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Costly Screening, Self Selection and the Existence of a Pooling Equilibrium in Credit Markets

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November 26th, 2011

Abstract

This paper presents a credit market model that embeds a costly, universal and imperfect screening technology in an otherwise simple model with borrower self-selection and costly lender screening. Contrary to the result in previous models, such as Wang and Williamson (1998) with random screening, the combination of universal screening and type I screening error produces pooling equilibrium as a non-trivial outcome. This result suggests that generalized lenders engaged in price rationing can sometimes compete with specialized lenders serving a single borrower type in credit markets that relies on costly lender screening as a sorting device.

JEL Classification: D82, D86, G20, L16

Keywords: costly screening, screening cost, self selection, pooling equilibrium, separating equilibrium

I am deeply indebted to Professor Anthony Yezer for his seasoned and altruistic advice throughout my dissertation research. I am grateful to Daniel Broxterman, Ana Fostel, Min Hwang, Summit Joshi and Harry Watson for valuable comments and advice. I also would like to thank Michael Bradley, Bryan Boulier, Warren Carnow, Paul Carrillo, Shahe Emran, Giovanni Favara, Alex Kapinos, William Larson, Wally Mullin, Donald Parsons, Jon Rothbaum, Tara Sinclair, Robert Van Order and participants at the department seminars at the George Washington University for helpful advice, comments and discussions at the early stage of this research. All errors are mine.
1 Introduction

Since the seminal paper by Rothschild and Stiglitz (1976), the literature on credit markets with adverse selection and screening\(^1\) has held that a pooling equilibrium does not exist unless there is sequential reasoning between lenders and borrowers in the equilibrium. This paper develops a simple credit market model without complex game-theoretic form, where risk-neutral borrowers self select because lenders make use of a costly, imperfect and universal screening technology. The assumptions of this credit market model appear to match conditions in some credit markets, mortgage markets for example. In contrast to expectations based on previous literature, a pooling equilibrium appears as a non-trivial outcome simply because of high screening cost.

This model contributes to the literature by demonstrating that making assumptions that are both reasonable and realistic about screening technology and behavior can change the previous presumption in the literature that competitive markets relying on costly screening as a sorting device must be served by specialized lenders. This finding that a pooling equilibrium possible is important because different types of lending strategies have been tried over the past twenty years as credit has been extended to diverse borrower risk categories. Particularly in the mortgage market, it is clear that lenders successfully pool across different borrower types. The results presented here show why and when such pooling equilibria are likely to be stable.

The model in this paper embeds a general screening technology in a similar environment to that in Wang and Williamson (1998). The model can reproduce the classic separating equilibrium result as a special case of a more general world of lending possibilities. In particular, when screening cost for each contract in the separating equilibrium becomes sufficiently high, the market switches to a pooling equilibrium.

Failure to find a sustainable pooling equilibria in previous models with borrower self selection and costly lender screening arises for two reasons. First, previous models assume that there is no need for universal screening because there are no fraudulent applicants. Second, there is no type I error in classifying applicants, i.e. good risks are never falsely

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\(^1\) In the literature of banking, mortgage and consumer finance, screening is often referred as underwriting.
rejected. Recent experience in mortgage lending has illustrated the importance of fraudulent applications, as documented in Jiang, Nelson and Vytlacil (2010). Therefore it is necessary for a lender in the separating equilibrium to screen all applicants instead of random sampling. But the environment of universal screening, combined with screening error of falsely rejecting some good borrowers, i.e. type I error, imposed an extra cost on good borrowers. When this cost is sufficiently high, a pooling equilibrium outperforms market separation by risk type.

2 Literature

The seminal paper by Rothschild and Stiglitz (1976) launched the literature on screening in an environment of adverse selection where the uninformed party takes the lead to reduce information asymmetries, as opposed to the canonical signaling literature where the informed party takes the lead. So far in the literature, scholars have investigated three types of screening devices\(^2\) used by the lender: first, combination of price and quantity; second, combination of price (interest rate) and collateral; and third, costly lender screening.

The literature about the first type of screening device, combination of price and quantity, starts from Rothschild and Stiglitz (1976), in which they explored the lender’s use of limited price and quantity contracts to promote self-selection of applicants in a competitive insurance market with risk-averse agents. They established that a separating equilibrium is the only possible equilibrium as a seller can always separate good customers from the bad by offering a range of contracts of different prices and quantities. Many papers have followed this pioneering work by adapting the model to various contexts. Dubey and Geanakoplos (2002) recast Rothschild and Stiglitz’s model to study financial markets in the framework of competitive pooling\(^3\), where borrowers self-select into pools with different prices and quantity-limits as

\(^2\)The term “screening device” first appeared in Stiglitz and Weiss (1981), in which they refer interest rate as one of the screening devices that a lender can use to distinguish different types of borrowers, in the environment where risky borrowers self-select loans with higher interest rates.

\(^3\)Here the concept of "pooling" means that lenders and borrowers do not trade bilaterally, but through pools of different quantities (set exogenously) and prices (determined by the market), where a lender can purchase shares of a pool and borrowers can sell promises of deliveries into the pool. To the extent that borrowers of different risks can sell promises to the same pool, it is a pooling equilibrium; otherwise, it is a
a screening device. They found that a separating equilibrium always exists and is unique. Martin (2007) changed the perturbations used in defining equilibria in Dubey and Geanakoplos’s model and found that there are cases where pooling equilibrium can Pareto dominate a separating equilibrium.

The second type of screening device, interest rate and collateral, was first studied by Stiglitz and Weiss (1981). In that paper, they studied screening in the credit market and examined the lender’s option of using interest rate or collateral as a device to induce different types of borrowers to self-select into different contracts. Bester (1985) established separating equilibrium using interest rate and collateral simultaneously as a screening device. Hellwig (1986) found pooling equilibrium when extending Bester’s (1985) screening environment to a three-stage game. Dell’Ariccia and Marquez (2006) applied the setup of using the combination of interest rate and collateral as a screening device to the study of the dynamics of bank competition, and found that equilibrium can switch between separating and pooling depending on the changing distribution of borrowers. Martin (2006) embedded the screening device of interest rate and collateral into a model of endogenous credit cycle, where collateral is linked to entrepreneurial wealth, and then the dynamic change of wealth is linked to the equilibrium regime switching between pooling and separating.

For the first two screening devices, screening is costless to the lender. The lender offers different contracts as a screening device, and the borrowers self-select accordingly hence producing a separating equilibrium. The device of price and quantity is free to both parties, while the device of price and collateral imposes cost of collateral on the borrower, but not the lender.

Relevant to the third type of screening device, it is common in credit markets that lenders employ costly and active screening. Lenders use a variety of underwriting techniques to actively screening borrowers, and borrowers self-select according to the expected cost of credit at different lenders. Underwriting applications is costly to the lender. One reason that formal underwriting is used in credit markets related to purchase of housing and automobiles
is that it is not feasible for the lender to design contracts with many different quantity or collateral requirements to be bundled with interest rate in order to separate borrowers. For example, unlike insurance or revolving credit cards, in the mortgage market, the size of the mortgage and the amount of collateral generally take on very restricted values so the lender has to rely on the device of costly screening. Another important reason to use costly screening is that there are fraudulent borrowers in the market who have no intention to repay the loan or who may misreport the collateral value. Consequently a lender needs to assess the collateral value, to check documents and to estimate the borrower’s creditworthiness to prevent any frauds, all of which incur costs to the lender.

There are very few papers that have examined the use of lender’s costly screening as a sorting device in the environment of borrower’s self-selection\(^4\). Wang and Williamson (1998) is the most notable. It is plausible that screening should not be free. And the costs that a lender spends on screening to prevent frauds will be reflected in the interest rate, which induces borrower’s self-selection. The Wang and Williamson model captures the information friction in the credit market caused by ex ante screening cost, as opposed to the ex post monitoring cost that was examined in earlier models (Townsend 1979, Bernanke and Gertler 1989)\(^5\).

Wang and Williamson (1998) derive the same no-pooling result as in Rothschild and Stiglitz (1976) although in a different model environment. In the model, there are two types of borrowers, good and bad, in a production economy. Lenders have a screening technology that allows them to identify borrower type perfectly, given a fixed expenditure for under-

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\(^4\)There is a relatively recent line of literature about the lender’s use of active and costly screening without borrower’s self selection. A primary aim of this literature is to show the negative externality in the quality of applicant’s pool caused by the competing bank’s screening activities (Broecker 1990, Cao and Shi 2001, Dierer 2008). Gehrig and Stenbacka (2004) examined this screening externality in a dynamic setting, in which pooling equilibrium becomes possible when there is a large share of good applicants in the market. Separately, Bubb and Kaufman (2010) provided a theory of lender’s cutoff rule in the mortgage market due to costly screening. These models do not have self-selection. Borrowers are passive agents who are subject to active screening by the lender.

\(^5\)Ex post monitoring cost refers to the cost a lender spends to verify the outcome of a project, particularly in the case when the borrower declare bankruptcy, so that a borrower will honestly report the outcome.
writing. Because applicants are aware of lender underwriting, it is only necessary to examine a proportion of applicants to deter bad risks from applying to lenders attempting to serve good borrowers. Thus screening is only applied to a fraction of applicants. Lenders in a pooling equilibrium accept all applicants and do not attempt to screen. In the separating equilibrium, the lender for the good borrowers conducts random and perfect screening sufficient to deter bad borrowers from applying. Wang and Williamson demonstrate that, if a pooling equilibrium does exist, one can always find a separating contract that is strictly preferred by the good borrower and earns a non-negative profit to the lender. Hence the pooling equilibrium will be broken.

This paper modifies two features of the previous model. First, it is necessary for lenders to underwrite all applicants, i.e. there is no random screening strategy. This change in assumptions can be motivated in many ways. Observation of lenders indicates that universal screening is common. The presence of fraudulent applicants, who have no intention of repaying, provides a theoretical rational for universal underwriting\(^6\). Fraudulent borrowers always have an incentive to apply as the cost of application is trivial compared to the potential gain. To a lender, the entire amount of loan approved to a fraudulent applicant becomes a loss, which can eat away the profit made from many good loans. In addition, legal requirements related to fair lending may mandate universal screening. Second modification of the previous model is the relaxation of the assumption of perfect identification of all applicants screened. Even with costly underwriting, it is possible to have type I error, i.e. rejection of good risks, and type II error, failure to reject poor risks.

Once the screening technology is changed to be universal and imperfect, i.e. every borrower is screened and both type I and type II errors can take place, a pooling equilibrium becomes a possible outcome when the screening cost is high. The intuition is similar to the case of using collateral as a screening device. In that environment, the use of collateral imposes a cost on the good borrower. When this cost is higher than the implicit subsidy that a

\(^6\)The experience with low and no documentation mortgage loans illustrates the importance of fraudulent applications. Initially loss rates on these loans were actually lower than full documentation mortgages but, over time, it appears that fraudulent applicants discovered this type of credit and default losses soared above those for other types of mortgage credit.
good borrower has to pay to the bad borrower in the pooling equilibrium, good borrowers will self-select the pooling contract and the equilibrium will be pooling. In the case of lender’s costly screening, it is the lender who bears the cost. When the cost of screening is too high, the specialized lender will have to charge the interest rate in the separating contract very high, and the good borrowers will self-select pooling contracts so the equilibrium will be pooling.

It is well known in the literature of screening (and signaling), as pointed out in Hellwig (1985), that the set of equilibria generally depends on the equilibrium defined in the model, dubbed as refinement of equilibrium in the game theory literature. For example, in the case of financial markets, Dubey and Geanakoplos (2002) found that the equilibrium is separating in the framework of competitive pooling. Subsequently Martin (2007) refined the definition of equilibria by changing the perturbation methods in the same framework and found pooling equilibrium is possible. In the case of financial intermediary, Hellwig (1986) interpreted models that produced separating equilibrium such as Rothschild and Stiglitz (1976), Wilson (1977) and Bester (1985) as a two-stage game. At the first stage, the uninformed agents offer some contracts; at the second stage, the informed agents choose among the offers. Hellwig further proposed a three-stage game based on Bester’s (1985) model environment, where at the third stage, the uninformed agents have the right to reject any contract applications from the informed agents, which in effect changes the expectations and strategies of the informed agents at the second stage. Hellwig discovered that in that three stage setting, the only stable equilibrium is pooling.

In this paper and in Wang and Williamson’s model (1998), a costly screening model can be regarded as a three-stage game as well. Lenders offer contracts of different combinations of screening intensity and interest rate first, borrowers choose among the offers and submit loan applications, then lenders conduct costly screening and have the right to reject the unqualified applicants. This paper refines the screening technology used in the previous costly screening model (instead of refining the definition of equilibrium), and changes the equilibrium result from separating to possible pooling. The existing literature has established pooling equilibrium in the environment of using the first two screening devices-
combination of price and quantity, and the combination of price and collateral. The result of this paper establishes the pooling equilibrium in the environment of costly screening, hence completing the proof for the existence of pooling equilibrium with all sorts of screening devices. Furthermore, unlike in the environment with the first two screening devices, the existence of pooling equilibrium in the environment with costly screening does not require formal construction of a three stage game with sequential reasoning between two parties, but simply arises from high screening cost, hence the result can be derived from a relatively simple model.

3 The model

The model environment\(^7\) has two periods. Investment takes place in period 1 and agents consume in period 2. There are four types of agents: lenders, type \(g\) borrowers, type \(b\) borrowers and fraudulent borrowers. Fraudulent borrowers have no intention to repay. They can be detected with certainty with a modest level of screening effort. In the credit market, there is a continuum of borrowers and lenders, with the measure of borrowers being strictly less than the measure of lenders, so competition drives the profit of lenders down to zero. Among the borrowers who receive credit, a fraction \(\alpha\) is type \(g\), and the remaining faction \(1 - \alpha\) is type \(b\). Both lenders and borrowers are risk-neutral.

Each lender makes one unit of investment good in period 1, either to a borrower in exchange for payment in period 2, or to an alternative risk-free investment project with a certain return of \(r\) units of consumption good in period 2.

Borrowers have no endowment in period 1 and each has access to an investment project that can generate \(x\) units of return of consumption good for every unit of investment good in period 1. The return of type \(i\) borrower, \(x\), is randomly distributed along its support at \([0, 1]\) according to the cumulative probability distribution function \(F_i(x)\), with corresponding probability density function \(f_i(x)\). Assume \(f_i(x) > 0\), and \(F_g(x)\) stochastically dominates

\(^7\)As the model environment in this paper is similar to that of Wang and Williamson (1998), notation in this paper is kept the same as in their paper insofar as possible.
\( F_0(x) \) in the first order.

Without screening, borrower type is private information. But each lender has access to a screening technology that allows her to observe a borrower’s type. It is assumed that a borrower can contact at most one lender in period 1, but a lender may contact many borrowers. So there is no negative screening externality as modeled in the banking competition literature. (Broecker 1990, Cao and Shi 2001, Gehrig and Stenbacka 2004, Dierer 2008)

Screening is costly and exhibits decreasing returns. If a lender spends \( C_0 \) on each application, she can perfectly screen out fraudulent borrowers. It is likely that \( C_0 \) is fairly low, because fraudulent borrowers lack valid documentation and can be more easily detected.\(^8\)

The cost of identifying fraudulent borrowers is the same for lenders specializing in good borrowers as it is in a pooling lender. The cost to separate good, \( g \), from bad, \( b \), borrowers is given by

\[
C = \beta (p - p_f)^i, \quad i \in [1, \infty)
\]  

(1)

where screening cost \( C \) is a quasi-convex function of the identification probability \( p \) (or screening accuracy). \( \beta \) is the parameter that determines the marginal cost of screening accuracy, or the slope of the cost curve. \( p_f \) is the identification probability that can be achieved based on the free information available in the market. The minimum of \( p_f \) is 0.5 when there is no free information in the market. When \( p < p_f, C = 0 \). In this model, we always assume \( p > p_f \). The form of equation (1) is based on the assumption that any lender will have to spend at least \( C_0 \) to screen out fraudulent borrowers. Therefore, the structure of the unit screening cost \( C \) by a lender who needs to identify type \( g \) borrowers is defined as following:

\[
C = \begin{cases} 
C_0 & \text{if } p \in [p_f, p_0] \\
\beta (p - p_f)^i, & \text{if } p \in (p_0, 1]
\end{cases}
\]  

(2)

where \( p_0 \) is the identification probability of type \( g \) borrower when the screening cost is \( C_0 \), i.e. \( C_0 = \beta (p_0 - p_f)^i \).

\(^8\)If \( C_0 \) is large, the pooling equilibrium dominates the separating equilibrium because all lenders have to engage in substantial screening, even those specializing in bad borrowers.
$p$: identification probability (or screening accuracy)

$C_0$: minimum screening cost required to exclude fraudulent borrowers

$p_0$: identification probability when the screening cost is $C_0$.

$p_f$: identification probability achieved based on the free information available in the market

$\beta$: parameter that determines the marginal cost of screening accuracy

$i$: parameter that determines the convexity of the screening cost curve

This definition of unit screening cost is illustrated in the following graph:

Figure 1. Unit screening cost as a function of identification probability

In the graph, curve OG represents the cost curve of identifying type g borrowers; curve OF represents the cost curve of identifying fraudulent borrowers. Curve OG is much steeper than OF as the marginal cost of accuracy is much higher in differentiating between good and bad borrowers than it is between good or bad borrowers and fraudulent borrowers. Before screening, all borrowers appear identical to a lender. Clearly, once a lender identified an application as fraudulent, she would not expend further effort underwriting the application. However, no equilibrium in which fraudulent borrowers are not deterred from applying is
possible so screening effort is always greater than or equal to $C_0$ and hence only good and bad borrowers are actually screened. When screening cost is $C_0$, a lender can perfectly identify frauds. The steeper is the curve $OF$, the higher will be $C_0$. In the credit market, a lender has to spend at least $C_0$ on screening, as the benefit of rejecting fraudulent borrowers is large. A fraudulent borrower has no intention to repay the amount of loan. Given that the cost of fraudulent applications is negligible, the probability of rejection must be essentially one in order to deter them. Lenders recognize this and engage in sufficient screening to perfectly deter all fraudulent borrowers. Therefore, the curve $C_0MG$ represents the screening cost of a specialized lender for type $g$ borrowers.\footnote{This definition of unit screening cost is much more general than that of Wang and Williamson's model, where unit screening cost is a fixed cost $\gamma$. By allowing screening to be imperfect, identification probability $p$ becomes a choice variable by which a lender can use to adjust cost that feeds into the loan interest rate.}

Imperfect screening opens the possibility to model both type I and type II screening errors. For example, if the lender for type $g$ borrower adopts identification probability $p_g$ in screening, and if the identification probability is symmetric between good borrowers and bad borrowers, then the two-way screening error is described as:

\begin{align*}
\text{Prob}(\text{good}|\text{good}) &= p_g \\
\text{Prob}(\text{bad}|\text{good}) &= 1 - p_g \\
\text{Prob}(\text{good}|\text{bad}) &= 1 - p_g \\
\text{Prob}(\text{bad}|\text{bad}) &= p_g
\end{align*}

Universal screening is the only way that a lender can exclude fraudulent borrowers from receiving credit. In the market where all lenders spent at least $C_0$ on screening each application, fraudulent borrowers will be deterred from applying as they know they will be screened out with certainty. If any lender spends less than $C_0$ on screening, she will be flooded with applications from fraudulent borrowers, so lender no has incentive to deviate from this screening scheme.

4 Equilibrium

Equilibrium contracts consist of payment schedule and identification probability pairs $[R_i(x), p_i]$, $i = g, b$, $x \in [0, 1]$, where $R_i(x)$ denotes the payment made by the borrower to the lender
when the return on the borrower’s investment is \( x \), and \( p_i \) denotes the identification probability that a lender adopts in screening to reveal a borrower’s type. There are two types of contracts: pooling contracts and separating contracts. A pooling contract is offered by a lender to all borrower types except for fraudulent borrowers, where a separating contract is offered by a lender to a particular borrower type. A pooling equilibrium is an equilibrium where both \( b \) and \( g \) borrowers are served by the same lender. Pooling will never involve fraudulent applications.

### 4.1 Pooling equilibrium

In a pooling contract, a lender lends to both type \( g \) and type \( b \) borrowers with the same payment schedule \( R(x) \). The lender still needs to engage in a minimum level of screening activity that costs \( C_0 \) to screen out fraudulent borrowers. When a lender spends \( C_0 \) on screening, all the fraudulent borrowers will be deterred from applying as they know they will be rejected with certainty. Fraudulent borrowers self select to stay out of the market. A pooling equilibrium is characterized by the payment schedule \( R(x) \) that satisfies the following properties:

\[
0 \leq R(x) \leq x; \quad x \in [0,1] \\
x \leq y \Rightarrow R(x) \leq R(y); \quad x, y \in [0,1] \\
\alpha \int_0^1 R(x)dF_g(x) + (1 - \alpha) \int_0^1 R(x)dF_b(x) \geq r + C_0 \tag{3}
\]

Condition (3) is the individual rationality (IR) constraint for the lender. It states that the expected return from the equilibrium pooling contract for a lender must be no less than the return on the alternative risk-free investment plus the minimum screening cost for excluding frauds, so that the lender can make a non-negative profit. \( \alpha \) is the fraction of type \( g \) borrowers in the credit market after excluding fraudulent borrowers. So \( \alpha \) here stands for the probability of lending to a good borrower in a pooling contract. In a competitive market, this constraint is binding and the equality holds.
4.2 Separating equilibrium

In general, a separating equilibrium is characterized by a pair of contracts \([R_i(x), p_i]\), \(i = g, b\) for different types of borrowers. In this environment, the lender specialized for type \(b\) borrowers will never want to reject type \(g\) borrower if type \(g\) seeks credit from her, as by including type \(g\) borrowers, the lender can lower the risk in the pool of accepted borrowers (and receive higher payments). In equilibrium, the lender specializing in type \(b\) borrowers only spends \(C_0\) on screening each application to deter fraudulent borrowers, and the lender in effect grants credit to everybody who submits a loan application (as fraudulent borrowers will not apply). In other words, identification probability \(p_b\) that the lender for type \(b\) borrowers adopts is 0. Therefore, in a separating equilibrium, equilibrium contracts are \([R_g(x), p_g]\) offered by the lender specialized for type \(g\) borrowers and \([R_b(x), 0]\) offered by the lender specialized for type \(b\) borrowers. the contracts must satisfy the following conditions\(^\text{10}\):

\[
0 \leq R_i(x) \leq x; \quad x \in [0, 1]; \quad i = g, b \\
x \leq y \Rightarrow R_i(x) \leq R_i(y); \quad x, y \in [0, 1]; \quad i = g, b \\
\]

\[
p_g \int_0^1 R_g(x)dF_g(x) + (1 - p_g)\mu_g \leq \int_0^1 R_b(x)dF_g(x) \\
\int_0^1 R_b(x)dF_b(x) \leq (1 - p_g) \int_0^1 R_g(x)dF_b(x) + p_g\mu_b
\]

\(^\text{10}\)In the general case, without the degenerating context described above, the equilibrium conditions can be written as following:

\[
0 \leq R_i(x) \leq x; \quad x \in [0, 1]; \quad i = g, b \\
x \leq y \Rightarrow R_i(x) \leq R_i(y); \quad x, y \in [0, 1]; \quad i = g, b \\
\]

\[
p_i \int_0^1 R_i(x)dF_i(x) + (1 - p_i)\mu_i \leq (1 - p_j) \int_0^1 R_j(x)dF_i(x) + p_j\mu_i; \quad i, j = g, b \\
\int_0^1 R_i(x)dF_i(x) \geq r + \frac{C_i}{p_i}; \quad i = g, b.
\]
\[
\int_0^1 R_g(x) dF_g(x) = r + \frac{C}{p_g} \quad (6)
\]

\[
\int_0^1 R_b(x) dF_b(x) = r + C_0 \quad (7)
\]

The conditions (4) and (5) are incentive compatibility (IC) constraints for borrowers to self-select their corresponding lenders. The left side of (4) is the expected cost of credit for borrower \( g \) if she seeks credit at her corresponding lender. At this lender, she is facing probability \( p_g \) of being correctly identified hence being accepted for credit with the correct payment schedule \( R_g(x) \), and with probability \( 1 - p_g \) of being falsely denied credit. If a loan is denied, borrower \( g \) cannot fund her project because a borrower can contact only one lender per time period, thus she consumes zero, and the loss in her linear utility is \( \mu_g \). The right side of (4) is the expected cost of credit if the borrower \( g \) seeks credit at the lender for type \( b \) borrowers, where type \( g \) will always be accepted as the lender only engages in minimum screening to deter fraudulent borrowers. The left side of (4) characterizes the type I error of screening. Condition (4) is always satisfied, as \( R_g(x) < R_b(x) \), this condition implies \( p_g \leq 1 \), which always holds. Which means type \( g \) borrower will never want to seek credit from a lender specialized in type \( b \) borrowers.

The left side of (5) is the expected cost of credit if borrower \( b \) seeks credit at her corresponding lender, where she will always be accepted. The right side of (5) is the expected cost of credit if the borrower \( b \) falsifies her type and seeks credit at the lender specialized in lending to type \( g \) borrowers. If borrower \( b \) is correctly identified with probability \( p_g \) hence

\[\int_0^1 [x - R_g(x)] dF_g(x) + (1 - p_g)(0 - M_g) \geq \int_0^1 [x - R_b(x)] dF_g(x)\]

so \( \mu_g = M_g + \int_0^1 x dF_g(x) \).
being rejected, her loss of utility is \( \mu_b \); if she is falsely accepted with probability \( 1 - p_g \), the payment schedule of her loan is \( R_g(x) \), which is lower than \( R_b(x) \). The right side of (5) characterizes the type II error of screening. Condition (5) means that when the lender specialized in type \( g \) borrowers maintains a certain level of screening accuracy \( p_g \), type \( b \) borrowers will be deterred.

The conditions (6) and (7) are the individual rationality (IR) constraints for lenders. They state that the expected return to a lender from each separating contract is equal to the return from the alternative risk-free investment opportunity \( r \) plus the average screening cost spent on each funded borrower, \( \frac{C}{p_g} \) or \( C_0 \). They are binding because competition drives the lender’s profit down to zero. Condition (7) is the IR constraint for the lender specialized in type \( b \) borrowers. Condition (6) is the IR constraint for the lender specialized in type \( g \) borrowers. \( \frac{C}{p_g} \) represents the fact that among all the applications from type \( g \) borrowers that a lender receives, only \( p_g \) fraction of applications are correctly accepted and \( 1 - p_g \) fraction of applications are falsely rejected. The lender needs to cover the screening cost being spent on all applicants from those who are accepted.

5 Existence of a pooling equilibrium?

Market equilibrium, if it exists, is either separating or pooling. If an equilibrium exists, it must be the case that the lender is making a non-negative profit and the borrowers have no incentive to deviate from the existing equilibrium contract. In previous literature, particularly in Rothschild and Stiglitz (1976) and Wang and Williamson (1998), a pooling equilibrium never exists, because if it does, a separating lender can always offer a non-negative-profit contract that makes the type \( g \) borrowers better off but not the type \( b \). In Wang and Williamson’s model, in the environment of random screening and with the absence of type I error, such separating contract is achieved by lowering the probability of screening so as to lower the average screening cost of each application and make the interest rate to type \( g \) borrower lower than the interest rate in the pooling equilibrium.

As shown in the following proof of Proposition 1, when screening is both universal and
imperfect with two-way errors, pooling equilibrium becomes a possible outcome. In this case, the choice variable that a lender can use to adjust average screening cost of each contract is screening accuracy \( p_g \). However, \( p_g \) is not a free variable in the presence of type I error. When \( p_g \) is lowered, the chance that a good borrower is falsely rejected increases, which imposes a cost on good borrowers seeking credit at the separating lender. When this cost becomes too high, it will push good borrowers away from the separating lender to pooling. So \( p_g \) is bounded from below. With this lower bound in \( p_g \), there will be a threshold for the parameter \( \beta \) in the unit screening cost, above which the screening cost will be too high for a separating contract to be profitable. So pooling becomes a possible outcome. This argument will become the basis for the proof of Proposition 1 below.

There is a key difference between the probability of random screening \( \pi_g \) and the probability of correct identification \( p_g \) (or accuracy), which is why random and perfect screening in Wang and Williamson’s model is not equivalent to universal and imperfect screening with two-way errors, even though it is appealing to think the two as the same for modeling purpose. A low screening probability \( \pi_g \) lowers the chance of a good borrower being falsely rejected, while a low screening identification probability \( p_g \) raises the chance of a good borrower being falsely rejected. That’s why type I error can only take effect in a model with universal screening.

The remaining of the paper first presents the proof for the existence of a pooling equilibrium in the environment of universal screening with both type I and type II screening errors. Then it follows by the proof that the environment of random screening with both type I and type II errors does not produce a pooling equilibrium, as a way to demonstrate the importance of universal screening. Furthermore, the proof for the non-existence of pooling equilibrium in the environment of universal screening with only type II error illustrates the importance of type I error on the result of pooling equilibrium.
5.1 Universal and imperfect screening with both type I and type II errors

Proposition 1 If screening is universal and imperfect (including both type I and II errors), then a pooling equilibrium exists.

Proof:

Proof of the existence of pooling equilibrium, requires the demonstration that, when the existing equilibrium is pooling, there is a case that no separating contract can outperform pooling; and that, when the existing equilibrium is separating, there is a case in which a pooling contract can outperform separating.\(^{12}\)

The imperfect screening technology has identification probability \(p_g\), symmetric between good type borrowers and bad type borrowers, and allows both type I and type II error.

\[
\begin{align*}
\text{Prob}(\text{good}|\text{good}) &= p_g \quad \text{Prob}(\text{bad}|\text{good}) = 1 - p_g \\
\text{Prob}(\text{good}|\text{bad}) &= 1 - p_g \quad \text{Prob}(\text{bad}|\text{bad}) = p_g 
\end{align*}
\]

By having unit screening cost as a function of identification probability, the model allows identification probability \(p_g\) to be a choice variable in the separating contract \([R_g, p_g]\), so that a lender can lower the average screening cost by lowering \(p_g\).\(^{13}\)

First, suppose there exists a pooling contract in the market characterized by (3).

The separating contract for good borrowers consists of pair \([R_g(x), p_g]\). If it can outperform pooling, it must satisfy the following three conditions:

\(^{12}\)In this environment, every borrower in the separating contract needs to be screened. Universal screening is necessary because there are fraudulent borrowers in the market whose opportunity cost or penalty of being caught cheating is very low, so only universal screening can deter them from applying. With universal screening, the optimal loan contract is still a debt contract. The proof for Proposition 2 in Wang and Williamson’s paper is not affected. The debt contract arises from the lender’s objective to minimize the screening identification probability \(p_g\) subject to the binding incentive compatibility constraint for bad borrowers and the zero expected profit constraint for the lender.

\(^{13}\)This is similar to Wang and Williamson’s model where a lender can lower the average screening cost by lowering the random screening probability \(\pi_g\).
1. zero expected profit constraint for the lender for good borrowers

\[
\int_0^1 R_g(x)dF_g(x) = r + \frac{C}{p_g}
\]  

(8)

where the unit screening cost \(C\) is defined as in (2).

2. incentive compatibility constraint for the bad borrower

\[
r_b \leq (1 - p_g) \int_0^1 R_g(x)dF_g(x) + p_g\mu_b
\]  

(9)

where the left side \(r_b \equiv \int_0^1 R(x)dF_b(x)\) (which is smaller than \(r + C_0\)) is the expected payment by a bad borrower in the pooling contract; the right side is the expected cost of credit of a bad borrower in the separating contract. This condition states that a bad borrower does not have enough incentive to deviate from the existing pooling equilibrium, so she self selects to stay out of the separating contract for good borrowers. She does not have enough incentive to deviate from pooling because type II error (error of false acceptance of bad borrowers as denoted by \(1 - p_g\)) is not big enough.

3. incentive compatibility constraint for the good borrower

\[
r_g \geq p_g \int_0^1 R_g(x)dF_g(x) + (1 - p_g)\mu_g
\]  

(10)

where the left side \(r_g \equiv \int_0^1 R(x)dF_g(x)\) (which is greater than \(r + C_0\)) is the expected payment by a good borrower in the pooling contract; the right side is the expected cost of credit of a good borrower in a separating contract. This constraint states that a good borrower should weakly prefer the separating contract over pooling and characterizes type I error as it indicates that a good borrower is facing the probability \(p_g\) of being falsely rejected.\(^{14}\)

Solve for \(p_g\) in the incentive compatibility constraint for bad borrowers in (9), it yields

\[
p_g \geq 1 - \frac{\mu_b - r_b}{\mu_b - \int_0^1 R_g(x)dF_g(x)} \equiv P_{DB}
\]  

(11)

Here \(P_{DB}\) is the minimum identification probability required to deter bad borrowers from deviating from the pooling contract to the separating contract for good borrowers. \(P_{DB}\)

\(^{14}\)Now, unlike in Wang and Williamson’s model, equation (8), (9) and (10) may not necessarily have a solution for \(R_g(x)\).
is decreasing in $R_g(x)$ because the higher is $R_g(x)$ the smaller $p_g$ is needed to deter bad borrowers. The minimum $R_g(x)$ takes place when the screening cost is at its minimum $C_0$, i.e. when $R_g(x) = \{R_g(x) : \text{arg}\int_0^1R_g(x)dF_g(x) = r + \frac{C_0}{p_0}\}$ (note $\frac{C_0}{p_0} = \beta p_0^{-1}$), the maximum $P_{DB} = P_{DB}(R_g(x))$; the maximum $R_g(x)$ is the same as the payment schedule in the pooling contract, in which case no need for screening as bad borrowers have no incentive to deviate from pooling, i.e. $R_g(x) = \overline{R}_g(x)$, the minimum $P_{DB} = 0$.

Solve for $p_g$ in the incentive compatibility constraint for good borrowers in (10), it yields

$$p_g \geq \frac{\mu_g - r_g}{\mu_g - \int_0^1 R_g(x)dF_g(x)} \equiv P_{IG}$$

(12)

Here $P_{IG}$ is the minimum identification probability required to induce good borrowers to deviate from pooling to separating. This boundary condition for $p_g$ is absent in Wang and Williamson’s model. $P_{IG}$ is increasing in $R_g(x)$ because the higher is $R_g(x)$ the less attractive is the separating contract compared to pooling hence the larger $p_g$ (or smaller type I error) is needed to attract good borrowers to the separating contract. when the minimum $R_g(x) = \{R_g(x) : \text{arg}\int_0^1R_g(x)dF_g(x) = r + \frac{C_0}{p_0}\}$ (note $C_0 = \beta(p_0 - p_f)^i$), the minimum $P_{IG} = P_{IG}(R_g(x))$; when the maximum $R_g(x) = \overline{R}_g(x)$, the maximum $P_{IG} = 1$ which means when the payment schedule in the separating contract equals the payment schedule in the pooling contract, screening has to be perfect to make the good borrower indifferent between the two.

The minimum $p_g$ has to be the bigger one among the two because both constraints (11) and (12) have to be satisfied,

$$p_g \geq Max[P_{DB}, P_{IG}] (> 0)$$

(13)

Therefore identification probability $p_g$ is contained in the following domain
\[
p_g \in [\text{Max}[P_{DB}(R_g(x)), P_{IG}(R_g(x))], 1]
\]

where \( R_g(x) : \text{arg} \int_0^1 R_g(x) dF_g(x) = r + \beta \frac{C_0}{p_0} \)
and \( P_{DB}(R_g(x)) = 1 - \frac{\mu_b - r_b}{\mu_b - \int_0^1 R_g(x) dF_b(x)} \)
and \( P_{IG}(R_g(x)) = \frac{\mu_g - r_g}{\mu_g - \int_0^1 R_g(x) dF_g(x)} \)

When the unit screening cost \( C = \beta(p_g - p_f)^i \), the zero-profit constraint (8) for the lender specialized for good borrowers becomes

\[
\int_0^1 R_g(x) dF_g(x) = r + \beta \frac{(p_g - p_f)^i}{p_g}
\]

In the above equation, when \( p_g \) is contained in the range as in (14), there must be a threshold for \( \beta \), above which no payment schedule \( R_g(x) \) can solve the equality, hence the conditions for separating contract to outperform the existing pooling contract are not satisfied\(^{15}\). It is critical to note that this lower bound of identification probability as specified in (14) can be greater than \( p_f \), so when the parameter \( \beta \) is reasonably large, it is not possible for a lender to freely lower the average unit screening cost \( \beta \frac{(p_g - p_f)^i}{p_g} \) by lowering \( p_g \) close to \( p_f \). Here \( p_g \) is not a free choice variable particularly because of the type I error embedded in constraint (10). As a result, there is a possibility that no separating contract can outperform the existing pooling contract, so pooling equilibrium is possible. As to be demonstrated in the numerical example, the threshold of \( \beta \) does not have to be very large to make pooling possible.

Now suppose the existing equilibrium in the market is separating, characterized by a contract of interest rate and screening accuracy pair \([R_g(x), p_g]\) by the lender specialized for the good type borrower and \([R_b(x), 0]\) by the lender for the bad type borrower, it needs to be shown that there is a possible pooling equilibrium that can outperform separating.

\(^{15}\)It is worth noting that \( p_0 \) does not establish the lower bound of the identification probability \( p_g \), because \( p_0 = (\frac{C_0}{\beta})^{1/i} + p_f \). When \( \beta \) increases, \( p_0 \) will decrease and will fall below the lower bound established in (14).
Comparing the zero-profit constraint (5) for the lender specialized for bad borrowers in the separating equilibrium and the zero-profit constraint (3) for the pooling lender, it is clear that $R_b(x) > \overline{R}(x)$, as in the pooling equilibrium there is an implicit subsidy from good borrowers to bad borrowers. Consequently, bad borrowers always strictly prefer a pooling equilibrium. Therefore a pooling equilibrium outperforms separating equilibrium if and only if good borrowers prefer the pooling contract over separating, i.e. in contrast to constraint (10), now the incentive compatibility constraint for good borrower should be

$$r_g \leq p_g \int_0^1 R_g(x) dF_g(x) + (1 - p_g)\mu_g$$

where $r_g \equiv \int_0^1 \overline{R}(x) dF_g(x)$

substituting into the zero-profit constraint of the specialized lender for good borrowers in (15), the above constraint becomes

$$r_g \leq p_g (r + \beta \frac{(p_g - p_f)^2}{p_g}) + (1 - p_g)\mu_g$$ (16)

since $r_g < \mu_g$, for any $p_g$ greater than $p_f$, there can possibly be a parameter $\beta$ that is sufficiently large to make the inequality hold. In another word, when $\beta$ is big enough, good borrowers prefer pooling, so there is a case that a possible pooling equilibrium can outperform separating.

QED

5.2 Numerical example

Assume the probability density function of project return of the good borrower $f_g(x)$ is $2x$; and the probability density function of the project return of the bad borrower $f_b(x)$ is $2(1 - x)$. So $f_g(x)$ stochastically dominates $f_b(x)$ in the first order, and $\int_0^1 f_i(x) dx = 1$, as shown in the following graph.

Figure 2. Probability of density function of project returns
The pooling contract characterized by the binding constraint (3), rewritten as

\[ \alpha \int_0^1 \overline{R}(x)f_g(x)dx + (1 - \alpha) \int_0^1 \overline{R}(x)f_b(x)dx = r + C_0 \]  \hspace{1cm} (17)

Let the payment schedule \( \overline{R}(x) = \overline{R}x \), where \( \overline{R} \) is the fixed percentage rate of return \( x \) to be paid to the lender in the pooling contract. So substitute \( \overline{R}(x) = \overline{R}x \), \( f_g(x) = 2x \) and \( f_b(x) = 2(1 - x) \) into (17), it yields

\[ \overline{R} = \frac{3(r + C_0)}{1 + \alpha} \]  \hspace{1cm} (18)

Let the risk free interest rate \( r = 0.02 \), the minimum screening cost \( C_0 = 0.001 \), and the proportion of good borrower \( \alpha = 0.5 \), then

\[ \overline{R} = 0.042 \]

Substitute the function of \( f_g(x) \) and \( f_b(x) \) into constraint (11) and (12), it yields that the minimum \( p_g \) in (13) becomes

\[ p_g \geq \max \left[ 1 - \frac{\mu_b - \frac{1}{3}\overline{R}}{\mu_b - \frac{1}{3}R_g}, \frac{\mu_g - \frac{2}{3}\overline{R}}{\mu_g - \frac{2}{3}R_g} \right] \]  \hspace{1cm} (19)
when $\overline{R} = 0.042$, and assume $\mu_g = 1$, and $\mu_b = 0.5$, then for any non-negative $R_g$, 

$$1 - \frac{\mu_b - \frac{1}{3}R_g}{\mu_b - \frac{1}{3}R_g} < \frac{\mu_g - \frac{2}{3}R_g}{\mu_g - \frac{2}{3}R_g},$$ therefore 

$$p_g \geq \frac{\mu_g - \frac{2}{3}R_g}{\mu_g - \frac{2}{3}R_g} = \frac{0.972}{1 - \frac{2}{3}R_g} \quad (20)$$

In the zero-profit constraint for the lender specialized for good borrowers (15), on the left hand side, $R_g \leq \overline{R}$, because if $R_g$ is greater than the pooling rate $\overline{R}$, all good borrowers will seek credit at the pooling lender. On the right hand side, the value is monotonically increasing in $p_g$. so the maximum $\beta$ must be paired with the minimum $p_g$ to keep $R_g$ on the left side within the feasible range. Substitute the lower bound of $p_g$ from (20) into the zero-profit constraint ((15), and assume $p_f = 0.5$, $r = 0.02$, $i = 2$, it yields 

$$\frac{2}{3}R_g = 0.02 + \beta \frac{(0.972 - 0.5)^2}{1 - \frac{2}{3}R_g} \quad (21)$$

By solving the above polynomial function of $R_g$, the relationship between $R_g$ and $\beta$ in the range $R_g \leq \overline{R}$ is illustrated in the following graph.

Figure 3. Upper bound of $R_g$ and $\beta$ 

It shows that when $\beta$ reaches 0.032, $R_g$ reaches the upper bound $\overline{R} = 0.042$. So in this numerical example, when $\beta$ goes beyond the threshold 0.032, equilibrium in the market is
pooling. This example demonstrates that $\beta$ does not have to be very large to make the market equilibrium switch from separating to pooling.

5.3 Random screening

It is now possible to demonstrate the importance of universal screening in Proposition 2 below, as it shows that when screening is random as in Wang and Williamson, and both type I and type II errors are allowed, a pooling equilibrium does not exist. In this environment, for any given level of screening accuracy $p_g$, the lender can choose to vary the screening probability $\pi_g$ in order to produce a separating contract to outperform pooling, the same way as she does in the Wang and Williamson’s model. As when the screening probability $\pi_g$ is lowered for any given screening precision $p_g$, the chance for a good borrower being falsely screened out is also lowered. So low $\pi_g$ does not impose any cost on good borrowers. Therefore two-way screening error alone, without universal screening, does not produce a pooling equilibrium. Wang and Williamson’s model setup is a special case of this environment, as shown in Lemma 1, where screening is random and perfect and no pooling exists.

Proposition 2 If screening is random and imperfect (including both type I and type II errors), a pooling equilibrium does not exist.

Proof: In this environment, there are two types of probabilities: in the separating contract, the lender for good borrowers randomly conducts screening with probability $\pi_g$; each time when a lender conducts screening, the screening has two-way errors with identification probability at $p_g$.

\[
\begin{align*}
\text{Prob}(\text{good}|\text{good}) &= p_g & \text{Prob}(\text{bad}|\text{good}) &= 1 - p_g \\
\text{Prob}(\text{good}|\text{bad}) &= 1 - p_g & \text{Prob}(\text{bad}|\text{bad}) &= p_g
\end{align*}
\]

As defined in (2), the unit screening cost $C$ is a function of $p_g$. 24
Suppose there exists a pooling contract in the market characterized by (3) except that \( C_0 = 0 \), because random screening implicitly assumes no fraudulent borrowers in the market hence no need to maintain the minimum level of screening.

The separating contract for good borrowers consists of pair \([R_g(x), \pi_g, p_g] \). If it can out-perform pooling, it must satisfy the following three conditions:

1. zero expected profit constraint for lender to good borrowers

\[
(1 - \pi_g + p_g \pi_g) \int_0^1 R_g(x) dF_g(x) = (1 - \pi_g + p_g \pi_g) r + \pi_g C \tag{22}
\]

where \( C \) is defined in (2). \((1 - \pi_g + p \pi_g)\) is the fraction of application being accepted. \( \pi_g \) is the fraction of applications that the lender screens.

2. incentive compatibility constraint for the bad borrower

\[
r_b \leq (1 - p \pi_g) \int_0^1 R_g(x) dF_b(x) + p_g \pi_g \mu_b \tag{23}
\]

where \( r_b \equiv \int_0^1 R(x) dF_b(x) < r \).

3. incentive compatibility constraint for the good borrower

\[
r_g \geq (1 - \pi_g + p \pi_g) \int_0^1 R_g(x) dF_g(x) + (1 - p_g) \pi_g \mu_g \tag{24}
\]

where \( r_g \equiv \int_0^1 R(x) dF_g(x) > r \).

We can always find a contract \([R_g^*(x), p_g^*, \pi_g^*] \) that satisfies the above three constraints. To see this result, rearrange (23), it yields

\[
p_g \pi_g \geq \frac{r_b - \int_0^1 R_g(x) dF_b(x)}{\mu_b - \int_0^1 R_g(x) dF_b(x)} \tag{25}
\]

rearrange (24), it yields

\[
(1 - p_g) \pi_g \geq \frac{r_g - \int_0^1 R_g(x) dF_g(x)}{\mu_g - \int_0^1 R_g(x) dF_g(x)} \tag{26}
\]

combine (25) and (26), it yields

\[
\pi_g \geq \frac{r_b - \int_0^1 R_g(x) dF_b(x)}{\mu_b - \int_0^1 R_g(x) dF_b(x)} + \frac{r_g - \int_0^1 R_g(x) dF_g(x)}{\mu_g - \int_0^1 R_g(x) dF_g(x)} \tag{27}
\]
When \( R_g(x) = \overline{R}(x) \), \( p_g \pi_g = (1 - p_g) \pi_g = \pi_g = 0 \); rearrange (22) it yields
\[
[1 - (1 - p_g) \pi_g] \int_0^1 R_g(x) dF_g(x) - r = \pi_g C
\] (28)

From (25), (26) and (27) we know that \( p_g \pi_g, (1 - p_g) \pi_g \) and \( \pi_g \) are decreasing in \( R_g(x) \). So after substituting (25), (26) and (27) into (28), we know that for any given \( p_g \), the left side of (28) is increasing and continuous in \( R_g(x) \) on \([0, 1]\), while the right side is decreasing and continuous in \( R_g(x) \) on \([0, 1]\). Therefore, given the values of the left and right sides of (28) at the endpoints of \([0, 1]\), there exists a unique solution for \( R_g(x) \). When \( R_g(x) = \overline{R}(x) \), the right side of (28) equals 0, while the left side equals \( r_g - r > 0 \). Therefore, for any given \( p_g \), there is a unique \( R_g^*(x) \) and this \( R_g^*(x) < \overline{R}(x) \). Therefore, there exists a separating equilibrium contract \([R_g^*(x), p_g^*, \pi^*]\) and the pooling equilibrium is broken. QED

The separating equilibrium exists regardless how large is the value of \( \beta \) in screening cost \( C \), because the lender can choose \( \pi_g \) to be close to zero to lower the average unit screening cost in order to have the payment schedule \( R_g(x) \) fall within a feasible range.

**Lemma 1** If screening is random and perfect, a pooling equilibrium does not exist.

**Proof:**
This is a special case of the above proposition. The random and perfect screening technology is the same as that in Wang and Williamson’s model.

There are no fraudulent borrowers, so in a pooling contract, the lender does not screen. The pooling contract is constructed to satisfy the zero profit constraint for the lender\(^\text{16}\)
\[
\alpha \int_0^1 \overline{R}(x) dF_g(x) + (1 - \alpha) \int_0^1 \overline{R}(x) dF_b(x) = r
\] (29)

\(^{16}\)Wang and Williamson proved in Proposition 4 of the paper that a pooling contract, \( R(x) \), is a standard debt contract with \( R(x) = \overline{R}, \ x \in [\overline{R}, 1] \); \( R(x) = x, \ x \in [0, \overline{R}] \); for some \( \overline{R} \in (0, 1) \). Correspondingly, the payment by a particular type of borrower \( i \), \( \int_0^1 R(x) dF_i(x) \), can be written as \( \overline{R} - \int_0^1 F_i(x) dx \). Here to simplify the notation, the basic payment notation \( R(x) \) is adopted while ignoring its debt feature. Same simplification is adopted in notations for separating contract as well.
In a separating contract, the lender who is specialized in loans to good borrowers engage in active screening, while the lender specialized in bad borrowers does not screen. Screening is random with screening probability at $\pi_g$ and unit screening cost for each contract being fixed at $\gamma$. The payment schedule for good borrowers is $R_g(x)$, for bad borrower is $R_b(x)$. So the equilibrium contract for good borrowers consists of pair $[R_g^*(x), \pi_g^*]$. 

Suppose the pooling contract exists, a separating contract needs to satisfy the following three conditions in order to outperform the pooling contract:

1. zero expected profit constraint for the lender to good borrowers

$$
\int_0^1 R_g^*(x)dF_g(x) = r + \pi_g^* \gamma
$$

2. incentive compatibility constraint for the bad borrower

$$
r_b = (1 - \pi_g^*) \int_0^1 R_g^*(x)dF_b(x) + \pi_g^* \mu_b
$$

where $r_b = \int_0^1 \overline{R}(x)dF_b(x) < r$

3. incentive compatibility constraint for the good borrower

$$
R_g^*(x) < \overline{R}(x) \text{ i.e. } \int_0^1 R_g^*(x)dF_g(x) < \int_0^1 \overline{R}_g(x)dF_g(x)
$$

This condition is different from the one in the environment of universal screening because there is no type I error. This is a key implicit assumption in Wang and Williamson’s model. The separating equilibrium only allows type II error because screening is random and perfect. With self selection, a bad borrower has incentives to apply for loans designed for good borrowers, while a good borrower will not apply for loans designed for bad borrowers, because the payment schedule $R_g(x) < R_b(x)$. So in effect, with the screening probability maintained by the lender for good borrowers at $\pi_g$, there is a likelihood of $(1 - \pi_g)$ that a bad borrower can be falsely accepted should he apply for loans to good borrowers (type II error). However, a good borrower will never be in a situation to be screened out when he applies for loans at the corresponding lender as the screening technology is perfect. So implicitly, this
screening scheme only incorporates type II error (false acceptance of bad borrower) while ignoring type I error (false rejection of good borrower).

Wang and Williamson showed in the proof for Proposition 5 that given the constraints in (30) and (31), condition in (32) can be satisfied. Consequently, suppose a pooling equilibrium exists, we can always find a separating contract characterized by \([R^*_g(x), \pi^*_g]\), which can make the good borrower better off, deter bad borrowers from applying for good contracts, and earn zero expected profit to the lender. Therefore, a pooling equilibrium does not exist.

Here the key for no-pooling result is that no matter how big is the fixed unit screening cost \(\gamma\), the lender can always lower the screening probability \(\pi_g\) in order to lower the average screening cost \(\pi_g^*\gamma\) and hence lowering the loan rate for type \(g\) borrowers \(R_g\) below the pooling rate \(\bar{R}(x)\). When type I error is absent, the screening probability \(\pi_g\) is a free choice variable.

QED

5.4 Universal and imperfect screening with only type II error

Proposition 3 demonstrates the role of type I screening error on the finding of a pooling equilibrium. It establishes that when screening is universal but only type II error is allowed, a pooling equilibrium does not exist. If type II error is the only error in the model, the lender is free to adjust the average screening cost by adjusting screening accuracy \(p_g\) in order to generate a separating contract that can outperform pooling. When accuracy \(p_g\) is lowered, the chance for a good borrower to receive credit is not affected as type I error is not in the model. Therefore, universal screening alone, without type I error, cannot lead to the result of pooling equilibrium.

Proposition 3 If screening is universal and imperfect (including type II error), a pooling equilibrium does not exist.

Proof:

In this environment, every borrower in the separating contract needs to be screened. The imperfect screening technology has identification probability \(p_g\), and allows only the error of
false acceptance of bad borrowers (type II error).

\[ \text{Prob}(\text{good}|\text{bad}) = 1 - p_g \quad \text{Prob}(\text{bad}|\text{bad}) = p_g \]

Unit screening cost for each contract \( C \) is defined as in (2).

Suppose there exists a pooling contract in the market characterized by (3).

The separating contract for good borrowers consists of pair \([R_g(x), p_g]\). If it can outperform pooling, it must satisfy the following three conditions:

1. zero expected profit constraint for lender to good borrowers

\[ \int_0^1 R_g(x)dF_g(x) = r + C \quad (33) \]

Here the screening cost on the right side is \( C \) instead of \( \frac{C}{p_g} \) because there is no type I error hence no good borrower is falsely rejected.

2. incentive compatibility constraint for the bad borrower

\[ r_b \geq (1 - p_g) \int_0^1 R_g(x)dF_b(x) + p_g \mu_b \quad (34) \]

where \( r_b \equiv \int_0^1 R(x)F_b(x)dx < r + C_0 \).

3. incentive compatibility constraint for the good borrower

\[ R_g(x) < \overline{R}(x) \quad (35) \]

Notice that when \( i = 1 \), i.e. when the unit screening cost \( C \) is a linear function of identification probability \( p_g \), the mathematical forms of constraints in this setup are identical to those in the Wang and Williamson’s set up. In another word, the environment of universal screening with type II error is equivalent to random and perfect screening. This equivalence highlights the hidden implicit assumption of one-way screening error in Wang and Williamson’s model.

Solve for \( p_g \) in the incentive compatibility constraint for bad borrowers in (34), it yields

\[ p_g \geq \frac{r_b - \int_0^1 R_g(x)dF_b(x)}{\mu_b - \int_0^1 R_g(x)dF_b(x)} \equiv P_{DB} \quad (36) \]

\( P_{DB} \) is decreasing in \( R_g(x) \) because the higher is \( R_g(x) \) the smaller \( p_g \) is needed to deter bad borrowers. The minimum \( R_g(x) \) takes place when the screening cost is at its minimum.
\(C_0\), i.e. when \(R_g(x) = \{R_g(x) : \arg \int_0^1 R_g(x)dF_g(x) = r + \frac{C_0}{p_0}\}\) (note \(C_0 = \beta(p_0 - p_f)^i\)), the maximum \(P_{DB} = P_{DB}(R_g(x))\); the maximum \(R_g(x)\) is the same as the payment schedule in the pooling contract, in which case no need for screening as bad borrowers have no incentive to deviate from pooling, i.e. when \(R_g(x) = \overline{R}_g(x)\), the minimum \(P_{DB} = 0\).

Plugging the form \(C = \beta(p_g - p_f)^i\) into (33), and substituting for \(p_g\) in (33) using (36), it yields the equation that solves for \(R_g\):

\[
\int_0^1 R_g(x)dF_g(x) = r + \beta\left[\frac{r_b - \int_0^1 R_g(x)dF_b(x)}{\mu_b - \int_0^1 R_g(x)dF_b(x)} - p_f\right]^i
\]  

(37)

The left side of (37) is increasing and continuous in \(R_g(x)\) on \([0, 1]\), while the right side is decreasing and continuous in \(R_g(x)\) on \([0, 1]\). Therefore, given the values of the left and right sides of (37) at the endpoints of \([0, 1]\), there exists a unique solution for \(R_g(x)\). When \(R_g(x) = \overline{R}(x)\), the right side of (37) equals \(r\), because when \(p_g < p_f\), \(C = 0\). The left side equals \(\int_0^1 \overline{R}(x)dF_g(x)\). Since \(\int_0^1 \overline{R}(x)dF_g(x) \geq r + C_0\) by the zero-profit constraint (3) in the pooling equilibrium. Therefore there is a unique solution\(^{17}\) \(R^*_g(x) < \overline{R}(x)\), the condition in (35) is satisfied. And the pooling equilibrium is broken. So pooling equilibrium does not exist.

A separating equilibrium can always exist because in the zero-profit constraint (33) (or (37)), when the parameter \(\beta\) is substantially large, in the absence of type I error, a lender can lower the identification probability \(p_g\) close to \(p_f\) to counter the effect of rising \(\beta\) on the payment schedule \(R_g(x)\). In addition, it is worth noting that \(p_0\) does not establish the lower bound of \(p_g\) in face of rising \(\beta\), because \(p_0 = \left(\frac{C_0}{\beta}\right)^{1/i} + p_f\), when \(\beta \to \infty\), \(p_0 \to p_f\).

QED

\(^{17}\)There are two cases for this unique solution.

When \(C = C_0\), solve for \([R^*_g(x), p^*_g]\) using constraints (33) and (34).

If \(p^*_g \leq p_0\), (where \(p_0 = \left(\frac{C_0}{\beta}\right)^{1/i} + p_f\)), then the solution of the separating contract is \([R^*_g(x), p_0]\), meaning minimum screening is good enough to deter both fraudulent borrowers and bad borrowers. And the zero-profit constraint is binding, while the incentive compatibility constraint holds inequality.

If \(p^*_g > p_0\), then the solution of the separating contract is \([R^*_g(x), p^*_g]\), solved from constraints (33) and (34) and making \(C = \beta(p_g - p_f)^i\) in constraint (33). In this case, both constraints are binding.
6 Conclusions

This paper demonstrates that in a credit market model where borrowers self-select and lender’s screening is costly, a pooling equilibrium can arise when lenders must use costly, universal and imperfect screening technology. This result is different from the previous view that pooling equilibria are unlikely established in the literature on costly screening models most notably by Wang and Williamson (1998). The finding that pooling equilibria can exist does not require sequential reasoning between lenders and borrowers as modelled in the screening literature based on other types of screening devices. Pooling equilibrium is simply a result of the high cost of screening, the possibility of both type I and II errors in the process, and the threat of fraudulent applicants which prompts universal screening.

The insight of possible pooling equilibrium in this paper has important implications on the organization of credit markets. It provides another explanation for cases in which credit markets pool borrowers with different risks. For example, it can explain why revolving credit markets pool large numbers of different borrower types into a single product and pricing structure. And it can shed lights on the long-standing debate in the mortgage market as to whether the mortgage credit should be supplied in a way where all qualified borrowers are pooled in the conventional loan market or borrowers should be separated into A, Alt-A and subprime types that all served by different lenders.

The model also suggests that, when either screening technology or the presence of fraudulent applicants changes, market equilibrium may switch from pooling to separating or back again. This implies one source of challenge foe stability in the organization and business models used in credit markets. Over the past 20 years, there have been dramatic changes in underwriting methods in consumer credit, particularly mortgages. Business models, such as low documentation lending, were provided in a separating equilibrium. Subsequently, it appears that a rise in fraudulent applications changed the viability of the low documentation loan market and it now appears that lenders are making all applicants provide substantial documentation, even if that is costly for them. Hopefully models in which screening costs are substantial but variable, the likelihood of falsely rejecting good risks is not zero, and the supply of fraudulent borrowers varies will allow a more complete model of possibilities for
organizing the market for credit.

References


