Human Capital, Weak Identification, and Asset Pricing

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I develop a new approach to accounting for human capital and essentially all stock and non-stock wealth in estimating the return on wealth portfolio. Using the estimated return and aggregate U.S. data in weak-identification robust tests of the Epstein and Zin (1991) and Weil (1990) model, I find that the model can simultaneously match the historical average equity premium, risk-free rate, and stock return volatility with reasonable parameter values.

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The Epstein and Zin (1991) and Weil (1990) state-noseparable recursive preferences disentangle risk aversion from intertemporal substitution of consumption. When the isolation of these two aspects of human preferences is conceptually important, it is usually a natural choice of preference specification for the researcher. It has, therefore, been widely used in economics and finance. However, the consensus view in the asset pricing literature on the Epstein-Zin-Weil (EZW henceforth) model is that it cannot resolve the equity premium puzzle presented in Mehra and Prescott (1985) and the related risk-free rate puzzle (see e.g. Weil, 1989, Kocherlakota, 1996, Campbell, 2003, Mehra and Prescott, 2003). The purpose of the present paper is to demonstrate that this consensus view may not hold.

1. The last three papers are excellent surveys on the asset pricing literature. So are Constantinides (2002) and Cochrane (2001). See the references therein for the contributions to this literature.

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The stochastic discount factor (SDF) of the EZW model includes the return on the optimal portfolio of the representative agent. This portfolio includes various forms of wealth, such as human capital, private equity, real estate, consumer durable goods, deposits, and bonds, in addition to stocks. The return on this optimal portfolio should therefore include the returns on all assets, despite that some of them are not observable (Roll 1977). However, most evaluations of this model in the literature, including Epstein and Zin (1991), have assumed that an aggregate return on stocks is adequate to proxy the return on the optimal portfolio of the representative agent. Such a practice may unduly affect the empirical performance of the model because the volatility of the optimal portfolio return is unlikely to be as high as that of stock return.

In the last decade, researchers have made important progress on accounting for human capital in econometric evaluations of asset pricing models (see e.g. Campbell, 1996, Jagannathan and Wang, 1996, Vissing-Jørgensen and Attanasio, 2003, Palacios-Huerta, 2003a). Since available estimates indicate that human capital is the largest component of the U.S. wealth portfolio, accounting for it is certainly the most important step toward addressing Roll’s critique. But to account for other often-ignored components aforementioned is also important in order to respond to Roll’s critique sufficiently. In this paper, I propose a new and simple approach to estimating the return on the optimal portfolio that consists of both human capital and essentially all forms of stock and non-stock wealth mentioned in the last paragraph.

This approach produces with ease an explicit estimate for the return on the optimal portfolio that reflects the returns on human capital, stocks, and non-stock wealth. It offers a benchmark for evaluating the usual proxy for this return used in previous papers. More importantly, it allows the EZW model to be tested without assuming lognormality of, and conditional homoskedasticity for, asset returns and consumption growth, and without having to use the log-linearized Euler equation. One undesirable consequence of using the log-linearized Euler equation is that the time discount factor and the relative risk aversion coefficient (RRA) cannot be identified.

Turning to econometrics, weak instruments, i.e. instruments that are weakly correlated with the endogenous variables or first order conditions in a model, has emerged as a major problem in empirical evaluations of economic models. They may cause model parameters to be weakly identified. Stock and Wright (2000, SW henceforth) provided evidence of weak identification in the major consumption-CAPMs
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(C-CAPM), including the EZW model. Examining the standard C-CAPM in detail, Neely, Roy, and Whiteman (2001) found that the curvature parameter capturing the RRA or the EIS was near non-identification due to the weakness of the usual instruments. Smith (1999) also reported Monte Carlo evidence for poor identification in estimating the EZW model. A recent survey by Stock, Wright, and Yogo (2002) emphasized that weak identification may invalidate statistical inference based on the standard asymptotic theory for the generalized method of moments (GMM).

When parameters are weakly identified, it is impossible to obtain precise point estimates for them, although the point estimates obtained by econometric methods that ignore weak identification may look precise. A researcher can, however, construct weak-identification robust confidence sets for these parameters using SW’s (2000) asymptotic theory for the continuous updating GMM estimator of Hansen, Heaton, and Yaron (1996). In this paper, I employ both SW’s (2000) approach and two standard GMM estimators to test the EZW model featuring the optimal portfolio return mentioned earlier.

The rest of the paper is organized as follows. In Section 1, I develop a new approach to addressing the Roll critique and compare the optimal portfolio return constructed by this approach with the usual proxy used in the literature, the value-weighted return on New York Stock Exchange (NYSE) stocks. In Section 2, I review the EZW model and its implications on equity premium, risk-free rate, and stock return volatility. I then explain in Section 3 the econometric methods used in the present study. I report the estimation and test results in Section 4, and compare them with those in the literature in Section 4.3. In Section 5, I conclude the paper and offer some suggestions for future research.

1. THE RETURN ON THE OPTIMAL PORTFOLIO

In this section, I first estimate the return on the optimal portfolio, compare it with the NYSE stock return, and then establish the reasonableness of my estimates.

1.1 Estimating the Optimal Portfolio Return

Consider the following constraint to the representative agent’s utility maximization problem

\[ W_{t+1} = (W_t - C_t)R_{N+1}^t, \]

where \( W_t \) is the agent’s total wealth, including human wealth and financial and real assets, such as deposits, stocks, bonds, real estate, private equity, and consumer durable goods, at period \( t \), \( C_t \) is conceptually his total consumption flow for the same period (but usually approximated by non-durables and services consumption when the purpose is to test a theory), and \( R_{N+1}^t \) is the stochastic gross return on his
portfolio from period $t$ to $t+1$. Since $W_t$ includes human wealth, $R_{t+1}^m$ is the weighted average of the return on financial assets and the return on human wealth.

It should be noted that whether human capital is tradable or not, and whether labor income is stochastic or not, Equation (1) is the relevant budget constraint for the agent. See Epstein (1988) and Epstein and Zin (1991, p. 267, footnote 3) for an explanation on why the constraint (Equation 1) accommodates non-tradable human capital and stochastic labor income. To recapitulate, a shadow value can be calculated for human capital when it is not tradable, and the shadow value can be included in $W_t$. They also wrote, “The problem of stochastic labor income is, therefore, a problem in the measurement of the return on the wealth portfolio.” The optimal consumption and portfolio choices of the representative agent, both of which affect wealth level, must satisfy the constraint (Equation 1). For this reason, and with a little abuse of notation, I henceforth use $C_t$ in the rest of the paper as the optimal consumption, and $W_t$ as the resulting wealth level from the representative agent’s optimal decisions. Naturally, $R_{t+1}^m$ from this point on should be interpreted as the return on his optimal portfolio.

The starting point of my approach to addressing the Roll critique is the observation that knowing the series $C_t/W_t$ is adequate for calculating the series $R_{t+1}^m$. To understand it, first rewrite Equation (1) as

$$
\frac{W_{t+1}}{W_t} = \left(1 - \frac{C_t}{W_t}\right) R_{t+1}^m .
$$

Then note that

$$
\frac{W_{t+1}}{W_t} = \frac{C_{t+1}}{C_t} \cdot \frac{C_t/W_t}{C_{t+1}/W_{t+1}} .
$$

Since the first term on the right-hand side of Equation (3), consumption growth, is readily available, I only need to estimate the $C_t/W_t$ series to calculate the wealth growth series. Using the $W_{t+1}/W_t$ estimates along with the $C_t/W_t$ series, I can then obtain the return series $R_{t+1}^m$ using Equation (2). Under special assumptions, such as independently and identically distributed (I.I.D.) asset returns and nonstochastic labor income, some consumption/portfolio choice models (including the model examined below) imply that the optimal $C_t/W_t$ is constant over time and therefore $W_{t+1}/W_t = C_{t+1}/C_t$ in the equilibrium. But the I.I.D. assumption does not hold in the data. With non-I.I.D. asset returns, the EZW model in general implies time-varying $C_t/W_t$ unless the EIS is 1 (see e.g. Giovannini and Weil 1989). Therefore, it is useful and important to consider time-varying $C_t/W_t$.

I now explain how to construct the $C_t/W_t$ series from Lettau and Ludvigson’s (2004, LL henceforth) $cay$ values that are recovered from the data and parameter estimates posted at their websites. The use of $cay$ in my approach does not involve

5. To most researchers, the term representative agent refers to a fictitious individual that consumes the average consumption, earns the average income, and holds the average household net worth in an economy. The behavior of the representative agent, being the average of all the individuals in an economy, can therefore differ substantially from that of a real-world individual.
its predictive power on the stock return premium, as is evident from the description below. So my approach to estimating the \( \frac{C_t}{W_t} \) series is immune to the recent debate on the usefulness of the \( cay \) variable in tests of stock return predictability.\(^6\)

For completeness I provide a brief introduction to LL’s \( cay \) variable and use their notation here. The representative agent’s wealth \( W_t \) is the sum of his financial and human wealth: \( W_t = A_t + H_t \), where \( A_t \) stands for financial and real assets and \( H_t \) stands for human wealth. Financial and real assets are measured as the household net worth and are from the \textit{Flow of Funds Accounts of the United States} compiled by the Federal Reserve System. It includes essentially all forms of assets: various forms of deposits, stocks, bonds, real estate, private equity, and consumer durable goods. Suppose that the steady-state share of \( H_t \) in \( W_t \) is \( \nu \), a constant. A loglinear approximation yields (ignoring a linearization constant)

\[
w_t = (1 - \nu) a_t + vh_t. \tag{4}\]

Here and henceforth, the lower-case variables are the natural logarithms of the corresponding upper-case variables unless otherwise noted. A key insight in LL (2004) is that the non-stationary component of \( h_t \) is captured by \( y_t \), the logarithm of labor income. This motivates the following representation for \( h_t \) in their paper

\[
h_t = \kappa + y_t + z_t, \tag{5}\]

where \( \kappa \) is a constant and \( z_t = E_t \sum_{j=0}^{\infty} \rho^j (\Delta y_{t+j} - r_{t+j}^h) \) is a stationary term.\(^7\) In the expression for \( z_t \), \( E_t \) denotes the conditional expectation based on information up to time \( t \), \( 0 < \rho < 1 \), \( \Delta \) is the first difference operator, and \( r_{t+j}^h \) is the logarithm of the gross return on the human capital that the representative agent owns. Substituting Equation (4) and then Equation (5) into \( c_t = w_t \) yields

\[
c_t - w_t = c_t - (1 - \nu) a_t - vh_t
= c_t - (1 - \nu) a_t - vy_t - \nu \kappa - vz_t \equiv cay_t - \nu (\kappa + z_t), \tag{6}\]

where \( cay_t \) is defined to equal \( c_t - (1 - \nu) a_t - vy_t \), and \( v \) is estimated from the cointegrating regression of \( c \) on \( a \) and \( y \).\(^8\) Now assume that \( r_{t+j}^h = \zeta + \Delta y_{t+j} + \eta_{t+j} \), where \( \zeta \) is a constant, \( \Delta y_{t+j} \) is the growth rate of (after-tax) per-capita labor

6. For example, Rudd and Whelan (2006) showed the non-stationarity of an alternative \( cay \) variable constructed with the total consumption expenditure (including durable goods purchase). But given the purpose of the present paper and the fact that durables are included in the wealth portfolio here, it is natural to follow the standard practice to interpret \( C_t \) as the consumption of nondurables and services (NDS) and the unobservable service flow from durables. Another point that they raised is that the deflating of consumption, financial assets, and labor income should be based on the same price index to be consistent with the budget constraint. Though this is conceptually correct, due to the fact that the durables purchase is only about 12% of the personal consumption expenditure (PCE), the price index for PCE and that for NDS are nonetheless close to each other. See also Note 20.

7. The mean of \( z_t \) is zero when \( Er_{t+j}^h = E\Delta y_{t+j} \) is assumed. But in more general (and more realistic) cases where \( Er_{t+j}^h - E\Delta y_{t+j} = \xi \neq 0 \), the mean of \( z_t \) is not zero any more. This latter fact justifies the human capital return assumption below.

8. LL’s estimates for \( v \) and \( 1 - \nu \) did not sum to 1 but was close. They pointed out that if the fraction of total consumption flow that is unobservable is \( u \), then the sum of these two estimates based on the observed consumption flow should be \( 1 - u \). See also Note 11 below. The fact that \( c \) is not the entire consumption flow, \( y \) is after-tax labor income and tax may not be stationary explains why the cointegrating relationship can exist among \( c, a, \) and \( y, \) instead of between \( c \) and \( y, \) or \( c \) and \( a \).
income, and \( \eta_{t+j} \) is a random disturbance with zero conditional mean. I call this assumption the human capital return assumption in the rest of this section for convenience. Under this assumption, \( z_t \) becomes a constant. Denote it as \( z \). It implies that labor income is a constant fraction of human capital over time, which can be seen when one moves \( y_t \) in Equation (5) to the left-hand side. As a result of the human capital return assumption, the consumption-wealth ratio is approximately proportional to \( \exp(cay_t) \), i.e.

\[
\frac{C_t}{W_t} = \exp(cay_t) \cdot \exp(-v(\kappa + z)) \equiv k \exp(cay_t),
\]

(7)

where \( k \) is defined to be \( \exp(-v(\kappa + z)) \), again ignoring the linearization constant in Equation (4). Hence the question now is how to pin down the constant \( k \).

If the U.S. economy has been in a steady state, as many economists believe, \( C_t/W_t \) should fluctuate around its steady state value. Since \( C/W = (C/A)(A/W) \), the steady state value of \( C/W \) can be obtained by plugging in the steady state values of the two constituent ratios. I assume that the steady state value of \( C/A \) is equal to the long-term average of \( C_t/A_t \). Using the consumption and the household net worth data in LL (2004) for \( C_t \) and \( A_t \) series, the average of \( C_t/A_t \) in annual terms is 0.1899 for the period of 1959-2001. According to LL's (2004) cointegration regression estimates, the steady state ratio of \( A/W \) is \( 1/v \). Therefore, I obtain the steady state value of \( C/W \) as \( 0.1899 \times 0.3022 = 0.05739 \) in annual terms, or 0.01436 in quarterly terms. Next I assume that the long-term average of the quarterly \( C_t/W_t \) series is equal to its steady state value 0.01436. This means that the time average of the right-hand side of Equation (7) is 0.01436. Given the series \( cay_t \) for the period mentioned above, \( k \) then can be solved as 0.00697 for quarterly \( C_t/W_t \) series.

With the \( C_t/W_t \) series available, the estimation of wealth growth using Equation (3) is straightforward. I then use Equation (2) to estimate the return on the optimal portfolio. Although approximations have to be used in this approach, and hence the \( R^m \) estimates cannot be expected to be perfectly accurate, they should nonetheless be much more precise than those obtained by using just aggregate stock return alone. In this sense, the Roll critique has been adequately addressed in the present paper.

9. If human capital return is defined as \( R_{t+1} = (H_{t+1} + Y_{t+1})/H_t \), then \( \eta_{t+1} \) becomes a constant over time. However, it can be shown that such a return definition is inconsistent with the human capital definition in LL (2004) that motivates Equation (5) in this paper. In addition, the human capital that the representative agent holds is the average human capital in the economy. So it is reasonable to relate its return to the growth of the per-person labor income, the labor income that the representative agent earns. See also Note 6.

10. But the return on human capital, which includes capital gains on human capital, is not constant.

11. Their estimate of \( 1 - v \) based on the observable consumption flow was 0.269 (rounded to 0.3 in their paper), and the estimate for \( v \) was 0.621, which are reported in their websites. This implies that the true \( 1 - v \) based on total (observable and unobservable) consumption flow should be \( 0.269/(0.269 + 0.621) = 0.3022 \). This value is only used to estimate the steady state \( A/W \) in the present paper.

12. The \( cay \) values recovered from LL's parameter estimates and data are annualized quarterly as their quarterly data were annualized. This fact has been taken into account in calculating \( k \).
The human capital assumption above is different from the assumptions used in Campbell (1996), Palacios-Huerta (2003a), and Vissing-Jørgensen and Attanasio (2003). They did not associate the return on human capital with labor income growth. But it is somewhat similar to the assumptions of Fama and Schwert (1977) and that of Jagannathan and Wang (1996) who employed labor income growth to capture the return on human capital. However, since here the labor income growth is not assumed to be unforecastable as in their papers, and the conditional mean of human capital return is not assumed to be the same as that of labor income growth, my assumption is also different from theirs.

1.2 The Reasonableness of the $R^m$ Estimates

For a comparison of the estimated $R^m$ series with the value-weighted real gross return series on the NYSE stocks, refer to Figure 1. The quarterly means of the two series are very close to each other: 1.01986 for NYSE stocks, and 1.01976 for the estimated $R^m$ (which by definition includes stock return). What is striking is how much the volatilities of these two series differ. The standard deviation of the real quarterly return on the NYSE stocks is almost ten times that of $R^m$: 7.976% vs. 0.838%. This difference is fairly robust. Adding two standard errors to LL’s (2004) estimated $1-\nu$ and subtracting two standard errors from their estimated $\nu$, which increases the share of financial assets in the wealth portfolio, only raises the standard deviation of the estimated $R^m$ series to 0.876%. In the following, I justify both the volatility and the mean of the estimated $R^m$. 

**Fig. 1.** The Real Value-Weighted Return of NYSE Stocks (Dashed Line) and $R^m$, the Real Return on Optimal Portfolio Incorporating Human Capital and Essentially All Stock and Non-stock Wealth (Solid Line). Both are quarterly rates.
The striking difference in volatility is due to two reasons. First, the dominating component of $R_m$ is the return on aggregate human capital, whose volatility has always been estimated to be smaller than that of stock or physical capital returns regardless of the particular human capital assumption adopted (see e.g. Baxter and Jermann, 1997, Palacios-Huerta, 2003c, Lustig and Van Nieuwerburgh, 2005). This result still holds here. For example, subtracting two standard errors from LL’s (2004) estimated $1 - \nu$ and adding two standard errors to their estimated $\nu$—which increases the share of human capital in the wealth portfolio—lowers the standard deviation of the estimated $R_m$ series to 0.811%. Since $\nu$ is about 70% as mentioned earlier, the return on human capital has a heavy weight that fluctuates around 70% in $R_m$. In contrast, the stock return has a small average weight of 6% in $R_m$. Consequently, its volatility can only affect that of $R_m$ in a very minor way. Second, there is evidence for a negative correlation between human capital return and financial asset return. Lustig and Van Nieuwerburgh (2005) used the EZW model, financial asset returns and consumption growth to infer the return to human capital. They found a robust negative correlation between the return on financial assets and the return to human capital.

It is instructive to compare the results above with those in Stambaugh (1982), who found that when the market portfolio was expanded from stocks to include bonds, real estate and durables, the volatility of the return on market portfolio remains dominated by the volatility of stock returns. As a result, he found that tests of CAPM were not sensitive to the variations in the measures of market portfolio. Here, the market portfolio is expanded even further, and human capital becomes the dominant component. Consequently, the test results may differ.

I now turn to the reasonableness of the estimated quarterly mean of $R_m$. One way to figure if it is sensible or not is to check what restriction such a mean value of 1.01976 puts on the mean of the return to human capital. If it requires an implausible mean of the return to human capital, the $R_m$ estimates are problematic. It can be shown that if the quarterly mean of the return to non-stock assets falls between 1.0025 and 1.01 (which I think is a reasonable range), the quarterly mean of the return to human capital must be between 1.0257 and 1.0231. So the question here is if a 9.6%–10.7% mean annual return on aggregate human capital is consistent with the empirical evidence in the labor/human capital literature. Card (2001) surveyed much of the 1990s empirical literature on the return to schooling and presented (in his Table II) various instrumental variable estimates that are consistent with such a magnitude. Palacios-Huerta (2003b) estimated the marginal return to human capital investment for various demographic groups in U.S. with different years of work experience. The weighted average of the estimates in his Table I is again about 10%.

2. THE EPSTEIN-ZIN-WEIL MODEL

In the EZW model, the representative agent is endowed with a recursive utility function that isolates his risk preference from his willingness to substitute...
consumption over time. He chooses consumption and portfolio weights on $N$ assets to maximize

$$U_t = \{(1-\beta)C_t^\rho + \beta(E_tU_{t+1}^\alpha)^{\rho/\alpha}\}^{1/\rho} \quad (8)$$

subject to the budget constraint (Equation 1) and the constraint that the portfolio weights add up to 1. In this formulation, $\beta$ is the time discount factor for the deterministic consumption path, $\alpha = 1 - \text{RRA}$, $\rho = 1 - 1/\text{EIS}$ and $\rho \neq 0$. Following Epstein and Zin (1991) and others, I define $\lambda = \alpha/\rho$ and $\gamma = 1/\text{EIS}$ to write the Euler equations for asset returns as follows $^{14,15}$

$$E_t\left[\beta^{\gamma}(C_{t+1}^\rho / C_t^\rho)(R_{t+1}^m)^{\lambda-1}R_{t+1}^j\right] = 1, \quad j = 1, \ldots, N - 1. \quad (9)$$

Here $R_{t+1}^j$ is the gross return on asset $j$ from period $t$ to $t + 1$. The $N^{th}$ asset is human capital, but it may not be tradable. Even if it is tradable, there may be substantial amount of market frictions that may fail Equation (9) for human capital return. For example, He and Modest (1995) demonstrated that in the presence of short sale constraint, borrowing constraint, or solvency constraint, Equation (9) becomes an inequality (see also Luttmer 1996). Recently, Palacios-Huerta (2003c) has shown that the market frictions in human capital are $4/\text{EIS}$ times greater than those in the financial market for different demographic groups. Consequently, Equation (9) may hold for returns on financial assets that are not subject to substantial market frictions but may fail for human capital return.

On the other hand, the consumption Euler equation is

$$E_t\left[\beta^{\gamma}(C_{t+1}^\rho / C_t^\rho)(R_{t+1}^m)^{\lambda}\right] = 1. \quad (10)$$

If Equation (9) holds for all the $N$ assets, Equation (10) should also hold because it is just a linear combination of the $N$ asset return Euler equations.

However, since the return on human capital may not satisfy Equation (9) as just explained, Equation (10) may not really be written as a linear combination of all the $N$ asset return Euler equations. Therefore, it is possible for Equation (9) to hold for stocks, bonds, and other tradable assets not subject to substantial market frictions, while in the mean time Equation (10) fails. This possibility, in fact supported by the empirical results below, sheds fresh light on why the EIS estimates in the aggregate data are small because it suggests that the standard EIS regression of consumption growth on an asset return has mis-identified this parameter (Zhang 2006). For further details, see pp. 892–893.

14. Equation (9) is the restriction on individual asset returns for given optimal consumption, so it is reasonable to call it the asset return Euler equations. Equation (10) is the restriction on optimal consumption for given return on the optimal portfolio (or portfolio weights), and is called the consumption Euler equation by some authors.

15. See p. 886 for a likely reason on why this is the standard practice.
Clearly, the presence of $R_{t+1}^m$ in the Euler equations (9) and (10) means that accounting for human capital and all financial assets in measuring the return on optimal portfolio is very important. Furthermore, the substantial difference between the volatilities of the estimated $R_{t+1}^m$ and its proxy should very significantly change the $\lambda$ estimate from those in Epstein and Zin (1991) and others because the small volatility in $R_{t+1}^m$ requires a large $\lambda$ (in absolute terms) to make the SDF sufficiently volatile.\(^{16}\) This result holds whether the sequence of $R^m$ includes elements that fall below 1 or not. A large $\lambda$ (in absolute terms) in turn influences the estimates of EIS and RRA, since $\lambda = (1-\text{RRA})/(1-1/\text{EIS})$.

The unconditional version of Equation (9) can be log-linearized using the joint lognormality assumption for consumption growth and asset returns to obtain a well-known decomposition for the average equity premium over the return on Treasury bill (T bill henceforth). Both the stock and T bill returns below are in logarithmic terms:

$$E(r^s - r^b) = \frac{\sigma_{\Delta c}^2 - \sigma_{\Delta r}^2}{2} + (1 - \lambda)(\sigma_{r^s, \Delta c}^2 - \sigma_{r^m, \Delta r}^2) + \lambda \gamma (\sigma_{\Delta c, r^s} - \sigma_{\Delta c, r^m}) .$$

(11)

See Epstein and Zin (1991) and Campbell (1996) for the conditional version of this equation. Here the superscripts $s$ and $b$ denote stocks and T bills, respectively. $\sigma^2$ denotes variance, and a $\sigma$ with double subscripts is for covariance. $\Delta c$ stands for consumption growth. As noted in Campbell (1996), the first term on the right-hand side of Equation (11) is due to Jensen’s inequality. The term $\sigma_{r^s, \Delta c}$ captures the “market risk” of holding stocks, i.e. the covariation of (log) stock returns with the (log) return on the entire wealth portfolio, and $\sigma_{\Delta c, r^s}$ captures the consumption risk of holding stocks.

When the consumption Euler equation is rejected, the value of the real gross risk-free rate $R_f$ can be computed from Equation (9) by setting $R_{j,t+1} = R_f$ as follows

$$R_f = \frac{1}{E[\beta^j(C_{t+1}/C)^{-\lambda}(R_{t+1}^m)^{\lambda-1}]} .$$

(12)

In addition, the joint log normality assumption above can be applied to a version of Equation (9) (i.e. the weighted average of Equation (9) over all individual stock returns) that holds for the logarithmic gross return on an aggregate stock portfolio, $r^u$

$$\sigma_r^2 = -2[\lambda \log \beta - \lambda \gamma E(\Delta c) + (\lambda - 1)E(r^m) + E(r^u)] - (\lambda - 1)^2 \sigma_{r^m}^2$$

$$- \lambda^2 \gamma^2 \sigma_{\Delta c}^2 + 2\lambda(\lambda - 1)\gamma \sigma_{\Delta r, r^m} + 2\lambda \gamma \sigma_{\Delta r, r^m} - 2(\lambda - 1)\sigma_{r^m, r^u} .$$

(13)

In the next section, I discuss how to estimate and test the EZW model using Equations (9) and (10) with the $R_{t+1}^m$ series calculated from Equation (2). In addition, I utilize the sampling counterparts to Equations (11)–(13) to both narrow down the

\(^{16}\) Var$(R^m)^2 = \lambda^2(1 + (\lambda - 1)^2)\mu^{2\lambda - 1}\text{Var}[R^m]$ by the second-order Taylor series expansion of $R^m$ around its mean $\mu$. This explains the need for a relatively large $\lambda$ in absolute terms.
parameter values and validate a confidence set of unknown parameters constructed using SW’s (2000) approach: is there any combination of $\lambda$, $\gamma$, and $\beta$ in a confidence set that is able to match the mean equity premium, the risk-free rate, and the variance of stock return at the same time?

As in Epstein and Zin (1991) and Campbell (1996), which of the two risks captured by the EZW model is more important in determining the equity premium obviously depends on the sizes of $\lambda$ and $\gamma$. First, if $\lambda = 1$, the model is reduced to the expected utility C-CAPM. Consequently, only the consumption risk matters. Second, if $\lambda = 0$, Equation (12) becomes a version of the CAPM extended to incorporate both human capital and financial assets, so only the “market risk” matters. Third, since the “market risk” $\hat{\sigma}_{m,r^2} / \hat{\sigma}_{r^2} = 3.979 \times 10^{-4}$ is almost one order of magnitude larger than the consumption risk $\hat{\sigma}_{\Delta c^2} / \hat{\sigma}_{c^2} = 4.907 \times 10^{-5}$, if the magnitudes of the “risk prices” $1/\lambda$ and $\lambda \gamma$ in Equation (11) are close to each other, the “market risk” dominates the consumption risk in determining the equity premium. Naturally, we would like to know if the empirical results will support such a possibility.

3. ECONOMETRIC METHODS

I now substitute the estimated $R_{t+1}^m$ as described in Section 1 into Equation (9). This yields the following Euler equations for asset returns:

$$E_t \left[ \beta^{\lambda} \left( \frac{C_{t+1}}{C_t} \right)^{\lambda(1-\gamma)-1} \left( \frac{\exp(cay_t - cay_{t+1})}{1 - k \cdot \exp(cay_t)} \right)^{\lambda-1} R_{j,t+1} - 1 \right] = 0, \quad j = 1, \ldots, N - 1.$$  

(14)

Recall $k = 0.00697$. Now define $\mu = (\beta, \lambda, \gamma)$, and denote the true value of $\mu$ by $\mu_0$. Let $\epsilon_{t,j+1}(\mu)$ be the bracketed term in Equation (14), and $\epsilon_{t,j+1}(\mu) = (\epsilon_{1,t+1}(\mu), \ldots, \epsilon_{m,t+1}(\mu))^\prime$, where $m$ is the number of assets used in a test. Let the $p$-vector $Z_t$ be a subset of the representative agent’s information set up to time $t$. Define $\phi_{t+1}(\mu) = \epsilon_{t+1}(\mu) \otimes Z_t$, where $\otimes$ is the Kronecker product. Then $E(\phi_{t+1}(\mu_0)) = \theta$ are the $m \times p$ orthogonality conditions that can be employed in estimating and testing asset pricing implications of the model. Let $\bar{\phi}(\mu) = (1/T) \sum_{t=1}^T \phi_t(\mu)$, where $T$ is the sample size. The GMM criterion function is a quadratic form in $\bar{\phi}(\mu)$,

$$S_T(\mu; \theta(\mu)) \equiv T \bar{\phi}(\mu) \bar{W}_T(\theta(\mu)) \bar{\phi}(\mu).$$

The efficient and heteroskedasticity-robust weighting matrix is written as $W_T(\theta(\mu))$ to accommodate different GMM estimators. When $\theta(\mu) = \mu$, the weighting matrix $W_T$ is continuously updated with $\mu$. The minimizer $\hat{\mu}$ obtained with such a weighting matrix is known as the continuous updating (CU henceforth) GMM estimator (see Hansen, Heaton, and Yaron 1996). The two-step GMM estimator, on the other hand, uses weighting matrices that do not update with $\mu$. 
It is well known that the small sample properties of the two-step GMM estimator and the associated test statistics are not satisfactory when they are used to test C-CAPM models.\textsuperscript{17} For example, the minimum $\chi^2$ test tends to over-reject in testing the time- and state-separable C-CAPM. On the other hand, Hansen, Heaton, and Yaron (1996) showed that the minimum $\chi^2$ test based on the CU GMM estimator has smaller size distortions in the finite sample than those based on the two-step and iterative GMM estimators. SW (2000) developed an alternative asymptotic theory for the CU GMM estimator that is robust to the presence of weak instruments. Since I am not aware of any other paper in asset pricing that uses their approach, I provide a brief, non-technical, introduction to it here.

They propose two methods to test jointly the model specification and the null hypothesis $\mathbf{\mu} = \mathbf{\mu}_0$ via the construction of a confidence set for the unknown parameters. The first method uses the following property of the criterion function of the CU GMM (i.e. Theorem 2 in their paper):

$$S_f(\mathbf{\mu}_0; \mathbf{\mu}_0) \xrightarrow{d} \chi^2_{m \times p}$$

where $m \times p$ is the degree of freedom. This result holds without any additional assumption on instrument validity except $E(\phi(\mathbf{\mu}_0)) = 0$. This is how the test above can accommodate weak instruments and, therefore, weakly identified models. The set of parameter values $\mathbf{\mu}_0$ that do not generate a large $S_f(\mathbf{\mu}_0; \mathbf{\mu}_0)$ relative to the $\pi\%$ critical value of the $\chi^2_{m \times p}$ distribution is called the $(1 - \pi)\%$ joint $S$ set in SW (2000).

On the other hand, it is possible that some model parameters are well identified while others are not. In such a case, a different confidence set for the weakly identified model parameters can be constructed according to Theorem 3 in SW (2000). The construction of this confidence set involves two steps. First, estimate the well-identified parameters for various values of the weakly identified parameters using the CU GMM. Second, evaluate the CU GMM criterion function using the same values of the weakly identified parameters (as in step 1) and the corresponding estimates for the well-identified parameters. The CU GMM criterion function thus evaluated converges in distribution to a $\chi^2_{m \times p - id}$ statistic, where $id$ is the number of well-identified parameters. The collection of the weakly identified parameters’ values that enable a model to pass the $\chi^2_{m \times p - id}$ test at the significance level $\pi\%$ is called a $(1 - \pi)\%$ concentrated $S$ set because the well-identified parameters are concentrated out in constructing the $S$ set in this case. This result relies on stronger assumptions than those for the $\chi^2_{m \times p - id}$ test above, and I refer the reader to SW (2000) for technical details. Importantly, the size distortions of these two $\chi^2$ tests of model validity are minimal in the Monte Carlo studies reported in their paper, and are much smaller than that associated with the standard GMM asymptotics in Hansen (1982).

If a model is correctly specified, when it is run through the entire parameter space at a certain significance level $\pi$, the $(1 - \pi)\%$ joint $S$ set or a $(1 - \pi)\%$ concentrated

\textsuperscript{17} See e.g. Ferson and Foerster (1994).
$S$ set, should not be null. A null $S$ set indicates the rejection of the over-identifying restrictions and, therefore, the rejection of the model being tested. A small $S$ set causes some ambiguity: it could indicate that the model is not rejected, and the parameters are precisely estimated, or that the data is too weak to reject the model completely. How to formally handle this ambiguity seems to be a gap in the literature. But intuitively speaking, it may be sufficient to use the sampling counterparts to Equations (11)–(13) to validate an $S$ set and to narrow down parameter values as mentioned on p. 883. The idea is that if a (joint) $S$ set contains the true values of the unknown parameters, at least one element of this $S$ set should be able to match, in a statistical sense, the average equity premium (and the average risk-free rate and the stock return volatility at the same time in the case of a joint $S$ set)—given that a non-empty $S$ set indicates the non-rejection of the model. Especially the elements that produce almost exact matches should be given more attention than those that cannot: the former are more likely to be the true parameter values than the latter. For example, the presence of the latter in an $S$ set could be due to instrument weakness, insufficient test power at these parameter values, or the gap between the finite-sample distribution and the asymptotic $\chi^2$ distribution of the test statistics.

To understand the empirical results in the next section, it is useful to know that the symptoms of weak identification summarized in SW (2000) are as follows: the parameter estimates from asymptotically equivalent GMM estimators are very different from each other, the estimates are not robust to the addition of instruments, inferences on model specification are sensitive to the particular GMM estimator used, and a confidence set for the 2-step GMM estimates has substantial areas of disagreement with a comparable $S$ set.

Since the goal of this paper is to investigate if the EZW model can solve the equity premium puzzle and the risk-free rate puzzle, I use two quarterly returns to test the asset return Euler equations (9): the value-weighted real return on NYSE stocks (rvwrq henceforth) and the real return on U.S. Treasury bills (rtbillq henceforth). The consumption measure that I use is real per person nondurable goods and services expenditure excluding clothing and shoes, seasonally adjusted and in 1996 chain-weighted dollars. Other measures of consumption are not

18. In other words, the elements that produce almost exact matches can, with a $p$ value close to 1, pass a test of the hypothesis that the equity premium produced by the population counterpart to Equation (11) is equal to the true equity premium, and likewise for the risk-free rate. In this sense, such elements are likely to be the true values of the model parameters. But those elements that produce inferior matches may fail such a test, and may therefore not be the true values of the model parameters. The difficulty with implementing such a test is that the covariance between the two covariance terms in Equation (11) is unknown and cannot be estimated, due to the fact that there is only one “observation” for each of these two terms.

19. Epstein and Zin (1991) cautioned that when the return on the optimal portfolio is proxied by rvwrq, it is usually not adequate to use just these two returns to test their model. This is not a problem here, given that I do not use this proxy.

20. The use of this measure of consumption means that the wealth portfolio in Section 1, $W$, should include the stock of clothing and shoes, which are not included in the household net worth measure reported by the Fed. This is not problematic because their share in $A$, and therefore $W$, is very small. For example, suppose that the stock of clothing and shoes per person was $5,000 by the end of 2001. It would be about 4.15% of $A$ by then. Since $A$ is only 30% of $W$, the stock of clothing and shoes would only be 1.25% of $W$. 
used because the *cay* estimates are based on this particular definition of consumption in LL (2004). The real after-tax labor income per person, used as an instrument in the present paper, is also the same as in their paper. The sample period is from the first quarter of 1959 to the fourth quarter of 2001.

I use four sets of instruments in testing the asset return Euler equations. See the notes beneath Table 1 for details. Among these instruments, the bond default premium, rvwrq, rtbillq, and real dividend yield are compounded from their monthly counterparts in Ibbotson Associates (2002). I include the *cay* variable in the fourth set of instruments because it has been used as an instrument in a few papers. Using this set of instruments in my empirical analysis hence facilitates the comparison of my results with those in the literature that used *cay*. The eight instruments used in estimating or testing the consumption Euler equation can be found beneath Table 4. Lastly, following the standard practice in the literature in dealing with possible time aggregation bias, I also lag each set of instruments by one more quarter in testing.

4. TEST RESULTS

I report the estimation and test results for Euler equations (9) and (10) in Section 4.1 below. Then I demonstrate in Section 4.2 how the historical equity premium, risk-free rate, and stock return volatility can be matched using the empirical results on Equation (9). In the third part of this section, I compare my results with those in the literature.

4.1 Results on Euler Equations

_Estimation of Equation (9) and evidence on weak identification._ Table 1 collects the estimation and test results produced by two GMM estimators using the standard asymptotics for the asset return Euler equations. In these results, λ and γ, instead of RRA and EIS, are directly estimated. A likely reason for such a standard practice is that it allows the variance-covariance matrix of \( \Phi_{t+1}(\mu_{0}) \) to be inverted without running into the problem of ill-conditioned matrix, which arises when RRA and EIS are estimated directly.

Comparing the 2-step GMM results with those of the CU GMM in this table, I find that two patterns emerge. First, the estimates of λ vary substantially across the two GMM estimators employed, and across different instrument sets in the CU GMM. For example, the λ estimates are −33.19 and −100.6, respectively, for the 2-step and CU estimators in panel 1. The corresponding RRA estimates are 2.33 and 11.06, respectively. In percentage terms, the change in RRA (475%) is far larger than that in λ (203%). On the other hand, the λ estimate produced by the CU

21. I am, however, skeptical about using *cay* as an instrument in testing the Euler equation (9). This is because in such a test, the instruments should be the variables that are included in the information set of the representative agent up to t, i.e. the information that is publicly available. Despite the predictive power of *cay*, its calculation requires the use of the entire sample, i.e. information not available up to t, for any t before the end of the sample period.
### Table 1: Two-Step and Continuous Updating GMM Estimation of Euler Equations for Stock and T Bill Returns Using Conventional Asymptotics

<table>
<thead>
<tr>
<th>Instrument Set</th>
<th>Method</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-step</td>
<td>0.9861 (0.0010)</td>
<td>-33.19 (8.92)</td>
<td>0.96 (0.18)</td>
<td>11.85 (0.037)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9863 (0.0012)</td>
<td>-100.6 (17.9)</td>
<td>0.90 (0.20)</td>
<td>8.94 (0.111)</td>
</tr>
<tr>
<td>2</td>
<td>2-step</td>
<td>0.9859 (0.0009)</td>
<td>-26.50 (4.51)</td>
<td>0.91 (0.17)</td>
<td>14.96 (0.092)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9859 (0.0009)</td>
<td>-59.94 (8.49)</td>
<td>0.95 (0.18)</td>
<td>19.07 (0.025)</td>
</tr>
<tr>
<td>3</td>
<td>2-step</td>
<td>0.9863 (0.0008)</td>
<td>-25.95 (4.47)</td>
<td>1.00 (0.16)</td>
<td>15.54 (0.159)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9862 (0.0009)</td>
<td>-60.15 (8.52)</td>
<td>1.00 (0.17)</td>
<td>20.08 (0.044)</td>
</tr>
<tr>
<td>4</td>
<td>2-step</td>
<td>0.9861 (0.0008)</td>
<td>-24.48 (3.92)</td>
<td>1.01 (0.14)</td>
<td>28.43 (0.008)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9858 (0.0008)</td>
<td>-98.03 (11.66)</td>
<td>0.96 (0.16)</td>
<td>27.96 (0.009)</td>
</tr>
<tr>
<td>1 Lagged</td>
<td>2-step</td>
<td>0.9864 (0.0016)</td>
<td>-45.14 (18.16)</td>
<td>0.97 (0.27)</td>
<td>10.46 (0.063)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9860 (0.0016)</td>
<td>-56.45 (19.41)</td>
<td>0.88 (0.27)</td>
<td>10.02 (0.075)</td>
</tr>
<tr>
<td>2 Lagged</td>
<td>2-step</td>
<td>0.9857 (0.0014)</td>
<td>-37.31 (14.62)</td>
<td>0.84 (0.25)</td>
<td>12.79 (0.172)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9856 (0.0014)</td>
<td>-60.55 (18.79)</td>
<td>0.80 (0.23)</td>
<td>11.97 (0.215)</td>
</tr>
<tr>
<td>3 Lagged</td>
<td>2-step</td>
<td>0.9843 (0.0013)</td>
<td>-29.11 (9.90)</td>
<td>0.59 (0.23)</td>
<td>15.51 (0.160)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9850 (0.0013)</td>
<td>-77.49 (19.92)</td>
<td>0.68 (0.19)</td>
<td>13.83 (0.243)</td>
</tr>
<tr>
<td>4 Lagged</td>
<td>2-step</td>
<td>0.9847 (0.0009)</td>
<td>-27.82 (4.84)</td>
<td>0.68 (0.17)</td>
<td>26.37 (0.015)</td>
</tr>
<tr>
<td></td>
<td>CU</td>
<td>0.9842 (0.0009)</td>
<td>-131.5 (16.9)</td>
<td>0.52 (0.16)</td>
<td>17.57 (0.175)</td>
</tr>
</tbody>
</table>

Notes: Set 1 includes four instruments: 1, and first lags of consumption growth, term premium, and real dividend yield. Set 2 includes Set 1 and the first lags of real T Bill rate and labor income growth. Set 3 combines Set 2 and the first lag of the default premium. Set 4 includes Set 3 and $cay$. Standard errors are in parentheses, except for the last column, which reports the minimum $\chi^2$ test of over-identifying restrictions and the corresponding $p$ values in parentheses. The degree of freedom for the $\chi^2$ test is the number of orthogonality conditions (i.e. the number of instruments $\times 2$) subtracted by 3, the number of parameters estimated.

Estimator changes from $-100.6$ in panel 1 to $-59.94$ in panel 2 when two additional instruments, lagged labor income growth and lagged rtbillq, are added to instrument set 1. This 40% change in $\lambda$ corresponds to a $-64\%$ change in the implied risk RRA varies from 11.06 to 4.00. Such high sensitivity of parameter estimates to the GMM estimator used and to the addition of instruments, in terms of either $\lambda$ or RRA, is a sign of weak identification. Second, the minimum $\chi^2$ tests associated with the two GMM estimators portray different pictures about the overall fit of the EZW model at the 5% level in four of the eight cases considered, even though these two estimators are asymptotically equivalent. For example, in panel 1, the $\chi^2$ statistic from the 2-step GMM suggests that the model is rejected at the 5% level. But the same statistic from the CU GMM, with a $P$ value of 11.1%, indicates that the model is not rejected at conventional significance levels. Such disagreement also occurs in panels 2, 3, and 8. Even at the 10% significance level, there are still disagreements in test results in three cases (see panels 1, 3, and 8). This is another symptom of weak identification I alluded to in Section 3.

In addition, the $\gamma$ estimates also vary a lot overall, though in the top four panels they seem to be around 0.96. See e.g. the $\gamma$ estimate of 0.52, 0.59, and 0.68 at

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22. The closeness of these estimates to 1 suggests that the EIS could just be 1. It is, however, difficult to test the restriction $\gamma = 1$ because an assumption underlying the Euler equations (9) and (10) is that $\gamma \neq 1$ (so that $\rho \neq 0$). Imposing $\gamma = 1$ changes the Euler equation to a form that includes an unknown function of the state of the economy (see Giovannini and Weil 1989). They also showed that with a Markovian and lognormal return on optimal portfolio, it is possible to derive the explicit Euler equation for the case of unitary EIS. But even in that case, the RRA cannot be identified without very strong assumptions. Furthermore, the Markovian assumption is not satisfied in the estimated $R^w$ series. An AR(4) regression for this series indicates that only the third lag of $R^w$ is significant at the 5% level.
the bottom two panels. Given that $\gamma$ is a part of the exponent of consumption growth in the SDF, and the variation in consumption growth is small, it seems difficult to see why it can be well identified. It may, therefore, be more appropriate to treat $\gamma$ as weakly identified along with $\lambda$. SW (2000), however, treated $\beta$ and $\lambda$ as well identified, and $\gamma$ as weakly identified. This is because the proxy for $\rho^m$ that they used, the weighted average of stock returns, is very volatile, making the identification of $\lambda$ possible.

I now present in Figure 2 more definitive evidence on weak identification. This figure plots for different sets of instruments the 95% confidence ellipses for the 2-step GMM estimates of $\lambda$ and $\gamma$, and the 95% concentrated $S$ sets (shaded area) for these two parameters. The $S$ sets are generated by running the model through the parameter space specified in Table 2 to search out the combinations of parameter values that are not rejected by the data. $S$ sets are not centered around the CU estimates, as pointed out by Kleibergen (2001). As such, the $S$ sets in plots in Figure 2 can include $\gamma$ values much smaller than 1 without those much larger than 1, even though the corresponding CU estimates are from 0.80 to 1.00 in Table 1. There is not a plot for the case of instrument set 4 because the corresponding 95% concentrated $S$ set is empty. For two of the seven cases in Figure 2, i.e. (C) and (G), there is no overlap between confidence region and $S$ set. For each of the remaining five cases, there is substantial area of disagreement between confidence region and $S$ set. Non-overlapping and substantial area of disagreement are both important signs of weak identification that SW (2000) emphasized. Therefore, it is necessary that weak identification be taken into account in my empirical analysis. Whether the evidence for weak identification here is stronger than that in their paper is, however, beside the point.

Results of $S$ set analysis on asset return Euler equations. I report the results of $S$ set analysis based on instrument sets 1, 2, and 4 in Table 3. The intervals in this table are meant to describe the boundaries of each $S$ set constructed using the parameter space specified in Table 2. They are not the 95% confidence intervals for

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**TABLE 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value Ranges</th>
<th>Increments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>[0.983, 1.01]</td>
<td>0.001</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>[-150, 2]</td>
<td>0.25</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>[0.401, 2.521]</td>
<td>0.02</td>
</tr>
</tbody>
</table>

---

23. I have tried in some tests wider ranges of parameter values than those reported in Table 2. But they did not turn out to be necessary except perhaps for $\lambda$, which in some cases took values smaller than $-150$. In addition, my goal here is not to construct the confidence interval for each individual parameter, which is technically difficult to do in this approach. Lastly, note that the $p$ values in Table 1 for CU GMM cannot be used to infer the size of $S$ sets because those $p$ values are based on the standard asymptotic theory.
Fig. 2. The 95% Concentrated $S$ Set (Shaded) for $\lambda$ and $\gamma$ and the 95% Confidence Ellipse for the 2-Step GMM Estimates of $\lambda$ and $\gamma$ Based on (A) Instrument Set 1; (B) Instrument Set 1 Lagged; (C) Instrument Set 2; (D) Instrument Set 2 Lagged; (E) Instrument Set 3; (F) Instrument Set 3 Lagged; and (G) Instrument Set 4 Lagged.
individual parameters (see also Note 23). The results based on instrument set 3 are similar to those based on instrument sets 2 and 1, and are not reported to conserve space.\textsuperscript{24}

In Table 3, the reader can see that when instrument sets 1 and 2 or their first lags are used, the \( S \) set analysis overall presents favorable evidence for the EZW model at the 5% level of significance. Out of the twelve \( S \) sets for these two sets of instruments, only one is null.\textsuperscript{25} The \( \beta \) values in the joint \( S \) sets are all smaller than 1, indicating that the EZW model is rejected for \( \beta \) values larger than 1. The ranges of \( \lambda \) and \( \gamma \) values in the 95% joint \( S \) sets are much wider than that of \( \beta \), reflecting the weak identification of these two parameters. Due to the wide ranges of \( \lambda \) and \( \gamma \), RRA values in all the \( S \) sets reported in this table also swing widely because \( RRA = 1 - \lambda(1 - \gamma) \) in the model. What is important here, though, is the fact that the RRA ranges in Table 3 all include what economists believe to be the reasonable values of this parameter, suggesting that the model is not rejected for these values. I will discuss the EIS values in Section 4.2.

The \( S \) set analysis using the fourth set of instruments and its lag, however, delivers mixed results. First, when this set of instruments is used, the 95% \( S \) sets (concentrated or not) are all empty. See the two rows corresponding to instrument set 4 in Table 3. This is evidence that the EZW model is rejected at the 5% level of significance. Second, when this set of instruments is lagged one more quarter, the \( S \) sets are not null any more. Two of them, the joint \( S \) set for \( (\beta, \lambda, \gamma) \) and the concentrated \( S \) set for \( (\lambda, \gamma) \), imply similar values of RRA and EIS to those implied by \( S \) sets based on the first three instrument sets. They are favorable evidence for the EZW model. However, since this set of instruments includes \( cay \) (see Note 21), the results based on it may not carry much weight either way.

To summarize, \( S \) set testing does not reject the asset pricing implications of the EZW model. This is in strong contrast with the results in Table 1. In that table, even for CU GMM, which has smaller size distortion in the finite sample than the 2-step GMM, the EZW model is rejected at 5% level for instrument sets 2 and 3 under the standard asymptotic theory. In addition, since none of the \( S \) sets above contain \( \lambda = 1 \), which is required by the standard C-CAPM, \( S \) set testing unequivocally rejects the standard model and favors the EZW model. In Section 4.2, I further demonstrate how matching the equity premium, the risk-free rate, and the stock return volatility help narrow down the parameter values.

Results of \( S \) set analysis on the consumption Euler equation. The 2-step and CU GMM estimates of the unknown parameters of the consumption Euler equation (10)
TABLE 3
$S$ SETS FOR EULER EQUATIONS FOR STOCK AND T BILL RETURNS

<table>
<thead>
<tr>
<th>Instrument Set</th>
<th>$95%$ Joint $S$ for $(\beta, \lambda, \gamma)$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>RRA</th>
<th>EIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Lagged</td>
<td>$[0.984, 0.988]$</td>
<td>$[-150, -17.25]$</td>
<td>$[0.401, 1.041]$</td>
<td>$[0.0025, 90.85]$</td>
<td>$[0.961, 2.494]$</td>
<td></td>
</tr>
<tr>
<td>2 Lagged</td>
<td>$[0.985, 0.987]$</td>
<td>$[-70, -10.25]$</td>
<td>$[0.401, 1.81]$</td>
<td>$[0.0025, 90.85]$</td>
<td>$[0.925, 2.494]$</td>
<td></td>
</tr>
<tr>
<td>4 Lagged</td>
<td>$[0.983, 0.988]$</td>
<td>$[-150, -3.75]$</td>
<td>$[0.401, 1.22]$</td>
<td>$[0.002, 74.5]$</td>
<td>$[0.819, 2.49]$</td>
<td></td>
</tr>
<tr>
<td>Null</td>
<td>$[0.983, 0.987]$</td>
<td>$[-150, -4.25]$</td>
<td>$[0.401, 1.001]$</td>
<td>$[0.8, 90.85]$</td>
<td>$[0.999, 2.49]$</td>
<td></td>
</tr>
</tbody>
</table>

$\lambda$ | $\gamma$ | RRA | EIS | $\lambda$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Lagged</td>
<td>$[-150, -26.25]$</td>
<td>$[0.401, 1.02]$</td>
<td>$[0.0025, 90.85]$</td>
<td>$[0.979, 2.494]$</td>
</tr>
<tr>
<td>2 Lagged</td>
<td>$[-74, -47.25]$</td>
<td>$[0.841, 1.001]$</td>
<td>$[0.93, 11.8]$</td>
<td>$[0.999, 1.189]$</td>
</tr>
<tr>
<td>4 Lagged</td>
<td>$[-150, -52.25]$</td>
<td>$[0.401, 1.001]$</td>
<td>$[0.85, 91]$</td>
<td>$[0.999, 2.494]$</td>
</tr>
</tbody>
</table>

Notes: This table presents the ranges of each parameter in a $95\%$ $S$ set and the ranges of implied RRA and EIS values. These ranges are, however, not the $95\%$ confidence intervals for individual parameters. The results when instrument set 3 is used are similar to those for instrument set 2 and are not reported to conserve space.

are very sensitive to the initial parameter values used. So are the test results of model specification produced by the CU estimator (not reported to conserve space). Therefore, it is natural to ask what SW’s (2000) approach will yield. I first test Equation (10) alone, then test Equations (9) and (10) jointly.

Table 4 presents the results of $S$ set analysis on Equation (10). There are two major differences between this set of results and those in Table 3. First, five of the six $S$ sets in Table 4 contain $\lambda = 1$, implying that the Euler equation of the expected utility C-CAPM as applied to $R^C$ cannot be rejected at 5% level, and the consumption implication of the EZW model is at odds with the data.26 Furthermore, this result is robust to variations in instruments. For example, dropping the default premium, the term premium, and the household net worth growth (or a subset of them) from the instruments list in the testing still does not reject $\lambda = 1$. Second, the range of $\lambda$ is now the whole range tested, $[-150, 2]$, in the joint $S$ sets for $(\beta, \lambda, \gamma)$ and the concentrated $S$ sets for $(\lambda, \gamma)$. This is a sign that the instruments are too weak, see SW (2000, the last three rows of p. 1064). Together, these two facts suggest that the results here are not discriminating enough to be meaningful.

A natural response to the ambiguity above is to test the consumption Euler equation and the asset return Euler equations jointly to shrink the $S$ sets in Table 4.

26. However, it may still be wrong to conclude that the expected utility C-CAPM holds because its Euler equation as applied to $R^C$ is also a linear combination of all the individual Euler equations, one of which is for the return on human capital and should not hold due to the nontradability or severe market frictions of human capital. This tension also points to the view below that the test results on the consumption Euler equation alone are not discriminating enough to be meaningful.
TABLE 4
S SETS FOR CONSUMPTION EULER EQUATION

<table>
<thead>
<tr>
<th>Instrument Set</th>
<th>95% Joint S Set for (β, λ, γ)</th>
<th>95% Concentrated S Set for (λ, γ)</th>
<th>95% Concentrated S Set for λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Lagged</td>
<td>[0.983, 0.989] [−150, 2] 0.401, 1.541 0.0018, 90.85 [0.649, 2.494]</td>
<td>[−150, 2] 0.401, 1.361 0.0018, 90.85 [0.735, 2.494]</td>
<td>[−150, −103.25]</td>
</tr>
<tr>
<td>5</td>
<td>[0.983, 0.992] [−150, 2] 0.401, 2.261 0.0018, 90.85 [0.442, 2.494]</td>
<td>[−150, 2] 0.401, 2.101 0.0018, 90.85 [0.476, 2.494]</td>
<td>[−7.25, 2]</td>
</tr>
</tbody>
</table>

Notes: The results in this table are obtained when Equation (10) is tested alone. The joint test of Equations (9) and (10) yields null S sets. The instrument set 5 includes 1, the first and second lags of real consumption growth, the first lag of household net worth growth, and the first lags of the real after-tax labor income growth, the term premium, the real dividend yield, and the default premium.

Such an exercise leads to a perhaps surprising finding: the joint S set for (β, λ, γ), the concentrated S set for (λ, γ), and the concentrated S set for λ, based on joint testing of Equations (9) and (10), are all null at 5% level! This result is also robust to the choice of instruments. Using instrument sets 1, 2, and 3 or their first lags, or using instrument set 5 or its first lag, all produces empty S sets in the test. Given the positive results on Equation (9) presented in the last subsection, the rejections here in the joint tests can only be interpreted as the failure of Equation (10). Since Equation (10) is a linear combination of Equation (9) across all assets including human capital, this failure can in turn be attributed to the fact that Equation (9) does not hold for the return on human capital due to reasons mentioned on pp. 881. As a result, existing EIS estimates obtained from the standard EIS regression of consumption growth on an asset return cannot be used to judge the EIS estimates in the present paper. See below for details. Along the same line, the risk-free rate should be calculated according to Equation (12), instead of solved using both Equations (9) and (10) as in Campbell (2003).

4.2 Matching Equity Premium, Risk-free Rate, and Stock Return Volatility

I now use Equations (11)–(13) to validate the S sets in Table 3 and to narrow down the parameter values. The results are in Table 5.27 The β values in this table all correspond to time discount rates of about 5.2% in annual terms. The λ and γ combinations in this table imply RRA values from 0.95 to 2.33, and EIS values from 0.999 to 1.030. While the RRA values are what economists would like to see, the EIS values may still be considered somewhat high by some quarters of our profession. To make these EIS values truly comparable to the existing low estimates

27. For this table, to further pin down the values of β, RRA, and EIS, I use smaller increments of 10⁻⁴ or 10⁻⁵ for β to rerun the S set analysis throughout the parameter space specified by the S sets in Table 3.
### TABLE 5
Matching Quarterly Historical Equity Premium, Risk-Free Rate, and Stock Return Volatility

<table>
<thead>
<tr>
<th>β</th>
<th>λ</th>
<th>γ</th>
<th>RRA</th>
<th>EIS</th>
<th>E.P.</th>
<th>r_{eq}</th>
<th>σ^2_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9867</td>
<td>-51.25</td>
<td>1.001</td>
<td>0.949</td>
<td>0.999</td>
<td>0.0149</td>
<td>0.0044</td>
<td>0.00709</td>
</tr>
<tr>
<td>0.9866</td>
<td>-51.50</td>
<td>0.981</td>
<td>1.979</td>
<td>1.019</td>
<td>0.0150</td>
<td>0.0047</td>
<td>0.00686</td>
</tr>
<tr>
<td>0.9864</td>
<td>-45.75</td>
<td>0.991</td>
<td>1.412</td>
<td>1.009</td>
<td>0.0130</td>
<td>0.0045</td>
<td>0.00675</td>
</tr>
<tr>
<td>0.9865</td>
<td>-45.75</td>
<td>0.991</td>
<td>1.412</td>
<td>1.009</td>
<td>0.0130</td>
<td>0.0044</td>
<td>0.00642</td>
</tr>
<tr>
<td>0.9867</td>
<td>-51.50</td>
<td>1.001</td>
<td>0.949</td>
<td>0.999</td>
<td>0.0149</td>
<td>0.0045</td>
<td>0.00643</td>
</tr>
<tr>
<td>0.9866</td>
<td>-51.50</td>
<td>0.981</td>
<td>1.979</td>
<td>1.019</td>
<td>0.0150</td>
<td>0.0047</td>
<td>0.00686</td>
</tr>
</tbody>
</table>

Notes: The first three columns report selected elements in the 95% joint S sets or concentrated S sets. The next two columns present the implied RRA and EIS values implied by the λ and γ values of each row. The column labeled “E.P.” shows the unconditional quarterly equity premium implied by the values of RRA and EIS in each row using Equation (11). The “r_{eq}” column reports the logarithmic quarterly risk-free rate implied by the values of β, λ, and γ of each row in the upper panel using Equation (12). The last column is the variance of stock return calculated using the parameter values in the first three columns and the sample moments on consumption growth, Rm, and other sample quantities using Equation (13).

for this parameter based on the aggregate data, Zhang (2006) shows that the former must be scaled by \([1 + w_j(λ - 1)]/λ\) due to the rejection of Equation (10), where \(w_j\) is the portfolio weight on the \(j\)th asset in the wealth portfolio. Given that the average weight of stocks in the entire wealth portfolio is about 6% as explained in Note 13, this scaling constant is about 0.04 for the λ values in Table 5. Therefore, the EIS values in this table imply comparable, but misidentified, “EIS” estimates of about 0.04 when consumption growth is regressed on stock return. This is consistent with the well-known claim in Hall (1988) and others that the “EIS” estimates based on aggregate data are much closer to 0 than to 1.

The last three columns of this table report (for quarterly frequency) the equity premium, the risk-free rate, and the variance of stock return produced by the parameter values of each row using sampling counterparts to Equations (11)–(13). For 28. At quarterly frequency the volatility of risk-free rate is 0.000028. Due to the large coefficient 2λ (in the order of 10^(-1)) for \(\log β\) in the counterpart of Equation (13) for the risk-free rate, to match the variance of risk-free rate requires the precision of \(\log β\) values that are in the order of 10^(-3). Such a match is therefore not attempted because the consumption and asset returns data are unlikely to be so informative about \(β!\)
the 1959–2001 period, the average real quarterly equity premium is 1.5%, the average real quarterly T bill return is 0.46%, and the variance of the quarterly NYSE stock return is 0.0065. It is clear that in five of the six $S$ sets in Table 5, there is at least one $\lambda$ and $\gamma$ combination reported in this table that is able to match the exact average equity premium in the data. For instance, in the joint $S$ sets for instrument sets 1 and 3, a combination of $\lambda = -51.50$ and $\gamma = 0.981$ generates the right equity premium of 1.5%. Coupled with a $\beta$ value of 0.9866, it also produces a risk-free rate of 0.47%, and a variance of stock return that is 0.00686.

The joint $S$ set for instrument set 2 includes elements that produce equity premium of 1.3% for very closely matched risk-free rate (e.g. 0.44%) and stock return volatility (e.g. 0.00642). Many of the other $S$ set elements that can match these three quantities are close to the parameter values reported in Table 5, though some others feature higher RRAs. The point here is, however, that the lower RRAs around 1 or 2 as presented in Table 5 cannot be rejected, and they very closely match the average equity premium, the average risk-free rate, and the stock return volatility at the same time. It is impossible to obtain this result by just using the conventional methods of estimation and testing or when the Roll critique is ignored.

Interestingly, none of the very large RRA values in the joint $S$ sets in Table 3 can match the three historical quantities above simultaneously. They produce negative risk-free rates, equity premia that are larger than 1.5%, and most of the time, negative stock return volatility. In the joint $S$ sets based on instrument sets 1, 2, and 3, all the elements implying RRA coefficients larger than 24.4, 8.7, and 10.5, respectively, produce negative risk-free rates. For example, in the joint $S$ set produced by instrument set 2, an RRA value of 12.91 corresponds to the element (0.9854, $-57$, 0.791). This element produces a quarterly equity premium of 1.74%, a quarterly risk-free rate of $-2.1\%$, and a stock return volatility of $-3.6\%$. Again, the reason that these very large RRA values (or the corresponding $\lambda$ and $\gamma$ values) emerge in an $S$ set could be due to instrument weakness, the discrepancy between the finite-sample distribution and the asymptotic distribution of the $S$ set test statistic, or insufficient test power at some parameter values. Thus, not every element in an $S$ set is expected to match the equity premium, the risk-free rate, and the volatility of stock return well at the same time. In fact, from elementary econometrics we know that a confidence set may not even contain the true values of the model parameters at all. Therefore, the fact that the elements in Table 5 perform so well suggest that they are either very likely to be, or at least very close to, the true values of model parameters.

So what drives the size of the equity premium in this model? Recall that in Equation (12) the third term for consumption risk is one order of magnitude smaller than the second (for “market risk”). Since the $\lambda$ and $\gamma$ values reported in Table 5 imply that the absolute values of $1 - \lambda$ and $\lambda \gamma$ are close to each other, the major

29. Furthermore, it can be shown that the parameter value combinations in the upper half of Table 5 can satisfy the two boundary conditions in Smith (1996) for the consumption and portfolio choice model in Svensson (1989) that features the EZW preferences.
determinant of the equity premium is the “market risk.” This finding is similar in spirit to the result in Campbell (1996) that the cross-sectional variation in asset returns is mainly explained by the market risk, though the market risk in his model is the covariance between the return of a portfolio and the aggregate stock return. Such a finding is also reminiscent of the CAPM (augmented to include human capital and non-stock assets in measuring market return). But formally $\lambda = 0$ must hold in Equation (12) for such a version of CAPM to be the true model. Since none of the $S$ sets reported in Table 3 contains $\lambda = 0$, this version of CAPM is still formally rejected.

4.3 Comparison with Related Results in the Literature

Epstein and Zin (1991) tested the asset return Euler equation and rejected it for the most part. Due to the volatile proxy that they used for $R_{mt+1}$, their $\lambda$ estimates fall between $-0.412$ and $0.141$, and their EIS estimates were always somewhat below 1. As a result, the RRA estimates in their paper were centered on 1. Their results can be compared with those in Table 1 of the present paper to understand the impact of addressing the Roll critique but not weak identification. For example, thirteen of the sixteen RRA estimates $1 - \hat{\lambda}(1 - \hat{\gamma})$ implied by the results reported in Table 1 here are larger than 1. These higher RRA estimates are consistent with the finding in Campbell (1996) that incorporating human capital raises RRA estimates in the conventional GMM framework.

Both Campbell (1996) and Vissing-Jørgensen and Attanasio (2003) accounted for human wealth (but not assets other than stocks and bonds) by relating the return on human capital to the returns on financial assets. Campbell (1996) estimated a multifactor asset pricing model motivated by the EZW model. His estimate of RRA was 5.5 in annual data, and 23 in monthly data when the share of human capital in total wealth was assumed to be $2/3$ (which is close to the 0.698 mentioned in Section 1 of the present paper). See Table 6 in his paper. Under the same assumption on human capital share, Vissing-Jørgensen and Attanasio’s (2003) RRA estimates in the EZW model were 10.2 and 6.3 for all the stockholders in their Consumer Expenditure Survey data when consumption was not substituted out. Their RRA estimate became 11.6 when consumption was substituted out. Their EIS estimate from the consumption Euler equation is 1.17. See Tables 1 (case 3) and 2 (cases 3 and 4 of panel A) of their paper. These RRA and EIS estimates imply $\lambda$ estimates of $-60.00$, $-37.06$, and $-68.24$, respectively, for their three RRA estimates above. These estimates, especially the EIS estimate, are somewhat close to what I have reported in Table 5. They did not test the asset return Euler equations. On the other hand, they did not reject the log-linearized consumption Euler equation using the CU GMM with the standard asymptotic theory. This seems to be somewhat counter-intuitive, given the nontradability of, or substantial market frictions in, human capital.

SW’s (2000) conventional GMM estimates of $\lambda$ are between 0 and 1. When they accounted for weak identification (but not the Roll critique) in testing the asset return Euler equation of the EZW model, their joint $S$ set for $(\beta, \lambda, \gamma)$ and their
concentrated $S$ set for $(\lambda, \gamma)$ produced with twice-lagged instruments contain $\lambda = 1$, implying that the standard C-CAPM cannot be rejected. Furthermore, when using instruments that are lagged only once, they obtained empty $S$ sets for the EZW model for both sets of instruments that they considered, thereby rejecting it. My results as described in Section 4.1 under “Results of $S$ set analysis on asset return Euler equations” are in sharp contrast with theirs. The difference between their results and mine attest to the importance of accounting for human capital and non-stock financial and real assets in evaluating models that involve the return on market portfolio and wealth growth.

Two recent calibration studies featuring the EZW preferences offer some clues on other possible solutions to the asset pricing puzzles when human capital is left out of the model. Melino and Yang (2003) found that it is crucial to allow the EIS to be state-dependent in order to match the historical equity premium and risk-free rate in the Mehra–Prescott (1985) environment. Bansal and Yaron (2004) reported that introducing consumption and dividend dynamics featuring an autoregressive component and conditionally heteroskedastic disturbances can help resolve a number of asset pricing puzzles if $\text{RRA} = 10$ and $\text{EIS} = 1.5$, including the two puzzles studied in the present paper.

To summarize, for three of the four sets of instruments, the $S$ set tests in the present paper do not reject the asset pricing implications of the EZW model at 5% level for reasonable values of $\beta$, RRA, and EIS after the Roll critique is addressed. The fourth instrument set includes a variable that was not publicly available, and the test results based on it may therefore be invalid. The non-empty $S$ sets obtained with the first three instrument sets contain combinations of parameter values that produce the right equity premium, risk-free rate, and stock return volatility. The dominating determinant of the equity premium is the “market risk”. However, the consumption Euler equation of the model is rejected at 5% level.

5. CONCLUSIONS

As is well known, in the standard C-CAPM with the power utility function, a very high RRA is needed to explain the 6% annual equity premium and, at the same time, a discount factor larger than 1 is needed to accommodate a low risk-free rate in the data. The RRA estimates based on other observed economic decisions are, however, around 2. In addition, it is difficult to accept a time discount factor larger than 1, given the evidence against it from other fields of economic research. These tensions give rise to the twin puzzles of equity premium and risk-free rate. After accounting for the Roll (1977) critique and weak identification, the present paper has demonstrated that in the EZW model, not only the historical equity premium is consistent with low RRA values around 2, and the low risk-free rate in the data does not require the time discount factor to be larger than 1, but also the stock return volatility is consistent with these parameter values. Therefore, disentangling EIS from RRA seems sufficient to resolve the two puzzles, contrary to earlier
findings. In addition, the correctly identified EIS is much closer to 1 than previous estimates based on aggregate data.

Two ideas suggest themselves for future empirical research. Given that the relevant values of EIS in the $S$ sets in this paper are often around 1, it should be useful to formally test the hypothesis of unitary EIS. It should also be interesting to study if the EZW model featuring the return on the optimal portfolio estimated in this paper can explain other puzzles in asset pricing and macroeconomics.

LITERATURE CITED


