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FCT-Based Convolution, Filtering and Correlation of Signals' Unified Structure

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Abstract

This paper focuses on constructing efficient algorithms of the main DSP discrete procedures: convolution, correlation functions and filtering of signals based on discrete cosine transform (DCT-II) and presenting them using unified structure. This structure is very useful in reducing the computational complexity by $2N-3$ real multiplications and $N-2$ real additions in comparison with indirect fast Fourier transforms (FFT) based algorithms calculated through fast cosine transform (FCT). The DCT and its fast calculation ways effectively can be used to calculate convolution, filtering and correlation of signals. For their calculating, the classic schema (two DCT + product of cosine spectrums + IDCT) will be saved.

Keywords – ***Convolution, Filtering and Correlation of Signals, Fast Fourier Transform (FFT), Fast Cosine Transform (FCT).***

1. Introduction

One of the most widely used complex techniques of Digital Signal Processing (DSP) is a convolution sum of two finite-duration sequences of length N (arbitrary input signal $x(n)$ and the response of the linear system $h(n)$) to generate an output sequence, which is denoted as $z(n)$ and represented compactly as $z(n)=x(n)\circ h(n)$ [1, 2, 3, 4, 5]. In particular, the filtering and correlation of signals that widely used in different DSP practical applications are convolution-based procedures [6, 7, 8, 9, 10, 11]. The procedures for computing the convolution can be calculated in two completely different ways. The first is a direct method, in which the problem can be approached directly in time-domain representation of the digital signals and it is called conventional or standard method. The second method is more practical and consists of indirect applying of a class of orthogonal transforms. In particular, we shall present here three such transforms, namely, the discrete Fourier transform (DFT), discrete Hartley transform (DHT) [6, 7, 12, 20, 24] and finally we well focus our attention as the paper concentrates on DCT [6, 13, 14, 15, 24]. It is important to remember that the above two approaches produce an identical output. The choice of the method in actual practice depends on the length of the convolution and the existence of computationally efficient algorithms (frequency-time transforms); convolution of small sequences is faster computed directly, and convolution of very large sequences faster computed in frequency-domain [17].

In this work the main instrument for computing the convolution is the second version of cosine transform known as DCT-II, the DCT and inverse DCT (IDCT) are widely used as a computational tools that play a very important role in many DSP applications, such as linear filtering, frequency analysis, image processing and image compression. The importance of the DCT and IDCT in such practical applications is due to the existence of computationally efficient algorithms [15, 17].

- **Frequency-domain based filtering and correlation of signals computation using orthogonal transforms**

In this section, the necessity of using the orthogonal transforms and their fast algorithms to compute the convolution will be demonstrated, due to the fact that many sources of information noticed that the direct computing of convolution can be accomplished with N^2 of arithmetic operations, as a result, the total number of operations to compute an N-point convolution increases very rapidly as N increases, so design effective algorithms is an important requirement.

The general form for fast computing of convolution based on orthogonal transforms has the following notation:

$$z(n)=IT\{T[x(n)]\circ T[h(k-n)]\}$$

Where $T[.]$ denotes some discrete transform of the quantity in brackets, $IT\{.\}$ denotes the inverse transform. Examples of these transforms are DFT, DHT, discrete Walsh transform (DWT) and others [18, 19, 20].

To be specific, suppose that $x(n)$ and $h(n)$, $n=0,1,...,N-1$ are two sequences and the real sequences $z(n)$ and $y(n)$, $n=0,1,...,N-1$ are the periodic convolution and correlation respectively, they can be represented as:

$$z(n) = x(n) \circ h(n) = \sum_{k=0}^{N-1} x(k)h(n-k),$$
$$n = 0,1,..., N-1 \qquad (1)$$

$$y(n) = x(n) \bullet h(n) = \sum_{k=0}^{N-1} x(k)h(n+k),$$
$$n = 0,1,..., N-1 \qquad (2)$$

In fact, the frequency-domain approach of the digital signals based on orthogonal transforms is computationally more efficient than time-domain approach; this is due to the existence of efficient algorithms for computing these transforms. This approach leads to different algorithms based on FFT, fast Hartley transform (FHT) and fast cosine transform (FCT). More simple interpretation of convolution and correlation algorithms is Fourier and Hartley based that will be explained in more details.

- **DFT-based filtering and correlation**

Since the DFT provides a discrete frequency representation of a finite-duration sequence in the frequency-domain and it has a low computational complexity using FFT, so it is interesting to explore its use as an efficient computational tool for linear system analysis and linear filtering using the convolution theorem [12, 17]. The convolution and correlation theorems for DFT are given as [7, 9, 12, 22]:

$$Z(k) = X(k) * H(k), k = 0,1,...,N-1 \qquad (3)$$

$$Y(k) = X^*(k) * H(k), k = 0,1,...,N-1 \qquad (4)$$

Where $Z(k)$ and $Y(k)$ are the $N-$point DFT of sequences $z(n)$ and $y(n)$ respectively (the spectrum result for convolution and correlation), $X(k)$ and $H(k)$ are the corresponding $N-$point DFT of sequences $x(n)$ and $h(n)$ respectively (spectrums of the input $x(n)$ and the impulse response of the system $h(n)$): $Z(k) = DFT_N\{z(n)\}$, $X(k) = DFT_N\{x(n)\}$, $H(k) = DFT_N\{h(n)\}$, $Y(k) = DFT_N\{y(n)\}$, and $[\ ]^*$ - sign of complex conjugate.

Now observe that the periodic convolution (1) (or correlation (2)) using the DFT transform can be implemented using of the following steps:

Step 1. Compute the $N-$point DFT of the sequences $x(n)$ and $h(n)$: $X(k) = DFT\{x(n)\}$ (Time-domain →frequency-domain)
$H(k) = DFT\{h(n)\}$ (Time-domain →frequency-domain)
Step 2. Compute the point-wise complex product $Z(k)$ using equation (3) for filtering (or complex product $Y(k)$ using equation (4) for correlation); the result of this step will be in frequency-domain.
Step 3. Compute the inverse $N-$point DFT of sequences $Z(k)$ (or $Y(k)$) to obtain the output sequences $z(n)$ (or $y(n)$), the result will be in time-domain.

This process is shown in Figure 1. In some applications, particularly in case of infinite-length sequences, this FFT-based approach may be more convenient to carry out than the standard approach [12].

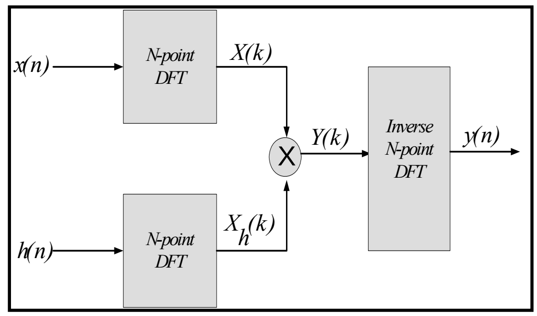

Fig.1: Block diagram representation of DFT-based convolution and correlation algorithms

Let us illustrate this with the help of an example, for instance if the DFT is used as a base transform, and for its calculations radix-2 FFT algorithm is used, the computational complexity of convolution or correlation requires $4N log_2^N$ real arithmetic operations, which means that the amount of arithmetic operations have been reduced by log_2^N times.

To calculate the linear aperiodic convolution or correlation, firstly the original finite-duration sequences $x(n)$ and $h(n)$ padded by N zeros, this zero padding process does not provide any additional information about the spectrum and secondly $2N-$point DFT can be computed [7].

- **DHT-based filtering and correlation**

The convolution and correlation theorems for DHT are given as [20, 22]:

$$Z_H(k) = [\ X_H(k) * H_H(k) - X_H(-k) * H_H(-k) + X_H(k) * H_H(-k) + X_H(-k) * H_H(k)\ ]/2,$$
$$k = 0,1,...N-1 \qquad (5)$$

$$Y_H(k) = [\ X_H(k) * H_H(k) + X_H(-k) * H_H(-k) + X_H(k) * H_H(-k) - X_H(-k) * H_H(k)\ ]/2,$$
$$k = 0,1,...N-1 \qquad (6)$$

Where $Z_H(k)$, $X_H(k)$, $H_H(k)$ and $Y_H(k)$ are $N-$point DHT of sequences $z(n)$, $x(n)$, $h(n)$ and $y(n)$ respectively; $Z_H(k) = DHT_N\{z(n)\}$,

$X_H(k) = DHT_N\{x(n)\}$, $H_H(k) = DHT_N\{h(n)\}$, $Y_H(k) = DHT_N\{y(n)\}$.

In a similar manner, comparing with the previous approach that uses DFT transform, this approach saves the same structure for computing $N-$point periodic convolution (or correlation function) , it leads to compute three $N-$point DHT, product of Hartley real spectrums (using equation (5) for filtering and (6) for correlation) and determine the inverse $N-$point DHT. For linear convolution three $2N-$point DHT should be used, this process is shown in Figure 2.

Let us now describe an interesting relationship between the DFT-based approach and the DHT-based approach, at the first glance, using DHT for convolution of real sequences $x(n)$ and $h(n)$ will be faster because:

- DFT is defined for complex- valued signals, and DHT is defined for real-valued signals, so sequences $Z(k)$, $Y(k)$ consist of complex numbers and $Z_H(k)$, $X_H(k)$ consist of real numbers, for this reason, the second approach allows twice reducing the amount of arithmetic operations.
- DHT has symmetry property, so

using this property; the direct and inverse transforms can be computed with the same FHT algorithm [18, 20, 21].

Therefore, as shown in [22], using special algorithms FFT of real (FFT_r) or Hermitian symmetric (FFT_h) sequences, DHT will not have an advantages in efficiency over FFT, in practical work, all of these algorithms (FFT_r, FFT_h and FHT) require the same amount of arithmetic operations and each of them can be used as the base algorithm for calculating other transforms. In the next section, the relation between effective algorithms of direct and inverse DCT and special variant of DFT will be explained.

2. Indirect algorithms (FFT→FCT)-based convolution and correlation

Traditional approach for calculating FCT algorithms is to build them using FFT or FHT algorithms, because historical FFT algorithms were build firstly, so all DSP applications, as possible, were solved through FFT.

Alternative approach, first time offered in [22], FCT is used as a base transform for DFT-DHT calculation, this approach explains the relationship between DCT and DFT, to describe this relationship, let $x(n)$, $n = 0,1,...,N-1$, some real input sequence, $L_N^{II}(k)$ and $X(k)$ are the $N-$point DCT and DFT respectively, which could be calculated using the following equations:

$$L_N^{II}(k) = DCT_{\mathbf{N}}^{II}\{x(n)\} = \sum_{n=0}^{N-1} x(n) C_{4N}^{(2n+1)k} \quad (7)$$

$$X(k) = DFT_{\mathbf{N}}\{x(n)\} = \sum_{n=0}^{N-1} x(n)(C_N^{nk} - jS_N^{nk}) \quad (8)$$

Where $C_K^r = cos(2\pi r/K)$, $S_K^r = cos(2\pi r/K)$ and $j = \sqrt{-1}$.

The inverse transforms IDCT and IDFT for (7) and (8) can be given by the following formulas:

$$x(n) = IDCT_{\mathbf{N}}^{II}\{L_N^{II}(k)\} = \sum_{k=0}^{N-1} p_k L_N^{II}(k) C_{4N}^{(2n+1)k}$$

$$x(n) = IDFT_{\mathbf{N}}\{X(k)\} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)(C_N^{nk} + jS_N^{nk})$$

Where $p_0 = 1/N$, $p_k = 2/N, k \neq 0$. Sequence $X(k)$ has a Hermitian symmetry: $X(k) \equiv X^*(N-k)$, because $x(n)$ is a real sequence [17].

To calculate DFT of the sequence $x(n)$ based on FCT or IDFT of sequence $X(k)$ based on IFCT, *Algorithm 1* or *Algorithm 2* are used respectively [22].

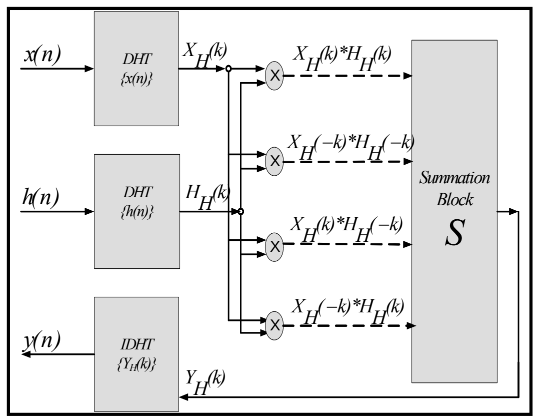

Fig.2: Block diagram representation of DHT-based convolution and correlation algorithms

Algorithm 1: Calculate DFT based on DCT

Step 1. Rearrangement the input sequence $x(n)$ to get a new sequence $x_T(n)$:

$$x_T(2n) = x(n),\quad x_T(2n+1) = x(N-1-n),\quad n = 0,1,...,N/2-1 \quad (9)$$

Step 2. Computation the DCT of $x_T(n)$ sequence using FCT algorithm:

$$L_{TN}^{II}(k) = DCT_N^{II}\{x_T(n)\}.$$

Step 3. Recovering the DFT_r of real sequence from DCT, to accomplish this task, the relationship between real part, imaginary part of DFTr and DCT is considered using formulas for getting the real and imaginary parts as in the equations below:

$$Re\{X(0)\} = L_{TN}^{II}(0),$$
$$Re\{X(N/2)\} = \sqrt{2}/2 L_{TN}^{II}(N/2) \quad (10)$$
$$\left.\begin{aligned} &Re\{X(k)\} = L_{TN}^{II}(k)\cdot C_{4N}^{k} + L_{TN}^{II}(N-k)\cdot S_{4N}^{k},\\ &-Im\{X(k)\} = L_{TN}^{II}(N-k)\cdot C_{4N}^{k} - L_{TN}^{II}(k)\cdot S_{4N}^{k},\\ &k = 1,2,...,N/2-1 \quad (11)\end{aligned}\right\}$$

This step is called *"Vector Rotation"* operations. The structural schema for this algorithm is shown in Figure 3, where R_N is the permutation block that implements the first step of the given algorithm.

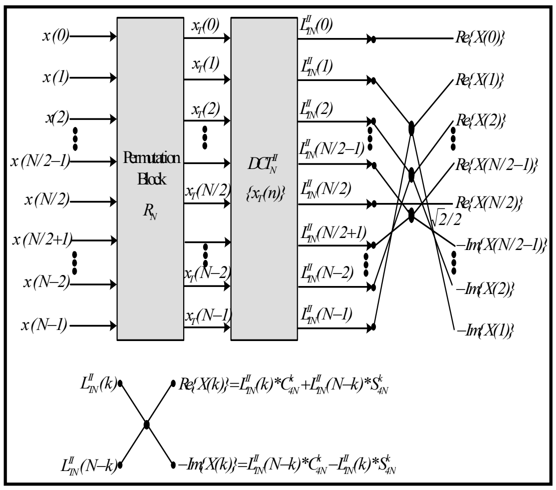

Fig.3: Structural schema for computing DFT through DCT-II

Algorithm 2: Calculate IDFT of Hermitian symmetric sequence based on DCT

Step 1. Generate the IDCT of real sequence from IDFT:

$$L_{TN}^{II}(0) = Re\{X(0)\},$$
$$L_{TN}^{II}(N/2) = \sqrt{2} Re\{X(N/2)\} \quad (12)$$
$$\left.\begin{aligned} &L_{TN}^{II}(k) = Re\{X(k)\}\cdot C_{4N}^{k} + Im\{X(k)\}\cdot S_{4N}^{k},\\ &L_{TN}^{II}(N-k) = -Im\{X(k)\}\cdot C_{4N}^{k} + Re\{X(k)\}\cdot S_{4N}^{k},\\ &k = 1,2,...,N/2-1 \quad (13)\end{aligned}\right\}$$

Step 2. Compute IDCT for $L_{TN}^{II}(k)$ using IFCT algorithm: $x_T(n) = IDCT_N^{II}\{L_{TN}^{II}(k)\}$.

Step 3. Reform the original sequence $x(n)$ from the sequence $x_T(n)$:

$$x(n) = x_T(2n),$$
$$x(N-1-n) = x_T(2n+1),$$
$$n = 0,1,...,N/2-1 \quad (14)$$

The structural schema for this algorithm is shown in Figure 4. Therefore, using indirect algorithms FCT and IFCT based on DCT and IDCT allow calculating convolution and correlation.

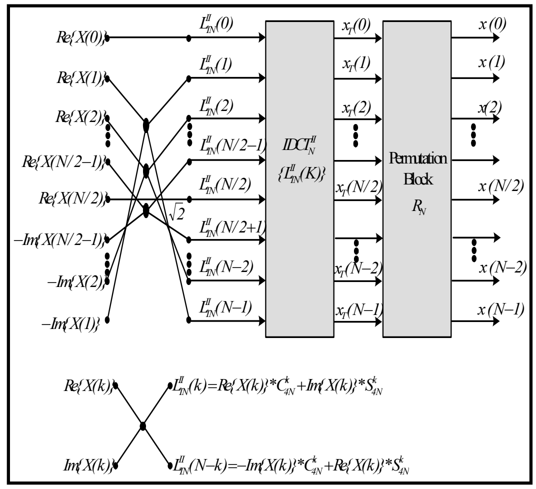

Fig. 4: Structural schema for computing IDFT through IDCT-II

Now, using *Algorithm 1* and *Algorithm 2*, the convolution and the correlation of two sequences can be calculated using *Algorithm 3* and *Algorithm 4*.

Algorithm 3: Indirect calculation of convolution

Step 1. Using *Algorithm 1*, calculate $N-$point DFT_r of real sequences $x(n)$ and $h(n)$, sequences $X(k)$, $X_h(k)$, $k = 0,1,...,N-1$ will be obtained.

Step 2. Calculate the product of spectrums $X(k)$, $X_h(k)$:

$$Re\{Z(m)\} = Re\{X(m)\} * Re\{X_h(m)\}$$
$$Im\{Z(m)\} = 0,\ m = 0, N/2;$$
$$Re\{Z(k)\} = Re\{X(k)\} * Re\{X_h(k)\} - Im\{X(k)\} * Im\{X_h(k)\}$$
$$Im\{Z(k)\} = Re\{X(k)\} * Im\{X_h(k)\} + Im\{X(k)\} * Re\{X_h(k)\};$$
$$Re\{Z(N-k)\} = Re\{Z(k)\}$$
$$Im\{Z(N-k)\} = -Re\{Z(k)\};$$
$$k = 0,1,...,N-1.$$

Step 3. Using *Algorithm 2*, calculate $N-$point IDFT of the Hermitian sequence $Z(k)$, $k = 0,1,...,N-1$, the required sequence $z(n)$ (the convolution of the two sequences $x(n)$ and $h(n)$, $n = 0,1,...,N-1$) will be obtained.

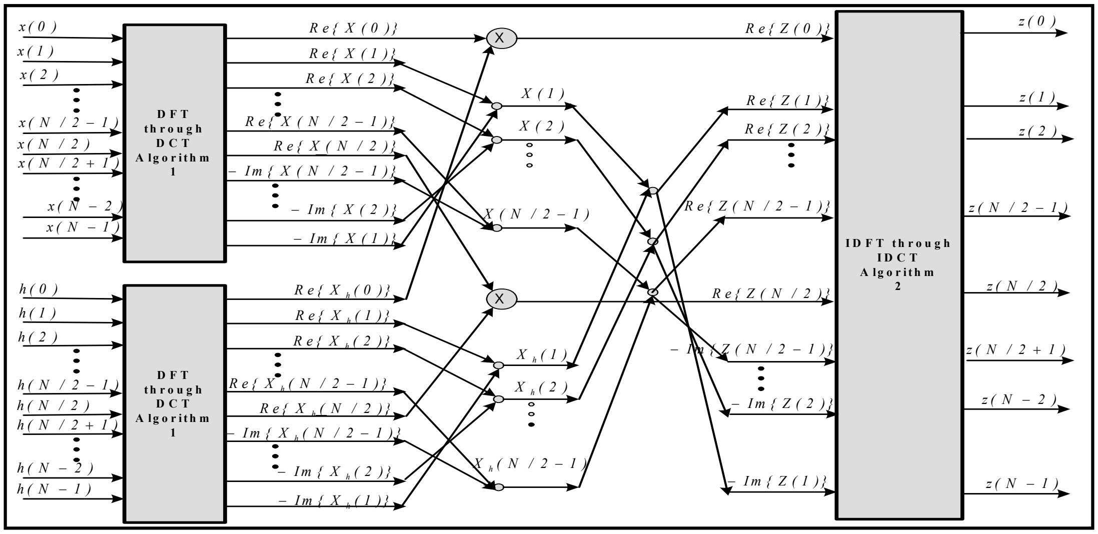

Fig.5: Block diagram representation of (FFT→FCT)-based convolution and correlation algorithms

Algorithm 4: Indirect calculation of correlation

Step 1. Using *Algorithm 1*, calculate $N-$point DFT_r of real sequences $x(n)$ and $h(n)$, sequences $X(k)$, $X_h(k)$, $k=0,1,...,N-1$ will be obtained.

Step 2. Calculate the product of spectrums $X(k)$, $X_h(k)$:

$$Re\{Z(m)\}=Re\{X(m)\}*Re\{X_h(m)\}$$
$$Im\{Z(m)\}=0\,,\ m=0,N/2\,;$$
$$Re\{Z(k)\}=Re\{X(k)\}*Re\{X_h(k)\}+ Im\{X(k)\}*Im\{X_h(k)\}$$
$$Im\{Z(k)\}=Re\{X(k)\}*Im\{X_h(k)\}- Im\{X(k)\}*Re\{X_h(k)\};$$
$$\left.\begin{aligned}Re\{Z(N-k)\}&=Re\{Z(k)\}\\ Im\{Z(N-k)\}&=-Re\{Z(k)\};\end{aligned}\right. \quad k=0,1,...,N-1\,.$$

Step 3. Using *Algorithm 2*, fulfill $N-$point IDFT of the Hermitian sequence $Z(k)$, $k=0,1,...,N-1$, the required sequence $y(n)$ (correlation of two sequences $x(n)$ and $h(n)$, $n=0,1,...,N-1$) will be obtained.

The graphical representation schema for these algorithms (*Algorithm 3* and *Algorithm 4*) is shown in Figure 5, the operations in the second step of these algorithms (called "*Vector Rotation Operations*") are realized, and the graphical schema of the base operation for calculating the product of two spectrums is presented in Figure 6. The analysis of this schema (Figure 5) shows that the classical algorithms of convolution and correlation through FFT are realized, with the difference, that indirect algorithms FFT_r and FFT_h through FCT and IFCT are used (see Figure 1). Using the below structure (Figure 5) have a sense when DSP computational system requires the fulfillment of the FCT, FFT_r, $IFFT_h$ and of course, convolution and correlation. If there isn't any need in DSP system to fulfillment the FFT_r and $IFFT_h$, then its possible to design more effective convolution and correlation algorithms of signals directly using FCT. These algorithms will be developed in the next section.

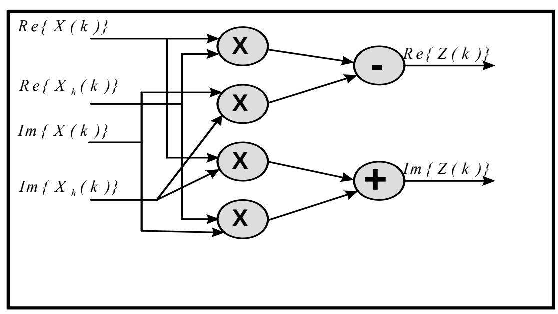

Fig. 6: Structural representation of the base operation for calculating the product of two spectrums

3. FCT-based convolution and correlation

The development of convolution's and correlation's theorems leads to effective using DFT and DHT to calculate correlation functions and filtering of signals, for DCT, until now such theorems doesn't exist, but if we turn our focus to the other side, i.e., to the presence of special computational means for FCT realization [23], it makes the development of DCT-based filtering and correlation algorithms an actual problem.

Therefore, using in the previous section, the simplest variant of solving the task of DCT-based filtering and correlation of signals consists in using the theorems of convolution and correlation, and the indirect algorithms for FFT-FHT calculation through FCT and vice verse. But this approach has theoretical and practical disadvantages, in theoretical aspect, we hope that the convolution of two signals in frequency-domain can be formulated directly through the product of their transform; practical disadvantage consists in the redundancy calculation because of the transfer from FFT-FHT to FCT and vice verse.

To solve these disadvantages, new variants of algorithms for DCT-based filtering and correlation are suggested in the following subsections (A and B).

Using *Algorithm 3* or *Algorithm 4,* the convolution or correlation of signals with the same above explained classic schema (*call it schema 1*):

- Two DFT through DCT + product of spectrums + IDFT through IDCT can be computed.
- To decrease the computational complexity and mainly to provide convolution and correlation to new schema: two DCT + product of spectrums + IDCT (*call it schema 2*), put together step 3 of *Algorithm1*, product of spectrums (3) for convolution or (4) for correlation, and step 1 of *Algorithm 2* in one global step.

A) *DCT-IDCT (FCT-IFCT)-based filtering*

Let sequences $X_{TN}^{II}(k)$ and $H_{TN}^{II}(k)$ are the DCT results of sequences $x_T(n)$ and $h_T(n)$ obtained from $x(n)$ and $h(n)$ after reordering (9) respectively. Then, putting together transforms (10), (11) for each ($X_{TN}^{II}(k)$ and $H_{TN}^{II}(k)$), spectrums product (3) and considering that

$$\left.\begin{aligned} X(k) &= Re\{X(k)\} + j\,Im\{X(k)\},\\ H(k) &= Re\{H(k)\} + j\,Im\{H(k)\} \end{aligned}\right\}$$

After non complex transforms, the following equations will be obtained:

$$\left.\begin{aligned} Re\{Z(k)\} &= \left((C_{4N}^k)^2 - (S_{4N}^k)^2\right)\cdot A_k + 2C_{4N}^k S_{4N}^k \cdot B_k,\\ -Im\{Z(k)\} &= \left((C_{4N}^k)^2 - (S_{4N}^k)^2\right)\cdot B_k - 2C_{4N}^k S_{4N}^k \cdot A_k,\\ k &= 0,1,\ldots,N-1 \qquad (15) \end{aligned}\right\}$$

$$A_k = X_{TN}^{II}(k)\cdot H_{TN}^{II}(k) - X_{TN}^{II}(N-k)\cdot H_{TN}^{II}(N-k) \quad (16)$$

$$B_k = X_{TN}^{II}(k)\cdot H_{TN}^{II}(N-k) + X_{TN}^{II}(N-k)\cdot H_{TN}^{II}(k) \quad (17)$$

Now, putting together (12), (13) and (15), and taking into consideration the cosine and sine double angel formulas, the following equations will be obtained:

$$\left.\begin{aligned} Z_{TN}^{II}(0) &= X_{TN}^{II}(0)\cdot H_{TN}^{II}(0),\\ Z_{TN}^{II}(N/2) &= \sqrt{2}X_{TN}^{II}(N/2)\cdot H_{TN}^{II}(N/2) \quad (18) \end{aligned}\right\}$$

$$\left.\begin{aligned} Z_{TN}^{II}(k) &= A_k\cdot C_{4N}^k + S_{4N}^k\cdot B_k,\\ Z_{TN}^{II}(N-k) &= B_k\cdot C_{4N}^k - A_k\cdot S_{4N}^k,\\ k &= 1,2,\ldots,N/2-1 \qquad (19) \end{aligned}\right\}$$

Where A_k and B_k are defined by equations (16) and (17). Using the obtained equations, the *Algorithm 5* could be used to find the convolution.

Algorithm 5: Convolution calculation

Step 1. According to equation (9), the additional sequences $x_T(n)$ and $h_T(n)$ will be formulated.

Step 2. Using FCT, calculate cosine spectrums of sequences $x_T(n)$ and $h_T(n)$:

$X_{TN}^{II}(k) = DCT_N^{II}\{x_T(n)\}$,

$H_{TN}^{II}(k) = DCT_N^{II}\{h_T(n)\}$.

Step 3. The multiplication of cosine spectrums can be done using the formulas (16)-(19), $Z_{TN}^{II}(k)$ will be obtained.

Step 4. Using IFCT, calculate IDCT sequence $Z_{TN}^{II}(k)$: $z_T(n) = IDCT_N^{II}\{Z_{TN}^{II}(k)\}$.

Step 5. Using formulas (14), get sequence $z(n)$.

B) (FCT-IFCT)-based correlation

In similar manner, let sequences $X_{TN}^{II}(k)$ and $H_{TN}^{II}(k)$ are the result of fulfillment DCT for sequences $x_T(n)$ and $h_T(n)$ obtained from $x(n)$ and $h(n)$ after permutation (9). Considering that

$$\left.\begin{aligned} X(k) &= Re\{X(k)\} + j\,Im\{X(k)\},\\ X^*(k) &= Re\{X(k)\} - j\,Im\{X(k)\},\\ H(k) &= Re\{H(k)\} + j\,Im\{H(k)\} \qquad (20) \end{aligned}\right\}$$

and using spectrums product formula (4) and transfer formulas from DCT to DFT of real sequence (10), (11) for each ($X_{TN}^{II}(k), H_{TN}^{II}(k)$), after non complex transforms, equations 21-23 can be obtained

$$Re\{Y(k)\} = A_k, \quad -Im\{Y(k)\} = B_k,\quad k = 0,1,\ldots,N-1 \quad (21)$$

$$A_k = X_{TN}^{II}(k)\cdot H_{TN}^{II}(k) + X_{TN}^{II}(N-k)\cdot H_{TN}^{II}(N-k) \quad (22)$$

$$B_k = X_{TN}^{II}(k)\cdot H_{TN}^{II}(N-k) - X_{TN}^{II}(N-k)\cdot H_{TN}^{II}(k) \quad (23)$$

Now, grouping transforms (21) and transfer formulas from IDFT to IDCT of real sequence (12), (13), equations (24), (25) can be obtained

$$Y_{TN}^{II}(0) = X_{TN}^{II}(0) \cdot H_{TN}^{II}(0)$$
$$Y_{TN}^{II}(N/2) = \sqrt{2} X_{TN}^{II}(N/2) \cdot H_{TN}^{II}(N/2) \quad (24)$$

$$Y_{TN}^{II}(k) = A_k \cdot C_{4N}^k - S_{4N}^k \cdot B_k,$$
$$Y_{TN}^{II}(N-k) = B_k \cdot C_{4N}^k + A_k \cdot S_{4N}^k,$$
$$k = 1, 2, \ldots, N/2-1 \quad (25)$$

Where A_k and B_k are defined by formulas (22) and (23). Using the obtained equations, the algorithm 6 could be used to calculate the correlation.

Algorithm 6: Correlation calculation

Step 1. Using formula (9), formulate the additional sequences $x_T(n)$ and $h_T(n)$.

Step 2. Using FCT, calculate cosine spectrums of the sequences $x_T(n)$ and $h_T(n)$:

$$X_{TN}^{II}(k) = DCT_N^{II}\{x_T(n)\},$$
$$H_{TN}^{II}(k) = DCT_N^{II}\{h_T(n)\}.$$

Step 3. Using formulas (22)-(25), calculate the product of cosine spectrums to get the sequence $Y_{TN}^{II}(k)$.

Step 4. Using IFCT algorithm, calculate IDCT sequence $Y_{TN}^{II}(k)$:

$$y_T(n) = IDCT_N^{II}\{Y_{TN}^{II}(k)\}.$$

Step 5. Using formula (14), get the $y(n)$ sequence.

The comparison between schema 1 and schema 2 shows that the second schema allows reducing one stage (transfer from DFT to DCT that contains $N/2$ vector rotation operations). The trivial variant of this operation is calculated using formula (12); it requires one real multiplication, the general variant of vector rotation operation (13) requires four multiplications and two additions. The above described stage consists of one trivial and $(N/2-1)$ general vector rotation operations. So $4(N/2-1)+1$ multiplications and $2(N/2-1)$ additions of real numbers are reduced.

DCT allows constructing the unification device for filtering-correlation of signals. Figure 7 illustrates the structural schema of FCT-based correlation and filtering of signals calculation. PB1 and PB2 are the permutations blocks that can be calculated according to (9) and (14). Note that these permutations in principal don't affect in the algorithm complexity because they can be effectively fulfilled during the required permutation for FFT bit-reverse indexing operation [22]. The DCT and its fast calculation ways effectively can be used to calculate the filtering and correlation of signals as shown in Figure 7. For calculating the convolution and correlation using DCT, the classic schema (two DCT + product of cosine spectrums + IDCT) is saved.

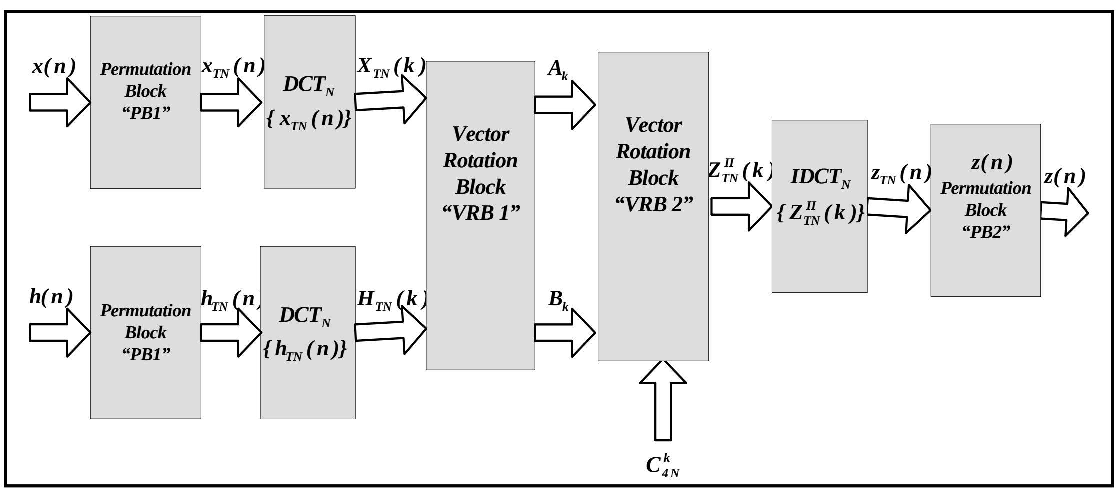

Fig.7: Structural schema for FCT-based correlation and filtering of signals algorithm

4. Conclusions

1. In this paper, the efficient algorithms of DSP procedures: convolution, correlation and filtering of signals based on DCT-II and their unified structure were presented.
2. DCT transform is extremely useful in computing convolution, correlation and filtering of signals, its FCT algorithms and their ways of calculation can be used effectively to fulfill the correlation and filtering of signals procedures using schema analog to FFT-FHT algorithms schema which means that it can be used as a base operational means for DSP applications.
3. Computational complexity analysis shows that the FCT-based algorithms for $N-$ point convolution and correlation allow reducing $2N-3$ real multiplications and $N-2$ real additions in comparison with FFT based algorithms calculated through FCT.

5. References

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