Technical Appendix to Supranational Agency: A Solution for Conflict in International Mergers?

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This note includes the general model and technical proofs of the paper titled “Supranational Agency: A Solution for Conflicts in International Mergers”. This paper develops a two Country three Firm model to compare the possibilities of conflict between competition authorities of different countries over the review of a merger under different welfare standards: national, consumer, and global. Use of transfer payment under the global welfare standard to ensure a conflict free efficient outcome is also explored.

General Model

Consider two identical countries $j = A, B$ with $N_A$ and $N_B$ number of firms, respectively. A Firm $i$ belonging to Country $j$ sells an amount $h_{ij}$ in the home country and exports an amount $e_{ij}$ to the other country. Then the demand function for Firm $i$ in Country $j$ is given by

$$P = a - bQ_j$$

where $Q_j = (\sum_{i=1}^{N_i} h_{ij} + \sum_{i=1}^{N_k} e_{ik})$, $j, k = A, B, j \neq k$

The profit of Firm $i$ belonging to Country $j$, is given by

$$\pi_{ij} = (P_j - c)h_{ij} + (P_k - c)e_{ij}$$

where $P_j$ is the price that the firm charges in its home country. $P_k$ is the price that the firm charges on its exportable to Country $k$.

National Welfare of Country $j$ is given by

$$W_j = \frac{1}{2} Q_j^2 + \sum_{i=1}^{N_j} \pi_{ij}$$
where Consumer Surplus in Country $j$ is

$$CS_j = \frac{1}{2} Q_j^2$$

and $Q_j$ is the total quantity demanded in Country $j$.

World/Global welfare is given by

$$W_w = \frac{1}{2} Q_w^2 + \sum_j \sum_{i=1}^{N_j} \pi_{ij}$$

where $w = A + B$ i.e. world in this model consists of two countries $A$ and $B$. $Q_w$ is the total quantity demanded in both Countries $A$ and $B$ together.

Assume that the firms choose their quantities simultaneously. Then the optimal quantities under Cournot competition is given by

$$e_{ij}^* = h_{ij}^* = \frac{a - c}{N + 1} \forall i, j \text{ where } N = N_A + N_B$$

Suppose now two firms want to merge. The incentive to merge arises from cost savings in terms of a decrease in post-merger marginal cost for the merging firms, which is the efficiency argument Assume that the marginal cost in the competitive fringe remains unchanged after the merger between two firms.

In this model, merger has two effects. Firstly, it has an anticompetitive effect since the number of firms in the industry is reduced after merger. Secondly, the merger leads to an increase in efficiency through cost savings as discussed above.
There are two possible cases in which two firms in this model can merge:

**Case (i) Firms belonging to the same country merge**

In particular, suppose two firms in Country \( A \) merge to form Firm \( M \). Then the demand function can be written as follows.

\[
P_A = a - b \left( \sum_{i=1}^{N_A-2} h_{iA} + h_M + \sum_{i=1}^{N_B} e_{iB} \right)
\]

\[
P_B = a - b \left( \sum_{i=1}^{N_B} h_{iB} + \sum_{i=1}^{N_A-2} e_{iA} + e_M \right)
\]

where \( h_m, e_m \) represent the production for the domestic market and export share of the merged firm’s output respectively.

The post-merger marginal cost of the merged firm is given by \( c_M < c \). The profit of the merged firm is given as

\[
\pi^M = (P_A - c_M)h_M + (P_A - c_M)e_M
\]

The profit of a representative non-merged Firm \( i \) in Country \( j \) after merger of the two firms is given by

\[
\pi^M_{ij} = (P_j - c)h_{ij} + (P_k - c)e_{ij}, \ i \neq M
\]
The welfare of Country $A$ and $B$ will then be

$$W_A^M = \frac{1}{2} Q_A^{M^2} + \sum_{i=1}^{N_A-2} \pi_{iA}^M + \pi^M$$

$$W_B^M = \frac{1}{2} Q_B^{M^2} + \sum_{i=1}^{N_B} \pi_{iB}^M$$

The Consumer Surplus being

$$CS^M = \frac{1}{2} Q_f^{M^2}$$

The World/Global welfare is

$$W_w^M = \frac{1}{2} Q_w^{M^2} + \sum_j \sum_{i=1}^{N_j-1} \pi_{ij}^M + \pi^M, j \neq M$$

Case (ii) Firms belonging to two different countries merge

In this case, the demand function for each country would be

$$P_A = a - b \left( \sum_{i=1}^{N_A-1} h_{iA} + h_{MA} + \sum_{i=1}^{N_B-1} e_{iB} \right)$$

$$P_B = a - b \left( \sum_{i=1}^{N_B-1} h_{iB} + h_{MB} + \sum_{i=1}^{N_A-1} e_{iA} \right)$$

where the total output of the merged firm is

$$h_{MA} + h_{MB} = h_M$$
Here it is assumed that when firms from two different countries merge, the merged firm operates as a domestic firm in both countries, implying \(e_M = 0\)\(^1\).

The profit of the merged firm is given as

\[
\pi^M = (P_A - c_M) h_{MA} + (P_A - c_M) h_{MB}
\]  

(18)

The profit of the non-merged firms and consumer surplus remain same as Case (i).

The welfare of Country \(j\) will then be

\[
W_j^M = \frac{1}{2} Q_j^M + \sum_{i=1}^{N_j-1} \pi_{ij}^M + \frac{1}{2} \pi^M
\]

(19)

The World/Global welfare is

\[
W_w^M = \frac{1}{2} Q_w^M + \sum_j \sum_{i=1}^{N_j-1} \pi_{ij}^M + \pi^M, \ j \neq M
\]

(20)

Again assuming that the merged and rival firms choose their quantities simultaneously after firms merge, the optimal quantities under Cournot competition can be obtained as follows.

\[
e'_{ij}^M = h'_{ij} = \frac{a - 2c + c_M}{N}, \ \forall \ i, j, i \neq M
\]

(21)

And the output of the merged firm is given as

Under Case (i), \(h_M = e_M = \frac{a - 2c_M + c}{N}\)

(22)

\(^1\) This assumption is made for the ease of calculation. However, relaxing this assumption would not affect our results since we assume the countries to be identical and the firms to be symmetric.
Under Case (ii), \( h_{MA} = h_{MB} = \frac{a - 2c + c}{N} \) \hspace{1cm} (23)

Substituting these values in the expression for profit, welfare and consumer surplus for the pre and post-merger cases, the corresponding optimized values are obtained. Next, define a variable

\[ X^* = X^M - X \]

where \( X = \pi, CS, W \) \hspace{1cm} (24)

At \( X^* = 0 \), the value of the post-merger marginal cost \( (c_M) \) corresponds to the point of indifference between the pre and post-merger.

For a three firm case, i.e. assuming \( N = 3 \)

Demand Function \( P = a - bQ \), where \( Q = q_1 + q_2 + q_3 \).

Before Merger

Profit of each firm

\[ \pi_1 = \pi_2 = \pi_3 = \frac{(a - c)^2}{16} \]

Consumer Surplus

\[ CS_A = CS_B = \frac{9(a - c)^2}{64} \]

Case 4.11: Firm 1 and 2 belong to Country A and Firm 3 belong to Country B

Pre-merger welfare in Country A

\[ W_A = \pi_1 + \pi_2 + CS_A = \frac{2(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{17(a - c)^2}{64} \]
Post-merger welfare of Country A

\[ W_A^M = \pi^M + CS_A^M \]

\[ = \frac{(a - 2c_M + c)^2}{9} + \frac{(2a - c_M - c)^2}{36} \]

\[ = \frac{4(a - 2c_M + c)^2 + (2a - 2c_M - c)^2}{36} \]

\( c_M \) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger welfare level of Country A, i.e. \( c_M \) is calculated at \( W_A^M = W_A \).

\[ \frac{4(a - 2c_M + c)^2 + (2a - 2c_M - c)^2}{36} = \frac{17(a - c)^2}{64} \]

\[ \Rightarrow 272(c - c_M)^2 + 320(a - c)(c - c_M) - 25(a - c)^2 = 0 \]

\[ \Rightarrow (c - c_M) = \frac{-320(a - c) \pm \sqrt{[(320(a - c))^2 + 4 \times 272 \times 25(a - c)^2]}}{272 \times 2} \]

\[ \Rightarrow (c - c_M) = \frac{-320(a - c) \pm (a - c)\sqrt{129600}}{544} \]

\[ \Rightarrow c_M = c - 0.07(a - c) \]

\[ \Rightarrow c_M = 1.07c - 0.07a \]

\[ \Rightarrow c_M - c = -0.7(a - c) < 0, \text{ since } a > c \]

\[ \Rightarrow c_M < c \]
And for $c_M > 0$ we require $c > 0.065a$

The other root is

$$\Rightarrow c_M = 1.25a - 0.25c > a$$

Hence this root is to be ignored.

Pre-merger welfare in Country B $W_B = \pi_3 + CS_B = \frac{(a - c)^2}{16} + \frac{9(a - c)^2}{32} = \frac{11(a - c)^2}{32}$

After merger marginal cost of the merging firm is $c_M < c$.

Profit of merged firm $\pi_M = \frac{(a - 2c_M + c)^2}{9}$

Profit of rival firm $\pi_{3M} = \frac{(a - 2c + c_M)^2}{9}$

Consumer Surplus $CS_A^M = CS_{BA}^M = \frac{(2a - c - c_M)^2}{18}$

Post-merger Welfare of Country B

$W_B^M = \pi_3^M + CS_B^M$

$$= \frac{(a - 2c + c_M)^2}{9} + \frac{(2a - c - c_M)^2}{18}$$

$$= \frac{2(a - 2c + c_M)^2 + (2a - c - c_M)^2}{18}$$
\[
\frac{2[(a - c) - (c - c_M)]^2 + [2(a - c) + (c - c_M)]^2}{18}
\]
\[
= \frac{6(a - c)^2 + 3(c - c_M)^2}{18}
\]
\[
= \frac{11(a - c)^2}{32}
\]

\[6(a - c)^2 + 3(c - c_M)^2 = \frac{11(a - c)^2}{32}\]
\[\Rightarrow 3(a - c)^2 = 48(c - c_M)^2\]
\[\Rightarrow 16(c - c_M)^2 = (a - c)^2\]
\[\Rightarrow c_M = c + \frac{(a - c)}{4}\]

Taking positive sign,
\[\Rightarrow c_M(1) = 1.55c - 0.55a\]

and

Taking negative sign,
\[\Rightarrow c_M(2) = 1.25c - 0.25a\]

In this case, we use both the threshold values of Country B.
Case 4.1.2: All firms belong to Country A

Pre-merger welfare of country A

\[
\pi_1 + \pi_2 + \pi_3 + CS_A = \frac{3(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{21(a - c)^2}{64}
\]

Post-merger welfare of country A

\[
\pi^M + \pi^M_3 + CS^M_A = \frac{(a - 2c_M + c)^2}{9} + \frac{(a - 2c + c_M)^2}{9} + \frac{(2a - c - c_M)^2}{36}
\]

\[
= \frac{12(a - c)^2 + 21(c - c_M)^2 + 12(c - c_M)(a - c)}{36}
\]

\(c_M\) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger welfare level of Country A, i.e. \(c_M\) is calculated at \(W^M_A = W_A\).

\[
\Rightarrow c - c_M = \frac{-192(a - c) \pm \sqrt{(192(a - c))^2 + 4 \times 336 \times 3(a - c)^2}}{336 \times 2}
\]

\[
\Rightarrow c - c_M = \frac{-192(a - c) \pm \sqrt{36864(a - c)^2 - 4032(a - c)^2}}{336 \times 2}
\]

\[
\Rightarrow c_M = c \mp \frac{10.804(a - c)}{2 \times 336}
\]

\[
\Rightarrow c_M = c \mp 0.016(a - c)
\]

\[
\Rightarrow c_M = 0.984c + 0.016a \text{ (Taking positive sign)}
\]
\[ \Rightarrow c_M - c = -0.05(a - c) \]

For \( c_M > 0, c > \frac{0.05}{1.05} a > \frac{a}{21} \)

The other root \( c_M = 0.44c + 0.56a \) is ignored as it is greater than \( c \).

Pre-merger welfare level of Country B \( CS_B = \frac{9(a - c)^2}{64} \)

Post-merger welfare level of Country B \( CS_B^M = \frac{(2a - c - c_M)^2}{36} \)

\( c_M \) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger welfare level of Country B, i.e. \( c_M \) is calculated at \( W_B^M = W_B \).

\[
\frac{(2a - c - c_M)^2}{36} = \frac{9(a - c)^2}{64}
\]

\[ \Rightarrow \frac{2a - c - c_M}{6} = \pm \frac{3(a - c)}{4} \]

\[ \Rightarrow 8a - 4c_M - 4c = 9(a - c) \text{ (Taking positive sign)} \]

\[ \Rightarrow c_M = \frac{5c - a}{4} = 1.25c - 0.25a \]

**Case 4.1.3: Firm 1 belongs to Country A, Firm 2 and 3 belong to Country B**

Pre-merger welfare level of Country A

\[ W_A = \pi_1 + CS_A = \frac{(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{13(a - c)^2}{64} \]
Post-merger welfare of Country A

\[ W_A^M = \frac{1}{2} \left( a - 2c_M + c \right)^2 + \frac{(2a - c - c_M)^2}{36} \]

\[ \Rightarrow W_A^M = \frac{6(a - c)^2 + 4(a - c)(c - c_M) + (c - c_M)^2}{36} \]

\( c_M \) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger welfare level of Country A, i.e. \( c_M \) is calculated at \( W_A^M = W_A \).

\[ 144(a - c)^2 + 192(a - c)(c - c_M) - 21(c - c_M)^2 = 0 \]

\[ \Rightarrow c_M = 1.1c - 0.1a \text{ (Taking the positive sign)} \]

Taking negative sign, we get the other root as

\[ c_M = 1.43a - 0.43c \geq a \geq c \]

Hence it is ignored.

Pre-merger welfare level of Country B:

\[ W_B = \pi_3 + CS_B = \frac{2(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{17(a - c)^2}{64} \]

Post-merger welfare of Country B:

\[ W_B^M = \frac{1}{2} \left( a - 2c_M + c \right)^2 + \frac{(a - 2c + c_M)^2}{9} + \frac{(2a - c - c_M)^2}{18} \]
is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger welfare level of Country B i.e. $c_M$ is calculated at $W_B^M = W_B$.

$$\frac{7(a - c)^2 + 4(a - c)(c - c_M) + 7(c - c_M)^2}{18} = \frac{13(a - c)^2}{32}$$

$$\Rightarrow 7(a - c)^2 + 208(c - c_M)^2 + 32(a - c)(c - c_M) = 0$$

This equation does not have real roots. Hence, for Country B, we take the threshold level to be equal to the threshold level of the merging firms, derived below.

**Threshold level of post-merger marginal cost for Merging Firms**

Pre-merger profit of the merging firms

$$\pi_1 + \pi_2 = \frac{2(a - c)^2}{16} = \frac{(a - c)^2}{8}$$

Post-merger profit of the merging firms

$$\pi^M = \frac{(a - 2c_M + c)^2}{9}$$

i.e. $c_M$ is calculated at $\pi^M = \pi_1 + \pi_2$

$$\Rightarrow \frac{(a - 2c_M + c)^2}{9} = \frac{(a - c)^2}{8}$$

$$\Rightarrow \frac{32(a - c)^2 + 32(a - c)(c - c_M) - (c - c_M)^2}{72} = 0$$
\[ c - c_M = \frac{-32(a - c) \pm \sqrt{\left[32(a - c)\right]^2 + 4 \times 32 \times 7(a - c)^2}}{72 \times 2} \]

\[ \Rightarrow c - c_M = \frac{1.03c - 0.03a}{1.03} \] (Taking positive sign)

The other root is

\[ c_M = 0.92a + 0.08c > c \]

Hence it is ignored.

**World/Global welfare Standard**

Pre-merger world/global welfare

\[ \pi_1 + \pi_2 + \pi_3 + CS = \frac{3(a - c)^2}{16} + \frac{9(a - c)^2}{32} = \frac{15(a - c)^2}{32} \]

Post-merger world/global welfare

\[ \pi^M + \pi^M_3 + CS^M = \frac{(a - 2c_M + c)^2}{9} + \frac{(a - 2c + c_M)^2}{9} + \frac{(2a - c - c_M)^2}{18} \]

\[ = \frac{8(a - c)^2 + 11(c - c_M)^2 + 8(c - c_M)(a - c)}{18} \]

\( c_M \) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger world welfare level, i.e. \( c_M \) is calculated at \( W^M_A = W_A \).
The roots are

\[ c - c_M = \frac{-128(a - c) \pm \sqrt{\left[(128(a - c))^2 - 4 \times 176 \times 7(a - c)^2\right]}{176 \times 2} \]

\[ c - c_M = \frac{-128(a - c) \pm \sqrt{\left[(16384(a - c))^2 - 4928(a - c)^2\right]}{176 \times 2} \]

The roots are

\[ c_M = 1.05c - 0.05a \]

and

\[ c_M = 0.22c + 0.78a \]

The second root \( c_M = 0.22c + 0.78a \) is ignored as it is greater than \( c \).

**Consumer Surplus Standard**

Pre-merger welfare level \( CS = \frac{9(a - c)^2}{32} \)

Post-merger welfare level \( CS^M = \frac{(2a - c - c_M)^2}{18} \)

\( c_M \) is the threshold level of post-merger marginal cost at which the authorities are indifferent between the pre and post-merger consumer surplus, i.e. \( c_M \) is calculated at \( CS = CS^M \).

\[ \frac{(2a - c - c_M)^2}{18} = \frac{9(a - c)^2}{32} \]
\[
\Rightarrow \frac{2a - c - c_M}{3} = \pm \frac{3(a - c)}{4}
\]

\[
\Rightarrow 8a - 4c_M - 4c = 9a - 9c \quad \text{(Taking the positive sign)}
\]

\[
\Rightarrow c_M = 1.25c - 0.25a
\]

The second root \(c_M = 4.25a - 3.25c\) is ignored as it is greater than \(a\). The values are given in the Table 1 in the paper.

Next we allow for the jurisdictions to review a merger using the National Welfare standard, defined as the sum of the profit of the domestic firms and the consumer surplus and analyze the possibilities of conflict. The result has been summarized in terms of Proposition 1.

**Proposition 1**

When consumer preferences and distributions are identical across both countries,

a. Conflict between national antitrust authorities is less when the rival firm and the two merging firms belong to the same country as compared to when the rival firm belongs to a different country.

b. Conflict between national authorities is less when the two merging firms lie in different countries as compared to when they lie in the same country.

**Proof of Proposition 1**

In Figure 1, the critical level of post-merger marginal cost for Country B is given by

\[
c_M^B(1) = 1.55c - 0.55a
\]

\[
c_M^B(2) = 1.25c - 0.25a
\]
For Case 4.1.1 i.e. Both the merging firms belong to Country A and the rival firm belongs to Country B \((N_A = 2, N_B = 1)\), the critical level of post-merger marginal cost for Country A is given by

\[ c_M^A = 1.07c - 0.07a \]

Subtracting, each of the critical levels of Country B from that of Country A, we get

\[ c_M^A - c_M^B(1) = 0.48(a - c) \]

\[ c_M^A - c_M^B(12) = 0.18(a - c) \]

Comparing the above two differences we get, Zone of conflict for Case 4.1.1 as

\[ 0.48(a - c) - 0.18(a - c) = 0.3(a - c) \]

For 4.1.2 i.e. All firms belong to country A \((N_A = 3, N_B = 0)\), Country A’s and B’s critical levels of post-merger marginal cost are given as

\[ c_M^A = 0.984c + 0.016a \]

\[ c_M^B = c_M^C = 1.25c - 0.25a \]

This is given in Figure 2. Subtracting the critical level of Country B from that of Country A for Case 4.1.2, we get the zone of conflict as

\[ c_M^A - c_M^C = 0.266(a - c) > 0 \]

Comparing zones of conflict in Case 4.1.1 and 4.1.2, we get
This proves Part a of Proposition 1.

Next compare the difference between the post-merger marginal cost of the merged firm in Country A and B for Case 4.1.2 and 4.1.3. For Case 4.1.3 i.e. Firm 1 belongs to Country A, Firm 2 and 3 belongs to Country B \((N_A = 2, N_B = 2)\), there is no real root for the equation representing the welfare difference curve of Country B i.e. the zone of conflict between Country A and B is undefined. For comparing Case 4.1.3 with Case 4.1.1 and 4.1.2, we take the effective zone of conflict for Case 4.1.3 which is the difference between the profit difference curve of the merging firms and the welfare difference curve. This is given as follows.

\[
c_M^B = c_M^M = 1.03c - 0.03a
\]

\[
c_M^A = 1.1c - 0.1a
\]

This is given in Figure 3. Subtracting critical value for Country A from that of Country B, we get the zone of conflict in Case 4.1.3 as \(c_M^A - c_M^B = 0.07(a - c)\).

Comparing the zone of conflict in Case 4.1.2 and 4.1.3, we get

\[
0.266(a - c) > 0.07(a - c)
\]

This implies that the zone of conflict is smaller in Case 4.1.3 as compared to Case 4.1.2.

Using the result of the first part of the proposition, we can say

\[
4.1.1 > 4.1.2 > 4.1.3
\]
Next we assume that jurisdictions adopt the World/Global welfare standard to review the mergers. The results are summarized in Proposition 2.

**Proposition 2**

World/Global welfare standard resolves conflicts between countries over merger but does not always yield an efficient outcome.

**Proof of Proposition 2**

The World Global welfare standard is the same for both countries, for all distributions. Hence the first part of the proposition is trivially true.

For Case 4.1.1, the post-merger marginal costs of the merging firm corresponding to the indifference point of Country B and A are respectively given as

\[ c^B_M(1) = 1.55c - 0.55a \]

\[ c^B_M(2) = 1.25c - 0.25a \]

\[ c^A_M = 1.07c - 0.07a \]

The critical level corresponding to the global indifference curve is given as

\[ c^W_M = 1.05c - 0.05a \]

Comparing

\[ c^B_M(1) < c^A_M < c^W_M \]
At $c_M^B(2)$, in Figure 1, the slope of $W_B^*$ is positive and hence for any $c_M$ greater than the threshold given by $c_M^B(2)$, Country B would approve the merger. Hence a movement from national welfare standard to global welfare standard would mean that Country B would now approve less number of mergers, while Country A would approve some mergers which are welfare reducing from national welfare perspective. Thus, the outcome is efficient for Country B but inefficient for Country A.

For Case 4.1.2, the post-merger marginal costs of the merging firm corresponding to the indifference points of Country A and B are given by

$$c_M^A = 0.984c + 0.016a$$

$$c_M^B = c_M^C = 1.25c - 0.25a,$$

respectively.

Comparing these two critical levels with global welfare indifference point, we get

$$c_M^C < c_M^W < c_M^A.$$

In this case, following the same logic as in Case 4.1.1, the global welfare standard yields efficient outcome for Country A and inefficient outcome for Country B. Refer Figure 2.

For Case 4.1.3, the post-merger marginal costs of the merging firm corresponding to the indifference point of Country A and effective indifference curve of Country B are given respectively by

$$c_M^A = 1.1c - 0.1a$$

$$c_M^B = 1.03c - 0.03a$$

Comparing these with the critical level corresponding to the global indifference level, we get

$$c_M^A < c_M^W < c_M^B.$$
Here the global welfare standard leads to an inefficient outcome for Country A and efficient outcome for Country B. Refer Figure 2.

Next it is assumed that jurisdictions adopt a consumer welfare standard. The results are summarized in terms of Proposition 3.

**Proposition 3**

When consumers are homogeneous across countries, adopting a consumer surplus standard resolves any conflict between countries. The outcome is efficient, in general but may turn out to be inefficient when the two merging firms and the rival firm belong to two different countries.

**Proof of Proposition 3**

The consumer surplus standard is the same for both countries for all distributions, since both countries are assumed to be identical. Hence the first part of the proposition is trivially true.

The level of post-merger marginal cost of the merging firm at which the pre and post-merger consumer surplus are equal is given as

\[ c_M^C = 1.25c - 0.25a \]

For Case 4.1.1 (refer Figure 1), the post-merger marginal costs of the merging firm corresponding to the indifference point of Country B and A are given by the following equations respectively.

\[ c_M^B(1) = 1.55c - 0.55a \]

\[ c_M^B(2) = 1.25c - 0.25a \]
$c^A_M = 1.07c - 0.07a$

Comparing these with the critical under consumer surplus standard, we get

$c^B_M(2) = c^C_M > c^B_M(1)$ and $c^A_M > c^C_M$.

The outcome is efficient. However, if $c^B_M(1) < c^C_M < c^B_M(2)$, Country B would face a loss in welfare if authorities in Country B approve mergers adopting the consumer welfare standard. Thus shifting to the consumer welfare standard is efficient in general except for the above mentioned region.

For Case 4.1.2 (refer Figure 2), the post-merger marginal costs of the merging firm corresponding to the indifference point of Country B coincides with the consumer surplus standard and that of Country A lies to the right of the consumer surplus difference curve as shown in Figure 2. The critical levels for Country A and B are respectively given by

$c^A_M = 0.984c + 0.016a$

$c^B_M = c^C_M = 1.25c - 0.25a$

Following the Case 4.1.1 we find that when Country A and B shift to the consumer surplus standard, the outcome turns out to be efficient.

For Case 4.1.3 (refer Figure 3), the post-merger marginal costs of the merging firm corresponding to the indifference point of Country A and B are given respectively by

$c^A_M = 1.1c - 0.1a$

$c^B_M = 1.03c - 0.03a$
Comparing these critical levels with the critical level corresponding to the point of indifference under the consumer surplus standard, we get

$$c^c_M < c^A_M < c^M_M$$

In this case the consumer surplus standard yields an efficient outcome, following the same logic as Case 4.1.1 and 4.1.2. This proves Proposition 3.

Next, we allow for the possibility of transfer payment from the parties who stand to gain from merger to parties who lose from merger in a Nash Bargaining framework to reach an efficient outcome under the global welfare standard.

The global welfare attained after a merger is denoted by $\Pi$.

$$\Pi = \frac{(a - 2c + c)^2}{9} + \frac{(a - 2c + c_M)^2}{9} + \frac{(2a - c - c_M)^2}{18}$$

$$= \frac{8(a - c)^2 + 11(c - c_M)^2 + 8(a - c)(c - c_M)}{18}$$

The disagreement points of the two jurisdictions are their pre-merger welfare levels. For Case 4.1.1, these are given by

$$d_A = \frac{2(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{17(a - c)^2}{64}$$

$$d_B = \frac{(a - c)^2}{16} + \frac{9(a - c)^2}{64} = \frac{13(a - c)^2}{64}$$

Following the Nash bargaining solution, the post-bargaining shares $S_i^{Wy}$ of each of the jurisdictions is given by
At
Hence, both the countries get their pre-merger welfare levels.

At any 0 < \( c_M \) < \( c_M^{w} \), \( [\Pi - d_A - d_B] > 0 \)

Therefore, \( S_A^{w} > d_A \) and \( S_B^{w} > d_B \)

The amount of transfer in general is \( S_i^{w} - W_i^M, i = A, B \)

We show the Nash Bargaining Solution calculation for Case 4.1.1.

Taking \( c_M = 1.06c - 0.06a \), we get,

\[
\Pi = \frac{(8 + 11 \times 0.0036 + 8 \times 0.06)(a - c)^2}{18} = \frac{8.5196(a - c)^2}{18}
\]

The disagreement points are given by

\[
d_A = \frac{17(a - c)^2}{64}, \quad d_B = \frac{13(a - c)^2}{64}
\]

We calculate

\[
[\Pi - d_A - d_B] = 0.4733 - 0.265625 - 0.23125 > 0
\]

Following the Nash bargaining solution, the post-bargaining shares of each of the jurisdictions is given by
Therefore, at $c_M = 1.06c - 0.06a$, $S_A^W > d_A$ and $S_B^W > d_B$

The amount of transfer is $t = W_B^W - S_B^M = 0.22939 - 0.2054 = 0.02399$

Thus, transfer payment is possible under Nash bargaining solution.

**Figure 1: Possibilities of Conflict when merging firms belong to the same country**

Case 4.1.1: Firm 1 and 2 belong to Country A, Firm 3 belongs to Country B

E refers to the threshold level at which Country B is indifferent between pre and post-merger level of domestic welfare of Country B
F refers to the threshold level at which Country B is indifferent between pre and post-merger level of domestic welfare of Country B and it also corresponds to the threshold level at which the countries would be indifferent between pre and post-merger levels of consumer surplus.

G refers to the threshold level at which Country A is indifferent between pre and post-merger level of domestic welfare of Country A.

H refers to the threshold level at which Country A and B are indifferent between pre and post-merger level of world welfare.
Region to the left of E and between F and G = No Conflict Zone.
Region between E and F and to the right of G = Zone of Conflict.

Figure 1 corresponds to data from Table 2.
Figure 2: Possibilities of merger when all firms belong to Country A

Case 4.1.2: All Firms belong to Country A

I corresponds to the threshold level at which Country B is indifferent between the pre and post-merger level of domestic welfare of Country B and also the threshold level at which Country A and B are indifferent between pre and post-merger level of consumer surplus

J corresponds to the threshold level at which Country A and B are indifferent between pre and post-merger level of world welfare level

K corresponds to the threshold level at which Country A is indifferent between the pre and post-merger level of domestic welfare of Country A

Region to the left of I: No Conflict Zone
Region between I and K: Conflict Zone

Figure 2 corresponds to data from Table 3

\[ W_A^*, W_B^*, W_W^*, CS^*, \pi_M^* \]

Figure 3: Possibilities of merger when merging firms belong to two different countries

Case 4.1.3: Firm 1 belongs to Country A and Firm 2 and 3 belongs to Country B

M corresponds to the threshold level at which Country A and B are indifferent between pre and post-merger levels of consumer surplus

N corresponds to the threshold level at which Country A is indifferent between pre and post-merger levels of domestic welfare of Country A
O corresponds to the threshold level at which Country A and B are indifferent between pre and post-merger levels of world welfare.

P corresponds to the threshold level at which Country B is indifferent between pre and post-merger levels of domestic welfare of Country B.

Region to the left of N: No Conflict Zone

Region between N and P: Conflict Zone

Figure 3 corresponds to the data in Table 4

Table 1: Critical Level of post-merger marginal cost of merged firms

<table>
<thead>
<tr>
<th>X *</th>
<th>( c_M^X ) (post-merger marginal cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.1.1</td>
</tr>
<tr>
<td>Country A welfare</td>
<td>1.07c – 0.07a</td>
</tr>
<tr>
<td>Country B Welfare</td>
<td>1.55c – 0.55a</td>
</tr>
<tr>
<td></td>
<td>1.25c – 0.25a</td>
</tr>
<tr>
<td>Global Welfare</td>
<td>1.05c – 0.05a</td>
</tr>
<tr>
<td>Consumer</td>
<td>1.25c – 0.25a</td>
</tr>
<tr>
<td>Surplus</td>
<td></td>
</tr>
</tbody>
</table>

* Threshold corresponding to the effective welfare difference curve
Table 2: Possibilities of Conflict when both the merging firms belong to Country A and the rival firm belongs to Country B.

<table>
<thead>
<tr>
<th>Region</th>
<th>National Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left of E</td>
</tr>
<tr>
<td></td>
<td>Between E and F</td>
</tr>
<tr>
<td></td>
<td>Between F and G</td>
</tr>
<tr>
<td></td>
<td>Between G and H</td>
</tr>
<tr>
<td>Country A</td>
<td>Allows merger</td>
</tr>
<tr>
<td></td>
<td>Allows</td>
</tr>
<tr>
<td></td>
<td>Allows</td>
</tr>
<tr>
<td></td>
<td>Does not Allow</td>
</tr>
<tr>
<td>Country B</td>
<td>Allows merger</td>
</tr>
<tr>
<td></td>
<td>Does not Allow</td>
</tr>
<tr>
<td></td>
<td>Allows</td>
</tr>
<tr>
<td></td>
<td>Allows</td>
</tr>
<tr>
<td>Points of conflict</td>
<td>No Conflict</td>
</tr>
<tr>
<td></td>
<td>Conflict</td>
</tr>
<tr>
<td></td>
<td>No Conflict</td>
</tr>
<tr>
<td></td>
<td>Conflict</td>
</tr>
<tr>
<td>Efficiency of mergers</td>
<td>Efficient</td>
</tr>
<tr>
<td></td>
<td>Inefficient</td>
</tr>
<tr>
<td></td>
<td>Efficient by global standard/ Inefficient by Consumer surplus standard</td>
</tr>
<tr>
<td></td>
<td>Inefficient by global standard/ Efficient by Consumer surplus standard</td>
</tr>
</tbody>
</table>

Table 3: Possibilities of Conflict when all firms belong to Country A

<table>
<thead>
<tr>
<th>Region</th>
<th>Left of I</th>
<th>Between I and J</th>
<th>Between J and K</th>
<th>To the Right of K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country A</td>
<td>Allows merger</td>
<td>Allows merger</td>
<td>Allows</td>
<td>No Merger Proposed</td>
</tr>
<tr>
<td>Country B</td>
<td>Allows merger</td>
<td>Does Not Allow</td>
<td>Does Not Allow</td>
<td>-</td>
</tr>
<tr>
<td>Points of conflict</td>
<td>No Conflict</td>
<td>Conflict</td>
<td>Conflict</td>
<td>-</td>
</tr>
<tr>
<td>Efficiency of mergers</td>
<td>Efficient</td>
<td>Efficient from welfare standard/inefficient from consumer standard</td>
<td>Not Efficient by Global and Consumer Welfare Standard</td>
<td>Efficient</td>
</tr>
</tbody>
</table>
Table 4: Firm 1 belongs to Country A, Firm 2 and 3 belong to Country B

<table>
<thead>
<tr>
<th>Region</th>
<th>Left of M</th>
<th>Between M and N</th>
<th>Between N and O</th>
<th>Between O and P</th>
<th>Right of P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country A</td>
<td>Allows</td>
<td>Allows</td>
<td>Does not Allow</td>
<td>Does not Allow</td>
<td>No Merger Proposed</td>
</tr>
<tr>
<td>Country B</td>
<td>Allows</td>
<td>Allows</td>
<td>Allows</td>
<td>Allows</td>
<td></td>
</tr>
<tr>
<td>Points of conflict</td>
<td>No Conflict</td>
<td>No Conflict</td>
<td>Conflict</td>
<td>Conflict</td>
<td>-</td>
</tr>
<tr>
<td>Efficiency of mergers</td>
<td>Efficient</td>
<td>Inefficient by Consumer Standard/ Efficient by Welfare Standard</td>
<td>Inefficient in world welfare terms/ Efficient by Consumer Standard</td>
<td>Efficient by both Global and Consumer Standard</td>
<td>Efficient</td>
</tr>
</tbody>
</table>