$ position error of 1.933 km and $3\sigma$ velocity error of 2.1375 m/s for one measurement every 5 minutes, while for one measurement every 2 seconds (time step), $3\sigma$ position error of 162.3 m and $3\sigma$ velocity error of 0.179 m/s is recorded.

6.4 Effect of varying Gradiometer Baseline Length

In this case, the change in filter performance is recorded after varying the gradiometer baseline length. The results have been generated using nominal modeling parameters (as given in Table 5), while gradiometer baseline length is varied.

From this study, it is noted that the $3\sigma$ error changes are absolutely negligible with respect to the change in the baseline length. This result is helpful to identify that for any practical application, and having a constraint on physical length, gradiometer baseline length cannot be
considered a significant modeling parameter for designing an autonomous orbital navigation (using gravity gradient measurements). However, it is acknowledged that for high precision in estimation, the measured baseline length must be known very accurately.

### 6.5 Effect of varying Orbital Regime

The results have been generated for different orbital regime, using the nominal parameters (as given in Table 5). Fixed-time step of integration has been varied for different orbits (as given in Table 6).

Results have been generated over two orbital periods, using 1500 measurements, and for a measurement frequency of one measurement every time-step. The orbital specification for different orbital regimes, implemented in the simulation are given in Table 6

#### 6.5.1 For Measurement Noise of 1 E\(\sigma\)

In this case, the measurements have been processed for measurement noise of 1 E\(\sigma\), and it has been found that the performance of the proposed concept has been nominal for low earth orbit, such as GOCE orbit, however, the 3\(\sigma\) error increases for higher altitude orbits. With measurement noise of 1 E\(\sigma\), and after 2 orbital periods, 3\(\sigma\) position error for GOCE orbit has been noted to be about 283.24 m, while corresponding 3\(\sigma\) velocity error is 0.635 m/s. This contrasts to 3\(\sigma\) position error of 614.32 m and 3\(\sigma\) velocity error of 0.0635 m/s for Vanguard-1 rocket. Similarly, for Ariane 5 orbit, 3\(\sigma\) position error of 2292.1 m and 3\(\sigma\) velocity error of 1.607 m/s is recorded, while for SES-16 GTO orbit, 3\(\sigma\) position error is 49641 m and 3\(\sigma\) velocity error is 9.6563 m/s.

#### 6.5.2 For Measurement Noise of 0.1 E\(\sigma\)

In this case, the measurements have been processed for measurement noise of 0.1 E\(\sigma\), and after 2 orbital periods, 3\(\sigma\) position error for GOCE orbit has been noted to be about 28.325 m, while corresponding 3\(\sigma\) velocity error being 0.03129 m/s. This contrasts to 3\(\sigma\) position error of 61.443 m and 3\(\sigma\) velocity error of 0.0635 m/s for Vanguard-1 rocket. Similarly, for Ariane 5 orbit, 3\(\sigma\) position error of 230.84 m and 3\(\sigma\) velocity error of 1.607 m/s is recorded, while for SES-16 GTO orbit, 3\(\sigma\) position error is 49641 m and 3\(\sigma\) velocity error is 9.6563 m/s.

### 6.5.2 For Measurement Noise of 0.1 E\(\sigma\) 1 – \(\sigma\)

In this case, the measurements have been processed for measurement noise of 0.1 E\(\sigma\) 1 – \(\sigma\). The 3\(\sigma\) error has been noted to vary by about a magnitude, when measurement noise is set to 0.1 E\(\sigma\) 1 – \(\sigma\), instead of 1 E\(\sigma\) 1 – \(\sigma\).

**Table 6: Orbital Specifications**

<table>
<thead>
<tr>
<th>Param.</th>
<th>SES-15</th>
<th>Vanguard1</th>
<th>Ariane-5</th>
<th>GOCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(^a)</td>
<td>34,874,137 meters</td>
<td>8,817,137 meters</td>
<td>24,247,137 meters</td>
<td>6,494,637 meters</td>
</tr>
<tr>
<td>e(^a)</td>
<td>0.4399832</td>
<td>0.2024625</td>
<td>0.7256533</td>
<td>0.000983</td>
</tr>
<tr>
<td>i(^a)</td>
<td>2.3115(^\circ)</td>
<td>34.2687(^\circ)</td>
<td>3.8115(^\circ)</td>
<td>96.5717(^\circ)</td>
</tr>
<tr>
<td>(\Omega^\circ)</td>
<td>217.0247(^\circ)</td>
<td>177.2534(^\circ)</td>
<td>218.7161(^\circ)</td>
<td>344.5256(^\circ)</td>
</tr>
<tr>
<td>(\omega^\circ)</td>
<td>214.6288(^\circ)</td>
<td>319.0597(^\circ)</td>
<td>331.6670(^\circ)</td>
<td>296.2811(^\circ)</td>
</tr>
<tr>
<td>(\nu^\circ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>dt(^\circ)</td>
<td>86.4758 s</td>
<td>10.9934 s</td>
<td>50.1337 s</td>
<td>6.9498 s</td>
</tr>
<tr>
<td>Total Steps</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
</tbody>
</table>

\(^a\)- Semi-major axis, e - Eccentricity, i - Inclination, \(\Omega\) - Ascending node, \(\omega\) - Argument of perigee, \(\nu\) - True Anomaly, dt - Integration time-step

With measurement noise of 0.1 E\(\sigma\) 1 – \(\sigma\), and after 2 orbital periods, 3\(\sigma\) position error for GOCE orbit has been noted to be about 28.325 m, while corresponding 3\(\sigma\) velocity error is 0.03129 m/s. This contrasts to 3\(\sigma\) position error of 61.443 m and 3\(\sigma\) velocity error of 0.0635 m/s for Vanguard-1 rocket. Similarly, for Ariane 5 orbit, 3\(\sigma\) position error of 230.84 m and 3\(\sigma\) velocity error of
0.1523 m/s is recorded, while for SES-16 GTO orbit, 3σ position error is 5757.7 m and 3σ velocity error is 1.431 m/s. These results have been verified using the Monte Carlo analysis, and the 3σ errors have been found to be within 95% confidence interval.

In Fig. 4, LVLH True Navigation error for 50 Monte Carlo runs for SES-15 orbit, using 0.1σ measurement noise, 15 seconds of integration step-size, and measurement frequency of 60 sec, with 8x8 Filter Gravity Model, and other nominal parameters (as given in Table 5) have been shown. Similar performance has been recorded, when Monte Carlo results have been analyzed for other orbital regimes.

7. CONCLUSION

The objective of this study was to provide a starting point for more rigorous analysis of the complex problem to design an autonomous orbital navigation system. The results, presented in this paper, have been helpful in recognizing the important modeling parameters and acknowledging the advantage, disadvantage and corresponding influence on the filter performance.

The characteristics of some of the important modeling parameters and their inter-relationship have been attempted and recorded, during this study. Though the results, for this study, are only conceptual and based on a very simple environmental model, they do provide an insight into the complexity of designing an autonomous orbital navigation system.

8. FUTURE WORK

This study has been important in recognizing and validating the efficacy and the potential of the proposed concept. However, the model presented here is very basic and low fidelity. Thus, further study and analysis of the proposed concept is required.

There is a need to recognize and include inherent uncertainties in the system and environment, and test the measurement model with prominent sources of error. This help to identify the sensitivity requirements and optimal sensor configurations, for autonomous orbital navigation.

9. ACKNOWLEDGEMENT

This is to acknowledge that partial financial support has been provided by the International Space Safety Foundation.

10. APPENDIX - MEASUREMENT PARTIAL DERIVATION

From the measurement model (as presented in Section 3, in this paper), it is known that

\[ \nabla_{gXX} = -2\frac{A_{D,14,X}}{L_X} - \omega_z^2 - \omega_x^2 \]  

(49)

Here, Eq. 49 is a first order approximation. On differentiating both sides of Eq. 49 with respect to position vector, yields

\[ \left( \frac{\partial \nabla_{gXX}}{\partial \mathbf{r}} \right)_{1x3} = -2 \frac{1}{L_X} \left( \frac{\partial (A_{D,14,X})}{\partial \mathbf{r}} \right)_{1x3} \]

\[ \frac{\partial (\omega_z^2)}{\partial \mathbf{r}} - \frac{\partial (\omega_x^2)}{\partial \mathbf{r}} \]  

(50)

Since \( \mathbf{v} = \left[ \begin{array}{ccc} \omega_x & \omega_y & \omega_z \end{array} \right] \) \( T^T \) is recorded, while for SES-16 GTO orbit, 3σ velocity error is 1.431 m/s. These results have been verified using the Monte Carlo analysis, and the 3σ errors have been found to be within 95% confidence interval.

In Fig. 4, LVLH True Navigation error for 50 Monte Carlo runs for SES-15 orbit, using 0.1σ measurement noise, 15 seconds of integration step-size, and measurement frequency of 60 sec, with 8x8 Filter Gravity Model, and other nominal parameters (as given in Table 5) have been shown. Similar performance has been recorded, when Monte Carlo results have been analyzed for other orbital regimes.

7. CONCLUSION

The objective of this study was to provide a starting point for more rigorous analysis of the complex problem to design an autonomous orbital navigation system. The results, presented in this paper, have been helpful in recognizing the important modeling parameters and acknowledging the advantage, disadvantage and corresponding influence on the filter performance.

The characteristics of some of the important modeling parameters and their inter-relationship have been attempted and recorded, during this study. Though the results, for this study, are only conceptual and based on a very simple environmental model, they do provide an insight into the complexity of designing an autonomous orbital navigation system.

8. FUTURE WORK

This study has been important in recognizing and validating the efficacy and the potential of the proposed concept. However, the model presented here is very basic and low fidelity. Thus, further study and analysis of the proposed concept is required.

There is a need to recognize and include inherent uncertainties in the system and environment, and test the measurement model with prominent sources of error. This help to identify the sensitivity requirements and optimal sensor configurations, for autonomous orbital navigation.

9. ACKNOWLEDGEMENT

This is to acknowledge that partial financial support has been provided by the International Space Safety Foundation.

10. APPENDIX - MEASUREMENT PARTIAL DERIVATION

From the measurement model (as presented in Section 3, in this paper), it is known that

\[ \nabla_{gXX} = -2\frac{A_{D,14,X}}{L_X} - \omega_z^2 - \omega_x^2 \]  

(49)

Here, Eq. 49 is a first order approximation. On differentiating both sides of Eq. 49 with respect to position vector, yields

\[ \left( \frac{\partial \nabla_{gXX}}{\partial \mathbf{r}} \right)_{1x3} = -2 \frac{1}{L_X} \left( \frac{\partial (A_{D,14,X})}{\partial \mathbf{r}} \right)_{1x3} \]

\[ \frac{\partial (\omega_z^2)}{\partial \mathbf{r}} - \frac{\partial (\omega_x^2)}{\partial \mathbf{r}} \]  

(50)

Since \( \mathbf{v} = \left[ \begin{array}{ccc} \omega_x & \omega_y & \omega_z \end{array} \right] \) \( T^T \) is recorded, while for SES-16 GTO orbit, 3σ velocity error is 1.431 m/s. These results have been verified using the Monte Carlo analysis, and the 3σ errors have been found to be within 95% confidence interval.

In Fig. 4, LVLH True Navigation error for 50 Monte Carlo runs for SES-15 orbit, using 0.1σ measurement noise, 15 seconds of integration step-size, and measurement frequency of 60 sec, with 8x8 Filter Gravity Model, and other nominal parameters (as given in Table 5) have been shown. Similar performance has been recorded, when Monte Carlo results have been analyzed for other orbital regimes.

7. CONCLUSION

The objective of this study was to provide a starting point for more rigorous analysis of the complex problem to design an autonomous orbital navigation system. The results, presented in this paper, have been helpful in recognizing the important modeling parameters and acknowledging the advantage, disadvantage and corresponding influence on the filter performance.

The characteristics of some of the important modeling parameters and their inter-relationship have been attempted and recorded, during this study. Though the results, for this study, are only conceptual and based on a very simple environmental model, they do provide an insight into the complexity of designing an autonomous orbital navigation system.

8. FUTURE WORK

This study has been important in recognizing and validating the efficacy and the potential of the proposed concept. However, the model presented here is very basic and low fidelity. Thus, further study and analysis of the proposed concept is required.

There is a need to recognize and include inherent uncertainties in the system and environment, and test the measurement model with prominent sources of error. This help to identify the sensitivity requirements and optimal sensor configurations, for autonomous orbital navigation.

9. ACKNOWLEDGEMENT

This is to acknowledge that partial financial support has been provided by the International Space Safety Foundation.

10. APPENDIX - MEASUREMENT PARTIAL DERIVATION

From the measurement model (as presented in Section 3, in this paper), it is known that

\[ \nabla_{gXX} = -2\frac{A_{D,14,X}}{L_X} - \omega_z^2 - \omega_x^2 \]  

(49)

Here, Eq. 49 is a first order approximation. On differentiating both sides of Eq. 49 with respect to position vector, yields

\[ \left( \frac{\partial \nabla_{gXX}}{\partial \mathbf{r}} \right)_{1x3} = -2 \frac{1}{L_X} \left( \frac{\partial (A_{D,14,X})}{\partial \mathbf{r}} \right)_{1x3} \]

\[ \frac{\partial (\omega_z^2)}{\partial \mathbf{r}} - \frac{\partial (\omega_x^2)}{\partial \mathbf{r}} \]  

(50)

Since \( \mathbf{v} = \left[ \begin{array}{ccc} \omega_x & \omega_y & \omega_z \end{array} \right] \) \( T^T \) is recorded, while for SES-16 GTO orbit, 3σ velocity error is 1.431 m/s. These results have been verified using the Monte Carlo analysis, and the 3σ errors have been found to be within 95% confidence interval.

In Fig. 4, LVLH True Navigation error for 50 Monte Carlo runs for SES-15 orbit, using 0.1σ measurement noise, 15 seconds of integration step-size, and measurement frequency of 60 sec, with 8x8 Filter Gravity Model, and other nominal parameters (as given in Table 5) have been shown. Similar performance has been recorded, when Monte Carlo results have been analyzed for other orbital regimes.
(a) LVLH true navigation position error (50 samples) and standard deviation as a function of time, for measurement noise: 0.1Eö

(b) LVLH true navigation velocity error (50 samples) and standard deviation as a function of time, for measurement noise: 0.1Eö

Figure 4: LVLH True Navigation error for 50 Monte Carlo runs for SES-15 orbit, using 0.1Eö 1-σ measurement noise, 15 seconds of integration step-size, and measurement frequency of 60 sec (with other nominal parameters)

\[
H_k = \begin{bmatrix}
-2 \frac{\partial (A_{D,14,X})}{\partial r} & \frac{1}{L_X} & \frac{1}{L_Y} & \frac{1}{L_Z} & 0_{1\times3} \\
-2 \frac{\partial (A_{D,25,Y})}{\partial r} & \frac{1}{L_X} & \frac{1}{L_Y} & \frac{1}{L_Z} & 0_{1\times3} \\
-2 \frac{\partial (A_{D,36,Z})}{\partial r} & \frac{1}{L_X} & \frac{1}{L_Y} & \frac{1}{L_Z} & 0_{1\times3} \\
\frac{1}{L_Y} \left( \frac{\partial (A_{D,25,X})}{\partial r} \right)_{1\times3} & \frac{1}{L_X} \left( \frac{\partial (A_{D,14,Y})}{\partial r} \right)_{1\times3} & \frac{1}{L_Y} \left( \frac{\partial (A_{D,36,X})}{\partial r} \right)_{1\times3} & \frac{1}{L_Z} \left( \frac{\partial (A_{D,14,Z})}{\partial r} \right)_{1\times3} & 0_{1\times3} \\
\frac{1}{L_X} \left( \frac{\partial (A_{D,25,Y})}{\partial r} \right)_{1\times3} & \frac{1}{L_Y} \left( \frac{\partial (A_{D,36,Y})}{\partial r} \right)_{1\times3} & \frac{1}{L_Z} \left( \frac{\partial (A_{D,25,Z})}{\partial r} \right)_{1\times3} & 0_{1\times3} \\
\frac{1}{L_Z} \left( \frac{\partial (A_{D,36,X})}{\partial r} \right)_{1\times3} & \frac{1}{L_Y} \left( \frac{\partial (A_{D,25,Z})}{\partial r} \right)_{1\times3} & \frac{1}{L_X} \left( \frac{\partial (A_{D,36,Y})}{\partial r} \right)_{1\times3} & \frac{1}{L_Z} \left( \frac{\partial (A_{D,25,X})}{\partial r} \right)_{1\times3} & 0_{1\times3}
\end{bmatrix}
\]

(57)

REFERENCES