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Insecure property rights and the missing middle

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Abstract

We analyze a theoretical model in which entrepreneurs' property rights to their firms are threatened by "raiders" who can challenge them to a contest for control of their firms. Entrepreneurs are heterogeneous with respect to their productivity, and decide how much capital to invest before raiders decide whom to attack. In equilibrium, low productivity entrepreneurs are unaffected by the existence of raiders, while mid- and high-productivity entrepreneurs suffer. However, while raiders essentially act like a tax for the highest productivity entrepreneurs, the investment behavior of mid-productivity entrepreneurs who try to avoid an attack is more drastically affected. Our model provides a novel theoretical explanation for the "missing middle" observed in many countries with insecure property rights.

Keywords: Property rights, rent-seeking, corruption, missing middle

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1 Introduction

In many countries, insecure property rights are a pervasive problem that can limit economic development. Specifically, insecure property rights may provide a severe disincentive for entrepreneurs to grow and develop their firms if they fear that doing so will attract the attention of other agents who can attempt to appropriate it.

Russia is an example of a country in which property rights insecurity is endemic. A well-known case is Mikhail Khodorkovsky, who, after the dissolution of the Soviet Union, built a successful business empire around the oil producing firm Yukos and was believed to be the richest man in Russia. In 2003, he was arrested and subsequently convicted of likely trumped up charges of tax evasion, thrown into prison and stripped of most of his assets, in particular, his stake in Yukos.

While Khodorkovsky's case is the most notorious, the general problem extends to the Russian economy at-large. The Russian term "reiderstvo" (a loanword derived from "raider") refers to illegal tactics from bribery, forgery and corruption to intimidation and violence employed to steal companies from their owners. Shelley and Deane (2016) describe the methods and scope of reiderstvo in Russia as follows:

> In the 21st century, the originators of raids have increasingly been government officials and businessmen [...] Reliable statistics can be hard to come by in Russia, but it is clear that raids are a big, underreported problem that is increasing rapidly [...] One known tactic of corporate raiders is to have business owners arrested on fabricated economic crime charges in order to take control of the company while the owners are tangled in court proceedings. Of the 200,000 economic crime cases [that investigative authorities had opened in 2014], only 15% resulted in a conviction, but a full 83% of businessmen still ended up losing control of their businesses.

A raider interviewed by Harding (2008) claimed that the profits from raiding were enormous: "It costs around \$120,000-\$170,000 to bankrupt an average company. But you can then make \$3-4 million profit." As a consequence, the raiding industry is flourishing. "According to the most conservative estimates, about 10000 raider attacks occur annually. [...] Only about 100 raiders are convicted each year, so the risk [for raiders] is minimal."[1]

While the Russian example of property right insecurity may be relatively extreme, similar issues certainly exist in other post-Soviet societies, and, more generally, many developing countries. Laying low and enjoying a quiet life rather than growing conspicuously may often

[1] See Center for Political Technologies (2008).

be a healthier strategy for entrepreneurs in environments with insecure property rights. We argue that this incentive structure is potentially very detrimental for economic development. In our model, entrepreneurs are heterogeneously productive and choose how much capital to invest in their firm. After the investment has occurred, criminal and/or politically connected agents – whom we call “raiders” – get to observe the firms’ potential profit and decide whether to challenge an entrepreneur for control of his firm. If the challenge is successful, the attacking raider takes over the entrepreneur’s firm and its profits go to the raider.

In order to initiate a challenge, raiders have to pay a fixed cost, which may be random from the point of view of entrepreneurs. If a challenge is initiated, the fight for control is modeled as a (possibly asymmetric) Tullock contest in which both the defending entrepreneur and the attacking raider spend resources in order to increase their respective winning probability.

In equilibrium, the lowest productivity entrepreneurs are unaffected by the existence of raiders because they do not provide a sufficiently attractive target. In contrast, mid- and high-productivity entrepreneurs suffer, but their reaction to the raider threat is quite different. Very high productivity entrepreneurs have to accept that they will most likely be challenged, and treat the existence of raiders essentially like a distortionary tax. Their optimal capital level is therefore somewhat reduced because some of the rewards may go to successful raiders while they do not get to offset their investment cost, but, fundamentally, the existence of raiders does not change their behavior all that much.

In contrast, mid productivity entrepreneurs also consider the option that, if they choose to stay small, their firms are less appealing targets for raiders, which reduces the probability that the firm is attacked. For many mid-productivity entrepreneurs, insecure property rights act as a particularly severe form of taxation, with marginal tax rates reaching more than 100 percent, in the sense that an entrepreneur would be worse off if his firm’s equilibrium profit exogenously increased by a dollar.

Furthermore, if a mid-productivity entrepreneur’s ability increases, their capital investment goes down. In contrast, among both low and high productivity entrepreneurs, productivity and capital investment are complements. The perverse effect for mid-productivity entrepreneurs stems from their equilibrium behavior of choosing capital investments that ensure that their firm does not attract an attack.

The fact that property rights and economic performance are related has been widely discussed in the literature; see Besley and Ghatak (2010) for an excellent discussion of the relevant literature. Our model is agnostic on whether the central government itself is the source of uncertainty of property rights, or whether it lacks sufficient enforcement mechanisms to protect entrepreneurs from the risk. Instead, we focus on the potential costs and consequences of this threat, in particular, the under-investment by entrepreneurs resulting in a "missing middle."

Using data from developing countries, the seminal work of Tybout (2000) analyzes the distribution of firm sizes, measured by their employment shares, for several developing countries and shows that they differ significantly from their OECD counterparts. In particular, Tybout (2000) argues that the middle of this distribution is "missing" in developing countries. While this diagnosis is not uncontested (see Hsieh and Olken (2014) and Tybout (2014) for a discussion of this issue), it has spawned a large empirical literature that analyzes the missing middle phenomenon for a large number of countries; see, e.g., Sleuwaegen and Goedhuys (2002); Pages et al. (2009); Pages (2010); Krueger (2013). Our contribution to this literature is that our model provides a positive explanation of how insecure property rights can generate a missing middle in the firm size distribution, and helps us understand why such an effect is indicative of a significant problem for the development of the economy.

Glaeser et al. (2016) emphasize the importance of property rights for development and point out that people in many developing countries do not believe that property rights are effectively protected by institutions such as courts and police. Their formal model focuses on the efficiency of different legal institutuions – injunctive relief versus compensation for property rights infringements – in a setting in which some agents are powerful and can subvert judicial actions.

The avoidance of taxes and regulations, such as the official minimum wage, has been suggested as a powerful incentive for many firms in the developing world to remain small and in the informal sector (Rauch, 1991; Levy, 2008; La Porta and Shleifer, 2014). In these "dual economy" models, entrepreneurs trade off these costs against the benefits of entering the formal economy, such as better access to credit and more efficient technology. Our model focuses on a different channel, namely the threats that arise from insecure property rights, although it is also possible to interpret this channel as related to taxes and regulations: The claim of non-compliance with some regulation or tax law may, in practice, provide the "raiders" with a pretext to attack a firm.

On the theoretical side, our model builds on the literature on contests (Tullock, 1967; Konrad, 2009), but in contrast to most of this literature, our main interest is not so much on the contest stage, but rather on the effect that the possibility of a conflict influences entrepreneurs' actions taken before the actual contest, thus endogenizing the size of the prize.

Substantively, our paper is also related to the literature on extortion and gang activity. Konrad and Skaperdas (1998) model extortion in a multi-stage game with three participants — gangs, shopkeepers and police. In contrast to the entrepreneurs in our model, shopkeepers in Konrad and Skaperdas (1998) cannot affect their risk of being targeted for extortion by the gangs (they only decide whether or not to pay off the gangs), and the main focus is on the interaction between gangs and the police. Choi and Thum (2004) focus on the effects that arise because gangs or corrupt officials may lack the ability to commit (i.e., after receiving a first installment of protection money, they may come back to demand further payment). In this context, they show that lower ability entrepreneurs may choose technologies with lower fixed costs because it provides them with more flexibility to exit the market if the protection money demand is higher than expected.

When considering the welfare loss that arises through contests, most of the contest literature focuses on the direct fighting costs that contestants have in equilibrium (see, e.g., the survey by Nitzan (1994)). While these costs also arise in our model, the central welfare effect in our model is related to the detrimental incentive effect that the presence of raiders has on the entrepreneurs' investment choice.

Zingales (2017) argues that the interaction of concentrated corporate power and politics is a threat to the functioning of the free market economy, and that economic theory should model firms, their behavior and objectives in an explicitly political framework. While Zingales's article focuses on the behavior of powerful "predator" firms that have the political ability to shape the game they are playing, our paper focuses on the "prey." We are particularly interested in the influence of the – often political – risk to entrepreneurs' property rights on their investment behavior.

There is also an important literature that explains why institutions that do not protect property rights effectively may survive or even be actively chosen in order to protect the interests of an oligarchic elite (Guriev and Sonin, 2009; Diermeier et al., 2017). This literature is complementary to our paper in that it explains why institutional problems such as insecure property rights protections may persist, while our contribution is to understand the economic consequences of insecure property rights.

Finally, our paper is more peripherally related to a literature that theoretically analyzes the determinants of the security of property rights in models where individual agents choose whether (and to which extent) to engage in productive, defensive and/or aggressive actions (Garfinkel, 1990; Skaperdas, 1992; Grossman and Kim, 1995; Hafer, 2006; Dal Bó and Dal Bó, 2011). In our model, the agents' identity as entrepreneurs or raiders is exogenously given, and our main focus is how differentially productive entrepreneurs respond to the insecurity of property rights, in a setting where the entrepreneurs' actions affect their likelihood of being in conflict.

The rest of this paper proceeds as follows. Section 2 builds the theoretical setup for the model, Section 3 solves for the equilibrium strategies of entrepreneurs and raiders, and analyzes welfare effects of raiders' presence. Section 5 discusses the main results and concludes.

2 The model

There is a continuum of entrepreneurs whose productivity parameter a is distributed according to density $\phi(\cdot)$, with associated cumulative distribution function $\Phi(\cdot)$.

If an entrepreneur with productivity a invests capital K (whose price we normalize to 1), the firm will later generate a cash flow of $\pi = a\sqrt{K}$, and thus a net profit of $a\sqrt{K} - K$.[2]

An (untaxed) entrepreneur with fully secure property rights to his firm would simply maximize

$$a\sqrt{K} - K \Rightarrow K^*(a) = a^2/4. \tag{1}$$

Unfortunately, the economy is also populated by "raiders" who do not produce anything, but can "challenge" an entrepreneur and, if successful, appropriate his firm (i.e., the cash flow $a\sqrt{K}$ specified above). Specifically, challenging first requires that the raider pays a fixed cost c; this cost is drawn from a cumulative distribution $F(\cdot)$ with associated density $f(\cdot)$ when the opportunity to attack a specific target arises, i.e. before the raider makes a decision whether to attack, but after the entrepreneur's effort choice and the corresponding realization of profit.

If a raider attacks, then the entrepreneur and the raider play a generalized Tullock contest game. That is, if the entrepreneur spends v and the raider spends w, then the probability that the entrepreneur will prevail and keep ownership of his firm is $\frac{\rho v}{\rho v+(1-\rho)w}$, while the raider wins with the complementary probability. Here, ρ is a parameter that measures the effectiveness of defense spending relative to attack spending. If $\rho = 1/2$, then both players' spending is equally effective, while if $\rho < (>)1/2$, the raider (entrepreneur) has an advantage.

Whoever wins the contest gets the firm's cash flow, and, of course, winner and loser both have to pay their respective fighting expenditures.

Discussion. While our model assumes that any raider challenge results in a fight between challenger and entrepreneur, this assumption is not required for our main results. Shelley and Deane (2016) detail that a raid typically begins with a preparation stage, where raiders collect information about the target, followed by a negotiation stage, when raiders intimidate the legal owner and try to achieve a settlement with the original owner. Only if negotiations break down, this is followed by an execution stage, where violence and quasi-legal activities play a more prominent role.

The essential characteristics of the situation modeled are the following: First, raiders face costs before the actual attack in terms of acquiring information and creating a credible

[2] One can think of this as a reduced form approach of a more elaborate model in which both capital and labor (or other short run factors) are used to produce output, but capital is chosen first. Adding a labor choice would complicate notation without adding qualitatively new insights.

threat scenario for the entrepreneur. This is captured by the fixed cost c in our model. Second, a challenge lowers the entrepreneur's (expected) continuation utility, relative to the case that he is not challenged, and also involves an efficiency loss, in the sense that the value of the reallocated assets to the raider is less than their value to the entrepreneur, and/or that fighting is costly at this stage. Our basic model generates these features by assuming that a challenge is followed by a costly fight,[3] but different assumptions would clearly work as well.

3 Analysis

3.1 The contest phase

We start with the analysis of the contest phase. When challenged, the optimization problem of an entrepreneur with cash flow π, is

$$\max_v \frac{\rho v}{\rho v + (1-\rho)w}\pi - v - K, \tag{2}$$

which yields the following first-order condition:

$$\frac{\rho(\rho v + (1-\rho)w) - \rho^2 v}{(\rho v + (1-\rho)w)^2}\pi = 1. \tag{3}$$

Similarly, the raider's problem is

$$\max_w \frac{(1-\rho)w}{\rho v + (1-\rho)w}\pi - w - c \tag{4}$$

which yields first-order condition

$$\frac{(1-\rho)(\rho v + (1-\rho)w) - (1-\rho)^2 w}{(\rho v + (1-\rho)w)^2}\pi = 1. \tag{5}$$

Solving equations (3) and (5) for v and w yields $v = w = \rho(1-\rho)\pi$. Since both contestants spend the same amount of money for fighting, the entrepreneur's winning probability is simply ρ, and substituting $v = w = \rho(1-\rho)\pi$ into the objective functions of the entrepreneur and of the raider, respectively, yields a continuation utility of

$$EU_{\text{attacked}} = \rho\pi - \rho(1-\rho)\pi = \rho^2\pi \tag{6}$$

[3]However, see Section 4.2 where we extend the model such that the firm is less valuable for the raider than the entrepreneur.

for the entrepreneur, and

$$EU_{\text{raider}} = (1-\rho)^2\pi - c \tag{7}$$

for the raider. Because fighting is wasteful, the combined share of cash flow that goes to the raider and the entrepreneur is $[\rho^2 + (1-\rho)^2]\pi = [1 - 2\rho(1-\rho)]\pi < \pi$. This will be part of the welfare loss associated with raiders, but we will see that there are additional negative welfare effects that might well exceed this effect.

A raider will spend the fixed cost for challenging an entrepreneur if and only if (7) is non-negative, hence with probability $F\big((1-\rho)^2\pi\big)$. This probability is decreasing in ρ and increasing in π.

One can think of ρ as being related to the quality of institutions in a country that determine how likely it is that the entrepreneur is victorious in the conflict between him and the raider. In a country with a perfect rule of law, ρ is close to 1, which not only implies that, if an attack were to happen, it would very likely end with a victory of the rightful owner, but also that attacks are, in expectation, very unattractive for raiders, and are therefore unlikely to occur. In contrast, in a country with low ρ, attacks occur more often and are, on average, more likely to be successful.

Unless $\rho = 1$, the probability of attack is weakly increasing in π, so an entrepreneur building a more successful firm increases the risk that he is targeted by a raider, and this exposure effect will potentially significantly diminish the marginal return of capital investment or even make it negative.

3.2 The investment decision

We now turn to the investment stage when the entrepreneur decides how much capital to invest in building his firm, knowing that this affects raiders' attack decision.

The simplest case to analyze arises if all raiders have the same cost of attacking $\bar{c}$; we call this the *point distribution* case. Clearly, there are only two different regimes for entrepreneurs to consider here: First, an entrepreneur can avoid an attack for sure by staying below the profit threshold that makes an attack worthwhile for raiders. By (7), this profit threshold is equal to $\bar{c}/(1-\rho)^2$. Alternatively, if the profit exceeds this threshold, an attack is certain to come.

If the entrepreneur's productivity is such that $a^2/2 < \bar{c}/(1-\rho)^2$, he can simply choose his optimal level of investment $K = a^2/4$, generate a cash flow of $a\sqrt{K} = a^2/2$, and be unconcerned about the existence of raiders, because his cash flow remains below the attack threshold.

If, instead, $(1-\rho)^2 a^2/2 > \bar{c}$, the entrepreneur has two choices: fight or hide. If the entrepreneur invests so much that the cash flow is above the attack threshold, then the entrepreneur's maximization problem is

$$\max_K \rho^2 a\sqrt{K} - K \Rightarrow K = \frac{\rho^4 a^2}{4}, \tag{8}$$

so that the entrepreneur's ex-ante expected utility is

$$EU_{fight} = \rho^2 a\sqrt{\frac{\rho^4 a^2}{4}} - \frac{\rho^4 a^2}{4} = \frac{\rho^4 a^2}{4}. \tag{9}$$

Alternatively, the entrepreneur can under-invest so that his cash flow remains just small enough not to draw any raider's attention, by choosing K such that[4] $a\sqrt{K} = \bar{c}/(1-\rho)^2$, so

$$K_{\text{hide}} = \frac{\bar{c}^2}{(1-\rho)^4 a^2}. \tag{10}$$

Choosing a capital level of K_{hide} yields an expected utility level of

$$EU_{hide} = \frac{\bar{c}}{(1-\rho)^2} - \frac{\bar{c}^2}{(1-\rho)^4 a^2} \tag{11}$$

for the entrepreneur.

Solving $EU_{fight} = EU_{hide}$ for a yields a critical value of $\hat{a}_{2,Point}$. Entrepreneurs with lower productivity than $\hat{a}_{2,Point}$ prefer to hide, while those with $a > \hat{a}_{2,Point}$ accept that they will be attacked by a raider and invest accordingly. This is shown formally in the proof of the following Proposition 1 in the Appendix.

Proposition 1. *Suppose that the raiders' cost is $\bar{c}$ with probability 1. Then, the optimal investment decision for an entrepreneur is given by the function*

$$K^*(a) = \begin{cases} \frac{a^2}{4} & \text{for } a \leq \hat{a}_{1,Point} \\ \frac{\bar{c}^2}{(1-\rho)^4 a^2} & \text{for } a \in (\hat{a}_{1,Point}, \hat{a}_{2,Point}] \\ \frac{\rho^4 a^2}{4} & \text{for } a > \hat{a}_{2,Point} \end{cases}, \tag{12}$$

where $\hat{a}_{1,Point} = \frac{\sqrt{2\bar{c}}}{1-\rho} < \hat{a}_{2,Point} = \frac{\sqrt{2\bar{c}\left(1+\sqrt{1-\rho^4}\right)}}{\rho^2(1-\rho)}$.

Figure 1 illustrates Proposition 1 for the case $\bar{c} = 1$ and $\rho = 0.75$. For these parameters, $\hat{a}_{1,Point} = 5.65$ and $\hat{a}_{2,Point} \approx 13.6$. The left panel shows the optimal choice of capital

[4] As mentioned, we assume that raiders do not attack when indifferent between attacking and not attacking. Nothing of importance would change for other tie-breaking assumptions, except that, in order to hide, entrepreneurs would choose profits that are close to the threshold, but do not quite reach it.

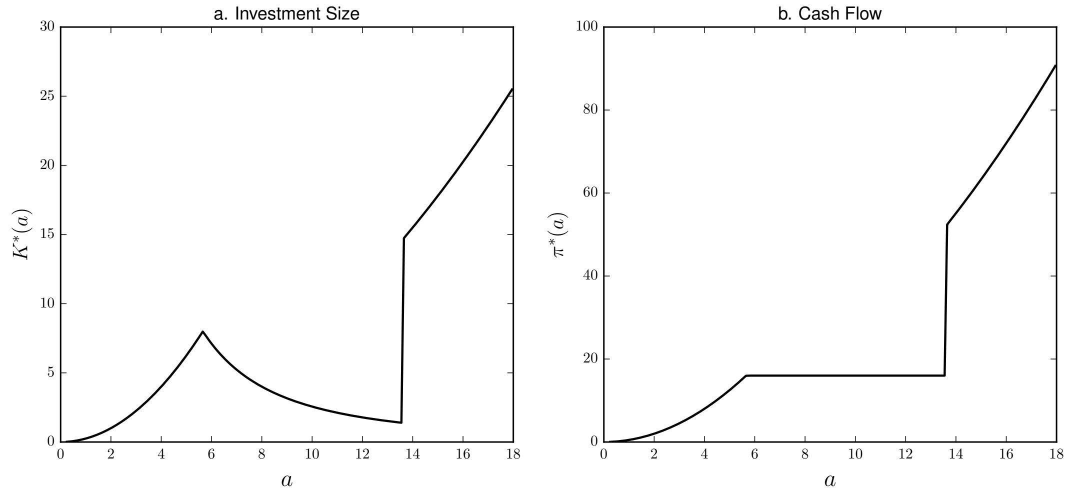

Figure 1: Capital investment and cash flow by productivity type for $\bar{c} = 1$ and $\rho = 0.75$ (Proposition 1)

investment for different types of entrepreneurs, and the right panel the corresponding cash flow values.

Between $\hat{a}_1$ and $\hat{a}_2$, capital investment is decreasing in a, and cash flow is flat. Intuitively, more productive entrepreneurs need to invest less in order to keep their firm's cash flow at a level that does not attract the raiders' attention.

Note that there is a large range of possible cash flows – between 18 and about 50 – that no entrepreneur chooses to have in equilibrium. This can be interpreted as the "missing middle" in the firm size distribution.

We now turn to the more complicated case that raiders have different costs distributed according to $F(c)$. Since an attack occurs if and only if $(1-\rho)^2 a\sqrt{K} > c$, the probability of this event is $F((1-\rho)^2 a\sqrt{K})$. The entrepreneur's objective function is then

$$\begin{aligned} \max_K F\left((1-\rho)^2 a\sqrt{K}\right)\rho^2 a\sqrt{K} + [1 - F\left((1-\rho)^2 a\sqrt{K}\right)]a\sqrt{K} - K = \\ \max_K \left[1 - F\left((1-\rho)^2 a\sqrt{K}\right)(1-\rho^2)\right] a\sqrt{K} - K \end{aligned} \tag{13}$$

The interpretation of the second line is that the entrepreneur gets to enjoy the fruits of his labor, unless he is attacked, which happens with probability $F\left((1-\rho)^2\pi\right)$, and in case of an attack, the entrepreneur loses, in expectation, a fraction $1-\rho^2$ of his profit.

Differentiating (13) with respect to K yields, as a first-order condition,

$$\frac{\left[1 - F\left((1-\rho)^2 a\sqrt{K}\right)(1-\rho^2)\right] a}{2\sqrt{K}} - 1 - f\left((1-\rho)^2 a\sqrt{K}\right)\frac{(1-\rho)^2(1-\rho^2)a^2}{2} = 0. \tag{14}$$

The first term in (14) is reminiscent of a standard tax term, where the expected "tax rate" to be paid due to the raider, is $F\left((1-\rho)^2 a\sqrt{K}\right)(1-\rho^2)$, the probability of being attacked times the fraction of profit lost in case of an attack. The second term, -1, is simply the marginal cost of investing capital.

The key effect, though, is embodied in the last term and does not correspond to standard effects in tax models. This term captures the effect that increased capital investment increases the probability of an attack by raiders. It is useful to write this term as

$$\left[(1-\rho)^2\frac{a}{2\sqrt{K}}\right] f\left((1-\rho)^2 a\sqrt{K}\right)\left[(1-\rho^2)a\sqrt{K}\right] = \frac{d(1-\rho)^2\pi}{dK} f\left((1-\rho)^2\pi\right)(1-\rho^2)\pi.$$

When the entrepreneur increases his investment, this leads to an increase of the raider's expected gross rent when fighting (the first term) which is then multiplied by the increase in the probability of attack that is due to a marginal increase in the raider's rent. Thus, the product of the first two terms captures the marginal increase in the probability of attack. Since the rent of the entrepreneur falls from π to $\rho^2\pi$ if attacked, $(1-\rho^2)\pi$ measures the utility impact of these marginal attacks on the entrepreneur.

Proposition 2 summarizes these results.

Proposition 2. *Suppose that the cumulative distribution of raider fixed costs $F(\cdot)$ is strictly increasing. Then, at the entrepreneur's optimal level of investment,* (14) *holds, and, if $f > 0$ at the point where* (14) *is satisfied, then the optimal level of investment is smaller than in the first-best.*

While (14) is, of course, a necessary condition that holds at the optimal level of K, the maximization problem (13) is, in general, not guaranteed to be globally concave, so (14) may be satisfied at several different levels of K. To see this, differentiate (14), which yields

$$-\frac{\left[1-F(\cdot)(1-\rho^2)\right]a}{4K\sqrt{K}} - f(\cdot)\frac{(1-\rho)^2(1-\rho^2)a^2}{2} - f'(\cdot)\frac{(1-\rho)^4(1-\rho^2)a^3}{4\sqrt{K}}$$

which may locally be positive if f' is sufficiently negative.

As an example for Proposition 2, suppose that the raider's c is drawn from a Pareto distribution with minimal value c_m and $\alpha = 1$, so that $F(c) = 1 - \frac{c_m}{c}$ for $c \geq c_m$ (and 0 otherwise). Using $f(c) = c_m/c^2$ for $c \geq c_m$ in (14), the first order condition becomes

$$a\frac{1-(1-\rho^2)\left(1-\frac{c_m}{(1-\rho)^2 a\sqrt{K}}\right)}{2\sqrt{K}} - 1 - \frac{c_m(1-\rho^2)(1-\rho)^2 a^2}{2(1-\rho)^4 a^2 K} = \frac{a\rho^2}{2\sqrt{K}} - 1 = 0 \qquad (15)$$

Obviously, there is a unique solution of the first-order condition, $K = \frac{\rho^4 a^2}{4}$, which is the same level as in (8). Note that this solution also satisfies the second-order condition, so the solution is a local optimum.

Observe that, for this distribution of c, the effect that more effort increases the probability of being challenged just exactly offsets the fact that not all firms with a profit larger than $\frac{c_m}{(1-\rho)^2}$ are challenged. Consequently, all the entrepreneurs for whom (15) characterizes an optimum choose their effort *as if they were sure to be challenged by a raider*, although each of them has a positive, and sometimes large, chance of escaping a challenge altogether.

While the optimal choice of entrepreneurs who accept a positive probability of being attacked is given by (15), there is another potentially attractive path of action available to entrepreneurs: Remember that the minimal possible raider fixed cost realization is c_m, which implies that staying at a level of profit not higher than $c_m/(1-\rho)^2$ guarantees that there is no challenge.[5] By choosing $K = \frac{c_m^2}{(1-\rho)^4 a^2}$, a challenge can be prevented for sure, and this investment level results in a utility level for the entrepreneur that is given by (11).

In the proof of the following Proposition 3, we show that this course of action is the optimal one for mid-productivity entrepreneurs.

Proposition 3. *Suppose that the raider's c is distributed according to $F(c) = 1 - \frac{c_m}{c}$ for $c \geq c_m$. Then, the entrepreneur's optimal investment is given by*

$$K^*(a) = \begin{cases} \frac{a^2}{4} & \textit{for } a \leq \hat{a}_{1,Pareto} \\ \frac{c_m^2}{(1-\rho)^4 a^2} & \textit{for } a \in (\hat{a}_{1,Pareto}, \hat{a}_{2,Pareto}] \\ \frac{\rho^4 a^2}{4} & \textit{for } a > \hat{a}_{2,Pareto} \end{cases}, \tag{16}$$

where $\hat{a}_{1,Pareto} = \frac{\sqrt{2c_m}}{1-\rho} < \hat{a}_{2,Pareto} = \frac{\sqrt{2c_m}}{\rho(1-\rho)}$.

Observe that, among mid-productivity types in (16), more productive types invest less capital, while the opposite is true among both low and high productivity entrepreneurs. In this sense, insecure property rights and the presence of the raider threat particularly affects the middle of the productivity distribution.

Figure 2 illustrates Proposition 2 for same parameters as Figure 1 above. For these parameters, $\hat{a}_{1,Pareto} \approx 5.6$ and $\hat{a}_{2,Pareto} \approx 7.5$. Again, the left panel shows investment, and the right panel the equilibrium cash flow level. For comparison purposes, Figure 2 displays the results for both Pareto and Point distribution.

For both distributions of raider costs, both entrepreneurs' utility and cash flows are the same for $a \leq \hat{a}_{1,Pareto}$ because these entrepreneurs are unconstrained. For $a \geq \hat{a}_{2,Pareto}$,

[5] As above, we assume as a tie breaking rule that raiders do not attack when indifferent.

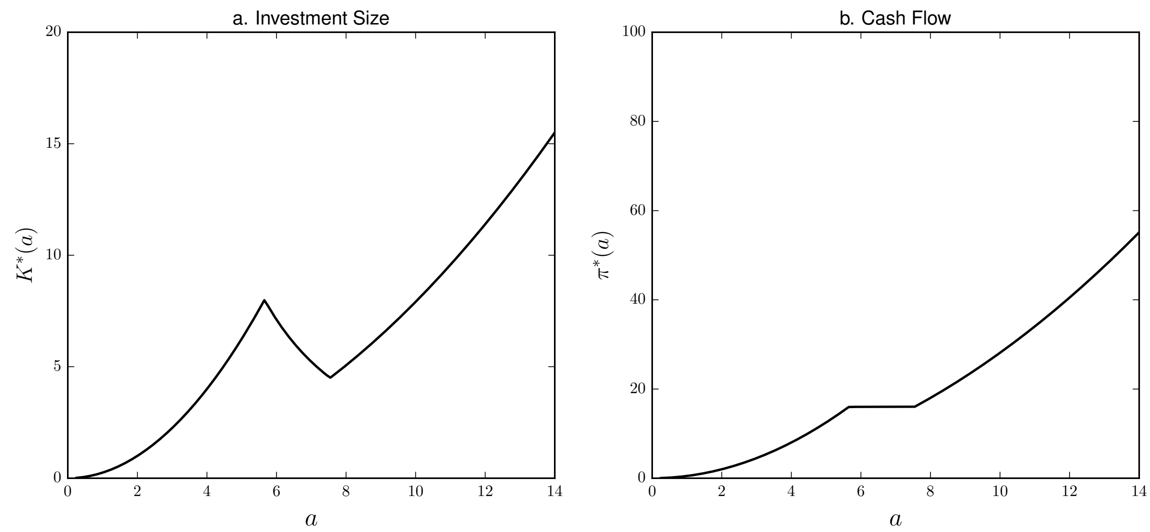

Figure 2: Capital investment and cash flow by productivity type for $c_m = 1$ and $\rho = 0.75$ (Proposition 3)

utility and cash flow is smaller under the point distribution than under the Pareto distribution because in the former case, all entrepreneurs with cash flow larger than the critical level are attacked, while that is not the case under the Pareto distribution. This implies that coming out of hiding is more attractive under the Pareto distribution, which in turn increases the hiding interval (since hiding, evidently, yields the same utility for both raider cost distributions, as no attacks take place in equilibrium in the hiding interval).

The distribution of firm sizes in the economy, in terms of the capital invested, follows in a straightforward way from combining the distribution of productivity in the economy with the optimal investment functions derived in Propositions 1 and 3, respectively. Corollary 1 provides the cumulative distribution functions of firm sizes when entrepreneurs' productivity is distributed according to $\Phi(\cdot)$, for both fixed and Pareto-distributed raider costs.

Corollary 1. *1. Suppose that the raiders' cost is $\bar{c}$ with probability 1. Then the resulting size distribution of firms has the following cumulative distribution function $G^p(\cdot)$.*

$$G^p(K) = \begin{cases} \Phi(2\sqrt{K}) & \text{for } K \leq \hat{K}_{1,Point} \\ \Phi(2\sqrt{K}) + \Phi(\hat{a}_{2,Point}) - \Phi(\hat{a}_{1,Point}) & \text{for } K \in \left(\hat{K}_{1,Point}, \hat{K}_{2,Point}\right] \\ \Phi(\hat{a}_{2,Point}) & \text{for } K \in \left(\hat{K}_{2,Point}, \hat{K}_{3,Point}\right] \\ \Phi(\frac{2\sqrt{K}}{\rho^2}) & \text{for } K > \hat{K}_{3,Point} \end{cases}, \quad (17)$$

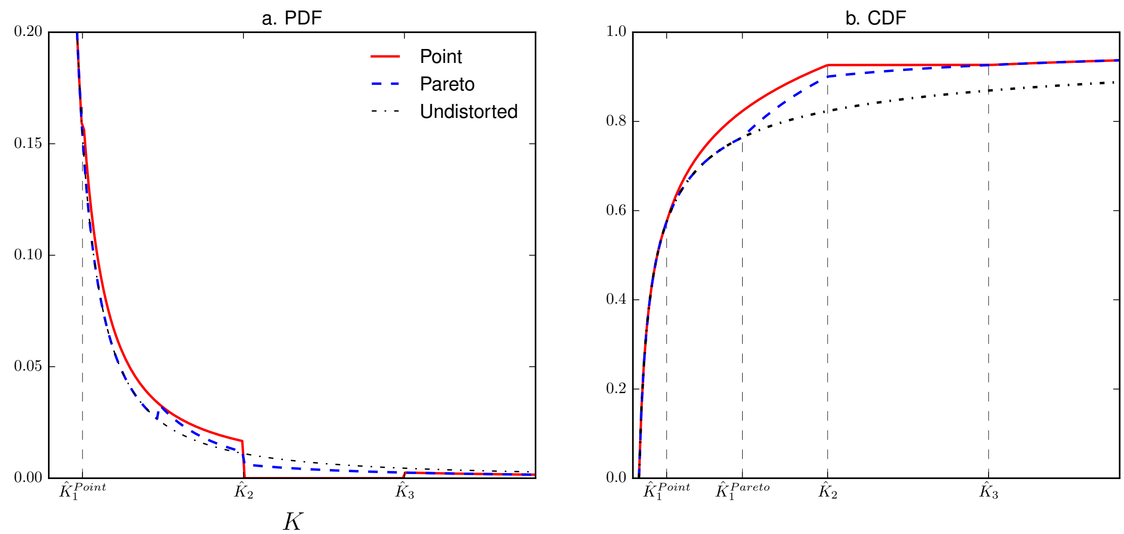

Figure 3: Probability density function (left) and cumulative distribution function (right) of the firm size for $\Phi(a) = 1 - \frac{1}{a}$

where $\hat{K}_{1,Point} = \frac{c\rho^4}{2(1-\rho)^2(1+\sqrt{1-\rho^4})}$, $\hat{K}_{2,Point} = \frac{c}{2(1-\rho)^2}$ *and* $\hat{K}_{3,Point} = \frac{c(1+\sqrt{1-\rho^4})}{2(1-\rho)^2}$.

2. *Suppose that the raiders' cost c is distributed according to $F(c) = 1 - \frac{c_m}{c}$ for $c \geq c_m$. Then the resulting size distribution of firms has the following cumulative distribution function $G^{Pareto}(K)$.*

$$G^{Pareto}(K) = \begin{cases} \Phi(2\sqrt{K}) & \textit{for } K \leq \hat{K}_{1,Pareto} \\ \Phi(2\sqrt{K}) + \Phi\left(\frac{2\sqrt{K}}{\rho^2}\right) - \Phi\left(\frac{c_m}{2(1-\rho)^2\sqrt{K}}\right) & \textit{for } K \in \left(\hat{K}_{1,Pareto}, \hat{K}_{2,Pareto}\right] \\ \Phi(\frac{2\sqrt{K}}{\rho^2}) & \textit{for } K > \hat{K}_{2,Pareto} \end{cases} \tag{18}$$

where $\hat{K}_{1,Pareto} = \frac{c_m\rho^2}{2(1-\rho)^2}$ *and* $\hat{K}_{2,Pareto} = \frac{c_m}{2(1-\rho)^2}$.

We illustrate Corollary 1 in Figure 3, by assuming that productivity a is distributed according to $\Phi(a) = 1 - \frac{1}{a}$. In an economy without raiders, this setting generates a size distribution of firms that follows "Zipf's law," a regularity well documented for many different developed economies.

Notice that the probability density function of the firm size distribution is 0 in the interval $[\hat{K}_2, \hat{K}_3]$ (and, accordingly, the cdf is flat) when all raiders have the same cost. While there is no such "gap" in the distribution of firm sizes when raiders' costs are Pareto distributed, there is still a marked difference between this distribution and the one that would arise in

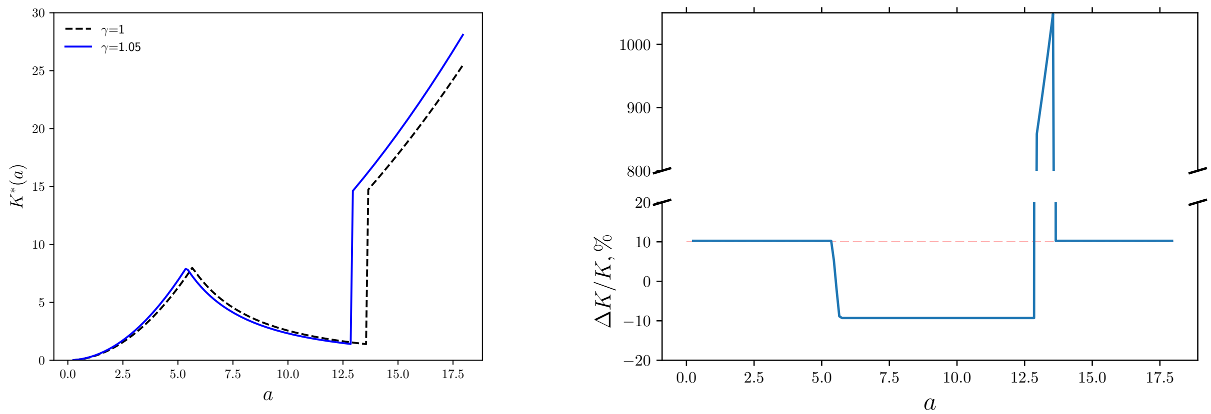

Figure 4: Investment size before and after $\gamma = 1.05$ shock(left) and the percentage difference in optimal investment size after the shock (right)

an economy without raiders. In particular, smaller firms are considerably over-represented in the economy with raiders, whereas large firms are not affected by the same magnitude.

It is instructive to compare the reaction of entrepreneurs to productivity or interest shocks in a raider economy on the one hand to that of entrepreneurs in an economy without raiders. Specifically, suppose that the productivity of capital of any type a entrepreneur is multiplied by a factor γ, so that the cash flow given investment K is $\gamma a\sqrt{K}$. In an economy without raiders, entrepreneurs would simply maximize

$$\gamma a\sqrt{K} - K,$$

by choosing $K = \frac{\gamma^2 a^2}{4}$. Thus, a particular increase in γ changes the optimal level of capital by the same percentage for all types of entrepreneurs.

In an economy with raiders, the same increase in γ leads to the same percentage change in optimal capital for those entrepreneurs who are too small to be bothered by raiders, and for those firms that are certain to be attacked, such as the group of the largest firms for the case where all raiders have deterministic costs.

In contrast, a small increase in γ will induce most of those entrepreneurs who would have chosen to hide before the shock to decrease their capital as a reaction to the productivity shock, in order to stay below the hiding threshold. However, the most productive entrepreneurs in the hiding group now find it attractive to switch to the "daring" regime instead, and thus to invest a lot more capital. Our model therefore generates the empirical prediction that, in a raider economy, the variability of the change in investment level should be higher among mid-productivity firms than among both small and large firms.

This prediction is illustrated in Figure 4, which depicts changes in the optimal choice of investment size after a uniform productivity increase to $\gamma = 1.05$. The entrepreneurs already daring to fight even before the productivity increase, change their investment size by a factor of $\gamma^2 - 1$. Most of those who were previously unconcerned can choose the same percentage increase in investment (i.e., by a factor of $\gamma^2 - 1$). The exception to this are those entrepreneurs with the very highest productivity in the unconcerned group. For them, the cashflow resulting from the previous choice of investment size and the new productivity level is now greater than the hiding threshold $\frac{\bar{c}}{(1-\rho)^2}$, which means they need to reduce their investment in order to hide from raiders.

A similar effect applies to those entrepreneurs who were already hiding before the change. If they want to avoid going over the hiding threshold, they need to reduce their investment to a proportion of $\frac{1}{\gamma^2}$ of the previous level.

However, for those entrepreneurs who were previously almost indifferent to coming out of hiding, the productivity increase makes it more attractive to switch to the "daring" regime instead. As Figure 4 shows, the highest productive entrepreneurs among hiding types, starting from the point where $\frac{\gamma^2 a^2 \rho^4}{4} > \frac{\bar{c}}{(1-\rho)^2} - \frac{\bar{c}^2}{(1-\rho)^4 a^2 \gamma^2}$, experience a large jump in the optimal investment size.

3.3 Measuring the burden from raiders through tax equivalents

One can view the presence of raiders in the economy as akin to a tax on entrepreneurs. Specifically, define the *utility equivalent tax rate*, t^U, as the tax rate on profit that would impose the same burden (in terms of utility reduction) as the presence of the raider threat. While a proportional tax on profits imposes the same proportional burden on entrepreneurs of different productivity types in a standard economy, the effect of raiders can be quite non-uniform for different productivity types.

Clearly, because the lowest types (those with $a \leq \hat{a}_1$) are not under any threat from raiders, their utility equivalent tax rate is equal to zero.

We now turn to the middle and high type entrepreneurs whose behavior is affected by the presence of raiders. Observe first that, if, instead of a raider threat, there was a proportional tax rate t, entrepreneurs would maximize $(1-t)[a\sqrt{K}-K]$, which is maximized at $K = a^2/4$. Substituting this back into the objective function yields

$$EU_{tax} = (1-t)\frac{a^2}{4} \tag{19}$$

as the entrepreneurs' indirect utility function. Consider an economy in which all raiders have a fixed cost of $\bar{c}$. Solving for the equivalent tax rate, we can find $t = 1 - \frac{4\bar{c}}{(1-\rho)^2 a^2} + \frac{4\bar{c}^2}{(1-\rho)^4 a^4}$, or

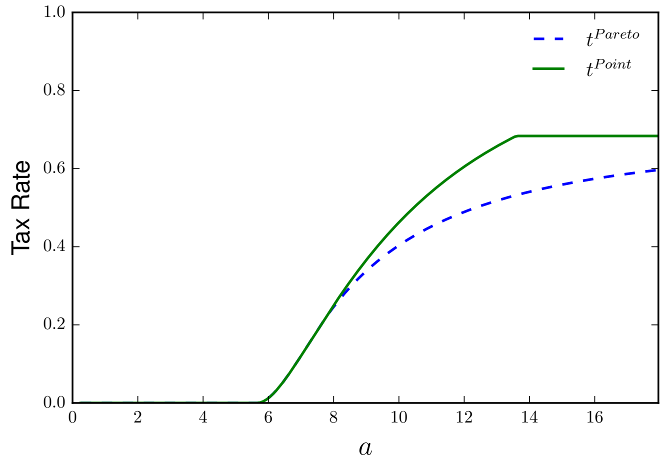

Figure 5: Equivalent tax rates for an economy with raiders with deterministic and Pareto distributed costs

$t = \left(1 - \frac{2\bar{c}}{(1-\rho)^2 a^2}\right)^2 > 0$, which is increasing for $a > a_{1,Point}$. Similarly, if raiders have Pareto distributed costs, hiding types face a tax rate of $t = 1 - \left(\frac{2c_m}{(1-\rho)^2 a^2)}\right)^2$. For all daring types, on the other hand, $t = 1 - \rho^4$.

Figure 5 displays the utility equivalent tax rates by productivity type, for the same parameter values as in the previous figures ($\rho = 0.75$, $\bar{c} = 1$, $c_m = 1$). Note that, whether the raiders' costs are drawn from a Pareto or a point distribution, does not affect the utility-equivalent tax rates of unconstrained and hiding types, while they differ for daring types. Specifically, because there always is a chance that a particular entrepreneur is not attacked when costs are Pareto distributed, the utility-equivalent tax rate for a given daring entrepreneur type in such an economy is smaller than in an economy where raiders have deterministic costs.

While the presence of raiders affects the *behavior* of mid-productivity entrepreneurs more severely than that of high-productivity entrepreneurs, the *economic burden* here is monotonically increasing in type. For example, for the point distribution, this is the case because all daring types have the same proportional burden from the presence of raiders in expected terms, while those types who choose to hide could choose an investment that leads to an attack (and thus the same burden as for the daring types), but optimally choose a different investment level and therefore must be better off.

However, as we will show in Section 4.1 below, the result that the burden from raiders is increasing in type depends on the assumption that all types have the same fighting efficiency. Once we allow for firms to invest in their defensive power, it may well be the case that the raider burden is highest for mid-productivity entrepreneurs.

An interesting – albeit quite obvious – observation is that the utility-equivalent tax rate is positive for all entrepreneurs who are hiding in equilibrium. Note that these types do not actually fight in equilibrium, so measuring the welfare loss from raiders merely by the fighting expenditures that occur is misleading. In fact, for a productivity distribution that puts considerably more of the probability mass on entrepreneurs in the middle rather than the high range, the actual fighting expenses can be an arbitrarily small percentage of the total welfare loss.

4 Extensions

4.1 Endogenizing Fighting Efficiency

So far, we have assumed that an entrepreneur's strength in a potential fight with a raider is exogenously given, and the same for everyone independent of the size of the respective firm. We now consider the case when entrepreneurs have an option to invest in fighting technologies. One can think of building up a legal department in the firm for defensive use in case of an attack, or investing in building up the goodwill of a local authority in the hope that this may provide some protection in case of a raider attack.

To see how much a given type of entrepreneur is willing to pay for an increase in ρ, it is useful to derive the value function of the entrepreneur's problem. For simplicity, we focus in this section on the case that all raiders have the same cost $\bar{c}$. Substituting the optimal investment choice K from (12) into the objective functions for the respective ranges yields

$$V(a,\rho,\bar{c}) = \begin{cases} \frac{a^2}{4} & \text{for } a \leq \hat{a}_{1,Point} \\ \frac{\bar{c}^2}{(1-\rho)^2} - \left(\frac{\bar{c}}{a(1-\rho)^2}\right)^2 & \text{for } a \in (\hat{a}_{1,Point}, \hat{a}_{2,Point}] \\ \frac{\rho^4 a^2}{4} & \text{for } a > \hat{a}_{2,Point} \end{cases}, \tag{20}$$

where $\hat{a}_{1,Point} = \frac{\sqrt{2\bar{c}}}{1-\rho} < \hat{a}_{2,Point} = \frac{\sqrt{2\bar{c}\left(1+\sqrt{1-\rho^4}\right)}}{\rho^2(1-\rho)}$.

We first analyze the value of a marginal increase in fighting efficiency. Differentiating (20) with respect to ρ, we get

$$V'_\rho(a,\rho,\bar{c}) = \begin{cases} 0 & \text{for } a \leq \hat{a}_{1,Point} \\ \frac{2\bar{c}}{(1-\rho)^3} - \frac{4}{(1-\rho)}\left(\frac{\bar{c}}{a(1-\rho)^2}\right)^2 & \text{for } a \in (\hat{a}_{1,Point}, \hat{a}_{2,Point}] \\ \rho^3 a^2 & \text{for } a > \hat{a}_{2,Point} \end{cases}. \tag{21}$$

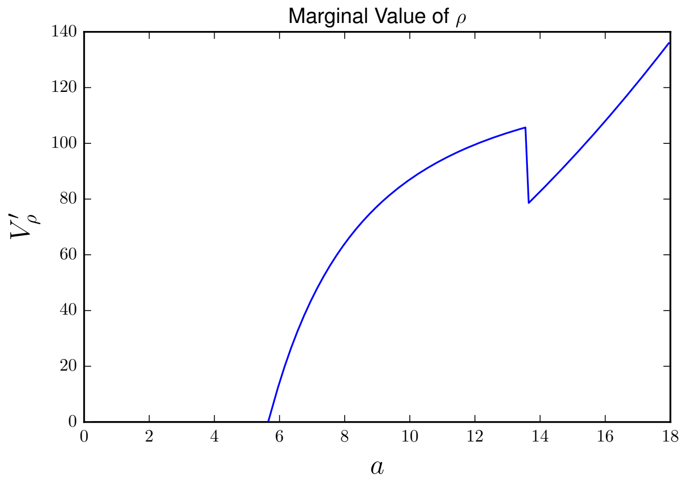

Figure 6: Marginal value of ρ for different productivity level.

Clearly, unconcerned entrepreneurs do not value an increase in fighting efficiency at all. More interestingly, while "hiding" types do not use a marginal increase in ρ for actually fighting, the higher ρ is valuable for them because it increases the amount of cash flow that is still safe from attack. Of course, "daring" types, i.e., those who are actually attacked in equilibrium, also value a marginal increase in ρ, and value it the more, the more productive they are.

Figure 6 depicts the marginal value of ρ by productivity level, for $\bar{c} = 1$ and an initial value of $\rho = 0.75$. Most interestingly, while the marginal value of an increase in ρ is increasing in productivity among both hiding and daring types, the highest productivity types among those who hide may value a marginal increase in ρ more than the lowest types of those who fight. Whether or not this occurs depends on parameters. In particular, there is a nonmonotonicity in the marginal value of ρ at sufficiently high initial values of ρ.

Specifically, the critical value of ρ is approximately 0.68 (independent of the level of $\bar{c}$). For values of ρ higher than this critical value, a marginal increase in ρ leads to an increase in the hiding threshold $\hat{a}_{2,Point}$. Intuitively, if the marginal value of ρ is larger for the highest hiding types than for the lowest daring types, hiding becomes more attractive relative to daring for the entrepreneur types in the neighborhood of the previous cutoff type, and hence the new hiding threshold $\hat{a}_{2,Point}$ shifts to the right.

We now consider what happens if there is an opportunity to buy, at a cost L, a discrete increase in fighting strength from ρ_0 to ρ_1. Note that an increase in ρ allows more entrepreneur types to be unconcerned, and both hiding and fighting also become more attractive for entrepreneurs with a higher ρ. Thus, whether the thresholds $\hat{a}_{1,Point}$ and $\hat{a}_{2,Point}$ increase or decrease depends on which individuals buy the improvement (determined by the cost L), and on which effects are stronger.

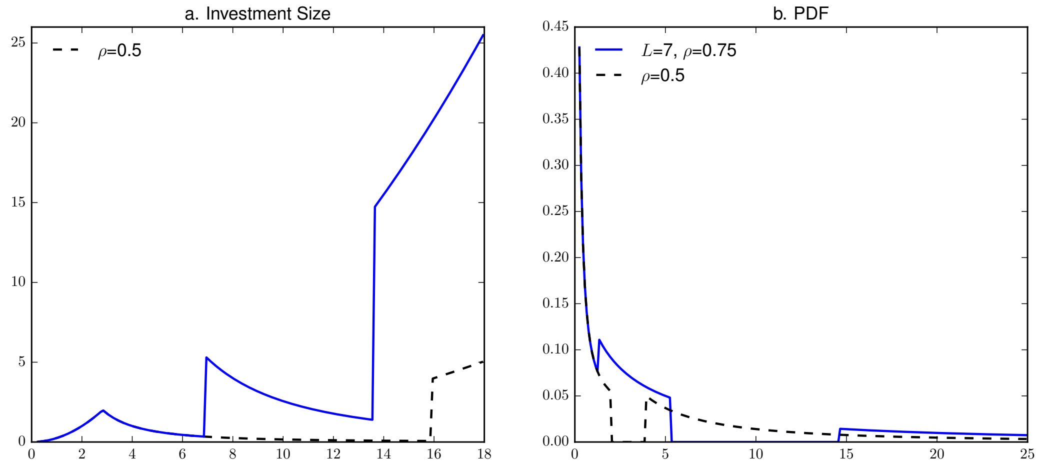

Figure 7: The opportunity to buy better fighting technology: Increasing ρ from $\rho_0 = 0.5$ to $\rho_1 = 0.75$ at a cost of $L = 7$

Moreover, the improved defense technology may either increase the size of the gap in the size distribution, or create a second gap, as Figures 7 and 8 demonstrate.

Figure 7 depicts a situation where entrepreneurs can buy, for a cost of $L = 7$, an increase in strength from $\rho_0 = 0.5$ to $\rho_1 = 0.75$. At this cost, the marginal entrepreneur who buys the increase is in the hiding range around $a = 7$, and he and some of his more productive neighbors remain in hiding – albeit at a higher level of capital – even after purchasing the strength increase. Relative to the case of $\rho = 0.5$, more productivity types choose to dare raiders, and since they jump to a significantly higher level of investment than in the case of $\rho = 0.5$, the gap in the distribution of firm sizes as measured by investment increases. The right-hand side depicts the resulting distribution of firm sizes, assuming that the distribution of productivity is given by a standard log-normal distribution.

Figure 8 is based on a situation with the same parameters, except that the cost is now $L = 18$. Again, the left panel depicts the optimal investment as a function of type, and the right panel the resulting distribution of investment size. At a cost of $L = 18$, the marginal entrepreneur who buys the increase is in the daring range, around $a = 16$. Consequently, there are two gaps in the distribution in this scenario: The first one is between hiding entrepreneurs and those who dare, but do not buy the improvement; and the second one is between daring entrepreneurs who do not buy the improvement, and those who do.

Figure 9 shows the effect on the equivalent tax rate, for an increase from $\rho_0 = 0.5$ to $\rho_1 = 0.9$ at a cost $L = 9$ (these parameters provide for better visibility of the effects).

The initial range where equivalent tax rates are increasing in type (i.e., from a level of 0 to the first peak) is the range of entrepreneurs who are hiding and do not find it worthwhile

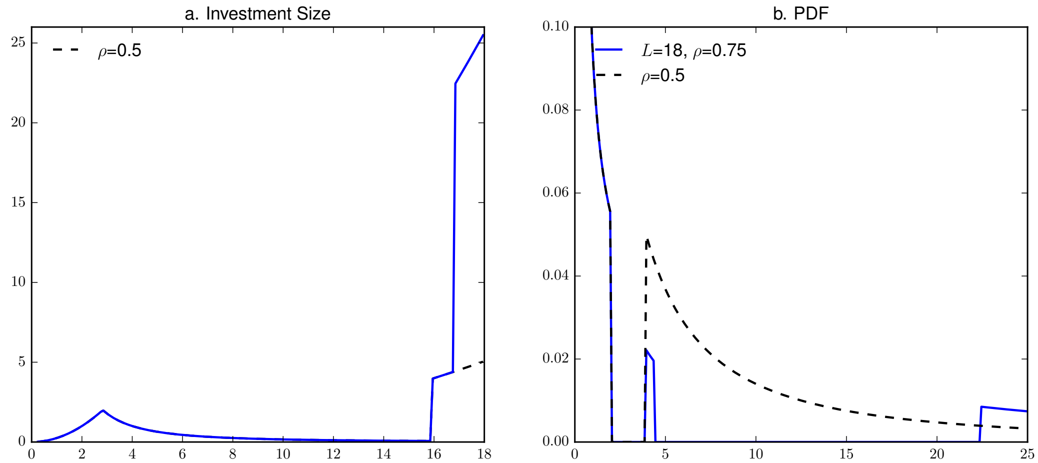

Figure 8: The opportunity to buy better fighting technology: Increasing ρ from $\rho_0 = 0.5$ to $\rho_1 = 0.75$ at a cost of $L = 18$

to invest in additional strength. The first peak occurs at a point where the entrepreneur is just indifferent between buying the strength increase and not buying it. This marginal entrepreneur and all more productive types up to $\hat{a}_{2,Point} \approx 22$ remain in hiding, but of course benefit by being able to invest more capital than before without attracting an attack. The rate at which these types benefit, in terms of their equivalent tax rate, is, however, very heterogeneous: Low types' net benefit is quite small because for them, the cost of $L = 9$ is relatively high compared to their benefit from being able to invest more. Thus, the tax rate is first decreasing in a after the peak. At the higher range of hiding types, the tax rate increases again, by the same arguments as in Section 3.3. Finally, for those types who are under attack even after the strength increase, the utility-equivalent tax rate decreases in type, towards a limit of $1 - 0.9^4 \approx 34\%$.[6]

4.2 The raider's efficiency in running a firm

So far, we have assumed that the raider, if he is successful in the contest for control of the firm, is able to run the firm as efficiently as the original entrepreneur, i.e., generates the same cash flow from the firm as the entrepreneur. This is probably approximately realistic in cases where the firm's productivity depends primarily on characteristics that are independent of the CEO's special skills – say, a firm focusing on natural resource extraction and, possibly to a slightly lower degree, ones engaged in simple manufacturing are likely to have approximately

[6] The utility-equivalent tax rate decreases in this range because the cost L is spread over an increased amount of profit as a increases.

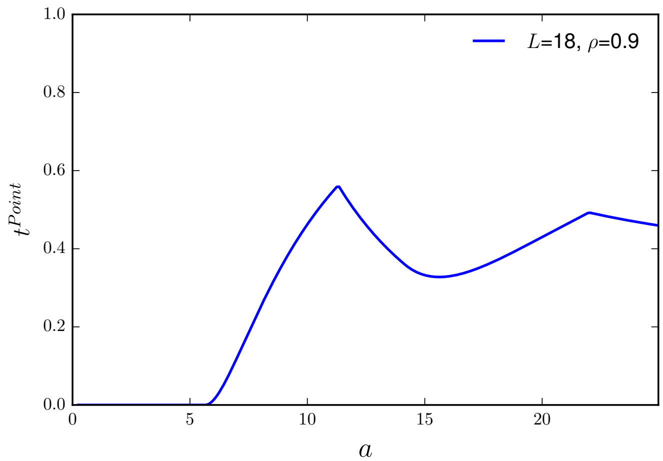

Figure 9: Equivalent tax rates for $\rho_0 = 0.5$, $\rho_1 = 0.9$ and $L = 9$.

the same value, whether managed by the original entrepreneur or the raider. In contrast, if the firm engages in a highly specialized activity such as software engineering, then the entrepreneur's identity may be essential for the success of the firm. To model such cases, let the fraction of the original entrepreneur's cash flow that a successful raider can obtain be denoted by $\theta \leq 1$.

In this case, the entrepreneur's optimization problem at the contest phase, given by (2) remains unchanged, but the maximization problem for the raider in contest phase now changes to

$$\max_{w} \frac{(1-\rho)w}{\rho v + (1-\rho)w}\theta\pi - w - c \tag{22}$$

The equilibrium in contest phase is now $v = w/\theta = \frac{\theta\rho(1-\rho)}{(\rho+\theta(1-\rho))^2}\pi$. In this equilibrium, the expected utility of the raider is

$$EU_{raider} = \frac{\theta^2(1-\rho)^2}{(\rho+(1-\rho)\theta)^2}\theta\pi - c, \tag{23}$$

and the challenged entrepreneur receives an expected payoff of

$$EU_{attacked} = \left(\frac{\rho}{(\rho+\theta(1-\rho))}\right)^2 \pi - K = \left(\frac{\rho}{(\rho+\theta(1-\rho))}\right)^2 a\sqrt{K} - K. \tag{24}$$

If an entrepreneur is certain to be attacked, the optimal K at the investment stage is therefore given by $K = \frac{a^2}{4}\left(\frac{\rho}{(\rho+\theta(1-\rho))}\right)^4$, which yields indirect continuation utility of

$$EU_{attacked} = \frac{a^2}{4}\left(\frac{\rho}{(\rho+\theta(1-\rho))}\right)^4. \tag{25}$$

Note that, for $\theta = 1$, the entrepreneur's capital investment choice and indirect utility are the same as in the basic case, while a decrease in θ increases both optimal investment and the entrepreneur's continuation utility.

Hiding types also benefit when $\theta < 1$. This follows because the safe cash flow level (i.e., the one that prevents an attack) is now $\frac{c(\rho+(1-\rho)\theta)^2}{(1-\rho)^2\theta^3}$, which is larger than the hiding threshold of $\frac{c}{(1-\rho)^2}$ in the basic model.

Finally, it is useful to compare changes in the entrepreneur's fighting ability ρ and those in the raider's efficiency after a successful take-over, θ. Qualitatively, it is clear that the effect on the entrepreneur and the raider of an increase in ρ is similar to the one from a decrease in θ: Both of these are helpful for the entrepreneur, while hurting the raider's ability to extract the firm's surplus.

They are, however, not entirely equivalent. To see this, rewrite (25) as

$$EU_{attacked} = \frac{a^2}{4}\left(\frac{1}{1+\theta\frac{(1-\rho)}{\rho}}\right)^4. \tag{26}$$

This implies that the entrepreneur's expected utility (and behavior) depends on $\kappa \equiv \theta\frac{(1-\rho)}{\rho}$.

In contrast, if we rewrite (23), we get

$$EU_{raider} = \left(\frac{1}{1+\frac{\rho}{(1-\rho)\theta}}\right)^2\theta\pi - c = \left(\frac{1}{1+\frac{1}{\kappa}}\right)^2\theta\pi - c, \tag{27}$$

which evidently depends on both κ and θ.

Figure 10 illustrates the difference. Specifically, $(\rho_1, \theta_1) = (1, 3/4)$, $(\rho_2, \theta_2) = (1/2, 1/3)$ and $(\theta_3, \rho_3) = (1/6, 1/3)$ lead to the same expected utility for attacked entrepreneurs, as $\kappa = 1/3$ for all three parameter combinations. In contrast, the raider's gross utility (i.e., before subtracting c) increases when θ increases. Consequently, attacking a firm is less attractive for raiders when the parameters are (ρ_3, θ_3) than in the first two scenarios. Thus, in this case, mid productivity entrepreneurs can invest more capital without triggering an attack, i.e., the hiding threshold increases. For the same reason, the utility of mid-productivity entrepreneurs is larger. The figure also shows that, the smaller is θ, the bigger is the jump in investment levels from the last hiding type to the first daring type, i.e. the range of the

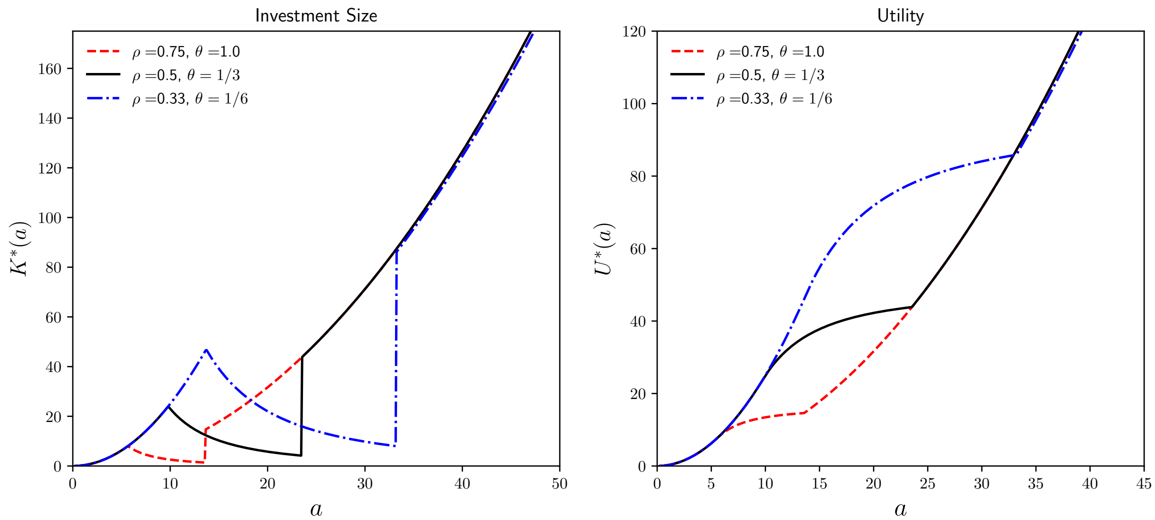

Figure 10: The effect of change in θ versus change in ρ on utility given fixed κ.

missing middle.

5 Discussion and conclusion

In this paper, we develop a theoretical framework to link insecure property rights to the investment decisions made by productive entrepreneurs. Specifically, we analyze a setting in which "raiders" may attempt to steal an entrepreneur's firm if his investment turns out to be too valuable.

In equilibrium, this problem affects most the investment behavior of entrepreneurs in the middle of the productivity distribution, because they can affect their risk of being targeted by staying small and less productive than otherwise optimal. In contrast, low productivity firms are unaffected because they are not sufficiently attractive targets, and large firms can be fatalistic and expect an attack to happen independent (at least at the margin) of their investment actions. Large firms may also be able to take more effective defensive measures than mid-level firms.

Our model provides a possible explanation for the phenomenon of the "missing middle" in the firm size distribution of many developing countries that has been documented extensively in the empirical literature. While, for some distributions of raider costs, our model produces an actual gap in the firm size distribution (even given a continuous distribution of entrepreneur productivity), the more robust and important point is that, for any raider cost distribution, the presence of raiders has a systematic effect on the firm size distribution. Indeed, as Tybout (2014) points out, the "missing middle" phenomenon should not necessarily be interpreted as a claim that the firm size distribution is necessarily bimodal.

Our model shows that an important welfare loss — and, possibly, the most important one — from the existence of raiders in the economy is not the amount of resources that raiders and attacked entrepreneurs spend in their fight about who gets to control the firm, but rather the detrimental effect that the raider threat has on the investment behavior of entrepreneurs.

Further research could take several directions. First, our model provides a framework in which to evaluate the effectiveness of anti-corruption measures aimed at reducing the threat to entrepreneurs' property rights. Second, and related, it is interesting to analyze the effects of economic shocks and/or different economic policies that affect the productivity distribution, or the cost of capital investment, in our framework. In particular, while the assumption of a given distribution of raider costs is without loss of generality in any given fixed situation, there may be (at least in the short term) "capacity constraints" on the raider sector, in the sense that, if the number of lucrative targets changes, raiders may not have the capacity to go after all of them, which could be captured by an endogenous shift in the raider cost distribution.

6 Appendix

Proof of Proposition 1. As argued in the text, entrepreneurs for whom $a^2/2 < \bar{c}/(1-\rho)^2$, thus for $a \leq \hat{a}_{1,Point}$ can simply choose the unconstrained optimal level of investment, $K = a^2/4$. Furthermore, those entrepreneurs who invest a sufficiently large amount of capital will be attacked, and therefore will choose according to (8).

It remains to derive the critical value of a such that entrepreneurs with lower productivity prefer to hide, while those with higher productivity accept that they will be attacked by a raider. Equating (9) and (11) yields

$$\frac{\rho^4 a^2}{4} = \frac{\bar{c}}{(1-\rho)^2} - \frac{\bar{c}^2}{(1-\rho)^4 a^2}. \tag{28}$$

This simplifies to

$$\rho^4(1-\rho)^4 a^4 - 4(1-\rho)^2 \bar{c} a^2 + 4\bar{c}^2 = 0,$$

so that the largest solution of this equation is $\hat{a}_{2,Point} = \frac{\sqrt{2\bar{c}\left(1+\sqrt{1-\rho^4}\right)}}{\rho^2(1-\rho)}$.

Furthermore, $\frac{\partial EU_{\text{fight}}}{\partial a} = \frac{\rho^4 a}{2} > \frac{\partial EU_{\text{hide}}}{\partial a} = \frac{2\bar{c}^2}{(1-\rho)^4 a^3}$ for all $a > \hat{a}_{2,Point}$. Thus, $EU_{\text{fight}} > EU_{\text{hide}}$ for all $a > \hat{a}_{2,Point}$. □

Proof of Proposition 3. The expected utility of an entrepreneur who accepts the possibility of a challenge is, using $K = \frac{\rho^4 a^2}{4}$ and $F(c) = 1 - \frac{c_m}{c}$ in (13),

$$\begin{aligned} EU_{\text{fight}} = \left[1 - (1-\rho^2)\left(1 - \frac{c_m}{(1-\rho)^2 a \frac{\rho^2 a}{2}}\right)\right] \frac{\rho^2 a^2}{2} - \frac{\rho^4 a^2}{4} = \\ \frac{(1+\rho)c_m}{(1-\rho)} + \frac{\rho^4 a^2}{4}. \end{aligned} \tag{29}$$

Solving for the largest a that equalizes (11) and (9) yields $\hat{a} = \frac{\sqrt{2c_m}}{\rho(1-\rho)}$.

To show that $EU_{\text{fight}} > EU_{\text{hide}}$ for all $a > \hat{a}$, observe that

$$\frac{\partial EU_{\text{hide}}}{\partial a} = \frac{2c_m^2}{(1-\rho)^4 a^3} < \frac{\sqrt{c}\rho^3}{(1-\rho)\sqrt{2}(1+\sqrt{2})^{1.5}} \text{ for all } a > \hat{a}$$

and

$$\frac{\partial EU_{\text{fight}}}{\partial a} = a\rho^4 > \frac{\rho^4}{\rho(1-\rho)} = \frac{\rho^3\sqrt{2(1+\sqrt{2})}\sqrt{c_m}}{2(1-\rho)} \text{ for all } a > \hat{a}.$$

Thus, $\frac{\partial EU_{\text{fight}}}{\partial a} > \frac{\partial EU_{\text{hide}}}{\partial a}$ for all $a > \hat{a}$, and since $EU_{\text{hide}}(\hat{a}) = EU_{\text{fight}}(\hat{a})$, this shows that $EU_{\text{fight}} > EU_{\text{hide}}$ for all $a > \hat{a}$. □

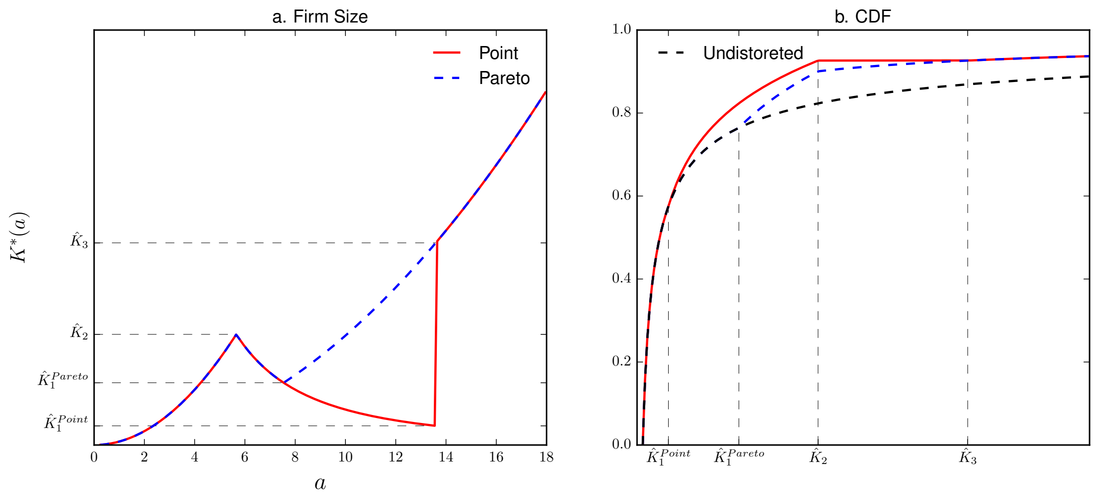

Figure 11: Implied values of investments for transition productivities

Proof of Corollary 1. Analytical distribution of K can be derived by substituting intervals of a into intervals of K and applying corresponding transformation. This can be visualized by looking at Figure 11. In particular, for Pareto distributed raiders, there are three thresholds for K. First, when Investment levels are less than $\hat{K}_{1,Pareto} = \frac{c^2}{(1-\rho)^4 \hat{a}^2_{2,Pareto}} = \frac{c_m \rho^2}{2(1-\rho)^2}$, there are only unconstrained entrepreneurs w.p. $G_K(K) = \Phi_a(2\sqrt{K})$. Next, within the range $K \in \left(\hat{K}_{1,Pareto}, \hat{K}_2\right]$[7], where $\hat{K}_2 = \frac{\hat{a}^2_{1,Pareto}}{4} = \frac{c}{2(1-\rho)^2}$ there are entrepreneurs from all three types: Unconcerned entrepreneurs w.p. $\Phi_a(2\sqrt{K})$, hiding and fighting entrepreneurs w.p. $\Phi\left(\frac{2\sqrt{K}}{\rho^2}\right) - \Phi\left(\frac{c_m}{2(1-\rho)^2\sqrt{K}}\right)$. After this point, all the entrepreneurs are under attack and fighting for property rights. Namely, when $K > \hat{K}_2$, $G(K) = \Phi(\frac{2\sqrt{K}}{\rho^2})$.

For Point distributed raiders, $G^p(K)$ can be constructed as a piecewise distribution from 4 parts. First part is populated by unconcerned entrepreneurs, with smaller productivity levels than any hiding one, $a \leq \hat{a}_{2,point}$ and have choice of investment $K = a^2/4$. From the fact that $K = a^2/4$ when $a \leq \hat{a}_{1,point}$ we can express $a = 2\sqrt{K}$. Respectively in range $K \leq \frac{c^2}{(1-\rho)^4 \hat{a}^2_{2,Point}} = \frac{c\rho^4}{2(1-\rho)^2(1+\sqrt{1-\rho^4})}$, $G^p_K(K) = \Phi_a(2\sqrt{K})$. In range $\hat{K} \in \left(\hat{K}_{1,Point}, \hat{K}_{2,Point}\right]$ there are unconstrained entrepreneurs w.p. $\Phi(2\sqrt{K})$ and hiding entrepreneurs w.p. $\Phi(\hat{a}_{2,Point}) - \Phi(\hat{a}_{1,Point})$. Moreover, $\hat{K}_{2,Point}$ corresponds to $\hat{K}_{2,Point} = \frac{\hat{a}^2_{1,Point}}{4} = \frac{c}{2(1-\rho)^2}$. In case of Point distributed raiders, there is a flat part in distribution, that lies in $K \in \left(\hat{K}_{2,Point}, \hat{K}_{3,Point}\right]$, where $\hat{K}_{3,Point} = \frac{\rho^4 \hat{a}^2_{2,Point}}{4} = \frac{c(1+\sqrt{1-\rho^4})}{2(1-\rho)^2}$. This happens w.p. $\Phi(\hat{a}_{2,Point})$. Finally, all the entrepreneurs under attack have productivity levels $a > \hat{a}_{2,Point}$, or equivalently investment

[7] Note that $\hat{K}_2 = \hat{K}_{2,Point} = \hat{K}_{2,Pareto}$

levels $K > \hat{K}_{3,Point}$. Since they choose $K = \frac{a^2\rho^4}{4}$, then $a(K) = \frac{2\sqrt{K}}{\rho^2}$ which implies $G^p(K) = \Phi(\frac{2\sqrt{K}}{\rho^2})$. Corollary 1 summarizes these findings.

$\square$

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