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Physical limitations and fundamental factors affecting performance of liquid crystal tunable lenses with concentric electrode rings

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A comprehensive analysis of fundamental factors and their effects on the performance of liquid crystal (LC)-based lenses is given. The analysis adopts numerical LC director and electric field simulation, as well as scalar diffraction theory for calculating the lens performance considering different variable factors. A high-efficiency LC lens with concentric electrode rings is fabricated for verifying and enriching the analysis. The measurement results are in close agreement with the analysis, and a summary of key factors is given with their quantitative contributions to the efficiency. © 2013 Optical Society of America

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1. Introduction
Electro-optical lenses based on liquid crystals (LCs) have been considered as potential candidates for replacing or simplifying bulky conventional optics with the advantages of electrically tunable power, small size and weight, low cost, low-power consumption, and high-speed switching. There are a number of approaches that have been reported with different design concepts including refractive lenses, diffractive lenses, and hybrid lenses [1–8]. Previous work has been done analyzing factors affecting the efficiency of cylindrical LC lenses with discrete strip electrodes, with a result that smoothing of the index and phase variations due to phase sampling and electrode gaps is important for a diffraction-limited performance [9,10]. However, a complete numerical analysis of all factors affecting the efficiency of a LC lens has not been done.

Independent of any specific design, there are fundamental factors that determine the LC lenses’ efficiency and performance. Generally speaking, they are scattering due to the components and materials, control of phase profile, phase sampling; and phase distortions caused by electrode discontinuity or gaps. The goal of this paper is to comprehensively discuss the effects of isolatable factors that contribute to the efficiency of the LC lenses with both numerical and experimental methods. More emphasis is given to the designs with discrete electrode rings, as they have shown the best performance [11,12].

The factors considered in this paper are: the basic structure of an LC cell (the indium-tin oxide (ITO), glass, alignment layer, LC material, and spacers); imperfect phase profiles due to discrete electrode structure and applied voltages, and the effect of gaps between electrodes and fringing electric fields.

2. Numerical Approaches
To accurately calculate the phase profile across the LC lens, a numerical model of the LC optical axis orientation is used, which takes into account the electrode pattern, applied voltages, cell thickness, and LC properties. From the acquired optical axis orientation, the resultant optical path difference...
(OPD) across the aperture is calculated. A high resolution calculation grid is adopted that is fine enough for fringing fields near the electrode edges being taken into account. In doing the numerical calculations, we consider an example LC lens design that has a diameter of 2.4 mm, and a thickness of 10 μm. It consists of 33 ring electrodes, with a sampling rate of 10 phase steps/wave (for \( f = 400 \) mm, design wavelength \( \lambda = 543.5 \) nm). The LC material considered has a birefringence \( \Delta n = 0.27 \). These parameters allow for a tunable focal length ranging from +400 mm to +infinity and from –400 mm to –infinity.

Having the phase profile across the LC lens, the light propagation is calculated from the scalar Rayleigh–Sommerfeld diffraction theory \([13]\). It considers an incident plane wave, takes into account the calculated phase profile for a lens, and predicts the intensity pattern in focal plane. In the numerical analysis, the Strehl ratio is used as a measure of the performance, and defined as the ratio of the peak intensity in a LC lens's focal plane to that for an ideal lens. In addition, the intensity distribution in the focal plane can be considered as the point spread function (PSF), and the modulation transfer function (MTF) is obtained by taking Fourier transform of the PSF \([13, 14]\). Details of our numerical modeling methods are given in Appendix.

### 3. Results of Analysis of Factors Affecting LC Lens Performance

By using our numerical model, we can consider the effect of possible factors on the reduction of efficiency of a LC lens.

A. Efficiency Loss due to Imperfect Phase Profile

Obtaining the desired parabolic phase profile of the LC layer with proper control of the electric field distribution is very important. An imperfect basic shape of the phase profile has a significant impact on the lens performance. It has been reported that a defocus aberration as small as \( \lambda/4 \) is significant enough to lower the Strehl ratio to 80% \([14]\). It is clear that the voltage profile needs to be well controlled to make the basic shape of phase profile a parabola.

A good feature of LC devices with a ring electrode structure is that it is anticipated that sufficient control of the basic shape of the phase profile will be possible.

However, for a LC lens with discrete electrodes, the continuous phase profile has to be sampled with phase steps. In principle, the analytical diffraction efficiency is proportional to the number of phase steps per wavelength of the light \( q \) \([15]\):

\[
\eta \propto (\sin(\pi/q)/(\pi/q))^2. \tag{1}
\]

Obviously, as the sampling rate has a significant effect on the efficiency, more electrodes are desired to reduce the phase step between them (Table 1). The phase profile of the lens with different sampling rates can be obtained with the numerical modeling, from which the Strehl ratio is calculated (Table 1). It is found that the phase smoothness between adjacent steps caused by the fringing field gives a slightly higher efficiency than the analytical prediction.

Also, it is interesting to simulate the effects of phase noise on the optical performance. Any non-uniformity, especially the possible scratch on the alignment layer caused during the rubbing process, or the spacers, can possibly induce a phase noise to the final lens OPD. To consider the effects of this phase noise, we calculate the light distribution in its focal plane as well as the MTF of an “ideal” lens having a perfect parabolic phase profile, to which random noise is added (random range 0 ~ 0.05λ and 0 ~ 0.1λ, \( \lambda = 543.5 \) nm). The results show that if the OPD has a noise with randomness 0 ~ 0.05λ, the Strehl ratio becomes 99.3% (normalized to the intensity peak in the focal plane of an ideal lens with a parabolic phase profile), and the MTF is almost the same as the ideal one. When the randomness increases to 0 ~ 0.1λ, the Strehl ratio drops to about 96.83%, and the MTF is noticeably lowered (Fig. 1).

From the analysis of this section, it is anticipated that while the requirements for an accurate phase profile are severe, this should not be a fundamental limiting factor in the efficiency of LC lenses.  

B. Efficiency Losses due to Gaps in the Electric Field Profile

Gaps are needed to separate electrodes to apply different voltages and tune the optical power.

| Table 1. Analytical Diffraction Efficiency and the Numerically Calculated Strehl Ratio for Different Phase Step Heights |
|---|---|---|
| Phase Step Height | Analytical Strehl Ratio (%) | Numerical Strehl Ratio (%) |
| 6 Steps/Wave | 91.2 | 92.4 |
| 8 Steps/Wave | 95 | 96 |
| 10 Steps/Wave | 96.8 | 98.5 |

Fig. 1. (Color online) Calculated MTF for the lens with noise in the OPD profile.
Obviously, as the electric field in gap areas is different from the adjacent electrodes, the director orientation distribution would be different, and an index of refraction variation is expected. To minimize the index or phase aberration, the gaps between electrodes should be small, compared to the thickness of the cell. The efficiency can be calculated as a function of gap width from both an analytical estimation and numerical modeling methods. An example is given for a LC lens with a diameter of 2.4 mm and 33 ring electrodes, and a focal length of 400 mm (10 phase steps per wave).

Analytically, the diffraction efficiency for different gap widths can be roughly expressed as:

\[ \eta \propto \left(1 - \frac{A_g}{\Lambda} \right)^2 \]  

Here, \( \Lambda \) is the total area of the lens, and \( A_g \) is the area of gaps. Using this expression, the efficiency can be roughly estimated for the specific lens design considered (Table 2).

However, the exact effect of gap width on the efficiency can only be obtained with accurate numerical modeling of phase variations in gap areas, by simulating electric field distribution and the resultant LC director profile.

Due to the concentric electrode ring structure, the fringing field between electrodes is always in the radial direction; however, the projection of the LC directors onto the plane of the cell is aligned uniformly in one direction [Fig. 2(a)]. Because of this, there are two typical lens areas where the tangent of the electrodes is perpendicular or parallel to the rubbing direction [Figs. 2(b) and 2(c)].

The calculation shows that for both cases, the basic shape of phase profile is a parabola (Fig. 3). However, in the gap areas between the electrodes, once the gap becomes larger than 1 \( \mu \)m, the calculated phase profile shows noticeable phase bumps, and a further increase in gap width induces higher bumps. Particularly, in the case of 3 \( \mu \)m gaps, the phase bumps in gaps of the outermost lens areas are larger than 0.2\( \lambda \); in lens center areas, the bumps are smaller than 0.1\( \lambda \).

The phase variation in the gap areas is due to the fact that the lower electric field in the gap area causes the director to be at a lower angle to the cell surface and to have a larger effective index of refraction. The magnitude of phase variation in gaps is dependent on the radial location of the cell, because the voltage on the electrodes near the outside of the lens (\( V_{edge} = 2.36 \) V) is much larger than that near the inside (\( V_{center} = 0.9 \) V), the relative decrease of the electric field and the phase variation due to gaps are larger.

The optical performance of the whole lens expressed by the Strehl ratio is calculated as the average of the previously discussed two cases, based on phase profiles as a function of gap width. It is found that the Strehl ratio in the focal plane drops significantly as the gap width increases to 3 or 4 \( \mu \)m (Table 3). In the table, along with the gap width used in our numerical calculation, the ratio of the electrode gap to the cell thickness (\( g/d \)) is also given as a useful and generalized parameter.

It is observed that the size of the center lobe in the focal plane for the LC lens is the same as ideal lens

![Fig. 2. (Color online) (a) Top view of lens areas (1 and 2) where a tangent to the electrodes is perpendicular or parallel to the rubbing direction. (b) LC director two-dimensional (2D) plane as a cross section of area 1, projection of LC directors on cell surface is perpendicular to the electrode axis. (c) LC director 2D plane as a cross section of area 2, projection of LC directors on cell surface is parallel to the electrode axis.](image)

![Fig. 3. (Color online) Calculated OPD profile as a good parabolic shape (in the director plane where electrodes are parallel to rubbing direction) when the gap is 3 \( \mu \)m.](image)

| Table 2. Analytical Diffraction Efficiency for Different Gap Widths |
|-------------------|-------------------|-------------------|-------------------|-------------------|
| Gap Width         | 1 \( \mu \)m Gap 2 | \( \mu \)m Gap 3 | \( \mu \)m Gap 4 | \( \mu \)m Gap |
| Area ratio (%)    | 2.67              | 5.33              | 8                | 10.67            |
| Analytical efficiency (%) | 94.6              | 89.3              | 84.2             | 79.2             |
[Figs. 4(a)–4(c)]; however, in the LC lens’s focal plane, the intensity minimums between lobes become brighter than that in ideal lens’s case, demonstrating that light is scattered to large angles for LC lens with large gaps [Figs. 4(a) and 4(b)], which is also indicated by the intensity spot profile of the LC lens in the focal plane: the center lobe drops, compared to the ideal lens; but the center lobe size remains the same, also consistent with the analytical prediction determined by the expression $1.22f \cdot \frac{\lambda}{D} = 110.5 \mu m$ as radius (here, $f$ is the focal length and $D$ is the lens diameter [18]) [Fig. 4(c)]. Accordingly, the cutoff frequency for the LC lens is the same as an ideal lens (indicating the same resolving ability), but there is a sharp drop at low frequencies in the MTF [Fig. 4(d)], due to the large angle light scattering from the effect of the electrode gaps.

The effect of the phase variations in gaps is interpreted as causing an increase in the dark level of the entire image. This can be considered as a haze due to the nonuniform diffraction caused by a variable-pitch diffraction grating resulting from the phase variation in the gaps.

Therefore, we expect the effect of an inhomogeneous electric field to be an issue of concern in a LC lens using the discrete electrode structure. For a better image quality, the electrode gaps should be as small as possible.

### 4. Summary of Factors and their Contribution to LC Lens Performance: Efficiency Analysis of the Actual LC Lens

Experimentally, an actual LC lens with concentric ITO ring electrodes is designed and fabricated to verify and enrich the factor analysis [Fig. 5(a)]. It has the same parameters as given above for the example lens in numerical calculations, including 33 discrete ITO ring electrodes with 3 µm gaps, a thickness of 10 µm, and a diameter of 2.4 mm. The tunable range is from +400 mm to +infinity and −400 mm to −infinity. Based on our previous design concept [19], there is an inter-ring resistor network between any pair electrodes, so that not all electrodes have to be addressed. A thin SiO$_2$ layer is deposited on the ITO layer and a small via is patterned through the layer over each electrode to be addressed. Then a thin layer of Nickel is deposited and patterned as bus lines connecting the addressed electrodes to the external voltage driver through the vias [Fig. 5(b)]. The LC lens is assembled with this patterned substrate and a uniform ITO glass substrate with spacers in

![Fig. 4. (Color online) (a) Calculated light distribution in focal plane (for display purpose, the contrast is boosted by taking the third root of the intensity before plotting) for ideal lens, and (b) LC lens with 10 phase steps per wave and 3 µm gaps. (c) Strehl ratio of LC lens (in the director plane where electrodes are parallel to rubbing direction), normalized to center lobe intensity peak of an ideal lens in its focal plane. (d) Calculated MTF for LC lens and ideal lens.](image-url)

<table>
<thead>
<tr>
<th>Ratio of gap to thickness ($g/d$) (%)</th>
<th>1 µm Gap</th>
<th>2 µm Gap</th>
<th>3 µm Gap</th>
<th>4 µm Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strehl ratio (director plane with electrodes perpendicular to rubbing direction) (%)</td>
<td>10.35</td>
<td>96.59</td>
<td>91.01</td>
<td>78.74</td>
</tr>
<tr>
<td>Strehl ratio (director plane with electrodes parallel to rubbing direction) (%)</td>
<td>97.78</td>
<td>95.59</td>
<td>87.73</td>
<td>71.89</td>
</tr>
<tr>
<td>Strehl ratio of whole lens (in average) (%)</td>
<td>98.07</td>
<td>96.09</td>
<td>89.37</td>
<td>75.32</td>
</tr>
</tbody>
</table>
between and filled with the LC material (property parameters are in Appendix A1).

As a first consideration of a real device, LC lenses consist of multilayer structures including ITO and polyimide (PI) alignment layers, which might scatter the incident light. Also, spacers and LC materials in the bulk can be the source of light scattering. To evaluate the effects of these potential causes of performance degradation, a test cell is made, consisting of the same components as the LC lens (rubbed PI alignment layer, spacers, LC material) except with a uniform ITO layer without ring electrodes.

To consider losses from the reflection and large-angle scattering of the LC material and internal cell components when no voltage is applied, a collimated on-axis and linearly polarized incident light beam (1 cm in width, \( \lambda = 543.5 \) nm) passes through the test LC cell. Reflection loss associated with LC lens's multilayer components and the glass-air interfaces is measured as about 16% by tilting the lens surface and reflecting the light beam to the detector. The reflection of glass-air interfaces of the glass lens is measured as about 8% of the total light. Therefore, double the amount of the light is lost due to internal reflections by the LC lens, compared to the glass lens. The transmitted light beam through the LC lens is measured with a transmission of 83.1% within the collection angle of the detector, giving the losses due to large-angle scattering within 1% of the total light. Therefore, the scattering effects of the main components of LC cell will be considered negligible.

Then, the optical performance of the actual LC lens is investigated. To drive the lens for a 400 mm focal length, the voltage profile for all electrodes is calculated, and the voltages to be applied to each addressable ring are shown (Table 4). Due to the inter-ring resistors, the voltages on the rings between the ones in the table vary linearly between them. Due to a controlled fabrication process, the thickness variation across the lens aperture is a small fraction of the wavelength of light. The calculated voltage profile is applied on the actual LC lens. Not requiring any voltage adjustment, the LC lens already has a good parabola-shape phase profile (focal length \( f = 400 \) mm in this example) that is compared with a reference high-quality glass lens of the same power (BK 7 optical glass, uncoated, 50.8 mm Dia with 2.4 mm aperture in center, Newport 20LKIT-1) [Figs. 5(c) and 5(d)], measured by a Mach–Zehnder interferometer. It also shows noticeable phase bumps across the aperture in the profile [Fig. 5(c)], similar to what is calculated due to the electrode gaps, which are not found in a glass lens's actual OPD [Fig. 5(d)].

Light distribution in the focal plane is measured by the CCD for both lenses with different exposure times [20]. The image of the LC lens's focal plane shows a large-area halo with a ring pattern. The scattering, which is actually very weak, can be displayed with the maximum exposure time when the central peak becomes extremely saturated [Figs. 5(e) and 5(f)]. The exact spot profiles for both lenses are measured in the focal plane. The Strehl ratio of the LC lens is about 80%, defined as the ratio of intensity peak measured in LC lens focal plane to that measured for the glass lens [Fig. 5(g)] and the light loss in the center lobe can explain the scattered large-area halo. The MTF for the LC lens is calculated from measured light distribution (PSF) in the focal plane. Note that the MTF is typically obtained by taking the Fourier transform of the PSF in the focal plane and normalizing it to the transform's zero frequency value (equal to the area under the PSF curve) [18]. However, experimentally, if the lens being considered has large angle scattering that results in a haze in the image, this method may not provide a useful quantification of the lens performance because the scattered light may not be completely collected by the limited detector acceptance angle.

To account for this possibility, we start with the idea that the sum of the area under the PSF curve, and the intensity of reflected light for each lens should be equal to the incident light intensity, and therefore the same for both lenses. With this thought, we normalize the curves for the LC lens to a number that is equal to the area under the glass lens's PSF (assuming no scattering measured for glass lens), corrected for the measured differences in the reflectivity of the glass and LC lens. Specifically, the normalization factor for the LC lens with our modified normalization is \( \text{Int(PSF, glass lens)} \times \frac{(1 - R_G)}{(1 - R_L)} \). Here, \( \text{Int(PSF, glass lens)} \) is the area under the PSF for the glass lens, \( R_G \) is the percentage of the total light reflected by the LC lens, and \( R_L \) is the percentage of the total light reflected by the glass lens. Therefore, this method assumes that the incident light intensity is the same and the measurements of intensity of the reflected light are quite precise. Because these assumptions can be questioned, we have plotted the MTF curves normalized by the typical method and our modified method [Fig. 5(h)]. In fact, the area of PSF of LC lens is measured as about 80% of that measured for glass lens, same as its measured Strehl ratio, indicating that the light is scattered out of the detector collection angle, the modified normalization method, should give more accurate results. With the typical normalization, the MTF curve of an LC lens is very similar to a glass lens; with the modified normalization approach, the MTF for an LC lens drops to about 0.85 at zero frequency, resulting from the scattering of the light. In addition, the cutoff frequency is measured as about 10.5 cyc/mm, close to the theoretical prediction \( D/(f \cdot \lambda) = 11 \) cyc/mm (here, \( f \) is the focal length and \( D \) is the lens diameter [14]).

To more directly isolate the effect of the electrode gaps on the actual LC lens, the scattering effects due to the phase variation across the aperture of the LC lens can be measured with a constant voltage applied to all the electrodes. For this, a collimated and linearly polarized incident light beam passes through the center area of a glass lens with a 2.4 mm aperture attached. A CCD is placed in the focal plane for measuring
light distribution. The LC lens with its rubbing direction along the light polarization axis is placed immediately behind the glass lens. Uniform voltages are applied to all electrodes, and light distribution is measured.

The data shows that the ITO electrode rings without applied field do not cause a large amount of scattering. When the voltages are applied, a scattered halo of rings are seen, which is the greatest when 2 ∼ 3 V are applied (Fig. 6), and the Strehl ratio is measured as the lowest (Table 5). It is very likely because the voltage versus phase retardation curve for the LC device happens to be very steep in the region near 2 v, which causes a larger phase variation in gaps affecting more lens area and scattering more light [21]. By factoring out the transmission loss caused by the reflections, the transmission efficiency from the phase variations, due to the gaps, as a function of different uniform voltages applied to the electrodes, can be obtained (Table 6).

In summary, we find the calculated value of the Strehl ratio for an LC lens with factors, such as 10 phase steps per wave (98.5% from Table 1, if no gaps) and 3 μm electrode gaps is 89.37% (Table 3). This can be compared with the measured Strehl ratio of 80% after correcting it by two factors to compensate for the higher reflectivity of the LC cell as compared with the glass lens (1/0.92) and for the measured light scattering by the internal components of the cell (1/0.99). Multiplying the measured Strehl ratio by the two measured correction factors yields 87.83%, as compared to the calculated value of

<table>
<thead>
<tr>
<th>Addressed Rings from Lens Center</th>
<th>NO. 1</th>
<th>NO. 5</th>
<th>NO. 10</th>
<th>NO. 14</th>
<th>NO. 19</th>
<th>NO. 23</th>
<th>NO. 28</th>
<th>NO. 33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated voltage (volt)</td>
<td>0.9</td>
<td>1.06</td>
<td>1.23</td>
<td>1.37</td>
<td>1.56</td>
<td>1.73</td>
<td>2.00</td>
<td>2.37</td>
</tr>
</tbody>
</table>

Fig. 5. (Color online) (a) Picture of the actual lens with flex connector. (b) Microscopic top view of the patterned substrate with reflected light. The bright lines are the Nickel bus lines with a width about 10 μm, and they are connected with ring electrodes through round vias (bright dots). The ring electrodes, inter-ring resistors, and gaps are shown as well. (c) Measured OPD of LC lens under optimized voltage profile. (d) Measured OPD of high-quality glass lens. (e) Original pictures captured by the CCD (22.2 mm × 14.8 mm in size) in the focal plane with 15 s exposure time for LC lens f = 400 mm, and (f) glass lens f = 400 mm. (g) Measured intensity spot profile in LC lens’s and glass lens’s focal plane, normalized to the intensity peak in glass lens’s focal plane. (h) Measured MTF of LC lens with typical and modified normalization approach, compared to glass lens.
Therefore, based on both the modeling and measurements, the main factors associated with the efficiency of the actual LC lenses and their quantitative contribution are given with the best approximation in Table 7.

Of course, there are some other factors that may not be as significant as ones already discussed for the optical performance, but should be concerned for broad applications, such as response time, dispersion of LC material, limited tunable range, etc. For our actual lens, the time response study is conducted by placing the lens between crossed polarizers with its rubbing direction 45° to them, and detecting the transmitted light intensity as a 5 V field is suddenly applied. The time duration for the 10 μm cell to switch from 90% to 10% of the maximal retardation is only about 35 ms. When the voltage is promptly removed, it takes about 0.7 s to relax back from 90% to 10% of the maximal retardation. In fact, the response time of LC cell is proportional to the square of the cell thickness. To reduce the response time, thinner cells can be used. Higher birefringence materials for lens application have been reported [22]. However, more chromatic dispersion could be caused. Dispersion of the LC material could potentially induce chromatic aberration and cause the focus points of different colors apart, particularly for a high-power lens with larger aperture. To minimize the aberration, LC mixtures with lower dispersion could be used while other parameters meet the optical performance requirement; also, combining lenses with different materials and voltage profiles could possibly compensate for the aberration with proper design. For the specific example lens in this paper, 400 mm focal length is another limitation for some applications. The lens is envisioned to be incorporated into a system with fixed lenses in applications where the F# needs to be smaller, or higher power can be obtained through the use of a Fresnel design.

5. Discussions and Conclusions

A systematic analysis of fundamental factors and their effects on the performance of LC-based electro-optical lenses is given. It has been found that electrode gaps, imperfect phase profile, and the phase sampling rate are three main factors affecting the LC lens performance. For these factors we find that the gaps between the electrodes should be less than 20% of the cell thickness, the basic shape of the phase profile should be accurate with discrepancy smaller than a quarter wave, and the phase step of the phase profile should be less than a tenth wave. It is noted that these factors are so important that new factors needed to be considered for an LC lens (the LC material, scattering for cell structure) are of lesser concern.

Appendix A

1. LC Director Modeling and Calculation of Voltage Profile

The LC director field two-dimensional (2D) calculation takes the electrode pattern, applied voltages, cell thickness, and the LC properties as input, and numerically calculates director orientation profile. Then, the lens OPD can be calculated by integrating the effective refractive index through the cell thickness.

In the modeling, a 2D grid is used; the LC director configuration is considered as a function of x and z coordinates. The x axis is defined along the lens diameter, intersecting with and perpendicular to the electrodes (strips in y direction); the z axis is
parallel to the cell normal. The modeling resolution is high as the unit length of grid is 0.25 μm in the z axis, and 0.5 μm in the x axis, so that the real electrode width and the gap between them can be taken into account to give accurate results.

The 2D electric field modeling can be achieved by solving the Laplace equation with the finite-difference method and numerical iterations [23]. In the calculation, the specific widths of discrete electrodes and the voltages applied are considered, and the electric field distribution is calculated across the cell thickness including fringing field in gaps.

Under a given voltage profile, the electric field can be calculated, and the equilibrium state of the director configuration can be obtained by minimizing the total free energy of the system. The LC director \( \vec{n}(|\vec{n}| = 1) \) can be specified by three components \((n_x, n_y, n_z)\). The elastic free energy density is expressed as a function of the director components and their spatial derivatives:

\[
f_{\text{elastic}} = \frac{1}{2} K_{11}(\nabla \cdot \vec{n})^2 + \frac{1}{2} K_{22}(\vec{n} \cdot \nabla \times \vec{n} + q_0)^2 + \frac{1}{2} K_{33}(\vec{n} \times \nabla \times \vec{n})^2.
\]  
(A1)

\( K_{11}, K_{22}, \) and \( K_{33} \) are the elastic constants representing splay, twist, and bend deformation of LC directors, and \( q_0 \) represents the chiral property of twist deformation. The electric free energy density for a constant voltage is given by

\[
f_e = -\frac{1}{2} \vec{E} \cdot \vec{D} = -\frac{1}{2} \vec{E} \cdot \left[ \varepsilon_0 \varepsilon_r \vec{E} + \varepsilon_r \Delta \varepsilon (\vec{E} \cdot \vec{n}) \vec{n} \right]
= -\frac{1}{2} \varepsilon_0 \varepsilon_r E^2 - \frac{1}{2} \varepsilon_r \Delta \varepsilon E_i n_i n_j, \quad i, j = x, y, z.
\]  
(A2)

The Gibbs free energy density \( f_G \) (the sum of the elastic and electric energy density) will be minimized when the Euler–Lagrange equation is satisfied, with the periodic boundary conditions and the fixed surface director orientation. The relaxation method calculation uses the director update formula given in Eq. (5) where \( \gamma \) is the viscosity coefficient [24]:

\[
\begin{align*}
\vec{n}_i^{\text{new}} &= \vec{n}_i^{\text{old}} - \frac{\Delta t}{\gamma} \left[ f_G \right]_{n_i}, \\
\left[ f_G \right]_{n_i} &= -\frac{\partial f_G}{\partial n_i} = \frac{\partial f_G}{\partial n_i} - \sum_{j=x,y,z} d_j \frac{\partial f_G}{\partial (n_j)}.
\end{align*}
\]  
(A3)

Here, the finite difference time derivatives are taken only in the forward direction, and the spatial derivatives are centered on grid locations by only considering the nearest neighbors. The relaxation process will scan all the grid points until the tolerance criterion for the equilibrium state is met \( (|f_G^{\text{new}} - f_G^{\text{old}}| < 10^{-6} \text{ N/m}^2) \). The electric field distribution \( V(x,y,z) \) is updated at each iteration once the dielectric tensor is regenerated by the contemporary LC configuration.

In the equilibrium state, assuming there is a linearly polarized light incident normal to cell surface (i.e., light propagates in the z direction), the OPL of the LC layers can be calculated as shown below, and \( \theta \) is the angle between individual directors to the z axis, \( d \) is the thickness of the cell:

\[
\text{OPL}(x) = \int_0^d n_{\text{eff}}(x, z) \cdot dz,
\]

\[
n_{\text{eff}}(x, z) = \frac{n_g n_e}{\sqrt{n_g^2 \cos^2 \theta(x, z) + n_e^2 \sin^2 \theta(x, z)}}.
\]  
(A4)

Here, subtracting OPL from lens center to edge gives lens OPD. The phase profile (OPD) of an ideal positive lens is approximately a parabola, expressed below \((r \) is the lens radius, and \( f \) is the desired focal length) [25]:

\[
\text{OPD}(r) \approx -\frac{r^2}{2f}.
\]  
(A5)

If the sampling rate \( f_s \) (number of phase steps per wave) is certain to represent the continuous phase profile, assuming the area of each ring electrode has an equal amount of the fraction of the unit wave, the total number of the rings is obtained as \( N = \text{OPD} \cdot f_s / \lambda \). Therefore, if the focal length is \( f = 400 \) mm, design wavelength \( \lambda = 543.5 \) nm, and 10 phase steps per wave is considered, there are totally 33 electrodes where the outer radius of each ring can be expressed as

\[
r_n = \sqrt{\frac{2nf_s}{f}}. \quad n = 1, 2, ..., N.
\]  
(A6)

Here, \( n \) is the index number of each ring electrode. In our lens, there is a 3 μm gap between any two neighboring electrodes. Therefore, the \((n+1)\) th ring’s inner radius is larger than the \( n \)th ring’s outer radius by the gap width. Once each ring’s inner and outer radius are determined, the width can be obtained. As the slope of a lens OPD increases from center to edge, the width of electrodes decreases accordingly. In this example, the central electrode disk has a radius about 200 μm, and the width of the outermost electrode is about 15 μm.

The correct voltage profile for a desired focal length can be obtained by a numerical optimization after the initial estimation. Taking a lens with focal length \( f = 400 \) mm as an example, the ideal parabolic phase profile can be calculated. As the location and the width of the electrodes are known, the OPD value for the center of each electrode can be obtained by interpolation. For the LC lens, the available OPD range is determined by the cell thickness and the material birefringence. The property parameters of the LC material used in the modeling are \( K_{11} = \)
11.0 \times 10^{-12}, \ K_{22} = 7.1 \times 10^{-12}, \ K_{23} = 30.5 \times 10^{-12}, 
 n_c = 1.8, \ n_o = 1.53, \ \Delta n = 0.27 (\lambda = 589 \text{ nm}, \ 20^\circ C), 
 e_3 = 21.9, \text{ and } e_1 = 6.1 (1 \text{ kHz}, \ 20^\circ C).

With this method, the optimized voltage profile for all electrodes is obtained as well as 2D director profile, from which, the resultant one-dimensional phase profile across the diameter of LC lens can be obtained.

2. Light Propagation and Lens Modeling

Light propagation and lens modeling algorithms are developed based on physical optics and scalar diffraction theory. The electric field distribution in the observation plane \( U_2(x, y) \) can be most accurately predicted by using the first Rayleigh–Sommerfeld diffraction solution [19]:

\[
U_2(x, y) = \frac{z}{j k} \int \int_U U_1(\xi, \eta) \exp(j k r_{12}) \frac{d\xi d\eta}{r_{12}},
\]

\[
r_{12} = \sqrt{z^2 + (x - \xi)^2 + (y - \eta)^2}. \quad (A7)
\]

Here, \((x, y)\) is the coordinate in the observation plane and \((\xi, \eta)\) is for the object plane, \( r_{12} \) is the distance between a point in object plane and one in observation plane, \( k \) is the wave vector, \( z \) is the distance between two planar planes, and \( i \) is the imaginary unit. Therefore, if the field distribution in one plane is given, the light distribution in another plane can be accurately simulated \((z \gg \lambda)\).

To simulate the propagation of the collimated plane wave through a thin lens, the aperture with the size of lens diameter is illuminated, and the field has unit amplitude within the aperture and zero outside, multiplied by a phase transmittance function in an exponential form of the lens OPD:

\[
U_1 = |U_1|e^{j\text{OPD}}. \quad (A8)
\]

The field distribution in any plane behind the lens can be calculated by using the Eq. (A8), and multiplying with its complex conjugate would give the light intensity distribution. Therefore, when the parabolic phase profile of ideal lens is used, the airy disk pattern in lens focal plane can be calculated by letting \( z = f \).

References