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Asset Pricing With Multiplicative Habit and Power-Expo Preferences

by

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Abstract

Multiplicative habit introduces an additional consumption risk as a determinant of equity premium, and allows time preference and habit strength, in addition to risk aversion, to affect "price of risk". A model combining multiplicative habit and power-expo preferences cannot be rejected.

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1. Introduction

Two formulations of habit formation have been used in macroeconomics and finance. One is the subtractive habit, popularized by Constantinides (1990). The other is the multiplicative habit proposed by Abel (1990). Carroll (2000) has especially argued for the multiplicative habit model.

Empirical analyses of habit formation have so far concentrated on subtractive habit [Ferson and Constantinides (1991), Heaton (1995), Campbell and Cochrane (1999), Otrok, Ravikumar and Whiteman (1999), Dynan (2000)]. Although there have been analytical and calibration studies of multiplicative habit [Abel (1999), Chan and Kogan (2002)], we are not aware of any econometric study that confronts it with data. We fill this gap by testing first the original Abel (1990) model of internal habit formation[1] and then an extended version that we develop below. In addition, we derive expressions for the equity premium to flesh out the asset pricing implications of both models.

Our extension of this model relies on a novel utility function called "power-expo" preferences. Introduced by Saha (1993), and initially estimated by Saha, Shumway, and Talpaz (1994, SST), these preferences have found applications in experiments [Holt and Laury (2002), HL] and in growth [Xie (2000)]. However, they have not been exploited in the asset pricing literature. The virtue of power-expo utility is that it allows relative risk aversion (RRA) to be flexible. This is important since there is empirical evidence for both increasing RRA [HL (2002), SST (1994)] and decreasing RRA (DRRA) [Ogaki and Zhang (2001), Zhang and Ogaki (2004)]. Guiso and Paiella (2000) used a utility function that is similar to the power-expo preferences in order to accommodate flexible risk aversion. But they did not explore its asset pricing implications. Nor did they combine it with habit formation.

[1] Campbell et al. (1997, pp. 328-329) argued that the external habit model of Abel's (1990) cannot help solve the equity premium puzzle and the riskfree rate puzzle.

2. Preferences

Let c_t denote the consumption of the representative agent at period *t*. He is endowed with the following period felicity function

$$u(c_t, c_{t-1}) = \frac{1 - e^{-a\frac{\left(c_t / c_{t-1}^{\alpha}\right)^{1-\gamma} - 1}{1-\gamma}}}{a}. \tag{1}$$

This subsumes two important special cases.

When $\alpha > 0$ and $a = 0$, (1) reduces to the multiplicative habit model [Abel (1990)]

$$u(c_t, c_{t-1}) = \frac{\left(c_t / c_{t-1}^{\alpha}\right)^{1-\gamma} - 1}{1-\gamma}. \tag{2}$$

Here RRA for given habit is the constant γ.

When $\alpha = 0$ and $a > 0$, (1) reduces to the power-expo utility function [Saha (1993)]

$$u(c_t) = \frac{1 - e^{-a\frac{c_t^{1-\gamma} - 1}{1-\gamma}}}{a}, \tag{3}$$

where $\gamma \geq 0$. RRA for this class of preferences is $R(c_t) = \gamma + ac_t^{1-\gamma}$, i.e. non-constant.

Similarly, RRA implied by (1) for given habit is

$$R(c_t, c_{t-1}) = \gamma + a\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}. \tag{4}$$

3. Asset Pricing

The representative agent can invest in n risky assets with net returns $r_{i,t+1}$, $i = 1,...,n$, and a riskless asset with net return r_f. Denote the conditional expectation based on information available at *t* by E_t. The Euler equation for the ith asset is (all derivations are available upon request)

$$E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] =$$
$$\beta E_t\left\{\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}}\left(1+r_{i,t+1}\right)\right\}. \tag{5}$$

Here β is the time discount factor of the representative agent. Since when $\alpha \neq 1$ this equation includes the nonstationary term c^{α} from three periods in the exponential functions and elsewhere, it is impossible to transform it into an equation that consists of stationary terms only. However, when the Abel (1990) model holds, i.e. $a = 0$ in (5), we can multiply $c_{t-1}^{(\alpha-1)(1-\gamma)}$ on both sides and rearrange terms to obtain the following equation that includes only stationary terms

$$E_t\left[1-\beta\alpha\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)}\right]$$
$$= \beta E_t\left\{\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}}\right)^{1-\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma-\alpha(1-\gamma)}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)}\right]\left(1+r_{i,t+1}\right)\right\}. \tag{6}$$

To shed some light on what (5) means for equity premium it is useful to first consider our two special cases. First, suppose that there is power-expo utility without habit formation.[2] In this case, the equity premium is approximately

$$E_t r_{i,t+1} - r_f \approx \left(\gamma + a c_t^{1-\gamma}\right) \cdot \mathrm{cov}_t\left(r_{i,t+1}, \Delta c_t / c_t\right). \tag{7}$$

Meyer and Meyer (2005) suggest that DRRA may be sufficient to explain the equity premium. However, (7) shows why this may not be true. If RRA is not constant, the equity premium should move systematically with the *level* of consumption, which is counterfactual. As Cochrane (2001, p. 467) asserts, "We cannot tie risk aversion to the *level* of consumption and wealth, since that increases over time while equity premia have not declined."

[2] We do not estimate this particular case because the corresponding Euler equation contains nonstationary terms similar to those in (5) that cannot be all transformed into stationary terms.

Next suppose that there is habit formation with constant RRA, as in Abel (1990). We suggest a novel method to see how habit may help explain the equity premium

$$E_t r_{i,t+1} - r_f \approx \left[\gamma - \frac{\beta\alpha^2(1-\gamma)}{1-\beta\alpha}\right]\mathrm{cov}_t\left(r_{i,t+1}, \frac{\Delta c_{t+1}}{c_t}\right) + \beta\alpha\frac{1-\gamma}{1-\beta\alpha}\mathrm{cov}_t\left(r_{i,t+1}, \frac{\Delta c_{t+2}}{c_{t+1}}\right). \tag{8}$$

The premium now depends upon *both* the contemporaneous covariance between equity return and consumption growth *and* the covariance between the return and consumption growth next period. This suggests why the multiplicative habit model might perform better than the standard consumption-CAPM based on constant RRA preferences: not only there is now a new risk factor but also the factor weights depend on time preference and habit strength, in addition to RRA.

Now consider the equity premium for the general case with both habit formation and non-constant RRA

$$E_t r_{i,t+1} - r_f = \frac{-\mathrm{cov}_t\left(r_{i,t+1}, \left(\frac{c_{t+1}}{c_t^\alpha}\right)^{-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^\alpha}\right)^{1-\gamma}}\right) + \beta\alpha\,\mathrm{cov}_t\left(r_{i,t+1}, \left(\frac{c_{t+2}}{c_{t+1}^\alpha}\right)^{1-\gamma}\frac{c_t}{c_{t+1}} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^\alpha}\right)^{1-\gamma}}\right)}{E_t\left\{\left[\left(\frac{c_{t+1}}{c_t^\alpha}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^\alpha}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^\alpha}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^\alpha}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}}\right\}}. \tag{9}$$

As in Abel's (1990) habit model, there are again two risks to holding equity. Power-expo preferences affect the equity premium through the exponential terms in (9). They can potentially raise the covariances between equity return and consumption by making the consumption terms in (9) more volatile. However, unless $\alpha = 1$, the equity premium in (9) is not guaranteed to be stationary due to the c^α terms in the exponential functions, mirroring the aforementioned nonstationarity issue in (5). In our empirical work, we therefore estimate a version of (5) that features $\alpha = 1$.

4. Empirical Results

Hansen, Heaton and Yaron (1996) report that the continuously updated GMM estimator (CUE) produces a model specification test with much smaller size distortion in the finite sample than the 2-step GMM. We hence use the CUE in our tests of the models above.

It is well-known that habit introduces a moving average structure, and therefore serial correlation, to the Euler equation disturbance. In our case, it is a first-order serial correlation. However, allowing for first-order serial correlation in our estimation of (6) and the version of (5) with $\alpha = 1$ always led to negative-definite weighting matrices in the CUE criterion function. This is a well-known problem in applications of GMM. Therefore, we adopt a solution to this problem described in Ogaki (1993, p. 468): Newey and West's (1987) Bartlett kernel estimator that does *not* impose zero weights on autocovariances of orders higher than 1. Specifically, we have used bandwidth values 3, 5, 9, 13, and 17 in the Bartlett kernel. We find that for all these values except 3, the negative-definite weighting matrix problem almost disappears. Furthermore, the estimation and test results are similar across these bandwidth values. Our empirical results below are based on setting this parameter to 9.

Our data are quarterly U.S. aggregate data on nondurable goods and services consumption, the value-weighted return on New York Stock Exchange (NYSE) stocks, and the return on U.S Treasury bills from 1958:IV to 2001:IV. All data are deflated by the Consumer Price Index. See the notes to Table I for the instruments that we use. To avoid possible time aggregation bias, we follow the standard practice to use twice-lagged instruments.

Table I presents the empirical results on Euler equations (6) and (5) using value-weighted NYSE stock return and Treasury bill return. The top panel of this table is for the special case of the Abel (1990) model, which requires $a = 0$. Here all the three parameters are precisely estimated, but the chi-square test of model specification rejects Abel's model decisively when once-lagged instruments are used: the *p*-value for this test is virtually zero. However, when

twice-lagged instruments are used, the rejection is reversed as in many tests of other models in the asset pricing literature.

A different set of results emerge in the next panel for the general model with multiplicative habit and power-expo preferences. First, the chi-square test no longer rejects model specification when once-lagged instruments are used: the *p*-value is now a comfortable 41.9%. Thus introducing power-expo preferences has substantially improved the model's fit with the data. Second, the highly significant *a* estimate of 0.989 (with a standard error of 0.163) indicates rejection of Abel's (1990) internal habit model, confirming the earlier rejection by the chi-squared statistic in the top panel based on once-lagged instruments. Finally, the bottom row shows that qualitatively similar results emerge when twice-lagged instruments are used.

The parameter estimates imply that the RRA in our general model increases with consumption growth, according to (4). The estimates based on once- (twice-) lagged instruments imply that RRA moves tightly around 0.9979 (0.9946) with a standard error of 0.0064 (0.0035). See Fig. 1 for the time series of RRAs. These small RRA values are impressive because both the standard consumption CAPM and the subtractive habit model in Campbell and Cochrane (1999) require RRA values that are one or two orders of magnitude larger to fit the historical average equity premium.

The intertemporal elasticity of substitution (IES) for consumption in this model is the inverse of RRA in the short horizon for which habit is given [Carroll et al. (2000)]. This IES measure has a mean value of 1.0021 (1.0054) with a standard error of 0.0065 (0.0035) for the first (second) set of parameter estimates. In an appendix, we show that the long-horizon IES is just 1 when $\alpha = 1$. Interestingly, Zhang (2006) found that the correctly identified EIS values in the Epstein and Zin (1991) and Weil (1990) model that simultaneously match the average equity premium, riskfree rate, and the stock return volatility are also very closely around 1.

5. Conclusions

We have found mixed evidence for the Abel (1990) model, but favorable empirical results for the general model that combines multiplicative habit formation with the power-expo preferences. Dropping multiplicative habit leads to counterfactual implications on the equity premium, and removing the power-expo preferences results in weakening or rejecting the model. The general model's implication on risk aversion is also sharply different from that of Campbell and Cochrane's (1999) subtractive habit model. There consumers become substantially more risk averse as consumption falls close to habit. Here they are slightly more risk averse when consumption rises relative to habit. However, the RRA and the IES in the general model are both estimated to be quite close to 1.

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Table I Continuously Updated GMM Estimation of Equations (9) and (5) Using Stock and T Bill Returns

Specification Test	β	γ	α	a
Abel (1990) Habit Model, Equation (6)				
A. *Instruments Lagged Once*				
43.50	0.997	1.099	1.003	--
(0.000)	(0.142)	(0.024)	(0.142)	--
B. *Instruments Lagged Twice*				
8.09	0.985	1.100	1.015	--
(0.525)	(0.170)	(0.035)	(0.175)	--
General Model, Equation (5) with $\alpha = 1$ Imposed				
A. *Instruments Lagged Once*				
9.20	1.000	0.004	1	0.989
(0.419)	(0.019)	(0.228)	--	(0.163)
B. *Instruments Lagged Twice*				
6.38	1.000	0.268	1	0.724
(0.702)	(.000)	(0.276)	--	(0.274)

Notes: 1. The first column reports chi-square statistics and associated *p*-values (in parentheses below the statistics). 2. The standard errors are in parentheses below parameter estimates. 3. The results here are based on using six instruments: 1, consumption growth, stock return, dividend yield, T bill return and the term premium on Treasury bonds, all in real terms. Results based on the first four instruments alone are similar.

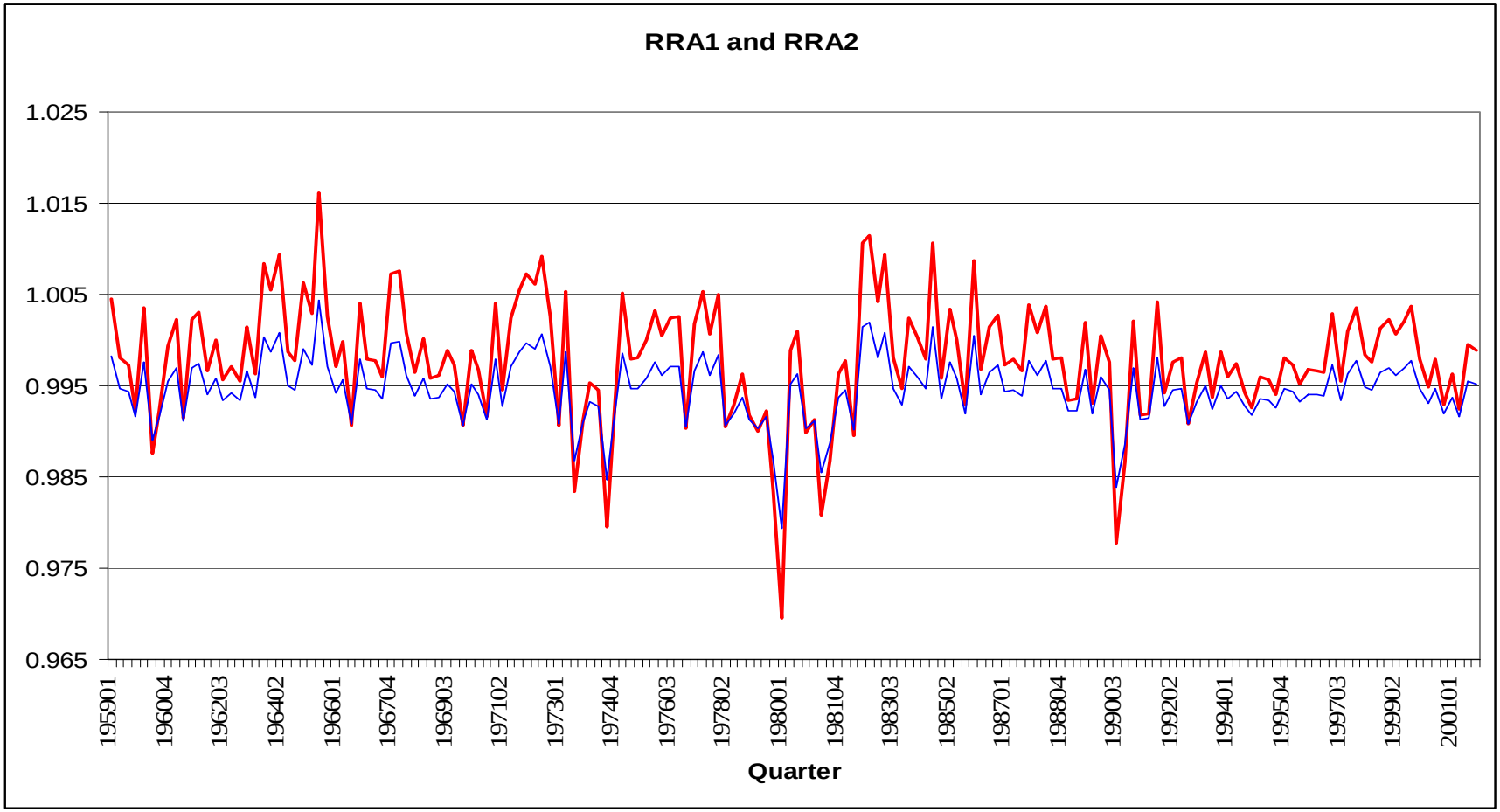

Fig. 1. Relative risk aversion calculated according to (4) using the estimates for the General Model. The upper, thicker, line (RRA1) is obtained using the estimates based on once-lagged instruments. The other line (RRA2) is based on twice-lagged instruments.

Appendix
Derivations of Equations (5), (7), (8) and (9) and IES

I. *The Euler Equation*

Lifetime utility is

$$U_t = \sum_{j=0}^{\infty} \frac{\beta^j}{a}\left[1 - e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+j}}{c_{t+j-1}^{\alpha}}\right)^{1-\gamma}}\right]. \tag{A.1}$$

The lifetime marginal utility of c_t is then

$$\frac{\partial U_t}{\partial c_t} = \frac{c_t^{-\gamma}}{c_{t-1}^{\alpha(1-\gamma)}} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha \frac{c_{t+1}^{1-\gamma}}{c_t^{\alpha(1-\gamma)+1}} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}. \tag{A.2}$$

Define $R_{i,t+1} = 1 + r_{i,t+1}$, and $R_f = 1 + r_f$. Following Abel (1990), the Euler equation for asset i is

$$E_t \frac{\partial U_t}{\partial c_t} = \beta E_t \frac{\partial U_{t+1}}{\partial c_{t+1}} R_{i,t+1}. \tag{A.3}$$

Evaluating this using Equation (A.2) yields

$$\begin{aligned} &E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] = \\ &\beta E_t\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}} R_{i,t+1} \end{aligned} \tag{A.4}$$

This is Euler Equation (5) in the text.

Since Equation (A.4) holds for all assets, including the risk-free asset, it follows that

$$\begin{aligned} &E_t\left[\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}\right] = \\ &\beta E_t\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}} R_f \end{aligned} \tag{A.5}$$

Combining Equations (A.4) and (A.5) yields

$$E_t\left[\frac{c_{t+1}^{-\gamma}}{c_t^{\alpha(1-\gamma)}}e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}}-\beta\alpha\frac{c_{t+2}^{1-\gamma}}{c_{t+1}^{\alpha(1-\gamma)+1}}e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\left(R_{i,t+1}-R_f\right)=0. \tag{A.6}$$

II. *Equity Premium*

The paper reports expressions for the equity premium in three cases.

II.A *Power-Expo Utility without Habit Formation*

When $\alpha=0$ Equation (A.6) reduces to

$$E_t c_{t+1}^{-\gamma}e^{-\frac{a}{1-\gamma}c_{t+1}^{1-\gamma}}\left(R_{i,t+1}-R_f\right)=0. \tag{A.7}$$

Define $g_{t+1}=\left(c_{t+1}-c_t\right)/c_t$. Equation (A.7) can then be expressed as

$$E_t\left(1+g_{t+1}\right)^{-\gamma}e^{-\frac{a}{1-\gamma}(1+g_{t+1})^{1-\gamma}c_t^{1-\gamma}}\left(r_{i,t+1}-r_f\right)=0. \tag{A.8}$$

Taking a Taylor series around $g_{t+1}=r_{i,t+1}=r_f=0$ yields Equation (7) in the text.

II.B *Habit Formation without Power-Expo Utility*

When $a=0$ Equation (A.6) reduces to the Euler equation in Abel (1990):

$$E_t\left[\frac{c_{t+1}^{-\gamma}}{c_t^{\alpha(1-\gamma)}}-\beta\alpha\frac{c_{t+2}^{1-\gamma}}{c_{t+1}^{\alpha(1-\gamma)+1}}\right]\left(R_{i,t+1}-R_f\right)=0. \tag{A.9}$$

Define $g_{t+2}=\left(c_{t+2}-c_{t+1}\right)/c_{t+1}$. Taking a Taylor series expansion around $g_{t+1}=g_{t+2}=r_{i,t+1}=r_f=0$ yields Equation (8) in the text.

II.C *Habit Formation with Power-Expo Utility*

The expression of the equity premium for this case, Equation (9) in the text, is obtained by rewriting Equation (A.6) using the definition of covariance.

III. *Long-Horizon Intertemporal Elasticity of Substitution (IES)*

This section derives the long-horizon IES in discrete time using the Euler equations in the text. To simplify, we follow the common practice of removing uncertainty and assume the existence of a balanced growth path (BGP). Section III.A below establishes the validity of our approach by showing that the IES in Carroll, Overland and Weil's (2000) continuous-time growth model featuring *AK* technology and multiplicative habit preferences can be recovered by our method. Section III.B then uses this method to solve the IES for our general model with $\alpha = 1$.

III.A *IES in the Model with Habit Formation But No Power-Expo Utility*

The Euler equation (6) in the text is now

$$\begin{aligned} &1 - \beta\alpha\left(\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)} \\ &= \beta\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}}\right)^{1-\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma-\alpha(1-\gamma)}\left(\frac{c_t}{c_{t-1}}\right)^{-\alpha(1-\gamma)}\right]R, \end{aligned} \tag{A10}$$

where R is now one plus the interest rate (which is a constant in the steady state for given *A* (in *AK*) and depreciation rate). Consider a BGP where consumption is growing at the constant rate g, so that e.g. $c_{t+1} = (1+g)c_t = (1+g)^2 c_{t-1}$. Note that *g* is constant over time *for given R*. A change in *R*, caused by e.g. a change in *A*, leads to a change in *g*. IES captures the response in 1 + *g* to a small change in *R*. To find IES, expressing Equation (A10) in terms of the growth rate and simplifying yields

$$1 = \beta R(1+g)^{(1-\alpha)(1-\gamma)-1}.$$

Therefore the IES is

$$\frac{\partial \ln(1+g)}{\partial \ln R} = \frac{1}{\alpha(1-\gamma)+\gamma}. \tag{A11}$$

This is exactly the same as the IES in Carroll et al. (2000, p. 347), allowing for the difference in notation (their γ is our α and their σ is our γ).

III.B *IES in the Model with Habit Formation and Power-Expo Utility*

The Euler equation (5), after removing uncertainty, is

$$\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_t}{c_{t-1}^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} =$$
$$\beta\left[\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+1}}{c_t^{\alpha}}\right)^{1-\gamma}} - \beta\alpha\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma} e^{-\frac{a}{1-\gamma}\left(\frac{c_{t+2}}{c_{t+1}^{\alpha}}\right)^{1-\gamma}}\right]\frac{c_t}{c_{t+1}}R.$$

Again consider a balanced growth path, so that the equation above becomes

$$\left[c_{t-1}^{(1-\alpha)(1-\gamma)} e^{-\frac{a}{1-\gamma}(1+g)^{1-\gamma} c_{t-1}^{(1-\alpha)(1-\gamma)}} - \beta\alpha c_t^{(1-\alpha)(1-\gamma)} e^{-\frac{a}{1-\gamma}(1+g)^{1-\gamma} c_t^{(1-\alpha)(1-\gamma)}}\right]$$
$$= \left[c_t^{(1-\alpha)(1-\gamma)} e^{-\frac{a}{1-\gamma}(1+g)^{1-\gamma} c_t^{(1-\alpha)(1-\gamma)}} - \beta\alpha c_{t+1}^{(1-\alpha)(1-\gamma)} e^{-\frac{a}{1-\gamma}(1+g)^{1-\gamma} c_{t+1}^{(1-\alpha)(1-\gamma)}}\right]\frac{1}{1+g}\beta R.$$

To ensure that this equation is consistent with the existence of BGP, we assume $\alpha = 1$. In that case, this equation reduces to $1+g = \beta R$, and IES is simply one.