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Party Polarization in Legislatures with Office-Motivated Candidates

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Abstract

We develop a theory of legislative competition in which voters care about local candidate valence and national party positions that are determined by the parties' median legislators. As long as election outcomes are sufficiently predictable, the only stable equilibria exhibit policy divergence between the parties.

If the degree of uncertainty about election outcomes decreases, and if voters place less weight on local candidates' valence, polarization between the parties increases. Furthermore, a systematic electoral shock makes the party favored by the shock more moderate, while the disadvantaged party becomes more extreme.

Finally, we examine data on state elections and the ideological positions of state legislatures and find patterns that are consistent with key predictions of our model.

Keywords: Legislative competition, polarization.

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1 Introduction

What is the nature of political parties in representative democracies? In the standard spatial model parties are teams, groups of politicians united to win control of government (Downs, 1957). All politicians within a party have the same induced preferences over the party's policy positions or ideology.

In practice, however, the politicians in many parties do not exhibit such extreme ideological unity. Dissent frequently arises within parties about the legislative agenda or party platform, in policy proposals, speeches, convention battles, and—at least in the U.S.—roll call votes.[1]

One possible reason for intra-party disagreement is that politicians care explicitly about policy and simply differ in their personal preferences. However, even if politicians care first and foremost about their own careers and re-election chances, disagreements may arise due to differences between the districts that these politicians represent. For instance, within the Democratic party, some incumbents represent liberal districts and some represent moderate districts, and they try to shift party policy in a direction preferred by their respective constituents—e.g., Democrats from liberal districts seek to pull their party to the left and those from moderate districts seek to pull it toward the center.

This paper develops and analyzes a simple model that adopts the second perspective, in which politicians care only about re-election but differ in the policy preferences of the districts they represent. Elected Democratic legislators collectively determine the national Democratic position, and similarly for Republicans. Local voters in a continuum of districts (differentiated by their median voters' policy preferences) vote on the basis of both these national party positions and local candidate quality.

There are different interpretations of which candidate characteristics constitute local candidate quality, often called "valence" in political economy models. On the most general level, we can think of valence as any attribute that is important to voters and not related to the main policy cleavage that is captured by the national party positions. This could be a candidate's effectiveness in steering government funds to the district, the quality of doing casework, but it could also summarize a candidate's previous experience in different sectors (such as the military, the private sector, public administration, etc.), or his performance in debates and campaign events, and so on. Which of the candidates' experiences and characteristics is particularly valued by the voters at the time of the election (relative to those of the opponent) depends, at least in part, on events that occur after the parties' nomination decisions, and therefore is appropriately viewed as a random variable.[2]

[1] As Robertson (1976) documents, this holds even countries with "strong party" systems such as the U.K.

[2] There is a large theoretical literature that incorporates valence in models of electoral competition (Myer-

The randomness of valence implies that some candidates are elected in territory that is ideologically challenging for their party, and these candidates exert a moderating force on their party, but are also electorally more vulnerable than those of their colleagues who hail from more ideologically aligned districts. As long as election outcomes are predictable enough (i.e., the average absolute value of candidate valence shocks is not too large), we show that the only stable equilibria exhibit policy divergence between the parties.

According to many measures, the degree of polarization between Republicans and Democrats has increased substantially over the past few decades, and is a salient feature of the political landscape in the U.S. today, both at the national level and in many states. There are, of course, many theoretical models of policy divergence, and the factors that can generate platform divergence.[3] By and large, papers in this literature aim to develop a theoretical understanding of the effects of specific assumptions about candidate objectives and the exact setting in which candidates compete on the political positions they adopt, and the degree of polarization. While assumptions and results are often motivated by certain stylized facts, these papers generally do not focus on generating and testing empirical predictions. This characterization is not meant as a criticism of this literature (to which we have contributed), but a central contribution of our model is that it makes clear comparative statics predictions about party positions: In particular,

(i) as the degree of uncertainty about election outcomes decreases, polarization between the parties increases;

(ii) as voters place less weight on local candidates' valence characteristics, polarization between the parties increases;

(iii) "electoral tides" (i.e., electoral shocks favoring one party over its opposition) move the favored party to a more moderate position, while the disadvantaged party becomes

son, 1993; Grossman and Helpman, 1996; Persson and Tabellini, 1999, 2002; Ansolabehere and Snyder, 2000; Groseclose, 2001; Aragones and Palfrey, 2002; Schofield, 2004, 2006; Messner and Polborn, 2004; Adams et al., 2005; Adams and Merrill, 2008, 2009; Kartik and McAfee, 2007; Ashworth and De Mesquita, 2009; Soubeyran, 2009; Bernhardt et al., 2011; Snyder et al., 2009; Snyder and Ting, 2011, and many more.).

The large empirical literature on U.S. congressional elections finds that the candidate "quality" has a significant effect on voting outcomes (see, e.g., Jacobson (1978); Green and Krasno (1988); Squire (1992); Ansolabehere et al. (2001)). The literature on U.S. presidential elections also finds a significant effect of economic growth on voting (and treats growth as a valence issue; see, e.g., Fair (1978); Hibbs (1987); Nadeau and Lewis-Beck (2001)). For an empirical estimate of the importance of valence relative to ideology based on a field experiment, see Kendall et al. (2014).

[3]These include policy motivation Wittman (1983); Calvert (1985); Londregan and Romer (1993); Roemer (1994); Osborne and Slivinski (1996); Besley and Coate (1997); Martinelli (2001); Gul and Pesendorfer (2009); entry deterrence (Palfrey (1984); Callander (2005)); incomplete information among voters or candidates (Castanheira (2003); Bernhardt et al. (2007); Callander (2008); and differential candidate valence (Bernhardt and Ingberman (1985); Groseclose (2001); Krasa and Polborn (2010, 2012); Bierbrauer and Boyer (2013).

more extreme.

The paper also finds that the available empirical evidence is consistent with our model. In particular, we show that data on party polarization from U.S. state legislatures and the U.S. Congress are strongly consistent with predictions (i) and (iii). The empirical section also presents anecdotal evidence consistent with predictions (ii). The findings regarding prediction (i) are especially interesting, because this prediction is exactly the opposite of the prediction generated by standard single-election models in which candidates care about policy as well as winning, such as Wittman (1983); Calvert (1985); Roemer (1994). In those models, polarization increases as uncertainty about electoral outcomes increases.

Clearly, the empirical evidence only provides suggestive support for our model because we have not investigated all plausible alternatives that might make similar predictions. We can be more confident, however, in asserting that the evidence appears to be inconsistent with some existing models of political competition with office- or policy-motivated parties or candidates. In fact, we would go one step further, and suggest that models that take into account the fact that parties compete across diverse constituencies should become a standard element of the political economy toolkit. It is time for the theoretical literature on party platform divergence to move beyond the fact that parties do not converge, and to pay more attention to the testable comparative statics predictions made by the different models. Our paper provides a first step in this direction.

2 The Model

There is a continuum of districts, indexed by M, the ideal policy position of the district median voter. Let F_M be the cumulative distribution function of M. We assume that F_M admits a continuous and positive probability density function, denoted f_M. Without loss of generality, we can normalize such that $F_M(0) = 1/2$, that is, the median value of M is zero.

The policy preferences of voter M are given by $u(x, M) = -|x - M|$.[4] We assume that voters care about the national party positions x_D (for Democrats) and x_R (for Republicans) when deciding whom to vote for.

Specifically, we assume that x_D and x_R are given by the median of the district medians represented by Democrats and Republicans in the legislature, respectively. We will defend this assumption below.

[4]In a previous version, available from the authors upon request, voters have quadratic preferences, given by $u(x, M) = -(x - M)^2$. The qualitative results are the same, but examples are easier to construct for absolute distance preferences.

For simplicity, the voters' utility function is assumed to be completely independent of the "positions" that their local candidates propose. The argument is that voters cannot expect that *their local representative* will select the policy for the nation at-large, but rather, the parties, made up by the representatives chosen in all districts, are crucial in the process of policy selection in the legislature. However, as we explain below, the results of the model are robust to the case that voters, in addition, care about their local candidates' positions.

In addition to policy, voters care about valence. If party k's candidate is elected in district i, then voters in that district receive a valence payoff $\alpha v_{k,i}$, where $v_{k,i}$ denotes the elected candidate's valence and α is a parameter measuring how important valence is for voters relative to policy.[5]

It is useful to define the Republican net valence in district i as $v_i \equiv v_{R,i} - v_{D,i}$, where $v_{R,i}$ and $v_{D,i}$ are the Republican and Democratic candidates' valences, respectively. The net valence v_i is a random variable that is distributed according to probability density function ϕ and cumulative distribution function Φ, assumed to be symmetric around 0. Valence shocks in different districts are independent.

Discussion. A crucial stylized fact in U.S. politics is that districts in which a party's presidential candidate wins by a substantial margin are usually considered "safe" districts for that party in the Congressional election, while those in which the Presidential election is close are also the ones that are "competitive" in Congressional elections in the sense that both parties' candidates have a significant probability of winning.

As we show below, the key assumptions that deliver this result in our model is that voters care about national party positions, and that those are determined by the median Democrat and median Republican in the legislature, respectively. This implies that all but the most moderate districts have a certain affinity to voting for the ideologically aligned party's candidate.

That voters care about national party positions appears uncontroversial. Even incumbents whose own position may fit their districts perfectly often have a hard time holding on to a district whose ideological leanings have moved away from their party. For example, Krasa and Polborn (2015) describe the case of Lincoln Chafee, the Republican U.S. senator from Rhode Island from 1999 to 2006, who had taken a number of moderate and liberal positions that brought him in line with voters in his state. While exit polls in the 2006 election gave Chafee a very high 62 percent personal approval rating, "most voters rejected him, many

[5]The reason why we do not combine α and v into one parameter is that we want to derive comparative static results with respect to the importance of valence for voters. For example, one can think of v_i as the overall value of the pork projects that candidate i would be able to attract, and of α as the fraction of those benefits that go to voters in the district (as opposed to voters in other districts).

feeling it was more important to give the Democrats a chance at controlling the Senate."[6] His Democratic challenger Whitehouse "succeeded by attacking the instances in which Chafee supported his party's conservative congressional leadership (whose personalities and policies were very unpopular, state-wide)."[7]

We do not explicitly model the process through which national party positions are determined, but rather assume that each party is identified by the voters with the position of their median representative in the legislature. One possible micro-foundation is that incumbents care only about winning their own seats. Whether a candidate wins also depends on her and her opponent's valence, which is stochastic. Because voters compare the valence difference between their local candidates with the difference in policy utility that they derive from the two parties' national positions, each incumbent has induced preferences over her party's platform that are single-peaked, with the peak located at her district's median. Therefore, if Republicans and Democrats adopt their national positions by majority vote among their respective caucuses, each party's position will be equal to the median of the districts represented by that party's legislators.[8]

Alternatively, the importance of national positions for voters may also arise because voters know the national party positions much better than the positions of their local candidates, as the mass media provides plenty of information about what national party leaders are doing or trying to do—e.g., what "the Democratic" or "the Republican" position is on key issues—but comparatively little information about most rank-and-file legislators. There is ample empirical evidence from surveys that voters are fairly good at distinguishing the relative ideological positions of politicians across parties, but poor at identifying the relative ideological positions of politicians within parties (see, e.g., Snyder and Ting 2002).[9] One clear reason for this is that, when the media cover politics, they naturally focus on current office-holders' actions, so that voters perceive the parties' positions as some aggregate of their current incumbents' positions (this point goes back to—at least—Key (1947)).

While national policy is important to voters, it is also important to stress that it is not the only determinant of voting behavior, as voters are willing to go against their ideological policy preferences if they sufficiently prefer the opposition candidate over the candidate of

[6] "A GOP Breed loses its place in New England", New York Times, November 27, 2006.

[7] See `http://en.wikipedia.org/wiki/Lincoln_Chafee`.

[8] For a different approach to intra-party bargaining about party positions, see Roemer (1999, 2001).

[9] Note that limited voter information about the local representative's position in Congress does not undermine the argument that representatives benefit from moving their party's position in the direction of their district. Voters do not "reward" the candidate for representing the district's position in that intra-caucus vote—which is, most likely, unobservable for voters anyway –, but they do observe the overall outcome of this process, and so each representative has perfect incentives to act as stipulated even if his voting behavior in the caucus cannot be observed by his voters. (If the voters could indeed observe their representative's behavior, this would provide an additional reason for representatives to "vote their district.")

their ideologically preferred party. Valence differences between local candidates imply that there are some Republicans who win in liberal districts (especially in moderately liberal ones, in which their ideological disadvantage is not too large), and vice versa for Democrats. From a technical point of view, the presence of valence uncertainty implies that elected candidates have a strict preference for moving their party's platform closer to the bliss point of their district's median voter because their respective winning probability strictly decreases with the distance between their party's position and their district's median voter.

Related literature. Our model borrows heavily from Snyder (1994); Ansolabehere et al. (2012), and Krasa and Polborn (2015), which, in turn, build on the early work by Robertson (1976) and Austen-Smith (1984, 1986).

In modeling how national party positions are determined, and how they matter for voters, we follow Snyder (1994) and Ansolabehere et al. (2012). In Snyder (1994) there are no valence shocks, and voters cannot distinguish between the parties' platforms if they are too close to each other. In Ansolabehere et al. (2012) there is a nationwide valence shock but no race-specific valence shocks. Consequently, the legislatures in these models are characterized by a simple cutoff such that all districts to the left of the cutoff have Democratic representatives, and all districts to the right have Republican representatives. Thus, these models display an extreme form of polarization without overlap between districts represented by the two parties, because there are no idiosyncratic factors that can influence the outcome of a local election. This is certainly counterfactual in general (and, in particular, during the less polarized eras of Congressional history), and since the models' equilibrium degree of polarization is the highest that is conceptually possible, they are not directly amenable to an empirical analysis of what factors determine cross-sectional or intertemporal changes in polarization.

Individual races in our model remain competitive between candidates because voters also care about the local candidates' valence (which is uncertain from an ex-ante perspective); thus, all districts have some positive probability of electing either party's candidate, though Democrats and Republicans are advantaged in liberal and conservative districts, respectively. The main focus of our theoretical analysis is how properties of the valence distribution (e.g., the degree of uncertainty about valence, or valence shocks that favor one party) affect the degree of polarization between the parties. Moreover, we test these empirical predictions, while Snyder (1994) and Ansolabehere et al. (2012) are purely theoretical.

Krasa and Polborn (2015) analyze another model of legislative competition and polarization in which national positions matter for voters and are determined by the median caucus member. However, their model differs from ours here in two crucial ways: First, their main focus is on the nomination process in which each candidate's position is assumed to be deter-

mined by policy-motivated primary voters, rather than by the reelection-seeking incumbents in our model. They show that the primary voters can, in almost all districts, exploit the preference of the district median voter for one party's national position, by nominating more extreme candidates than the median voter prefers. Second, voters in their model choose local candidates calculating how the election of the local Democrat or Republican affects the expected national policy (which is the median of the majority party in the legislature), taking into account both the effect on the likelihood that either party wins a majority, and the position of the majority party. In contrast, our modeling of the voters' choice is more reduced form, but—we would argue—is behaviorally plausible and much more tractable.

Other papers that examine the electoral consequences of intra-party differences in constituency preferences include Calvert and Isaac (1981); Ingberman and Villani (1993); Snyder and Ting (2002, 2003); Castanheira and Crutzen (2010); Crutzen et al. (2010).

3 Basic Analysis

3.1 District winning probabilities and equilibrium party positions

The median voter in district M prefers the Republican if and only if

$$-|x_D - M| \leq -|x_R - M| + \alpha v, \tag{1}$$

where x_R and x_D are the Republican and Democratic platforms, respectively. Without loss of generality, let $x_D \leq x_R$, i.e., the Democratic platform is weakly to the left of the Republican one.[10]

Note that, while we assume that voters care only about national positions, we show at the end of this subsection that the results of our model remain unchanged if voters instead care both about national platforms and their local representative's position.

Rearranging (1), the Republican candidate wins if $\alpha v \geq |x_R - M| - |x_D - M|$, thus with probability

$$W(M) \equiv Prob(\text{R wins in district } M) = \begin{cases} 1 - \Phi\left(\frac{x_R - x_D}{\alpha}\right) & \text{if } M \leq x_D \\ 1 - \Phi\left(\frac{x_R + x_D - 2M}{\alpha}\right) & \text{if } M \in (x_D, x_R) \\ 1 - \Phi\left(\frac{x_D - x_R}{\alpha}\right) & \text{if } M \geq x_R \end{cases} \tag{2}$$

Since there is a continuum of districts, it is useful to think of $W(M)$ as the (approximate)

[10] This is without loss of generality because, if $x_D > x_R$, we can simply rename the parties.

proportion of districts close to M that are won by Republicans. Consequently, the density of districts represented by Republicans is given by

$$\frac{f_M(M)W(M)}{\int_{-\infty}^{\infty} f_M(M)W(M)dM}, \tag{3}$$

and the density of districts represented by Democrats is given by

$$\frac{f_M(M)[1-W(M)]}{\int_{-\infty}^{\infty} f_M(M)[1-W(M)]dM}. \tag{4}$$

As explained above, the Republican and Democratic positions x_R and x_D are given by the medians of (3) and (4), so that x_R satisfies

$$\frac{\int_{-\infty}^{x_R} f_M(M)W(M)}{\int_{-\infty}^{\infty} f_M(M)W(M)dM} = \frac{1}{2} \tag{5}$$

and, analogously, x_D satisfies

$$\frac{\int_{-\infty}^{x_D} f_M(M)[1-W(M)]}{\int_{-\infty}^{\infty} f_M(M)[1-W(M)]dM} = \frac{1}{2} \tag{6}$$

There is always an equilibrium in which the parties' positions are both at 0. To see this, note that substituting $x_D = x_R = 0$ in (2) yields $W(M) = 1 - \Phi(0) = 1/2$ for all M. Substituting this, (5) and (6) are satisfied since 0 is the median of f_M. Thus, we have

Proposition 1. *There exists an equilibrium in which $x_D = x_R = 0$.*

Intuitively, if the party positions are identical, then no party's candidate has an advantage in any district, and therefore the densities of districts won by Democrats and Republicans are indeed identical, so that both parties have the same position. Note that Proposition 1 applies even if $\phi(\cdot)$ is not symmetric because all that is needed for the conclusion is that W is constant as a function of M.

An important caveat is that the district with the median voter exactly at 0 is the expected median district in both the Democratic and the Republican caucus in this equilibrium, but, evidently, cannot be the *realized* median district in both the Democratic and the Republican caucus. Thus, a key question with the no differentiation equilibrium is whether it is stable if voters' expectations are formed dynamically by observing the parties' realized positions in the legislature. We address this issue in Section 3.2 below.

We now turn to the question whether there exists an equilibrium with policy divergence. In order to increase tractability, we focus on the case that the distribution of districts is

uniform on $[-k, k]$ and the distribution ϕ is single-peaked and symmetric around 0. This setting allows us to focus on a symmetric profile in which $x_R = x = -x_D$.

Proposition 2. *Suppose that the distribution of districts is uniform on $[-k, k]$ and the distribution ϕ is single-peaked and symmetric around 0. A symmetric equilibrium with policy differentiation, $x_R = x = -x_D$, exists if and only if $\phi(0) > \frac{\alpha}{4k}$. Moreover, if such an equilibrium exists, it is unique.*

Proof. See Appendix. □

Figure 1 illustrates the type of divergent equilibrium described in Proposition 2. In such an equilibrium, Democrats have higher chances being elected in liberal districts because their national policy position is more popular in those districts, and vice versa for Republicans in conservative districts. Given the higher electoral success rate of Democrats in liberal than in conservative districts, the density of Democratic districts in the legislature is downward-sloping, and thus the median Democrat hails from a relatively liberal district. Conversely, Republicans are concentrated in relatively conservative districts, implying that the median Republican comes from a conservative district, and thus justifying why voters associate Democrats with liberal positions, and Republicans with conservative ones.

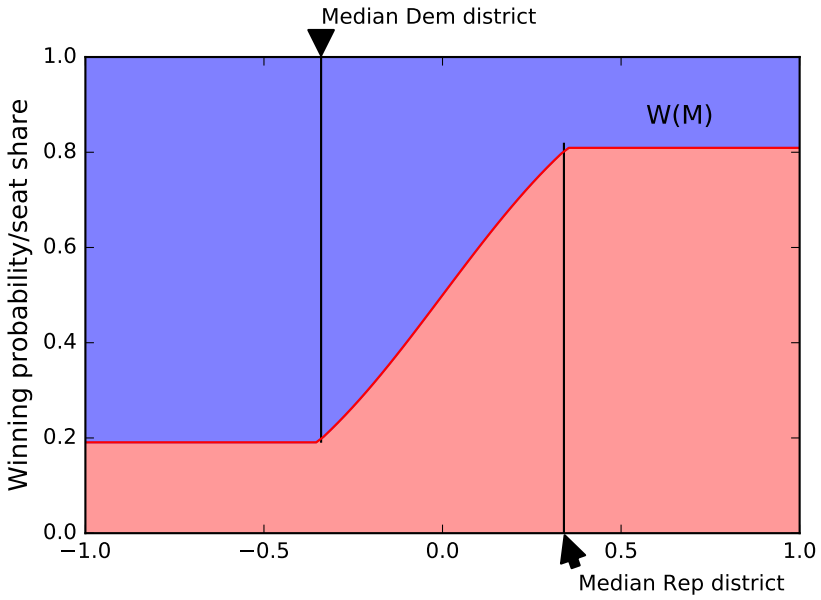


Figure 1: Republican winning probability $W(M)$ by district and the determination of party positions

The existence of an equilibrium with policy differentiation depends on the net valence shock being sufficiently concentrated around zero (i.e., $\phi(0)$ is sufficiently large). If the

valence shocks are generally small, then most liberal and conservative districts will not experience shocks that are large enough for the median voter to go against her policy preference, and so liberal districts are mostly Democratic, and conservative districts mostly Republican, supporting equilibrium policy differentiation. In contrast, if the valence shocks are too large, then these shocks will be almost all that matters for who gets elected. In this case approximately half of the liberal districts will be won by Republicans, and half of the conservative districts by Democrats, so the medians of the two caucuses will be the same.

Before we proceed with the analysis, it is useful to show that the results of our model remain unchanged if voters instead care both about national platforms *and* their local representative's position (as claimed above). To see this, suppose that voter M's policy utility is $u(x, M) = -|x - M| - \gamma|x_M - M|$, where x stands for the national policy as before, and x_M is the position chosen by the local representative. The weight parameter $\gamma > 0$ depends on the weight voters put on the local policy component, relative to the party position, and could reflect the proportion of legislative votes in which candidates are free to "vote their district," relative to those votes in which he has to follow the party line. Since candidates are office-motivated, standard arguments imply that both local candidates set their equilibrium position to the ideal position of the district median voter.[11] Consequently, these terms would simply cancel against each other in (1), and all results would therefore be unaffected.

3.2 Stability

Propositions 1 and 2 characterize equilibria without and with policy divergence between parties. Equilibria are, by definition, profiles where voters in each district choose candidates based on their expectations about the parties' positions in Congress, and their choices yield party caucuses that exactly justify the voters' expectations.

For two separate, but interrelated, reasons, it is important to think about the stability of these equilibria. First, in practice, voters may base their decision whom to vote for in an election not on their rational expectations of party positions (which may be difficult to form), but rather on their observations of the parties' positions in the outgoing legislature. In this case, the party positions form a dynamic system, and the decisive question is whether this system converges to one of the equilibria.

Second, in a legislature with finitely many representatives, the composition of the two parties' caucuses is uncertain. Thus, the system is never exactly at the equilibrium, and

[11] This assumes (just like in the basic model) that candidates choose their positions before valence is realized. If valence is known before positions are chosen, then one candidate can (generically) win with any position in a certain interval.

therefore the question whether dynamic forces move the system in the direction of an equilibrium, or away from it, is particularly relevant.[12]

Suppose that the legislature is elected based on the belief that the Republican position is at x and the Democratic position is at $-x$. Substituting this in (6), and rearranging, we obtain that an equilibrium is characterized by a zero of the following function $Z(x)$.

$$\begin{aligned} Z(x) &= \frac{1}{4k}\left\{\int_{-k}^{-x}[1-W(M)]dM - \int_{-x}^{k}[1-W(M)]dM\right\} \\ &= \frac{1}{4k}\left\{(k-x)\Phi\left(\frac{2x}{\alpha}\right)dM - \int_{-x}^{x}\Phi\left(\frac{-2M}{\alpha}\right)dM - (k-x)\Phi\left(-\frac{2x}{\alpha}\right)dM\right\} \qquad (7) \\ &= \frac{1}{4k}\left\{(k-x)\left[2\Phi\left(\frac{2x}{\alpha}\right)-1\right]-x\right\} \end{aligned}$$

When $Z(x)$ is positive, then more Democrats hail from districts between $-k$ and $-x$ than from the remaining districts, so that the median Democratic caucus position is to the left of $-x$. Conversely, the median Republican caucus position is to the right of x. Thus, both parties' caucuses end up more extreme than expected, and if expectations are adaptive, a positive $Z(x)$ should lead to increased policy divergence in the next period, and vice versa.

An equilibrium is stable if deviations from the equilibrium lead to adjustments that lead back to the equilibrium. Formally,

Definition 1. *Let x_0 denote the degree of policy differentiation at an equilibrium, i.e., $Z(x_0) = 0$. An equilibrium is (globally) stable if and only if $Z(x) > 0$ for all $x \in (0, x_0)$ and $Z(x) < 0$ for all $x > x_0$.*

In the proof of Proposition 2, we show that $Z(\cdot)$ is globally concave. Therefore, if $Z'(0) > 0$ — such as in the left-hand panel of Figure 2 — so that an equilibrium with $x > 0$ exists, it must be the case that $Z'(x) < 0$ at the interior equilibrium $\bar{x}$, and this implies that this equilibrium is stable. Conversely, if $Z'(0) < 0$, such as in the right-hand panel of Figure 2, then only the equilibrium without policy differentiation exists, and it is stable. Summarizing this discussion, we have

Corollary 2. *Suppose that the distribution of districts is uniform on $[-k, k]$ and the valence distribution ϕ is single-peaked and symmetric around 0. Then, there is a unique stable equilibrium. If $\phi(0) > \frac{\alpha}{4k}$, this stable equilibrium involves divergent party platforms, while if $\phi(0) \leq \frac{\alpha}{4k}$, the unique stable equilibrium has $x_D = x_R = 0$.*

[12]The convention in most of the existing literature on political competition is to focus on one-shot models of elections with the (usually unstated) presumption that the equilibrium repeats in practice. Of course, this approach is only justified if the equilibrium is indeed stable.

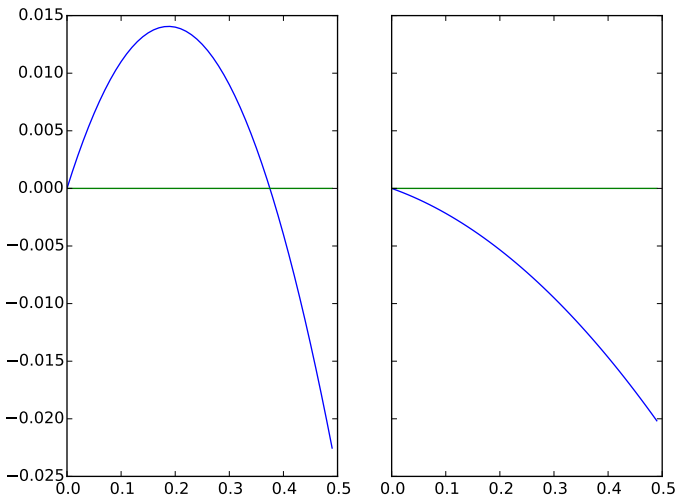


Figure 2: The function $Z(x)$ and equilibrium policy divergence

Intuitively, the extent of uncertainty affects the stability of the full convergence equilibrium as follows. Consider a small deviation such that Republicans are a bit more conservative than Democrats. If uncertainty is sufficiently small, then net valence realizations are close to zero in almost all districts, and all voters follow their policy preference in choosing whom to elect; in most liberal districts, this will be the Democrat, and in most conservative districts, it will be the Republican. The resulting legislature will therefore have more liberal Democrats and more conservative Republicans, so the political system will move farther away from the initial full convergence equilibrium, which is therefore unstable.[13]

In contrast, if valence uncertainty is very large in every district, then the small initial policy difference between the parties has hardly any effect on who gets elected in liberal or conservative districts, because that is primarily determined by the realization of net valence. Consequently, the parties look more similar after the next round of voting, and the party positions converge back to the full convergence equilibrium.

3.3 Comparative statics

We now show that, as valence becomes less important to voters, the extent of policy differentiation between the parties increases in equilibrium. Intuitively, as the importance of valence decreases for voters, a district is less likely to vote for the ideologically disadvantaged

[13]In addition, this argument shows that the divergence equilibrium is stable for sufficiently small uncertainty: If there is a deviation to smaller levels of divergence, then the above argument implies that divergence will grow next period.

party's candidate. This effect leads to ideological stratification of Congress, and that in turn implies that the median party member now comes from a more extreme district.

Proposition 3. *Suppose that the distribution of districts is uniform on* $[-k, k]$ *and the valence distribution* ϕ *is single-peaked and symmetric around 0. Furthermore, suppose that* $\phi(0) > \frac{\alpha}{4k}$*, and consider the symmetric equilibrium with policy differentiation where the parties' equilibrium positions are* $x_R = x = -x_D$*. An increase in* α *leads to less policy differentiation:* $\frac{dx}{d\alpha} < 0$*.*

Proof. Applying the implicit function theorem to the equilibrium condition $Z(x) = 0$ yields

$$\frac{dx}{d\alpha} = -\frac{\frac{\partial Z}{\partial \alpha}}{Z'(x)} = \frac{\frac{-x(k-x)}{k\alpha^2}\phi\left(\frac{2x}{\alpha}\right)}{-Z'(x)} < 0, \tag{8}$$

because $x \in (0, k)$ and $Z'(x) < 0$ at an interior equilibrium (see proof of Proposition 2). □

Proposition 3 predicts that reforming the organization of the legislature in a way that affects how much an individual legislator can do for her constituents also has an effect on polarization. For example, consider a reform that makes it harder for individual legislators to acquire "pork barrel" projects that benefit his district—say, the total amount of pork available for the legislature as a whole is reduced by half, and legislators now compete with each other about this smaller prize. Then, the utility that voters in the district have from a legislator with a given ability to attract pork diminishes. As α decreases, the argument for holding on to an incumbent whose party is a bad ideological fit for the district is diminished. Consequently, we would expect that a reduction in the importance or availability of pork projects leads to ideological polarization.

Another feature that potentially may affect the importance of valence for voters has to do with the geographical shape of districts. We may think of legislators' valence being related to their ability to bring local public goods or employment on public projects to their district. However, a part of the benefit of these projects will spill over into other districts—say, a firm in the legislator's district that gets a federal grant may employ some citizens who are residents in other districts, or residents of other districts may benefit from road construction in the district. Such spillovers, and the concomitant reduced incentive to provide such local public goods, are likely more significant when the ratio of district boundary to district area is high, such as in many skillfully gerrymandered districts.

It is also interesting to compare the result of Proposition 3 with the effect that would arise in the deterministic valence advantage model of Aragones and Palfrey (2002). In the latter model, an increase in the importance of a fixed valence advantage increases the need for the weaker candidate to differentiate himself more starkly from his stronger opponent.

This effect leads to larger polarization, the opposite from the comparative static prediction of Proposition 3.

The same effect arises in a model framework analyzing multiparty spatial competition with probabilistic voting, pioneered by Schofield and others in a series of articles and a book.[14] In the model, high-valence parties adopt positions near the mean of the voter ideal point distribution, while low-valence parties are forced to adopt extremist positions (and hope for a large preference shock in their favor). If valence becomes more important, then the low valence party is forced to move farther away from the center, thereby increasing polarization.[15]

We now turn to an effect that is very similar from a formal point of view, namely the degree of uncertainty about the shock. One of the main explanations in the literature for why we observe policy divergence in elections is the policy-motivated candidates model, following the seminal contributions of Wittman (1983) and Calvert (1985). In those models, candidates choose their position to trade off an increased probability of winning with a more moderate policy against the lower satisfaction for the candidate that comes from implementing a less preferred policy in case the candidate wins.[16]

However, if the preference of the median voter is known, models with policy-motivated candidates have the same equilibrium as the original Downsian model with office-motivated candidates. Therefore, uncertainty (either about the median voter's preferred policy, or about his evaluation of the candidates' valences) is essential in these models in order to generate policy divergence. Intuitively, larger uncertainty implies that it is less likely that the election outcome is very close and therefore that a candidate could affect it by compromising and moving to a more moderate position. Thus, the more uncertainty there is in an election about the voters' preferences, the greater will be the degree of political polarization between the candidates' policies in equilibrium in these models of policy-motivated candidates. See, for example, Smirnov and Fowler (2007).

In contrast, we will now show that increased uncertainty about valence in our model of political competition in legislative elections reduces polarization. The intuitive reason is that large valence shocks help some Republicans win in liberal districts, and some Democrats

[14]Schofield (2003, 2004, 2006, 2007), Schofield and Sened (2006), and Schofield and Miller (2007).

[15]It is also interesting to note that in both the Schofield and Aragones and Palfrey models, there is a natural symmetry of the policy space: For example, there is no reason for the weaker parties to get "stuck" on one side of the median (or mean) voter for an extended period of time, so these models predict that which party is, say, the left party, fluctuates over time.

[16]Wittman (1983) assumes that electoral uncertainty arises because candidates are not perfectly informed about voter preferences and voters are not perfectly informed about candidates' policy platforms. Calvert (1985) assumes each candidate has a "subjective probability of winning" function. Roemer (1994) derives electoral uncertainty by modeling candidate uncertainty about voter preferences.

win in conservative districts. These representatives have a strong interest in moving their party to a more moderate position in order to stay competitive in their district. In contrast, when valence shocks are generally small, then the overwhelming majority of liberal and conservative districts are won by Democrats and Republicans, respectively, and the two parties thus represent essentially disjunct sets of districts, so that inter-party polarization is large.

To formally analyze the effect of a change in the distribution of the valence shock on the equilibrium level of policy divergence, consider the following definition of increased valence uncertainty.

Definition 3 (Increased risk of the valence shock.)**.** *We say that β parametrizes a* **higher risk of the valence shock** *if the following conditions hold:*

1. *$\Phi(v, \beta)$ is symmetric around 0 for all β; and*
2. *$[\Phi(v, \beta_1) - \Phi(v, \beta_0)]v \leq 0$ for $\beta_1 > \beta_0$;*

Note that the second condition says that $\Phi(v, \beta_1) \geq \Phi(v, \beta_0)$ for $v < 0$ and $\Phi(v, \beta_1) \leq \Phi(v, \beta_0)$ for $v > 0$. This higher risk definition is slightly stronger than a standard mean-preserving spread because we assume that the Φ-distributions for different values of β intersect only once, at 0. However, many common families of distributions satisfy this assumption. For example, it is satisfied if Φ is a uniform distribution on $[-\beta, \beta]$, or if Φ is a normal distribution $N(0, \beta^2)$ with standard deviation β.

Proposition 4 shows that a higher of the valence shock leads to an equilibrium with less polarization.

Proposition 4. *Let β parametrize a higher risk of the valence shock, and let $\beta_1 > \beta_0$. Suppose that the distribution of districts is uniform on $[-k, k]$ and the distribution ϕ is symmetric around 0 and single-peaked. Furthermore, suppose that $\phi(0, \beta_0) > \frac{\alpha}{4k}$, and consider the symmetric equilibrium with policy differentiation where the parties' equilibrium positions are $x_R = x(\beta_0) = -x_D$ for the valence distribution $\Phi(v, \beta_0)$. Then $x(\beta_1) < x(\beta_0)$.*

Proof. Proceeding like in the proof of Proposition 3, we have

$$\frac{dx}{d\beta} = -\frac{\frac{\partial Z}{\partial \beta}}{Z'(x)} = \frac{\frac{(k-x)}{2k}\frac{d\Phi\left(\frac{2x}{\alpha}, \beta\right)}{d\beta}}{-Z'(x)} \leq 0, \tag{9}$$

since $\frac{d\Phi\left(\frac{2x}{\alpha}, \beta\right)}{d\beta} \leq 0$ for all $x > 0$ by Definition 3, and $Z'(x) < 0$. □

Figure 3 illustrates Proposition 4. Staring from a case of relatively large valence shocks, a lower risk of the valence distribution leads to a steeper cumulative distribution function. The resulting regions between the two cdfs depict Democratic gains in liberal districts and Republican gains in conservative ones, both of which make the two parties more extreme.

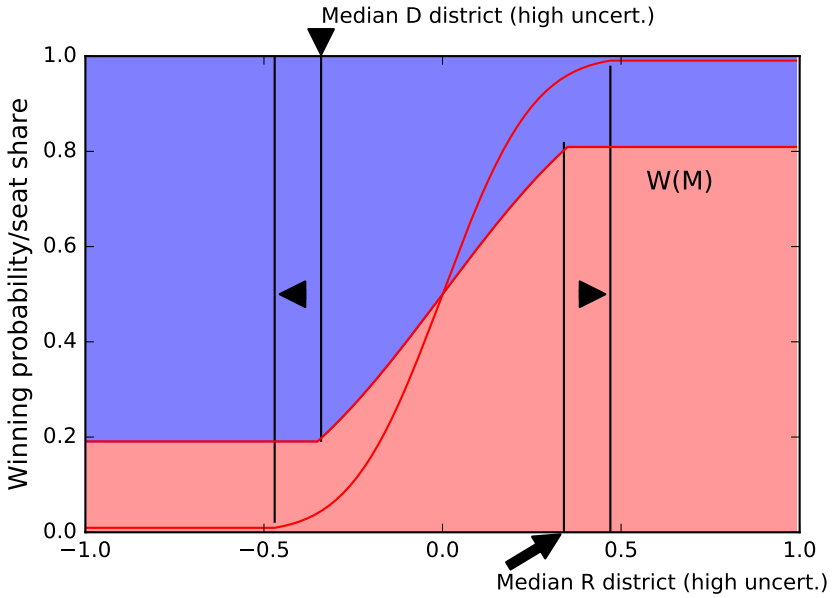


Figure 3: Valence uncertainty and party positions

Finally, it is important to point out that our model and the Wittman (1983) and Calvert (1985) models also differ in whether "unexpected" uncertainty has an effect on polarization. In the Wittman-Calvert model, uncertainty only affects the candidates' positions if they expect the uncertainty and therefore choose divergent positions. If candidates believed that there was no uncertainty, they would choose the same position, and so there is no polarization even if it turns out, ex-post, that the candidates' prediction was wrong (i.e., if the realized median voter's position was different from the candidates' belief, or if the median voter has a strict valence preference for one of the candidates). In contrast, in our model, the *realized* level of uncertainty in all districts matters for polarization, as it affects the composition of the legislature.

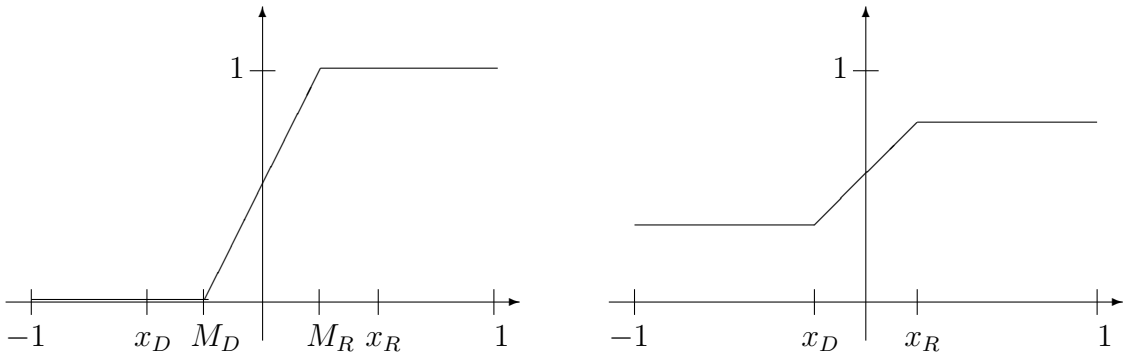
Also, if we are thinking of campaigns in real-time, an increase in valence uncertainty during the campaign would lead to a readjustment in candidates' positions in the campaign in the Wittman-Calvert model, while in our model, the effect of this increased uncertainty (if it applies to all districts) would be seen after the campaign in the newly elected set of incumbents. While certainly empirically very challenging, this is, in principle, a testable

difference between the models.[17]

3.4 A numerical example

In this subsection, we provide a brief numerical example of the model, for the case that net valence is distributed uniformly on $[-\beta, \beta]$, so that the cumulative distribution is given by $\Phi(t) = \frac{\beta+t}{2\beta}$. Also, we assume that $k = 1$.

It is useful to remember that in all districts to the left of x_D, the Republican's net valence must be greater than his policy disadvantage in these districts, $x_R - x_D$, for the Republican to win, and analogously for the districts to the right of x_R. Thus, if $x_R - x_D = 2x \leq \alpha\beta$, then the Democrat's winning probability in the extreme liberal districts is $\Phi(2x/\alpha) = \frac{\beta+2x/\alpha}{2\beta} = \frac{1}{2} + \frac{x}{\alpha\beta}$, while if $2x > \alpha\beta$, then the Democrat is certain to win in extremely liberal districts (beyond M_D in Figure 4a). Of course, this applies analogously in the conservative districts (beyond M_R) for the Republican candidate.



(a) Small valence shocks

(b) Large valence shocks

Figure 4: Republican winning probability

If $\alpha\beta$ is sufficiently small, then the Republican winning probability remains 0 even in some districts that are more moderate than x_D (see Figure 4a). In contrast, if shocks can be sufficiently large, then winning probabilities remain interior for all districts. In fact, because the policy preference for the Republican is the same in all districts that are more conservative than x_R, the winning probability is the same for all these districts, and similarly for all districts that are more liberal than x_D.

In the Appendix, we show that the Republican equilibrium position in this example is

[17] We are grateful to a referee for proposing this interpretation.

given by

$$x_R = -x_D = \begin{cases} \frac{1}{2} & \text{if } \alpha\beta \in (0,1] \\ 1 - \frac{\alpha\beta}{2} & \text{if } \alpha\beta \in (1,2] \\ 0 & \text{if } \alpha\beta > 2 \end{cases}. \tag{10}$$

Note that this example displays the properties described in Propositions 1, 3 and 4: The equilibrium features polarization unless the valence uncertainty is too large, and increases in uncertainty or in the importance of valence (weakly) reduce the level of polarization.

However, note that, if valence shocks are sufficiently small (i.e., in the case of $\alpha\beta < 1$), marginal changes in α or β have no effect on polarization. Intuitively, such changes affect the positions of M_D and M_R in Figure 4a and how steeply the winning probability increases, and thus leads to different seat distributions *among the more moderate half* of each caucus; however, these changes do not affect at all the more extreme half of each caucus, and therefore do not change the median.

3.5 The Effect of Wave Elections

So far, we have interpreted valence as an exclusively candidate-specific property. We now analyze the effects of an additional (and deterministic) party-specific valence shock favoring, say, all Republicans candidates. One can think of such a party valence effect as stemming from anything that makes all Republicans more popular relative to Democrats. In particular, there is a widespread notion that some midterm elections are a "referendum on the President" (even though the President is not on the ballot), implying that the popularity of the incumbent President can create a systematic shock. For example, the approval ratings of President Bush in 2006 and President Obama in 2010 were the lowest in recorded history, and both were associated with large swings in votes and seats against the presidents party, large enough to change party control of Congress.

Intuitively, the effect of a positive shock for the Republicans is that some Democrats will be replaced by Republicans. Moreover, those Democrats who hail from conservative and moderate districts are more endangered by the resulting Republican tide than those located in liberal districts. Thus, most freshman Republicans in the new legislature represent relatively moderate districts, and therefore their self-preservation interest is to draw the Republican party position towards moderation. In contrast, the surviving Democrats will, on average, represent more liberal districts than before, because those from moderate and conservative districts will have been defeated in disproportionate numbers. We would therefore expect that the Democratic position in the new legislature becomes more liberal.

To analyze this setting formally, we assume that the Republican candidates' valence

is now drawn from a uniform distribution on $[-\beta + s, \beta + s]$, for $s > 0$. For notational convenience, we normalize $\alpha = 1$, which is without loss of generality as the results of the last section have shown that only the value of the product $\alpha\beta$ matters for results.

We first turn to the small valence shock case from Figure 4a. The location of M_D and M_R is determined by the following equations which say, respectively, that the voter at M_D must be indifferent between the Democrat and the best possible Republican, and that the voter at M_R must be indifferent between the Democrat and the worst possible Republican.

$$-(x_R - M_D) + s + \beta = -(M_D - x_D) \tag{11}$$

$$-(x_R - M_R) + s - \beta = -(M_R - x_D) \tag{12}$$

Observe that the Republican winning probability between M_D and M_R is linear in M, so that the average Republican winning probability in that range is simply $1/2$. This yields the following two equations for x_D and x_R.

$$x_D + 1 = M_D - x_D + \frac{M_R - M_D}{2} \tag{13}$$

$$1 - x_R = \frac{M_R - M_D}{2} + x_R - M_R \tag{14}$$

For example, (14) says that the number of seats to the right of x_R (which are all won by the Republican candidate) must equal the number of districts between M_R and x_R (which are also all won by Republicans) plus half the number of districts between M_D and M_R (since those districts are won by Republicans only half of the time).

Solving the equation system (11) – (14) yields

$$x_D = -\frac{1+s}{2}, \quad x_R = \frac{1-s}{2}, \quad M_D = -\frac{\beta}{2} - s, \quad M_R = \frac{\beta}{2} - s. \tag{15}$$

Note that we derived the equation system (11) – (14) based on the assumption that $x_D \leq M_D < M_R \leq x_R$. The solutions in (15) satisfy this assumption if and only if $s \in [\beta - 1, 1 - \beta]$. Since $\beta < 1$ is anyway assumed for the case of small valence shocks, the relevant constraint here is $s \leq 1 - \beta$. Within this range, it is obvious that both x_D and x_R are decreasing functions of s.

We now turn to the case that still $\beta < 1$, but $s > 1 - \beta$. In this case, the Republican winning probability will remain positive even in districts in $[-1, x_D]$.

Specifically, a voter located at x_D (or to the left of that point) will prefer the Republican if

$$-(x_R - x_D) + s + v \geq 0,$$

hence, with probability $1 - \frac{\beta - s + x_R - x_D}{2\beta}$. On the conservative side, in contrast, there will still be a district M_R at which the Republican winning probability reaches 1, and it continues to be determined by (12).

The equations for the Democratic and Republican median have to be adjusted as follows:

$$(1 + x_D)\frac{\beta - s + x_R - x_D}{2\beta} = (M_R - x_D)\frac{\beta - s + x_R - x_D}{4\beta} \quad (16)$$

$$(1 + x_D)\left[1 - \frac{\beta - s + x_R - x_D}{2\beta}\right] + (M_R - x_D)\frac{3\beta + s + x_D - x_R}{4\beta} + x_R - M_R = 1 - x_R \quad (17)$$

The left-hand and right-hand sides of (16) and (17) are the Democratic and Republican seats to the left and to the right of the respective medians. Observe that a party's average winning probability in the competitive range $[x_D, M_R]$ is equal to the average of their respective winning probability at x_D and at M_R (because the winning probability is linear in M).

Solving (12), (16) and (17) yields

$$x_D = \frac{5\beta - 4 - \sqrt{\beta\,(17\beta + 8s - 8)}}{4}, \quad x_R = \frac{21\beta - 4(1 - s) - 5\sqrt{\beta\,(17\beta + 8s - 8)}}{4}, \quad M_R = \frac{15\beta - 4 - 3\sqrt{\beta\,(17\beta + 8s - 8)}}{4}. \quad (18)$$

One can check that $x_R \geq M_R$ is indeed always satisfied. The condition that x_D must be larger than -1 simplifies to $s \leq 1 + \beta$. If s is larger than this value, the entire Democratic caucus will be wiped out, and the median of the Republican caucus is then at 0 (as Republicans represent all districts).

For solutions given by (18), x_D is clearly decreasing in s. Differentiating x_R with respect to s yields

$$\frac{\partial x_R}{\partial s} = 1 - \frac{5\sqrt{\beta}}{\sqrt{17\beta + 8s - 8}} \quad (19)$$

This is negative whenever $s > 1 - \beta$, i.e., for all values in the relevant range.

Finally, consider what happens when β is large so that a case similar to the one in Figure 4b arises, where both parties have a positive winning probability in each district.

In this case, there are only two conditions, namely the ones for the median x_D and x_R, which are modified as follows:

$$(1 + x_D)\frac{\beta - s + x_R - x_D}{2\beta} = (x_R - x_D)\frac{\beta - s}{2\beta} + (1 - x_R)\frac{\beta - s - (x_R - x_D)}{2\beta} \quad (20)$$

$$(1 + x_D)\frac{\beta + s - (x_R - x_D)}{2\beta} + (x_R - x_D)\frac{\beta + s}{2\beta} = (1 - x_R)\frac{\beta + s + (x_R - x_D)}{2\beta} \quad (21)$$

The solution of this equation system is given by

$$x_D = -\frac{\beta + s}{2\beta^2}\left[s^2 + 2\beta\left(1 - \frac{\beta}{2}\right)\right], \quad x_R = \frac{\beta - s}{2\beta^2}\left[s^2 + 2\beta\left(1 - \frac{\beta}{2}\right)\right] \tag{22}$$

We must have $x_D \geq -1$, which is equivalent to $s^3 + \beta s^2 + (2\beta - \beta^2)s - \beta^3 < 0$. While an explicit solution of this inequality is tedious, it is clear that the solution set is non-empty. For example, for $\beta = 2$, all $s < 1.5$ satisfy this condition.

Differentiating the solutions in (22) with respect to s yields

$$\frac{\partial x_D}{\partial s} = \frac{\partial x_R}{\partial s} = -\frac{3s^2 + 2\beta\left(1 - \frac{\beta}{2}\right)}{2\beta^2} < 0. \tag{23}$$

Thus, in all cases, a positive shock s for the Republicans leads to a leftward shift of both parties, making Democrats more extreme and Republicans more moderate.

Summarizing these results, we have proved the following proposition.

Proposition 5. *Suppose that districts are distributed uniformly on* $[-1, 1]$*, and that individual net valence shocks are distributed uniformly on* $[-\beta + s, \beta + s]$*, with* $s > 0$*.*

Whenever the original situation has an equilibrium with differentiation, and both parties are represented in the legislature, then an increase in s will move both party positions to the left.

We conclude this section by discussing an alternative way of modeling shocks that are favorable for Republicans, namely to assume that the ideal points of all voters move to the right. Note that it is less clear how to fit this approach in our model framework: If every voter's ideal point shifts, both Republican and Democratic representatives now stand for more conservative policies, and so such a shock would not necessarily translate into an electoral advantage for Republicans.

Furthermore, if the shock is a policy preference shock, one would have to separate the resulting shift in equilibrium party positions into one arising from popular demand for different policies (because of the preference shift) and another one working through a change of party representation in the legislature. Our modeling approach with a valence shock has the advantage that voters' policy preferences remain constant, and so the equilibrium policy shift is due only to the effect that the shock influences the composition of party caucuses in the legislature.

4 Empirical Analyses

4.1 Incumbent District Ideology vs. Party Ideology

At a basic level our model implies that there should be a strong, positive relationship between (i) the ideological distance between districts held by Republican and Democratic incumbents and (ii) the ideological gap between the parties' platforms. One reasonable proxy for district ideology, at least for relatively recent years, is the presidential vote—districts with a higher Republican share of the two-party presidential vote are more conservative than those with a lower share. As in our model, we proxy for each party's platform by the median ideology of the party's Congressmen based on their roll call voting records. When we do this for the U.S. House of Representatives for the period 1976-2012 we obtain Figure 5.

We construct the "conservative ideology score" of each district i in each year t, C_{it}, by averaging the Republican share of the two-party vote in the two presidential elections closest to year t.[18] We then define the *Polarization of Incumbents' Districts* in year t as $(\sum_{i\in\mathcal{R}_t} C_{it}/N_t^R) - (\sum_{i\in\mathcal{D}_t} C_{it}/N_t^D)$, where $\mathcal{R}_t$ and $\mathcal{D}_t$ are the sets of districts held by Republicans and Democrats in year t, and N_t^R and N_t^D are the numbers of districts held by Republicans and Democrats in year t. This is the gray curve in the figure, and the scale is given by the axis on the right.[19]

We use the well-known DW-NOMINATE scores to measure ideological positions.[20] Specifically, we define the degree of polarization in year t, *Polarization in US House*$_t$, is simply the difference between the median DW-NOMINATE score of Republican incumbent members of congress in year t and the median DW-NOMINATE score of Democratic incumbent members of congress in year t. This is the black curve in the figure, and the scale is given by the axis on the right.

Evidently, the two curves both trend sharply upward, and the two variables are therefore highly correlated ($\rho = 0.97$). Of course, factors other than those featured in our model might explain the common trends, but, at least, the evidence points in the right direction.

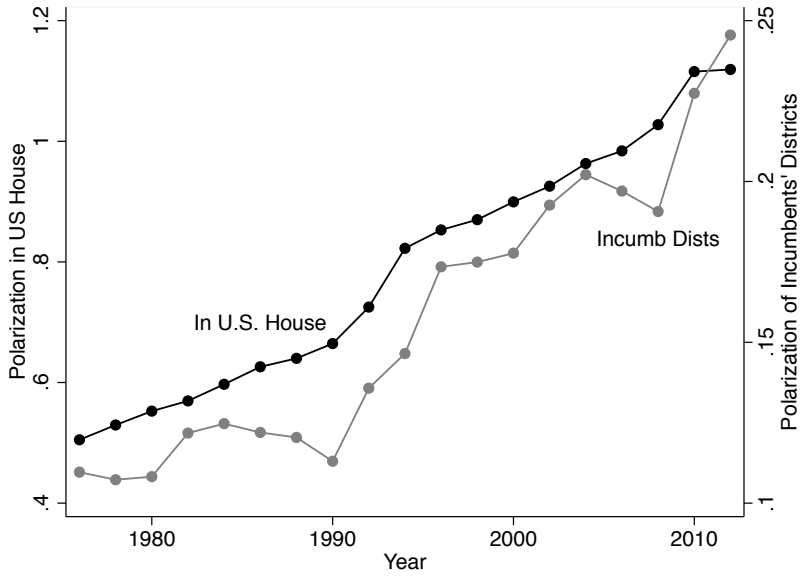
The same basic relationship holds for state legislatures. We again use the presidential vote as a proxy for district ideology, and for simplicity we focus on lower chambers. In this case we only have the results of the 2000 presidential election at the state legislative district

[18]Using the two closest Presidential elections under the same district lines smooths the potential effect of blowout presidential elections on this measure of district ideology. Note also that, since Congressional districts are redistricted after each decennial census, $C_{i,2002}$, for example, is calculated based on the 2004 and 2008 Presidential election returns in district i.

[19]The figure looks very similar if we use medians rather than means.

[20]The data is at http://voteview.com/dwnomin.htm. The scores range from about -0.75 to 1.35; the overall mean is 0.03 and the overall standard deviation is 0.44. For Democrats overall, the mean is -0.32 and the standard deviation is 0.16, and for Republicans the mean is 0.46 and the standard deviation is 0.24.

Figure 5: Polarization in U.S. House vs.
Polarization of Incumbent Districts



level. Thus, the "conservative ideology score" of district i in state j in year t, C_{ijt}, is the Republican share of the two-party vote in 2000, and the *Polarization of Incumbents' Districts* for state j in year t is $(\sum_{i\in\mathcal{R}_t} C_{ijt}/N_{jt}^R) - (\sum_{i\in\mathcal{D}_t} C_{ijt}/N_{jt}^D)$, where $\mathcal{R}_{jt}$ and $\mathcal{D}_{jt}$ are the sets of state lower house districts held by Republicans and Democrats in state j in year t, and N_{jt}^R and N_{jt}^D are the numbers of state house districts held by Republicans and Democrats in state j in year t. We can construct this measure for 122 state houses for the period 1992–2000 and for 123 state houses for the period 2002-2010.[21]

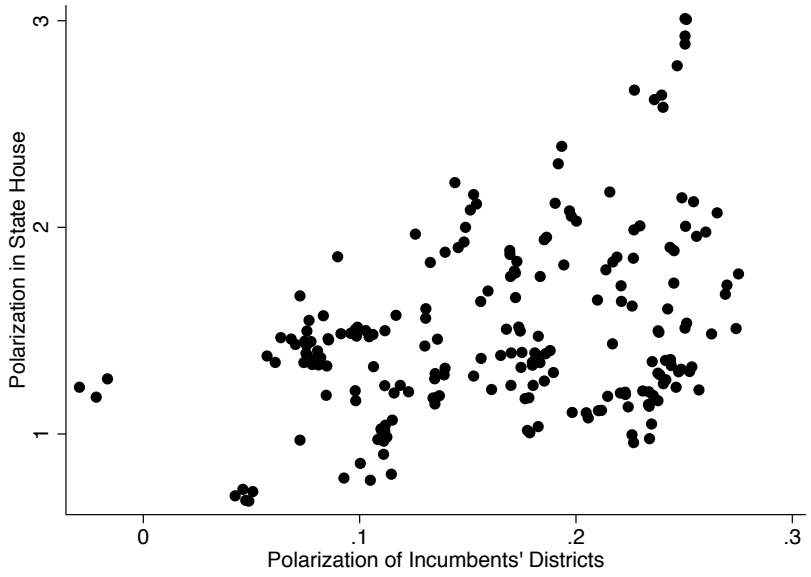
Similarly, we measure legislative polarization using roll-call data. McCarty and Shor estimated ideological scores for almost 21,000 state legislators from all 50 states elected over the period 1993 to 2014.[22] They calibrated the scores using the results of a large-scale survey (NPAT) in order to make the scores comparable across states. The scores are oriented so that more conservative legislators have higher scores.[23] We define the *Polarization in State House*$_{jt}$ as the difference between the median roll-call score of Republican incumbent state house members in state j in year t and the median roll-call score of Democratic incumbent state house members in state j in year t.

[21]The results are similar if we use medians rather than means. They are also similar if we use both upper and lower chambers and average the gaps in the two chambers.

[22]The data is at http://americanlegislatures.com/data/. See Shor and McCarty (2011) for more details. Data is missing for some years for some states.

[23]The scores range from about -3.5 to 5; the overall mean is 0.02 and the overall standard deviation is 0.89. For Democrats overall, the mean is -0.74 and the standard deviation is 0.53, and for Republicans the mean is 0.75 and the standard deviation is 0.44.

Figure 6: Polarization in State House vs.
Polarization of Incumbents' Districts



The correlation between *Polarization in State House* and the *Polarization of Incumbents' Districts* in Figure 6 is 0.42.

4.2 Electoral Uncertainty and Polarization in State Legislatures

One of the key predictions of the model is that if electoral uncertainty increases, then the degree of polarization between the parties should decrease (Proposition 4). Recall that this is the opposite of the prediction from other models, such as the Calvert-Wittman model. We can test this prediction, at least in a crude fashion, by studying U.S. state legislatures. We find that polarization is strongly and negatively correlated with measures of electoral uncertainty.

We define the *Polarization in State House*$_{jt}$ as above—the difference between the median roll-call score of Republican incumbent state house members in state j in year t and the median roll-call score of Democratic incumbent state house members in state j in year t.

Updating the data used in Ansolabehere and Snyder (2002) and Hirano and Snyder (2014), we have constructed a dataset of election results for all offices elected statewide in each state— U.S. senator, governor, lt. governor, attorney general, secretary of state, state treasurer, etc.—as well as state-level presidential election results, for the period 1988–2014. Let *State Electoral Uncertainty*$_{it}$ be the standard deviation of the Democratic vote-share

across all offices elected in state i between years $t-3$ and t.[24] This is a proxy for the electoral uncertainty facing candidates. When *State Electoral Uncertainty* is large, then it is likely that idiosyncratic factors specific to particular races, such as incumbency, candidate attributes and positions on particular issues have a substantial effect on the vote in the district. Furthermore, it could be that the partisan composition of the district electorate exhibits large swings from election to election. In contrast, when *State Electoral Uncertainty* is small, then the vote is probably driven more by stable partisan loyalties.[25]

Figure 7 shows a scatterplot of *Polarization in State House* and *State Electoral Uncertainty*.[26] The correlation between the two variables is -0.44.

Figure 7: Polarization in State House
vs. State Electoral Uncertainty

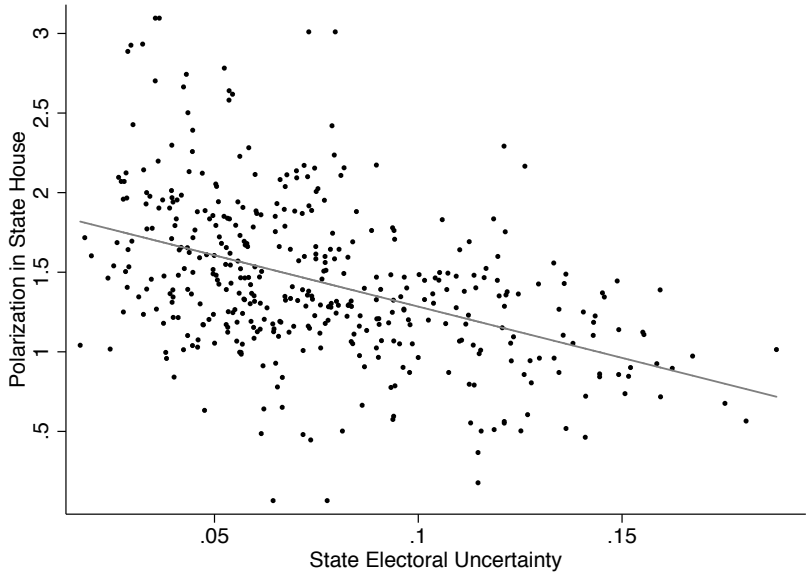


Table 1 presents regression results showing that the correlation between the two variables is not only highly significant both substantively and statistically, but also that the correlation remains even after controlling for a time trend. This is true for each chamber separately as

[24]Note that we use state-wide offices only, which have the same constituency—i.e. the entire state. We do not use districted offices such as U.S. House or state legislative seats. We only include cases with 4 or more races. Note also that we are using the elections from mostly non-legislative offices; as Ansolabehere and Snyder (2002) show, the electoral results for these offices show patterns similar to those for the U.S. House and Senate in terms of incumbency advantages, midterm slumps, and overall variation, so we feel comfortable using the results for these offices to proxy for the uncertainty faced by a state's legislators.

[25]We investigated alternative measures of electoral uncertainty, such as the standard deviation of the Democratic vote-share across all offices elected in state i in year t alone, and find results quite similar to those reported below.

[26]We match *State Electoral Uncertainty* for state i and year t to polarization among legislators in state i elected in year t and serving in years $t+1$ and $t+2$.

well (columns 3–6). We do not claim the results in Table 1 establish causality of any sort, of course, but they do indicate a surprisingly large, negative, correlation.[27]

Table 1: **Polarization vs. Electoral Uncertainty in State Legislatures**

Variable	House/Senate Average		State House Only		State Senate Only	
State Electoral Uncertainty	-6.230	-5.874	-6.442	-6.111	-5.978	-5.697
	(0.581)	(0.586)	(0.647)	(0.660)	(0.653)	(0.663)
Year	–	0.011	–	0.009	–	0.009
		(0.003)		(0.004)		(0.004)
# Observations	460	460	413	413	417	417

Dependent variable = Polarization in state legislature or state legislative chamber. Standard errors are in parentheses.

4.3 Electoral Uncertainty and Polarization in the U.S. House

We can perform a similar analysis of the effects of electoral uncertainty on polarization for the U.S. Congress. This has one advantage and one disadvantage relative to state legislatures. The advantage is that we can measure electoral uncertainty at the *district* level, which corresponds more closely to parameter β in the model. The disadvantage is that we can only exploit variation over time.

We focus on the U.S. House for the period 1972 to 2012. For each district i and year t, let *Electoral Uncertainty*$_{it}$ be the standard deviation of the Democratic vote-share across all presidential and congressional elections held in the district between years $t-6$ and $t+6$.[28] Let *Electoral Uncertainty*$_t = (1/435)\sum_{i=1}^{435}$ *Electoral Uncertainty*$_{it}$ be the average standard deviation across districts for each year t. This is a proxy for the electoral uncertainty facing candidates in year t.

We define the *Polarization in US House*$_{jt}$ as above—the difference between the median DW-NOMINATE score of Republican incumbent members of congress in year t and the median DW-NOMINATE score of Democratic incumbent members of congress in year t.

We find a very strong negative correlation of -0.94 between *Polarization in US House*

[27]For example, the coefficients might reflect causation in the opposite direction—i.e., as the parties become more polarized voters might engage in more purely partisan voting, which would reduce *State Electoral Uncertainty*.

[28]We are careful not to cross redistricting periods in defining districts, including "special" redistricting episodes resulting from court challenges or extra state legislative action (e.g., in Texas in 2003). Thus, for example, for $t = 2004$ we only use elections between 2002 and 2010.

and *Electoral Uncertainty* for the U.S. House, which parallels the results for the case of state legislatures.[29]

The reason we restrict attention to the period 1972-2012 is that measuring *Electoral Uncertainty* accurately for earlier years is problematic. Specifically, (i) prior to 1952, for many districts, we do not have data on presidential election outcomes; (ii) during the 1950s and early 1960s, a large number of congressional races were uncontested—in particular, almost 60% of U.S. House races were uncontested in the ten states of the "solid south" during the period 1950-1962 (so we would have to rely almost entirely on the the presidential vote for these cases); and (iii) the Supreme Court's reapportionment decisions of the mid-1960s (*Baker v. Carr* and *Wesberry v. Sanders*) produced multiple redistricting episodes for most states for the 1960s.

4.4 Which Seats Change Hands in Elections?

According to the model, during wave elections the advantaged party should gain mainly moderate districts and districts previously held by the opposing party. More generally, in any election the seats that change hands as a result of valence shocks should be concentrated among the moderate districts. This is what we find in the data.

Consider, for example, the 1994 election. In this election there was a clear pro-Republican tide—for example, the Republicans picked up 52 seats in the U.S. House, winning control of the chamber for the first time in 40 years. The average (normalized) Republican presidential vote in the state house districts held by Republicans both before and after the election was 57.9%. The average Republican presidential vote in the state house districts newly won by Republicans was just 53.0%. In 2006, by contrast, the tide favored the Democrats. In that year the average Democratic presidential vote in the state house districts held by Democrats both before and after the election was 61.1%. The average Democratic presidential vote in the state house districts newly won by Democrats was just 47.1% (that is, on average the newly won seats were in Republican-leaning districts).

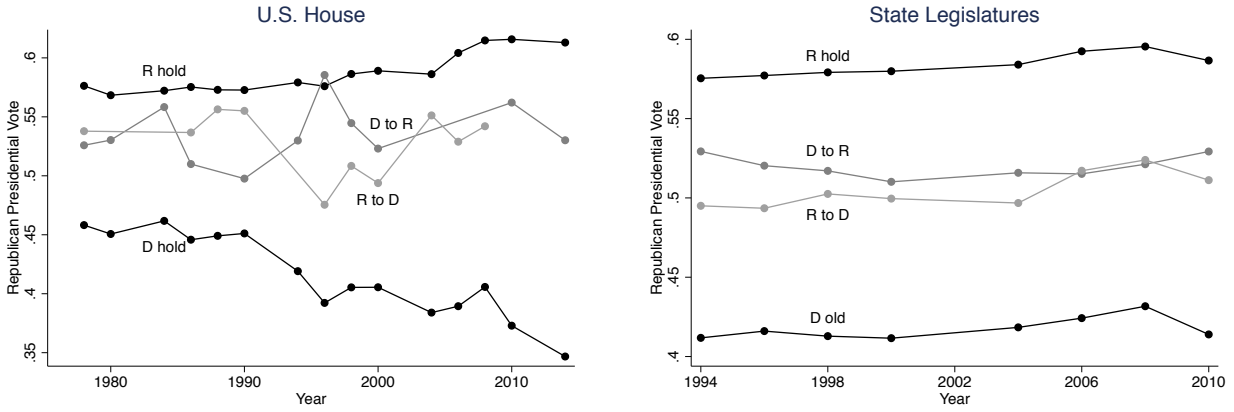
Looking across all elections shows a clear pattern: In *every* year the seats that switch from one party to the other are on average more centrist (less partisan) than the districts that do not change hands. Averaging across all years we have the following. In the districts held by Democrats in both t and $t+1$ the Democratic presidential vote was 58.4%; in districts that switched from Republican at t to Democratic at $t+1$ the Democratic presidential vote was 47.1%; in districts that switched from Democratic at t to Republican at $t+1$ the Democratic

[29] Again, we match *Electoral Uncertainty* for year t to polarization among representatives elected in year t and serving in years $t+1$ and $t+2$.

presidential vote was 46.3%; and in the districts held by Republicans in both t and $t+1$ the Democratic presidential vote was 41.3%.

Figure 8 shows the patterns for all years for which we have reliable data. The graph on the left is for the U.S. House, while the graph on the right is for state legislatures. Each figure has four curves, one for the districts held by Republicans in both t and $t+1$ (labeled R Hold), one for the districts held by Democrats in both t and $t+1$ (D Hold), one for districts that switched from Democratic at t to Republican at $t+1$ (D to R), and one for districts that switched from Republican at t to Democratic at $t+1$ (R to D). The U.S. House figure covers the period 1976 to 2012, and the figure for state legislatures covers the period 1994 to 2010.[30] The vertical axis shows the Republican share of the two-party presidential vote. In both figures it is clear that the districts held by each party are generally much safer for that party than either the districts it wins or the districts it loses in any given election cycle.[31]

Figure 8: Ideology of Districts Held, Won and Lost by Each Party



Finally, we also checked whether the seats that change hands as a result of valence shocks tend to be held by legislators with moderate voting records. Again, for state legislatures, we use the McCarty and Shor data, and for the U.S. House, the DW-NOMINATE scores. We measure how moderate a legislator is in each year by comparing the legislator to his or her partys delegation – Democrats are more moderate to the degree they are more conservative than the average Democrat in their delegation, and Republicans are more moderate to the

[30]For the U.S. House we pool midterm years with the preceding presidential years, and we only show cases with more than five observations. For the state legislatures we pool odd-numbered years with the subsequent even-numbered years.

[31]Halberstam and Montagnes (2015) also analyze the linkage between national political issues and local (district-level) elections. They find that U.S. senators elected in presidential election years have more extreme voting records than those elected in midterm elections, and interpret this as evidence that the moderate positions of presidential candidates benefit down-ballot candidates of both parties, which allows more extreme candidates to win district-level elections that they would lose if the voters' attention was focused solely on them. They also provide a simple non-strategic model of this behavior.

degree they are more liberal than the average Republican in their delegation. We then compare the moderateness of new members who represent seats that changed hands with that of old and new members who represent seats that did not change hands. In the state legislatures, those members representing seats that changed hands are 0.05 points more moderate than other legislators, and in the U.S. House they are .03 points more moderate. Both of these differences are statistically significant at the .01 level,[32] and, while not huge, they are still substantively meaningful. For example, the average standard deviation of moderateness in the state legislative data is 0.29, and the average standard deviation in the U.S. House data is 0.16.

4.5 Party Positions and Seat Shares

Another prediction of the model is that there should be a positive correlation between a party's seat share and how "moderate" the party is, even controlling for the underlying average ideology of voters. This is true whether a party's seat-share advantage is the result of a partisan tide, i.e., a positive party-wide valence shock (as in Proposition 4), or due to a skewed distribution of district medians, possibly due to gerrymandering. If the Democratic Party has an advantage in seats, then the median position among Democratic legislators should be further to the right. The median position among Republican legislators should also be further to the right. And, the midpoint between the parties should be further to the right. If the Republican Party has an advantage in seats, then the median position among legislators in both parties, and the midpoint between the parties, should be further to the left.[33]

To check this prediction, we return to the state legislatures. As above, let x^H_{Rit} be the median location of Republicans in the lower house in state i in year t, let x^H_{Dit} be the median location of Democrats, and define x^S_{Rit}, x^S_{Dit} analogously for state senates. Also, for state i and year t, let $Midpoint^j_{it} = (x^j_{Rit} + x^j_{Dit})/2$ be the midpoint between the two parties in chamber j.

We constructed a dataset with the partisan composition of each state legislative chamber using Dubin (2007) and data from the National Conference on State Legislatures.[34] Let $Dem\ Seat\ Share^j_{it}$ be the share of seats held by Democrats in chamber j of state i in year t.

We use voter partisanship to proxy for the average voter ideology in each state and year. More specifically, using the data on statewide offices described above, let $Avg\ Dem\ Vote_{it}$

[32] In computing the p-values we cluster the standard errors by legislator.

[33] Note, this is the opposite of the prediction in Smirnov and Fowler (2007), who analyze a dynamic version of the Calvert-Wittman model.

[34] See, e.g., http://www.ncsl.org/research/about-state-legislatures/partisan-composition.aspx for the last few years of data.

be the average Democratic vote-share across all offices elected in state i between years $t-3$ and t.[35] We also include second-order and third-order polynomial terms of *Avg Dem Vote* in the regressions, to capture possible non-linearities in the relationship between ideology and partisanship. We also include the variables t and t^2, to capture the possibility of trends in the national ideological "mood" (e.g., Stimson 1991).

Table 2 presents the results of regressions of the chamber party medians or midpoints on the corresponding Democratic seat shares and the variables to control for average voter ideology. The first row is for state lower houses and the second row is for state senates. In the third row we average the dependent and independent variables across the two chambers in each state. For example, in the first column the dependent variable is $(x_D^H + x_D^S)/2$, and the independent variable is $(\textit{Dem Seat Share}^H + \textit{Dem Seat Share}^S)/2$.

Table 2: **Median Party Positions and Midpoints vs. Democratic Seat Shares**

Case	Democratic Median	Republican Median	Midpoint
State House	1.254	0.051	0.681
	(0.409)	(0.312)	(0.252)
State Senate	1.310	0.308	0.627
	(0.338)	(0.314)	(0.256)
House/Senate	1.472	0.255	0.793
Average	(0.418)	(0.305)	(0.273)

Dependent variable = Party Median or Midpoint. Cell entries are estimated coefficients on Democratic Seat Share. Standard errors are in parentheses. Number of observations = 403 in all cases.

As the model predicts, all of the estimated coefficients are positive. For the Democrat party medians and midpoints the coefficients are substantively large and statistically significant. For the Republican party, on the other hand, the coefficients are relatively small and not statistically significant. We are not sure why this is the case and it surely deserves further exploration. In any case, we must of course add the caveat that we do not place any causal interpretation on the estimates. We simply note that they are (broadly) consistent with the model.

[35] Again, we only include cases with 4 or more races.

4.6 Other Evidence

The prediction in Proposition 3 is broadly consistent with recent work on the U.S. Congress. Hall and Shepsle (2014) argue that as power shifted from committee leaders to party leaders in the House of Representatives beginning in the 1970's, voters should have started to place less value on the seniority of their representative, because they would have understood that it was less valuable to have a senior representative serving as—or next in line as—a committee or subcommittee chair. They document that the electoral value of seniority is significantly lower in the "strong party" regime post 1976 compared to the "weak party" regime before 1976.[36] If we take seniority as one of the main components of valence in this context, then Proposition 3 predicts that polarization in the U.S. House should increase after 1976. This is what the standard time-series plots of polarization based on roll call scores, such as NOMINATE, show.[37]

Finally, we can conduct a limited analysis of the correlates of polarization in state legislatures in 1960 using data from LeBlanc (1969). Leblanc collected roll call data for 26 state senates for 1959-1960, and calculated the Rice index of "Party Likeness" for each case.[38] Since likeness measures the degree to which the parties vote together, we take *Party Disagreement* $= 1 -$ *Party Likeness* as a crude measure of polarization.

We consider two independent variables. The first is *State Electoral Uncertainty*, the standard deviation of the Democratic vote-share across all offices elected in each state between 1955 and 1958. This is a measure of electoral uncertainty. The second variable is *High TPO*, a dummy equal to 1 for states classified by Mayhew (1986) as having strong "traditional party organizations"—i.e., strong, patronage-based, electoral organizations.[39] As argued in Primo and Snyder (2010), it is likely that voters in states with strong party organizations vote more on the basis of party affiliation rather than the candidate characteristics. If so, then states with *High TPO* $= 1$ will tend to be states where voters place less weight on candidate valence (i.e., states with relatively low values of α).

[36]Relatedly, Ansolabehere and Pettigrew (2013) show that in 2009 the job approval ratings and electoral support of incumbents was unrelated to their seniority.

[37]See, e.g., http://voteview.com/political_polarization_2014.htm. Note that this is the opposite of the prediction in Londregan and Romer (1993).

[38]For each roll call j, $Likeness_j = 1 - |D_j - R_j|$, where D_j is the percentage of Democrats voting yes and R_j is the percentage of Republicans voting yes. Averaging across all roll calls yields the Party Likeness index. We drop three states—Connecticut, New Jersey, and Rhode Island—because they had so few roll calls (6, 13 and 12, respectively) that we do not trust their indices.

[39]Mayhew (1986, p. 19-20) defines a traditional party organization as a state or local party organization with the following five characteristics: (1) it has substantial autonomy, (2) it lasts a long time, (3) its internal structure has an important element of hierarchy, (4) it regularly tries to bring about the nomination of candidates for a wide range of public offices, and (5) it relies substantially on "material" incentives, and not much on "purposive" incentives, in engaging people to do organization work or to supply organization support. His scores range from 0 to 5. We set *High TPO* $= 1$ for states with scores of 4 or 5.

The correlation between *Party Disagreement* and *State Electoral Uncertainty* is -0.43. The sign is consistent with the prediction in Proposition 4—i.e., greater electoral uncertainty is associated with less inter-party polarization. The correlation between *Party Disagreement* and *High TPO* is 0.53. The sign is consistent with Proposition 3—i.e., a lower weight on candidate valence is associated with a higher degree of inter-party polarization. Both correlations are statistically significant at the .05 level.

5 Discussion

This paper presents and analyzes a novel model of legislative competition in which political parties are not treated as unitary actors, but rather as coalitions of self-interested incumbents that try to influence party positions to further their own reelection chances. In equilibrium, parties are ideologically differentiated and consequently have some districts in which they are favored, some where they are underdogs, and others where they are roughly equally matched. The degree of polarization between the parties depends on how important candidate-specific characteristics and qualities are for voters relative to their policy preferences, because this trade-off affects how many representatives of each party hail from districts that are not ideologically aligned with the party, and that therefore exert a moderating influence on their party.

The model generates several predictions for the relationship between measures of electoral uncertainty and electoral success on the one hand, and the parties' positions on the other hand. Using data from both U.S. Congress and state legislatures, we find evidence consistent with these key predictions, but inconsistent with the canonical model of policy-motivated candidates in isolated elections.

Our model derives these results from a setting that, compared with most models in the existing literature, has only very rudimentary strategic elements. The predictions emerge rather mechanically which can be seen as a strength because it shows that a few, relatively simple and realistic properties, working together can generate a rich set of predictions that accord closely with reality.

While understanding the strategic motivations of political actors is, of course, important for our understanding of our political system, we would argue that the existing literature has been relatively unbalanced. Scholars have developed a plethora of sophisticated theoretical channels for polarization, but this has not been matched with a corresponding empirical literature that tests these theories and assesses whether they can explain the observed change in political polarization, primarily because the fundamental parameters of these theoretical models, as well as their development over time, are frequently unobservable.

By simplifying some of the potentially strategic aspects through making plausible behavioral assumptions, our model is not only able to accommodate both an election stage and a stage at which the parties' national policy positions are determined in the legislature, but also remains tractable enough to derive clear comparative static results on the effects of electoral uncertainty and electoral waves. By taking these predictions to the data, our model takes an important step in the direction of empirically testing theories of polarization.

The model can be extended in various interesting ways. First, one could introduce heterogeneity in goals across incumbents—e.g., some incumbents might care strongly about the fortunes of the party as a whole in addition to their own re-election (this might be the case for those in line to be party or committee leaders, if majority party leaders wield much more power than minority party leaders).

Second, one could alter the way the party platforms are chosen, moving away from a system in which only the median matters. A weighted majority rule is one possibility, with higher weights for those who are in line to be party leaders. This might give extra weight to incumbents who represent extreme districts, since they are likely to be those with the most seniority in office and therefore the most experience (and it is likely that political parties, like most organizations, value experience).[40] Relatedly, a more explicit model of how party positions in the legislature determine implemented policies may lead to interesting insights.

Third, the parties in our model are weak central organizations in terms of their ability to influence which candidates their party nominates in each district. This is by design, as we believe it captures the current state of affairs in the United States where nomination decisions are very decentralized. Yet, because spillovers are important here, there are certainly incentives for coordination through a central organization, and it could be interesting to develop a model in which parties play a strategic role here.

Finally, one could incorporate other key actors in the model, such as interest groups that fund the parties. This might be especially interesting, since it is not clear whether such actors would serve to increase or decrease polarization, or how they would interact with the incumbent politicians in each party.

[40]It could also be the case that the degree of cohesion within parties (say, measured by the standard deviation of positions within one party) matters for voters.

Appendix

Proofs

Proposition 2. *Suppose that the distribution of districts is uniform on $[-k, k]$ and the distribution ϕ is symmetric around 0 and single-peaked. A symmetric equilibrium with policy differentiation, $x_R = x = -x_D$, exists if and only if $\phi(0) > \frac{\alpha}{4k}$. Moreover, if such an equilibrium exists, it is unique.*

Proof. An equilibrium is characterized by a zero of the function $Z(x)$ as defined in (7). Clearly, $Z(0) = 0$ because then $\Phi(\cdot)$ is constant in both integrals; this corresponds to the equilibrium without divergence. Also, $Z(k) = -\frac{1}{4} < 0$. A sufficient (and, as we will show, also necessary) condition for an equilibrium with policy divergence is therefore that $Z'(0) > 0$.

Differentiating (7) with respect to x yields

$$\begin{array}{l} Z'(x) = \frac{1}{4k}\left[\frac{4}{\alpha}(k-x)\phi\left(\frac{2x}{\alpha}\right) - \left[2\Phi\left(\frac{2x}{\alpha}\right) - 1\right] - 1\right] = \\ \frac{1}{2k}\left[\frac{2}{\alpha}(k-x)\phi\left(\frac{2x}{\alpha}\right) - \Phi\left(\frac{2x}{\alpha}\right)\right] \end{array}. \tag{24}$$

Differentiating (24) again gives

$$Z''(x) = \frac{1}{2k}\left\{-\frac{2}{\alpha}\phi\left(\frac{2x}{\alpha}\right) - \frac{2}{\alpha}\phi\left(\frac{2x}{\alpha}\right) + \frac{4}{\alpha^2}(k-x)\phi'\left(\frac{2x}{\alpha}\right)\right\} < 0, \tag{25}$$

because $\phi'(t) \geq 0$ for all $t \leq 0$ by the assumption that $\phi(\cdot)$ is single-peaked at 0. Thus, a necessary and sufficient condition for an equilibrium with policy differentiation to exist is $Z'(0) > 0$ (remember that $Z(0) = 0$ and $Z(k) < 0$).

Substituting $x = 0$ into (24) yields

$$Z'(0) = \frac{1}{2k}\left\{\frac{2k}{\alpha}\phi(0) - \frac{1}{2}\right\} \tag{26}$$

Thus, a necessary and sufficient condition for the existence of an equilibrium with policy differentiation is that $\phi(0) > \frac{\alpha}{4k}$, i.e., the valence shock is sufficiently concentrated around 0. Furthermore, $Z'' < 0$ guarantees that there is at most one equilibrium with policy differentiation. □

Derivation of equation 10. The most moderate of the districts that are secure for Democrats is given by

$$-(M_D - x_D) = -(x_R - M_D) + \alpha\beta, \tag{27}$$

because then even the best Republican valence realization only makes the voter indifferent to the Democrat. Solving for M_D and using symmetry (i.e. $x_D = -x_R$) yields $M_D = -\alpha\beta/2$, and analogously, $M_R = \alpha\beta/2$.

In this case, $x_R = -x_D = 1/2$. To verify this, observe that the number of Republican districts to the right of $1/2$ is simply $1/4$, as Republicans win all of these districts. To the left of x_R, Republicans win all the districts in $[M_R, x_R]$, and, on average, half of the competitive districts in $[M_D, M_R]$. Together, this is

$$\frac{1}{2} \cdot 1(1/2 - \alpha\beta/2) + \frac{1}{2} \cdot \frac{1}{2}[\alpha\beta/2 - (-\alpha\beta/2)] = 1/4.$$

Here, the first fraction, $1/2$, is the density of districts – remember that districts are uniformly distributed on $[-1, 1]$, and the second term is the average winning probability in these districts.

Thus, $x_R = 1/2$ is indeed the median of the Republican caucus whenever $\alpha\beta \leq 1$. Analogously, one can show that $x_D = -1/2$.

If $\alpha\beta \in [1, 2]$, we are in the case described by Figure 4b, and have to show that $x_R = -x_D = 1 - \frac{\alpha\beta}{2}$ if $\alpha\beta \in [1, 2]$.

The number of Republican districts to the right of $1 - \frac{\alpha\beta}{2}$ is

$$\frac{1}{2}\left[1 - (1 - \frac{\alpha\beta}{2})\right]\left[\frac{1}{2} + \frac{x}{\alpha\beta}\right] = \frac{\alpha\beta}{4}\left[\frac{1}{2} + \frac{x}{\alpha\beta}\right] = \frac{1}{4},$$

after substituting $x = 1 - \frac{\alpha\beta}{2}$. The number of Republican districts to the left of $1 - \frac{\alpha\beta}{2}$ is

$$\frac{1}{2}\left[-(1 - \frac{\alpha\beta}{2}) - (-1)\right]\left[\frac{1}{2} - \frac{x}{\alpha\beta}\right] + \frac{1}{2}\left[(1 - \frac{\alpha\beta}{2}) + (1 - \frac{\alpha\beta}{2})\right]\frac{1}{2} = \frac{1}{4}$$

so that $x_R = 1 - \frac{\alpha\beta}{2}$ is, in fact, the median of the Republican caucus.

Finally, if $\alpha\beta > 2$, then $x_R = x_D = 0$. This follows immediately from Proposition 2. □.

References

Adams, J. and S. Merrill (2008). Candidate and party strategies in two-stage elections beginning with a primary. *American Journal of Political Science 52*(2), 344–359.

Adams, J. and S. Merrill (2009). Policy-seeking parties in a parliamentary democracy with proportional representation: A valence-uncertainty model. *British Journal of Political Science 39*(03), 539–558.

Adams, J. F., S. Merrill, and B. Grofman (2005). *A unified theory of party competition: a cross-national analysis integrating spatial and behavioral factors*. Cambridge University Press.

Ansolabehere, S., W. Leblanc, and J. M. Snyder (2012). When parties are not teams: Party positions in single-member district and proportional representation systems. *Economic Theory 49*(3), 521–547.

Ansolabehere, S. and S. Pettigrew (2013). The electoral value of seniority: Does incumbent tenure affect the attitudes of voters? Mimeo, Harvard.

Ansolabehere, S. and J. M. Snyder (2000). Valence politics and equilibrium in spatial election models. *Public Choice 103*(3-4), 327–336.

Ansolabehere, S. and J. M. Snyder (2002). The incumbency advantage in US elections: An analysis of state and federal offices, 1942-2000. *Election Law Journal 1*(3), 315–338.

Ansolabehere, S., J. M. Snyder, and C. Stewart III (2001). Candidate positioning in us house elections. *American Journal of Political Science 45*, 136–159.

Aragones, E. and T. R. Palfrey (2002). Mixed equilibrium in a downsian model with a favored candidate. *Journal of Economic Theory 103*(1), 131–161.

Ashworth, S. and E. B. De Mesquita (2009). Elections with platform and valence competition. *Games and Economic Behavior 67*(1), 191–216.

Austen-Smith, D. (1984). Two-party competition with many constituences. *Mathematical Social Sciences 7*(2), 177–198.

Austen-Smith, D. (1986). Legislative coalitions and electoral equilibrium. *Public Choice 50*(1), 185–210.

Bernhardt, D., O. Câmara, and F. Squintani (2011). Competence and ideology. *The Review of Economic Studies 78*, 487–522.

Bernhardt, D., J. Duggan, and F. Squintani (2007). Electoral competition with privately-informed candidates. *Games and Economic Behavior 58*(1), 1–29.

Bernhardt, D. and D. Ingberman (1985). Candidate reputations and the incumbent effect. *Journal of Public Economics 27*, 47–67.

Besley, T. and S. Coate (1997). An economic model of representative democracy. *Quarterly Journal of Economics 112*, 85–114.

Bierbrauer, F. J. and P. C. Boyer (2013). Political competition and mirrleesian income taxation: A first pass. *Journal of Public Economics 103*, 1–14.

Callander, S. (2005). Electoral competition in heterogeneous districts. *Journal of Political Economy 113*, 1116–1145.

Callander, S. (2008). Political motivations. *Review of Economic Studies 75*(3), 671–697.

Calvert, R. L. (1985). Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. *American Journal of Political Science 29*, 69–95.

Calvert, R. L. and R. M. Isaac (1981). The inherent disadvantage of the presidential party in midterm congressional elections. *Public Choice 36*(1), 141–146.

Castanheira, M. (2003). Why vote for losers? *Journal of the European Economic Association 1*(5), 1207–1238.

Castanheira, M. and B. S. Crutzen (2010). Comparative politics with endogenous intra-party discipline. Mimeo, Erasmus University Rotterdam.

Crutzen, B. S., M. Castanheira, and N. Sahuguet (2010). Party organization and electoral competition. *Journal of Law, Economics, and Organization 26*(2), 212–242.

Downs, A. (1957). *An economic theory of democracy*. Harper.

Dubin, M. J. (2007). *Party affiliations in the state legislatures: A year by year summary, 1796-2006*. McFarland.

Fair, R. C. (1978). The effect of economic events on votes for president. *The Review of Economics and Statistics 60*, 159–173.

Green, D. P. and J. S. Krasno (1988). Salvation for the spendthrift incumbent: Reestimating the effects of campaign spending in house elections. *American Journal of Political Science 32*, 884–907.

Groseclose, T. (2001). A model of candidate location when one candidate has a valence advantage. *American Journal of Political Science 45*(4), 862–886.

Grossman, G. M. and E. Helpman (1996). Electoral competition and special interest politics. *The Review of Economic Studies 63*(2), 265–286.

Gul, F. and W. Pesendorfer (2009). Partisan politics and aggregation failure with ignorant voters. *Journal of Economic Theory 144*, 146–174.

Halberstam, Y. and B. P. Montagnes (2015). Presidential coattails versus the median voter: Senator selection in U.S. elections. *Journal of Public Economics 121*, 40–51.

Hall, A. B. and K. A. Shepsle (2014). The changing value of seniority in the us house: conditional party government revised. *The Journal of Politics 76*(01), 98–113.

Hibbs, D. A. (1987). *The American political economy: Macroeconomic and electoral politics in the United States*. Cambridge, Harvard University Press.

Hirano, S. and J. M. Snyder (2014). Primary elections and the quality of elected officials. *Quarterly Journal of Political Science 9*(4), 473–500.

Ingberman, D. and J. Villani (1993). An institutional theory of divided government and party polarization. *American Journal of Political Science 37*, 429–471.

Jacobson, G. C. (1978). The effects of campaign spending in congressional elections. *American Political Science Review 72*(2), 469–491.

Kartik, N. and R. P. McAfee (2007). Signaling character in electoral competition. *American Economic Review 97*(3), 852–870.

Kendall, C., T. Nannicini, and F. Trebbi (2014). How do voters respond to information? evidence from a randomized campaign. *American Economic Review 105*(1), 322–353.

Key, V. (1947). *Politics, Parties, and Pressure Groups* (2nd ed.). New York: Thomas Y. Cromwell.

Krasa, S. and M. K. Polborn (2010). Competition between specialized candidates. *American Political Science Review 104*(4), 745–765.

Krasa, S. and M. K. Polborn (2012). Political competition between differentiated candidates. *Games and Economic Behavior 76*, 249–271.

Krasa, S. and M. K. Polborn (2015). Political competition in legislative elections. Mimeo, University of Illinois.

LeBlanc, H. L. (1969). Voting in state senates: Party and constituency influences. *Midwest Journal of Political Science 13*, 33–57.

Londregan, J. and T. Romer (1993). Polarization, incumbency, and the personal vote. In W. Barnett, M. Hinich, and N. Schofield (Eds.), *Political economy: Institutions, competition, and representation*, pp. 355–377. Cambridge Univ. Press Cambridge.

Martinelli, C. (2001). Elections with privately informed parties and voters. *Public Choice 108*(1), 147–167.

Mayhew, D. R. (1986). *Placing parties in American politics*. Princeton, NJ: Princeton University Press.

Messner, M. and M. K. Polborn (2004). Paying politicians. *Journal of Public Economics 88*(12), 2423–2445.

Myerson, R. B. (1993). Effectiveness of electoral systems for reducing government corruption: a game-theoretic analysis. *Games and Economic Behavior 5*(1), 118–132.

Nadeau, R. and M. S. Lewis-Beck (2001). National economic voting in us presidential elections. *Journal of Politics 63*(1), 159–181.

Osborne, M. and A. Slivinski (1996). A model of political competition with citizen candidates. *Quarterly Journal of Economics 111*, 65–96.

Palfrey, T. R. (1984). Spatial equilibrium with entry. *Review of Economic Studies 51*, 139–156.

Persson, T. and G. Tabellini (1999). The size and scope of government:: Comparative politics with rational politicians. *European Economic Review 43*(4), 699–735.

Persson, T. and G. E. Tabellini (2002). *Political economics: explaining economic policy*. MIT press.

Primo, D. M. and J. M. Snyder (2010). Party strength, the personal vote, and government spending. *American Journal of Political Science 54*(2), 354–370.

Robertson, D. B. (1976). *A theory of party competition*. John Wiley & Sons.

Roemer, J. E. (1994). A theory of policy differentiation in single issue electoral politics. *Social Choice and Welfare 11*(4), 355–380.

Roemer, J. E. (1999). The democratic political economy of progressive income taxation. *Econometrica 67*, 1–19.

Roemer, J. E. (2001). *Political competition: Theory and applications*. Harvard University Press.

Schofield, N. (2004). Equilibrium in the spatial valence model of politics. *Journal of Theoretical Politics 16*(4), 447–481.

Schofield, N. (2006). Local political equilibria. In D. Austen-Smith and J. Duggan (Eds.), *Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks*, pp. 43–77. Springer.

Schofield, N. (2007). The mean voter theorem: necessary and sufficient conditions for convergent equilibrium. *The Review of Economic Studies 74*(3), 965–980.

Schofield, N. and G. Miller (2007). Elections and activist coalitions in the united states. *American Journal of Political Science 51*(3), 518–531.

Schofield, N. and I. Sened (2006). *Multiparty democracy: elections and legislative politics*. Cambridge University Press.

Shor, B. and N. McCarty (2011, 8). The ideological mapping of american legislatures. *American Political Science Review 105*, 530–551.

Smirnov, O. and J. H. Fowler (2007). Policy-motivated parties in dynamic political competition. *Journal of Theoretical Politics 19*(1), 9–31.

Snyder, J. M. (1994). Safe seats, marginal seats, and party platforms: The logic of party platform differentiation. *Economics & Politics 6*(3), 201–213.

Snyder, J. M., S. Hirano, and M. Ting (2009). Distributive politics with primary elections. *Journal of Politics 71*(4), 1467–1480.

Snyder, J. M. and M. M. Ting (2002). An informational rationale for political parties. *American Journal of Political Science 46*, 90–110.

Snyder, J. M. and M. M. Ting (2003). Roll calls, party labels, and elections. *Political Analysis 11*(4), 419–444.

Snyder, J. M. and M. M. Ting (2011). Electoral selection with parties and primaries. *American Journal of Political Science 55*(4), 782–796.

Soubeyran, R. (2009). Does a disadvantaged candidate choose an extremist position? *Annals of Economics and Statistics/Annales d'Économie et de Statistique 93/94*, 327–348.

Squire, P. (1992). Challenger quality and voting behavior in us senate elections. *Legislative Studies Quarterly 17*, 247–263.

Stimson, J. A. (1991). *Public opinion in America: Moods, cycles, and swings*. Westview Pr.

Wittman, D. (1983). Candidate motivation: A synthesis of alternative theories. *American Political Science Review 77*, 142–157.