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Uneven Development and the Dynamics of Distortion[¤]

Peter Skott[y] Rajiv Sethi[z]

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Abstract

This paper develops a two-country, two-sector model of international trade with increasing returns to scale in one sector. Free trade leads to asymmetric equilibria even when both countries are identical with respect to technology, tastes and factor endowments. For some parameter values, trade leads to a uniform welfare improvement in both countries, while for others it can give rise to uneven development in the sense of persistent disparities in wages, income and welfare. In the latter case, distortionary industrial policy by the less developed country may be welfare enhancing. If the dynamics of policy changes are endogenized, the model gives rise to periodic changes in industrial leadership or leapfrogging. Implications of this phenomenon for the empirical literature on convergence are discussed.

JEL Classi...cation: F12, F13, O19

Keywords: Uneven Development, Scale Economies, Trade, Distortion, Convergence, Growth Regressions

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1 Introduction

This paper presents a stylized model of the interaction between standard economic forces and changes in the policy regime. This interaction is modelled as a two way process: policy regimes shape the economic outcome, but economic outcomes in turn lead to induced changes in policy regimes.

Our starting point is a model of international trade under increasing returns which gives rise to uneven development, in the sense of persistent income disparities between otherwise identical countries. The idea that trade under increasing returns can give rise to uneven development goes back at least to the work of Allyn Young (1928) and the development literature from the 1940s onwards, e.g. Rosenstein-Rodan (1943), Myrdal (1957) and Kaldor (1970). According to this tradition the economic forces of the market system generate uneven development. Some countries prosper while others stagnate, and this tendency to polarization provokes changes in policies and institutions in the countries that lag behind. Thus, the endogenous generation of income inequalities tends to undermine a premise of exogenously given policies and institutions. The development process involves the interaction between economic forces and wider social and political change, and theories that take the institutional framework as exogenously given may turn out to be seriously misleading.

With the recent revival of interest in growth theory, theories of uneven development now …nd support in a number of contemporary models. The introduction of non-decreasing returns to the reproducible factors into models with spatial disaggregation has been used by several writers to generate growth patterns with some form of uneven development (early contributions include Krugman (1981) and Lucas (1988)). Despite the similarities between the old literature on uneven development and the recent work, however, large di¤erences remain between the two traditions. Perhaps the most striking di¤erence is the insistence in the 'new' growth theory that optimization subject to well-speci…ed structures of preferences and technology be at the centre of the analysis. A great advantage of this approach is that it forces one to be quite explicit about the links in the argument, where, in the past, too often economists may have settled for vague verbal descriptions of the processes involved. But the optimizing approach easily diverts attention from crucial aspects of the growth process. Preferences may not be constant, technological possibility sets can be unknown, and the growth process takes place within a changing political and institutional environment. The new growth literature implicitly denies or overlooks the interaction between economic performance and institutional change.

The present paper takes a di¤erent approach. We shall set up a small structural model with distinct and well-de…ned policy regimes. Free trade gives rise to uneven development and makes distortionary policy attractive in the lagging region. To the extent that these policies are successful, income leadership is transferred from one region to the other. This

sets up pressures for distortion in the country that …nds itself pushed into second place, even as it builds pressure for the removal of distortions in the country that has attained leadership. Periodic changes in income leadership and in the degree of distortion in various countries may be the result. The argument has implications for the interpretation of the empirical literature on economic 'convergence', some of which are discussed below.

The paper is in six sections. Section 2 presents a simple two-country model with free trade. The model includes two traded goods, and increasing returns in one of the traded good sectors implies the existence of multiple equilibria, an unstable symmetric equilibrium and at least two locally stable asymmetric equilibria. Section 3 introduces in interventionist policy regime. Countries may choose to tax one traded good sector in order to subsidize the other. Policy changes occur in response to dissatisfaction with economic performance, and this process of endogenous policy change is analyzed in Section 4. Section 5 relates the argument to empirical issues, with particular attention paid to the literature on economic convergence, and Section 6 concludes. All proofs are collected in an Appendix.

2 A model of uneven development

2.1 Basic assumptions

There are 2 countries. The countries are identical with respect to technology, and there is only one input, labor. Labor is homogeneous and the two countries have the same, inelastic labor supply L, which is set equal to 1 by choice of units. There are two traded goods, a 'manufacturing' good (M) with (external) increasing returns to scale and an 'agricultural' good (A) with constant returns to scale. The precise speci…cation of the production functions a¤ects the quantitative results but is not critical for the qualitative conclusions stressed in this paper. For analytical convenience the following simple formulations are used:

$$M_i = B_i x_i \tag{1}$$

$$B_i = x_i^{\circ}; \qquad \circ > 0 \tag{2}$$

$$A_i = 1 \text{ ¡ } x_i \tag{3}$$

where subscript i indicates country and x_i denotes the share of the labor force employed in manufacturing. Equations (1–2) describe the manufacturing sector. The increasing returns are external to the individual …rm, but country speci…c, and enter the manufacturing sector through the coe¢cient B_i. There is no open unemployment, so the share of the labor force in agriculture is simply $1 \text{ ¡ } x_i$.

There are no costs of transportation and the 'law of one price' holds for all traded goods. All producers act as price takers and, by assumption, the individual producer faces constant

returns to scale. It follows that there will be no pro…ts and that the wage rate in a sector will be equal to the value of the average product. Letting the price of the agricultural good act as numeraire, with p representing the relative price of the manufactured good, we have

$$w_i = pB_i = px_i^{\circ} \tag{4}$$

where w_i is the manufacturing wage in country i. Since the price of the agricultural good is set equal to 1, with constant returns to scale the agricultural wage will also equal 1. The mean wage, therefore, is

$$\hat{w}_i = x_i w_i + (1 - x_i) = px_i^{1+\circ} + 1 - x_i: \tag{5}$$

In order to make meaningful welfare comparisons across equilibria with di¤erent relative prices, it is necessary to de‡ate nominal wages by the appropriate de‡ator. Assuming Cobb-Douglas preferences with weight $\bar{}$ on the manufactured good, the appropriate de‡ator is given by $p^{\bar{}}$ and the real wage is therefore

$$!_i = \hat{w}_i p^{i\,\bar{}}: \tag{6}$$

Under the assumed preference structure, the demand pattern is characterized by …xed shares of income allocated to each good, the share $\bar{}$ going to manufacturing and $1 - \bar{}$ to the agricultural good. This gives the following equilibrium conditions:

$$p(M_1 + M_2) = \bar{}(Y_1 + Y_2) \tag{7}$$

$$A_1 + A_2 = (1 - \bar{})(Y_1 + Y_2) \tag{8}$$

$$Y_i = pM_i + A_i; \qquad i = 1;2 \tag{9}$$

where Y_i is total (nominal) income in country i.

International migration is excluded, but labor moves freely among sectors. In the short term, and with …nite adjustment speeds, it is assumed that the rate of migration between sectors is a function of the di¤erences in wage rates:

$$\hat{x}_i \acute{} \frac{\dot{x}_i}{x_i} = \frac{,(w_i - \hat{w}_i)}{\hat{w}_i}; \qquad , > 0; \qquad i = 1;2: \tag{10}$$

The equations (10) de…ne a two dimensional system of di¤erential equations with state variables x_1 and x_2. This completes the basic model.

2.2 Autarchy

Consider …rst the case of autarchy. Here it is possible for prices to vary independently in the two countries, and equations (7–8) must be satis…ed for each country separately:

$$p_i M_i = \bar{}Y_i \tag{11}$$

$$A_i = (1 - \bar{})Y_i \tag{12}$$

In this case we have:

Proposition 1 There is a unique and asymptotically stable equilibrium of the dynamics (10) under autarchy. In each country, the equilibrium employment share is $x = \bar{\ }$, the relative price is $p = \bar{\ }^{i\,\circ}$, and the real wage is $\ddot{\ } = \bar{\ }^{-\circ}$.

Despite the existence of scale economies, equilibrium under autarchy is unique. Since both countries are identical, they have the same prices, wages, incomes, and welfare. This case will serve as a benchmark for purposes of comparison with the case of free trade, which we analyze next.

2.3 Free trade dynamics

Under free trade, both countries face the same prices for both tradeable goods. From (7) and (8) we have

$$p = \frac{\bar{\ }(2\ \ i\ x_1\ i\ x_2)}{(1\ i\ \bar{\ })\ x_1^{1+\circ} + x_2^{1+\circ}} \tag{13}$$

Depending on parameter values, it turns out that the system (10) will have between three and seven equilibria with ...nite and strictly positive prices. The symmetric autarchy equilibrium remains an equilibrium under free trade, since wages in all sectors are equalized for each country. However, it need not be stable and other (asymmetric) equilibria may exist. De...ne an interior equilibrium to be one in which both countries are producing both tradeable goods, and a boundary equilibrium to be one that is not interior. An equilibrium with complete specialization is one in which one country specializes in manufacture while the other specializes in agriculture.

First observe that the only interior equilibrium is the symmetric, autarchy equilibrium. To see why there cannot be an asymmetric interior equilibrium, note that in an asymmetric interior equilibrium under free trade, productivity in the manufacturing sector would be greater in the country in which more labor is allocated to manufacturing. Since agricultural wages are identical in both countries, it is impossible for wages to be equalized across all sectors in both countries. Hence all interior equilibria must be symmetric. But there is no trade in a symmetric equilibrium since preferences are identical in the two countries, so the only symmetric equilibrium is the one obtained under autarchy.

In addition to the unique interior equilibrium, there exist a number of asymmetric equilibria.

Proposition 2 The dynamics (10) have the following set of equilibria:

(a) There is exactly one interior equilibrium, and it is identical to the unique autarchy equilibrium.

(b) There is a pair of equilibria with complete specialization. The country specialized in manufacturing has higher wages if and only if $2\bar{} > 1$.

(c) If $2\bar{} < 1$, there is a pair of equilibria in which one country specializes in agriculture while the other produces both goods. Wages in the two countries are equal.

(d) If inequality (14) below is satis…ed, then there exist two pairs of equilibria in which one country specializes in manufacture while the other produces both goods.

$$\frac{1}{1+\circ}\left(\frac{\circ}{1+\circ}\right)^{\circ} > \frac{1-\bar{}}{\bar{}^{1+\circ}} \tag{14}$$

The inequality is satis…ed only if $2\bar{} > 1$.

Asymmetric equilibria come in pairs because, given any asymmetric equilibrium, another equilibrium may be obtained by relabelling the countries. Employment shares, wages and prices at the various equilibria described in Proposition 2(a)–2(c) are given in Table 1. Equilibria which are identical up to a relabelling of countries have been grouped together in pairs.

Type	x_1	x_2	p	w_1	w_2
(a)	$\bar{}$	$\bar{}$	$\bar{}^{-\circ}$	1	1
(b)	1 0	0 1	$\bar{}(1-\bar{})^{-1}$	$\bar{}(1-\bar{})^{-1}$ 1	1 $\bar{}(1-\bar{})^{-1}$
(c)	$2\bar{}$ 0	0 $2\bar{}$	$(2\bar{})^{-\circ}$	1 1	1 1

Table 1: Equilibria under Free Trade.

Note that equilibria in which one country specializes in agriculture while the other produces both goods cannot occur simultaneously with equilibria in which one country specializes in manufacture while the other produces both goods, since inequality (14) holds only if $2\bar{} > 1$. The four equilibria described in part (d) of the Proposition collapse to two when the parameter values are such that (14) becomes an equality, but this case is not generic and is neglected hereafter. In addition to the equilibria enumerated in the proposition, there are two 'equilibria' at $(x_1; x_2) = (0; 0)$ and $(x_1; x_2) = (1; 1)$. However, these correspond to a relative price p that is either in…nite or zero. Moreover, these rest points are unstable for all parameter values, and will also be neglected hereafter. Subject to these caveats the model will have three, …ve, or seven equilibria. For any given parameter constellation, however, only a subset of these will be stable.

2.4 Stability

The unique interior equilibrium which corresponds to the state of autarchy cannot be stable under free trade.

Proposition 3 The unique interior equilibrium is a saddle-point.

The intuition underlying this saddle-point instability is straightforward. If the initial position is one of complete symmetry between the two countries then the dynamics must produce convergence to the symmetric equilibrium. Assume, however, that there is a small initial asymmetry such that, compared with the symmetric equilibrium, country 1 has a slightly lower proportion of its work force in manufacturing and a higher proportion in agriculture, while for country 2 the opposite is the case. As a result of this asymmetry, manufacturing wages will be higher in country 2 than country 1 (output is sold at the uniform world price, and productivity is higher in country 2). Agricultural productivity and wages, on the other hand, are the same in the two countries. Thus, the initial asymmetry must produce sectoral wage di¤erences within the countries, and the resultant process of sectoral migration increases the asymmetry: in country 1 workers move away from manufacturing while in country 2, which had the high initial share of manufacturing employment, the associated high productivity and income in manufacturing attracts more workers to the sector. This leads to local divergence away from the equilibrium from any initial point that is not perfectly symmetric.

A direct implication of Proposition 3 is that there are no limit cycles in the model.[1] Hence all trajectories must converge to one of the equilibria enumerated in Proposition 2. The stability of these boundary equilibria depends on parameter values.

Proposition 4 Equilibria with complete specialization are unstable if $2^{-} < 1$ and asymptotically stable if $2^{-} > 1$.

Figure 1 depicts a phase diagram illustrating Propositions 3 and 4. In the …gure shown, all trajectories which originate at a point at which the labor allocations in the two countries are unequal converge to an equilibrium with complete specialization. There are no boundary equilibria with incomplete specialization. The phase diagram for the case in which boundary equilibria with incomplete specialization exist looks somewhat di¤erent, with the isoquants consisting of two disconnected segments, but in either case equilibria with complete specialization are locally asymptotically stable.

[1] This follows from the Poincaré-Hopf index theorem, which requires that a closed orbit must enclose at least one equilibrium point, and if it encloses exactly one, then the equilibrium must be either a sink or a source (Guckenheimer and Holmes, 1983, p.51).

Figure 1: Phase diagram for the case $2^{-} > 1$.

Since equilibria in which one country specializes in manufacture while the other produces both goods do not exist when $2^{-} < 1$, and equilibria with complete specialization are unstable, all trajectories converge to an equilibrium in which one country specializes in agriculture while the other produces both goods whenever $2^{-} < 1$. Although such an equilibrium is asymmetric with regard to the allocation of employment, it is symmetric with regard to income, wages and welfare. Both countries have the same wage rate, face the same prices, and consume the same amounts of each good. Free trade cannot give rise to uneven development in this case. Trade does however, lead to an increase in e¢ciency on a global scale and an improvement in overall welfare in both countries as returns to scale in manufacturing are exploited. Relative to autarchy, the global allocation of labor in the manufacturing sector remains unchanged at 2^{-}, but since it is now concentrated in one country, productivity in manufacturing and total world production is higher. The total production of the agricultural good is unchanged, and income is the same in both countries, so welfare is enhanced uniformly. These remarks be stated formally as follows.

Proposition 5 If $2^{-} < 1$, then one country specializes in agriculture at any stable equilibrium while the other country produces both goods. Welfare in both countries is uniformly higher than under autarchy.

The more interesting scenario from the perspective of this paper occurs when $2^{\bar{}} > 1$. In this case, convergence must occur to an equilibrium in which one country is completely specialized in the production of the manufactured good. Furthermore, uneven development in the sense of unequal income, wages and welfare is a necessary outcome in this case. Wages in the country which produces a positive amount of the agricultural good must equal 1 in all sectors with positive production. Whether convergence occurs to an equilibrium of type (b) or one of type (d), the country which specializes in manufacturing will have wages that exceed 1. This is obvious for equilibria of type (b), since the wage in the manufacturing country is simply $^{\bar{}}(1 ¡ ^{\bar{}})^{i\,1}$ which must exceed 1 if $2^{\bar{}} > 1$. In equilibria of type (d), employment and hence productivity in the manufacturing sector of the country which is specialized in manufacturing will be greater than employment and productivity in the manufacturing sector of the country which is not specialized. Since the price of the manufactured good is the same in both countries, wages in manufacturing must be higher in the country that is specialized. Hence when $2^{\bar{}} > 1$, income will be higher in the country that is specialized in manufacturing. Since both countries face the same prices for tradeables, and a …xed share of nominal income is spent on each of them, the country specialized in manufacturing will enjoy higher consumption of both tradeable goods. Hence, disregarding unstable equilibria, $2^{\bar{}} > 1$ is a necessary and su¢cient condition for uneven development in the present model.

Proposition 6 If $2^{\bar{}} > 1$, then one country specializes in manufacturing at any stable equilibrium while the other country either specializes in agriculture or produces both goods. The country specializing in manufacturing has strictly higher welfare than does the other country.

The above result states that if the manufacturing good is a su¢ciently important component of consumption ($2^{\bar{}} > 1$) free trade will lead to a deterioration in relative material well-being for the residents of the country that fails to specialize in manufacturing. This need not imply, of course, that there will be an absolute deterioration in welfare relative to autarchy. The following numerical example illustrates that if $^{\bar{}}$ is su¢ciently high, such an absolute deterioration can indeed occur.

Example 1 Suppose $2^{\bar{}} > 1$ and $° = 0{:}2$. Then from Proposition 6 there exists a stable equilibrium with complete specialization. At this equilibrium, using equation (6) together with the entries in Table 1, it may be veri…ed that the country which specializes in agriculture experiences an absolute decline in welfare relative to autarchy if and only if $^{\bar{}} > ½^5 ¼ 0{:}53$, where ½ is a root of $Z^6 + Z^5 ¡ 1$.

The possibility that free trade may lead to a decline in welfare for one of the countries makes it likely that steps will be taken to reverse this decline by the imposition of some form

of industrial or trade policy. Direct imposition of tari¤s is more likely to invite retaliation than a domestic subsidy to the sector with increasing returns, and in the next section we shall focus on the implications of this kind of industrial policy.

3 Distortionary Policy

There are many ways in which a country with a poor economic performance may try to change its policies or institutions. This paper looks at just one of them, a distortionary domestic industrial policy. To simplify it is assumed that the countries choose between two options: a regime of 'laissez-faire' and one of 'distortion'. Under 'laissez-faire' individual …rms/workers face the market clearing world market prices. 'Distortion', on the other hand imposes a tax on the agricultural sector and gives a subsidy to employment in manufacturing. The government budget is assumed to be balanced. Recalling that the agricultural wage has been set equal to 1, this balanced budget condition is then

$$t_i (1 - x_i) = s_i x_i$$

where t is the rate of taxation and s the subsidy per worker. For the moment, consider the tax per-worker t as an exogenous parameter, with $0 < t < 1$. In this case the total tax collected will vary with the amount of agricultural employment, falling from a maximum value of t when agricultural employment is at its highest level, to zero when no workers are employed in agriculture. The subsidy per-worker in the manufacturing sector will also vary with the sectoral composition of employment as follows:

$$s_i = t_i \left(\frac{1 - x_i}{x_i} \right)$$

The policy of distortion gives rise to the following industrial wage rate:

$$w_i = p x_i^{\circ} + t_i \left(\frac{1 - x_i}{x_i} \right) \tag{15}$$

with the agricultural wage now equal to $1 - t$. Mean wages are

$$\hat{w}_i = x_i w_i + (1 - x_i)(1 - t_i) = p x_i^{1+\circ} + 1 - x_i \tag{16}$$

For a given sectoral composition of employment, the existence of the tax has no e¤ect on the mean wage in the economy. The tax does, however, alter relative wages since it involves a transfer from agriculture to manufacturing. This e¤ect can cause a sectoral composition of labor which is an equilibrium point in the absence of the policy to become a point of disequilibrium, inducing an intersectoral movement of labor. More signi…cantly, it can cause

the stability properties of various equilibria to be transformed, giving rise to non-negligible welfare e¤ects. Note, for instance, that as x_1 approaches zero, the subsidy per worker tends to in…nity. As a result, equilibria with zero manufacturing employment can never be stable in a country that adopts a policy of distortion and chooses a positive tax rate.

In this section we consider the set of equilibria and their stability properties in the transformed model, under the assumption that country 1 adopts a policy of distortion while country 2 retains a policy of laissez-faire. Furthermore, we investigate the e¤ects of the policy only under the restriction that $2^{-} > 1$, since it is this case alone which gives rise to uneven development under free trade.

Since $x_1 = 0$ is impossible in a stable equilibrium, we neglect this case. Equilibria in which country 1 is specialized in manufacturing ($x_1 = 1$) are exactly as in the case of free trade, since, there being no employment in the agricultural sector, there is no tax and no subsidy in equilibrium. The only case in which equilibria with distortion di¤er from those under free trade arises, therefore, when the distorting country produces both goods. Country two may also produce both goods (yielding an interior equilibrium), or may specialize in one of the tradable goods.

It may be shown that if the degree of distortion is su¢ciently high, no equilibrium in which the distorting country produces both goods can be stable.

Proposition 7 Suppose $2^{-} > 1$, and that country 1 adopts a distortionary policy with tax rate t 2 $(0;1)$ while country 2 adopts a policy of laissez-faire. For any given °, there exists a number ¿(°) 2 $(0;1)$, such that if $t >$ ¿(°), country 1 is specialized in manufacturing at any stable equilibrium.

With a su¢ciently high tax rate, all trajectories converge to an equilibrium in which the distorting country is specialized in manufacturing, and therefore enjoys higher wages and income. A su¢ciently aggressive distortionary policy pursued by a country that is initially specialized in agriculture can therefore give rise to a favorable outcome provided that the other country maintains a policy of laissez faire.

What if the tax rate is insu¢ciently high? Then convergence may occur to an equilibrium in which the distorting country produces both goods while the other country is specialized in manufacturing. Wages in the distorting country then equal $1 ¡ t$ so they decline in terms of the numeraire agricultural good. However, the increase in global output of the manufactured good leads to a decline in its price, thus shifting the terms of trade in favor of the distorting country. The welfare e¤ects of the distortion are ambiguous for the distorting country but unambiguously negative for the country which maintains a policy of laissez faire. Thus, it is quite possible for a unilaterally adopted distortionary policy to give rise to a decline in welfare in both countries. But even in this case, where the tax rate is too low to ensure

that the distorting country becomes specialized in manufacturing, the tax may still produce a reduction in income disparities.

To summarize, for a country that is initially specialized in agriculture the unilateral imposition of the policy considered here can lead to a variety of outcomes depending on the magnitude of the tax. A tax that is su¢ciently high can lead to complete specialization in manufacturing, and a rise in both nominal income and real consumption which is su¢cient to leapfrog its trading partner. Too low a tax can have a negligible and possibly detrimental e¤ect on domestic welfare in the distorting country, even though the country's relative position is improved due to a fall in foreign income. Intermediate levels of the tax lead to higher domestic income and welfare without necessarily leading to complete specialization in manufacturing. In either case an improvement in the distorting country's relative standing is attained. The attraction of such a policy to a country that …nds itself specialized in the 'wrong' industry is apparent.

4 Endogenous Policy Dynamics

In this section we model the dynamics of distortionary policy. The endogenization of policy is central to the literatures on public choice and political business cycles. These theories see policy makers as rational individuals that maximize their own well-de…ned utility functions. While this view may be a healthy antidote to naive beliefs in the complete benevolence of policy makers, the optimizing approach strains credulity, specially when large-scale changes in policies and institutions are considered. The implications of large changes in areas like industrial or trade policy, exchange rate regime or labor market institutions are hard to predict, and reforms in these areas are usually the results of a complex political process involving groups with con‡icting interests and perceptions of the world. The approach taken here is that policy adjustments are essentially adaptive responses to prevailing economic conditions.

Let t_1 and t_2 represent the tax rates in the two countries. Our speci…cation of tax rate dynamics captures the basic intuition that a country will increase (decrease) its tax rate if the movement of labor towards manufacturing will raise its relative position as measured by the ratio of real per-capita incomes. Speci…cally, it is assumed that:

$$t_i = f\left(\frac{@(!_i/!_j)}{@x_i}\right); \qquad f^0 > 0; \qquad f(0) = 0 \qquad i = 1;2: \tag{17}$$

One attractive feature of this speci…cation is that at any rest point of these dynamics (when $t_1 = t_2 = 0$), no country can improve its relative position by a marginal, unilateral change in its tax rate. Hence equilibria of the system satisfy a minimal rationality requirement.

Equations (10), together with equations (17) constitute a four-dimensional system in the variables $x_1; x_2; t_1; t_2$.[2]

The resulting system is too complex for its equilibria and their stability properties to be completely characterized by analytical methods, although a number of partial results may be obtained.

Proposition 8 The system (17), (10) has a symmetric interior equilibrium with labor allocation in each country given by $x = (\bar{} + \bar{}°)=(1 + \bar{}°)$. At this equilibrium the Jacobian of the system has two characteristic roots with real part equal to zero for any admissible parameter values. If $° \cdot 1$ and $\bar{}=(1 ¡ \bar{}) ¸ 1 + °$, the system contains no other interior equilibria.

Note that the symmetric solution for x is larger with distortion ($x = (\bar{} + \bar{}°)=(1 + \bar{}°)$) than in the undistorted case ($x = \bar{}$, cf. Proposition 2). The two equilibria can be Pareto ranked, and the distorted equilibrium is unambiguously Pareto superior. The presence of external scale economies implies that without taxation or other forms of intervention too little labor is employed in the industrial sector. Taxation corrects this problem. In fact, using the expression for real wages, it is readily seen that the distorted symmetric equilibrium Pareto-dominates all other symmetric allocations with $x_1 = x_2 = x$.

A partial characterization of the existence and stability properties of boundary equilibria is as follows.

Proposition 9 The system (17), (10) has no stable boundary equilibria with $x_i = 0$ or $x_i = 1$: Furthermore, if $° \cdot 1$; the system has no boundary equilibria with $t_i = 0; t_j \not= 0$ or $t_i = 1; t_j \not= 0$:

Given the parameter restrictions there are no stable boundary equilibria[3] but the stability properties of the interior equilibrium cannot be determined by local linearization methods since the Jacobian of the system has two characteristic roots with real part equal to zero. Instead, computer simulations can be used to examine the system in greater detail. It turns out that for a variety of parameter values and initial conditions, trajectories converge not to a ...xed point but to a periodic orbit: the share of industrial employment in each country and the tax rate ‡uctuate persistently over time. One such trajectory is depicted in Figure 2,

[2] The equations (17) do not guarantee that the tax rates t_i remain within the unit interval, though this can be ensured by adding the appropriate boundary conditions. In the simulation results to be reported below, the only boundary condition required is $t_i ¸ 0$ when $t_i = 0$.

[3] The parameter restrictions in Propositions 8-9 are su¢cient, rather than necessary, and since plausible values of ° are much smaller than one, we have made no attempt to relax the restrictions.

which shows the projection of the four–dimensional dynamics onto x_1–t_1 space.[4] Movement around the cycle is clockwise.

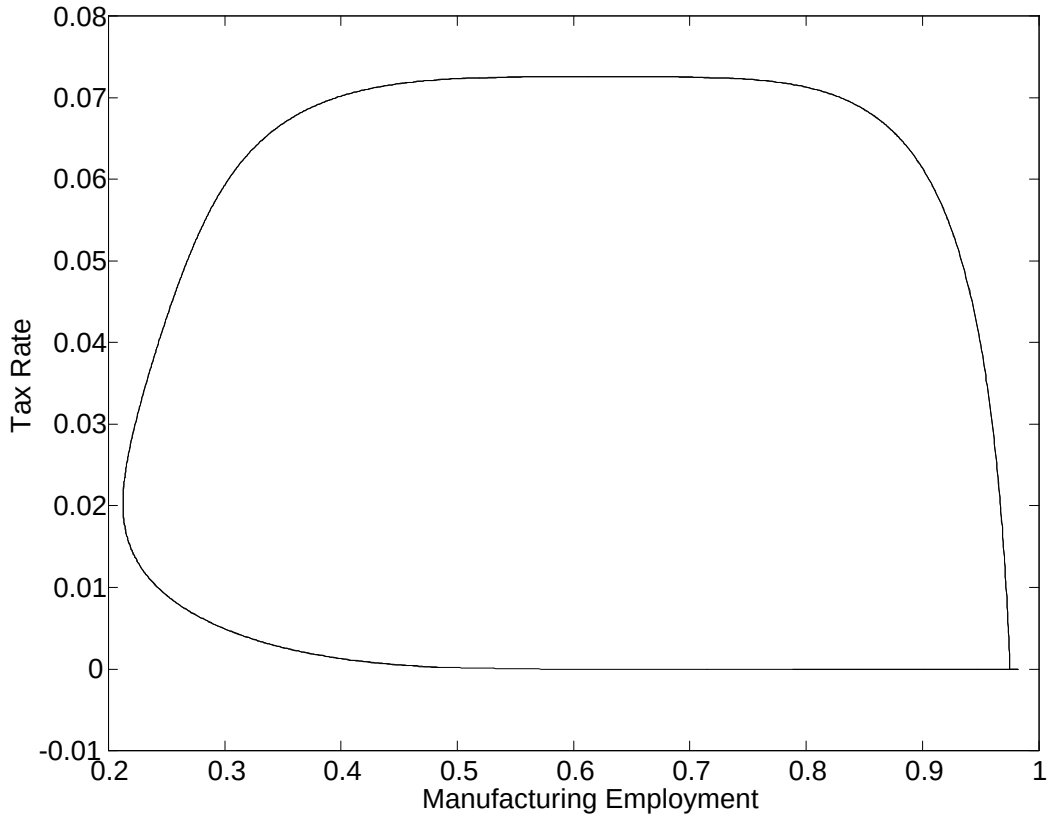

Figure 2: A Stable Limit Cycle

The dynamics for the tax rate and manufacturing employment in country 2 have the same qualitative properties, though the two countries are in di¤erent phases of the cycle at any given point in time. This is seen most clearly in Figure 3, which plots the manufacturing employment in the two countries. When one country is at its peak with respect to the share of employment in manufacturing, the other is near its trough. The country that leads at this point has a declining tax rate, while the follower is increasing its rate of distortion. This tends to raise manufacturing employment in the latter even as it declines in the former. When the two countries have broadly similar shares of manufacturing employment, they have very unequal tax rates as one country is in the process of leapfrogging the other. As the cycle is traversed further, the countries end up with their positions exactly reversed and the process begins anew.

[4] The simulation uses the parameter values ° = 0:1, ¯ = 0:6, f(z) = 0:01z^3, and ¸ = 10.

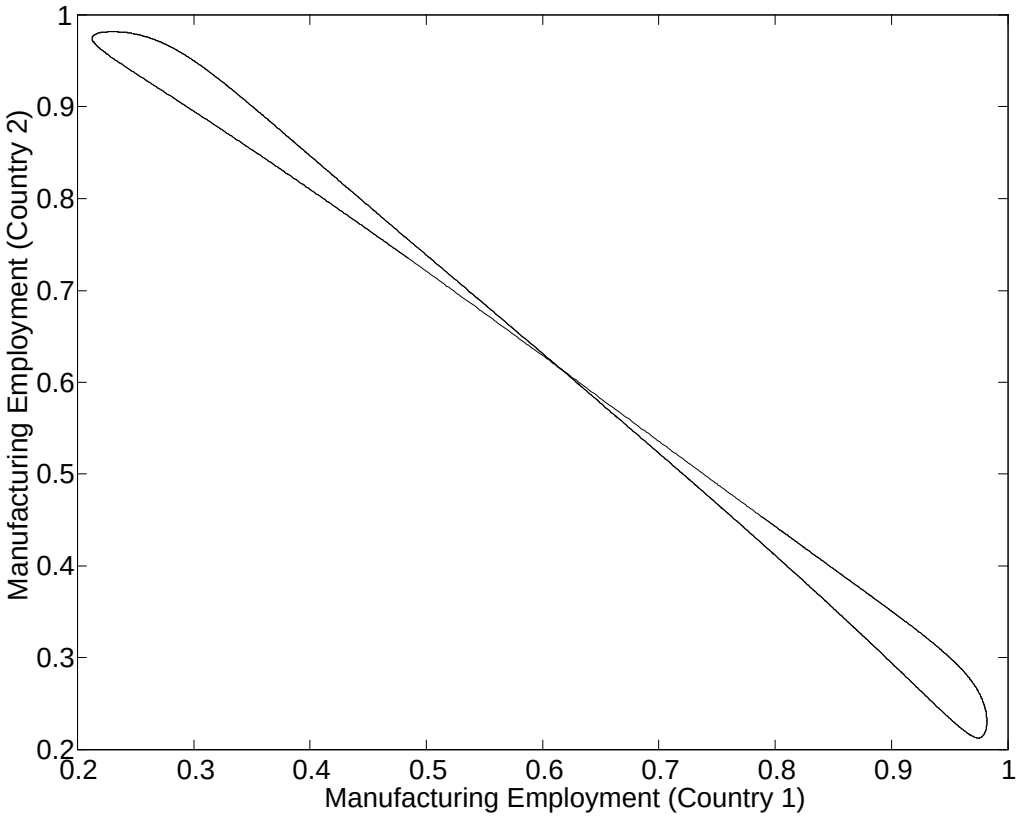

Figure 3: Changes in Manufacturing Leadership

The movement of the real mean wage in Country 1 relative to that in Country 2 is shown in Figure 4. Movements in this relative wage are rather more erratic than the corresponding movements in the tax rate and in manufacturing employment. The pattern of uneven development that arises under exogenously given distortionary policy is replaced, when policy is endogenized, by a pattern of periodic changes in leadership, with the dominance of one country giving way over time to the dominance of the other.

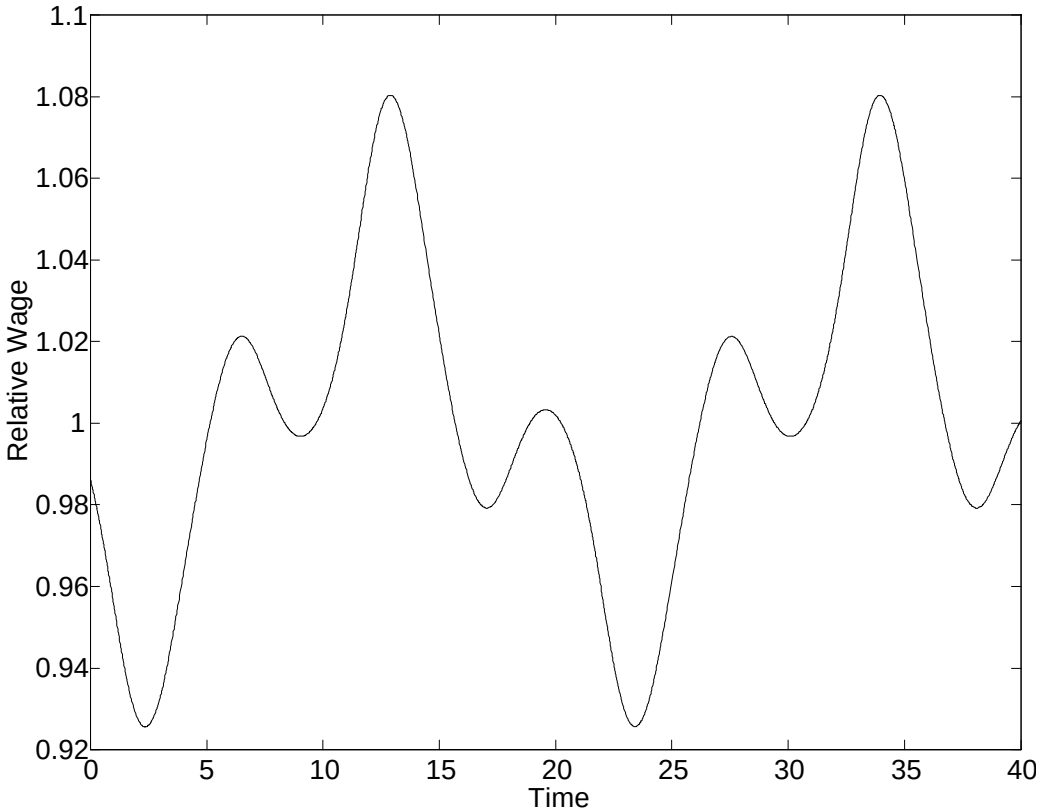

Figure 4: Leapfrogging in Relative Incomes

5 Convergence

One key implication of our model is that induced policy changes may convert a process of uneven development into one of persistent ‡uctuations and leapfrogging. This result, which we believe to be quite robust, may have important implications for the interpretation of the evidence on economic convergence. Consider a data set for a set of countries where each country forms part of a two-country economy or trading bloc, and assume that the model in Section 4 can be used to describe the evolution of these two-country economies. With these assumptions, simulations of the model can provide the data for a regression of the growth rate of relative income on the initial level of relative income.

The regressions show no evidence of unconditional convergence. Depending on the precise sample, the estimated coe¢cient in the unconditional regression can be either positive or negative, and for large samples it is almost invariably insigni…cant. Introducing the country's own tax rate as an additional explanatory variable, however, leads to a very di¤erent picture. The coe¢cient on initial income now becomes negative and highly signi…cant. In other words, there are clear indications of conditional convergence.

As an illustration, consider a typical simulation using 100 data points. The unconditional regression produced the following result (t statistics are in parentheses)

$$y = \underset{(0:21)}{¡1:56} + \underset{(0:24)}{1:82z_1}$$

where z_1 denotes the country's level of relative income and y the growth rate of its relative income. The F–statistic for the regression is $F_{1;98} = 0:059$, which is insigni…cant even at 10%. In this particular case the coe¢cient on the initial level happened to be positive, that is, there appears to be divergence. Using the same data set, this picture of divergence is transformed into one of strong conditional convergence. The regression now shows

$$y = \underset{(6:71)}{51:38} ¡ \underset{(6:90)}{56:34z_1} + \underset{(9:54)}{123:07z_2}$$

where z_2 is the tax rate. The t-values suggest a very strong e¤ect of tax rates on growth. Furthermore, the coe¢cient on initial income is now signi…cant even at the 1% level, and the F–statistic for the regression is $F_{2;97} = 45:57$, also signi…cant at 1%.

These implications of the model are similar to the …ndings of most empirical studies: absolute convergence is rejected by the international data while conditional convergence …nds strong support (even if many of the detailed results may be "fragile" and the precise speci…-cations sometimes hard to interpret; Levine and Renelt (1992) and Andrés, Doménech and Molinas (1996)). But the mechanism behind the appearance of conditional convergence in the model is fundamentally di¤erent from the standard interpretation of conditional convergence. According to the standard interpretation, each country approaches its own steady growth path. Di¤erences between the steady growth paths are accounted for by the conditioning variables, and the speed of convergence re‡ects the parameters of the utility and production functions (e.g. Barro and Sala-i-Martin (1991, 1995), Mankiw, Romer and Weil (1992), Sala-i-Martin (1996) and, for a critique, Quah (1996)). Although simple and stylized, our model highlights some of the possible problems with this interpretation. The presence of "conditional convergence" in our model is illusory. If the conditioning variables – that is, the tax rates – were to be kept constant, the growth rates of the two countries would not, in general, exhibit conditional convergence. As an example, consider the case where both tax rates are set equal to zero and where the initial position satis…es $x_1 + x_2 \cdot 2^{\bar{}}$. The analysis in Section 2 implies that the subsequent dynamics will be characterized by conditional divergence: the country with the higher initial income will have the highest growth rate throughout the transitional process towards long-run equilibrium. The presence in the regressions of conditional convergence is the result of active policy. It is the induced changes in t_1 and t_2 which generate persistent ‡uctuations in the model and which ensure that the average long-run income is the same in the two countries. In this sense it is policy intervention that stabilizes an otherwise unstable system.[5]

[5] This point is related to the so-called "Lucas critique". Convergence regressions estimate reduced-form

6 Conclusions

The economic historian Paul Bairoch has complained about the "historical amnesias of economics" (1989, p.225). The US maintained high rates of protection until the 1950s; England did not adopt free trade until 1846, and except for an interlude in the 1860s and 1870s most continental European countries maintained protectionist policies throughout the period from 1800 to about 1960. The future Third World economies, by contrast, were subject to "compulsory economic liberalism" (p.238). The historical evidence shows that the currently advanced countries in Europe and North America did not achieve this status as a result a laissez-faire trade policy. It is almost certain, Bairoch argues, "that during the XIXth century, contrary to the classical model, for most of the now developed countries, if not all those countries except the leader (U.K.), free trade meant depression and protection meant growth and development. On the other hand, it is certain that for the future less developed countries free trade meant, as we have seen, the acceleration of the process of economic underdevelopment" (p.241). More recently, non-tari¤ obstacles and interventionist industrial policy may have played a critical role in the economic success of countries like Japan and South Korea (e.g. Westphal (1990), Rodrik (1995), Wade (1990), World Bank (1993)).

Economic policy and economic performance interact in a way that is both complex and dynamic. The present model, which is consistent with Bairoch's interpretation of the historical evidence, analyses a highly stylized example of this kind of interaction. There are only two countries, for instance, and we did not include capital accumulation. Uneven development under free trade therefore involved (the asymptotic convergence towards) constant – but di¤erent – levels of per capita incomes in the two countries.

When both countries adopt laissez-faire policies the outcome is determined entirely by initial conditions: the dynamics favor the country that is most industrialized initially. Not surprisingly, the introduction of distortionary policies could change this. Assuming that the other country adopts a laissez-faire policy, a country may use taxes and subsidies to achieve high levels of industrialization and income in the long run. It should be noted, in particular, that the intervention in this paper is neutral in the sense of Krueger (1978) and Bhagwati (1978). The intervention favors a product – manufacturing output – but exports and domestic sales of the product are treated symmetrically. There is no anti-export bias.[6] Similar results could have been obtained using a combination of import tari¤s and export

regressions that may not be robust if the economic policy rule or the institutional environment were to change.

[6] Empirical work on trade and growth often uses measures of anti-export bias as an indication of trade orientation (Edwards (1993) surveys the literature). A positive correlation between trade orientation in this sense and economic growth does not contradict the predictions of the model.

subsidies for manufacturing goods, and the precise form of intervention is not likely to be critical to the outcome.

As a depiction of the actual, highly complex interaction between economic performance and changes in economic policies and institutions, the model has obvious limitations. These limitations call for further work, both theoretical and empirical, but do not, we believe, a¤ect the thrust of the argument: economies may converge in the long run not because market forces necessarily favor convergence, but rather because persistent divergence induces changes in policies that eventually cause patterns of uneven development to be reversed.

Appendix

Proof of Proposition 1. Combining (1–3) with (11–12), we have

$$p_i = \frac{\bar{\ }(1 - x_i)}{(1 - \bar{\ }) x_i^{1+\circ}} \tag{18}$$

Hence from (4),

$$w_i = x_i^{\circ} p_i = \frac{\bar{\ }(1 - x_i)}{(1 - \bar{\ }) x_i}:$$

Substituting this in (5) yields:

$$\hat{w}_i = x_i w_i + (1 - x_i) = \frac{1 - x_i}{1 - \bar{\ }}:$$

The dynamics (10) reduce to two independent one dimensional systems:

$$\dot{x}_i = \, x_i \left(\frac{w_i}{\hat{w}_i} - 1 \right) = \, (\bar{\ } - x_i)$$

each of which has a unique asymptotically stable equilibrium at $x_i = \bar{\ }$. The values for p and ! follow from (18), (4), and (6). ■

Proof of Proposition 2. Part (a) is proved in the text. Equilibrium requires that $\dot{x}_1 = \dot{x}_2 = 0$ which from (10) implies that either $x_i = 0$ or $w_i = \hat{w}_i$. The condition $w_{im} = \hat{w}_i$ is satis...ed whenever $x_i = 1$, so $(x_1; x_2) = (0; 1)$ and $(x_1; x_2) = (1; 0)$ are equilibria, proving part (b). Now suppose $x_1 = 0$ (country 1 specializes in agriculture) and that x_2 2 $(0; 1)$ (country 2 produces both goods). Since the wage in agriculture is 1, this can be an equilibrium only if $w_2 = 1$. From (13) and (4) this yields $x_2 = 2\bar{\ }$. But since $x_2 < 1$, there is an equilibrium at $(x_1; x_2) = (0; 2\bar{\ })$ if and only if $2\bar{\ } < 1$. By symmetry, there is a corresponding equilibrium at $(x_1; x_2) = (2\bar{\ }; 0)$, proving part (c). Now suppose that $x_1 = 1$ (country 1 specializes in manufacture) and that x_2 2 $(0; 1)$ (country 2 produces both goods). Again, this can be an equilibrium only if $w_2 = 1$. From (13) and (4) this yields $x_2^{\circ}(\bar{\ } - x_2) = 1 - \bar{\ }$. The proposition then follows from Lemma 1 below. ■

Lemma 1 If

$$\frac{1}{1+\circ} \left(\frac{\circ}{1+\circ} \right)^{\circ} > \frac{1 - \bar{\ }}{\bar{\ }^{1+\circ}}$$

then $2\bar{\ } > 1$ and there exist exactly two solutions, z_1 and z_2, to the equation $z^{\circ}(\bar{\ } - z) = 1 - \bar{\ }$.

Proof. To see that inequality (14) implies that $2\bar{\ } > 1$, observe that if $2\bar{\ } \cdot 1$, then the RHS of (14) is strictly greater than 1. The LHS is bounded above by 1 for all values of $\circ > 0$. Hence (14) cannot hold if $2\bar{\ } \cdot 1$. To prove the remainder of the claim de...ne the function

$f(z) = z^{\circ}(\bar{} \text{ ¡ } z)$. Note that $f(z) > 0$ whenever $0 < z < \bar{}$, and that $f(0) = f(\bar{}) = 0$. Since $f(z)$ is continuously di¤erentiable, it has a maximum in the range $0 < z < \bar{}$ at which $f^{0}(z) = °z^{\circ \text{ i } 1}(\bar{} \text{ ¡ } z) \text{ ¡ } z^{\circ} = 0$. This …rst order condition yields a unique maximum at $z = \bar{}°/(1 + °)$ at which point $f(z)$ is given by

$$f(z) = \bar{}^{1+°} \left(\frac{°}{1+°}\right)^{\circ} \left(\frac{1}{1+°}\right)$$

Equation (14) will have exactly two solutions if and only if this maximum value exceeds $1 \text{ ¡ } \bar{}$, which yields the result. ■

Proof of Proposition 3. We need to …nd the Jacobean matrix at the equilibrium point. A lengthy but straightforward derivation (available upon request from the authors) yields the following Jacobean:

$$J = \frac{1}{2} \begin{pmatrix} °(1 \text{ ¡ } \bar{}) \text{ ¡ } 1 & \text{ ¡ } °(1 \text{ ¡ } \bar{}) \text{ ¡ } 1 \\ \text{ ¡ } °(1 \text{ ¡ } \bar{}) \text{ ¡ } 1 & °(1 \text{ ¡ } \bar{}) \text{ ¡ } 1 \end{pmatrix}$$

with determinant $¢ = \text{ ¡ } °¸^{2}(1 \text{ ¡ } \bar{}) < 0$. Since the determinant is negative for all admissible parameter values, the real parts of the two eigenvalues have opposite signs. Hence the symmetric equilibrium is a saddle point. ■

Proof of Proposition 4. Consider an equilibrium $e = (x_1; x_2)$ with complete specialization and suppose without loss of generality that $x_1 = 0$ and $x_2 = 1$. Recall that the agricultural wage is equal to 1. From (4) $w_1 = 0$. From Table 1, $w_2 = \bar{}(1 \text{ ¡ } \bar{})^{\text{ i } 1}$. Suppose …rst that $2\bar{} > 1$. Then $w_2 > 1$. By continuity of the wage functions, there exists a neighborhood N of the equilibrium such that $w_1 < 1$ and $w_2 > 1$ at all points in N. It follows that $w_1 < \hat{w}_1$ and $w_2 > \hat{w}_2$ at all points in $N \text{ ¡ } e$. Hence from (10) $\underline{x}_1 < 0$ and $\underline{x}_2 > 0$ at all points in $N \text{ ¡ } e$. All trajectories initially in N therefore converge to e and e is asymptotically stable. Now suppose that $2\bar{} < 1$. In this case $w_2 < 1$ at e and by continuity of payo¤s $w_2 < 1$ at all points $(x_1; x_2) = (0; 1 \text{ ¡ } ")$ for $"$ su¢ciently small. It follows that $w_2 < \hat{w}_2$ and hence from (10) $\underline{x}_2 < 0$ while $\underline{x}_1 = 0$ at all such points. Trajectories initially at any such point diverge monotonically from e, and e is unstable. ■

Proof of Proposition 7. Since equilibria with $x_1 = 0$ are necessarily unstable when country 1 distorts, we need only show that there exists $¿(°) \text{ 2 } (0; 1)$ such that equilibria with $x_1 \text{ 2 } (0; 1)$ are unstable if $t > ¿(°)$. There are three cases to consider: (i) equilibria with $x_2 = 0$, (ii) equilibria with $x_2 = 1$, and (iii) interior equilibria. In each case, since x_1 is interior, equality of wages in country 1 yields

$$1 = x_1^{\circ} p + \frac{t}{x_1} \tag{19}$$

First consider the case $x_2 = 0$. Then

$$p = \frac{\bar{\ }(2 \text{ ¡ } x_1)}{(1 \text{ ¡ } \bar{\ }) x_1^{1+°}}$$

So from (19)

$$1 = \frac{\bar{\ }(2 \text{ ¡ } x_1)}{(1 \text{ ¡ } \bar{\ }) x_1} + \frac{t}{x_1}$$

which simpli…es to $x_1 = 2\bar{\ } + t(1 \text{ ¡ } \bar{\ })$. This is inconsistent with $x_1 < 1$ for any t, since $2\bar{\ } > 1$. Hence $x_2 = 0$ cannot hold. Next, consider x_2 interior. Equality of wages in country 2 implies $1 = x_2^{°}p$. Substituting for p in (19) yields

$$1 \text{ ¡ } t = \frac{x_1^{°}}{x_2^{°}} + t \left(\frac{1 \text{ ¡ } x_1}{x_1}\right)$$

for which the solution is

$$x_2 = \left(x_1^{°} \frac{x_1}{x_1 \text{ ¡ } t}\right)^{\frac{1}{°}}$$

Since $x_2 > 0$ it must be the case that $x_1 > t$. Since $x_2 < 1$, it must be the case that

$$x_1^{°} \frac{x_1}{x_1 \text{ ¡ } t} < 1$$

or

$$g(x_1) = x_1 (1 \text{ ¡ } x_1^{°}) > t \tag{20}$$

Note that $g(0) = g(1) = 0$. The function g is continuous and has a unique maximum on the interval $[0;1]$. To see why, observe that $g^0(x) = 1 \text{ ¡ } x_1^{°}(1 + °) = 0$ yields

$$x_1 = \left(\frac{1}{1 + °}\right)^{\frac{1}{°}}$$

The maximum value is

$$¿(°) = \left(\frac{1}{1 + °}\right)^{\frac{1}{°}} \left(\frac{°}{1 + °}\right)$$

which lies in the interval $(0;1)$. If $t > ¿(°)$, there is no x_1 satisfying $g(x_1) > t$, and hence no equilibrium with x_2 interior. Finally consider $x_2 = 1$. In this case stability of equilibrium requires that the manufacturing wage in country 2 exceeds the agricultural wage there, or $w_2 = x_2^{°}p = p > 1$. But from (19),

$$t = (1 \text{ ¡ } x_1^{°}p) x_1 < (1 \text{ ¡ } x_1^{°}) x_1 = g(x_1)$$

where the function g is as de…ned in (20) above. But $t < g(x_1)$ cannot be satis…ed when $t > ¿(°)$ as shown above. Hence, if $t > ¿(°)$ there can be no stable equilibrium with x_1 interior, and country 1 specializes in manufacturing at all stable equilibria. ■

Proof of Proposition 8. Differentiating (13) yields

$$\frac{\partial p}{\partial x_i} = \frac{\beta(1-\gamma)\left(x_1^{1+\sigma}+x_2^{1+\sigma}\right) - \beta(1-\gamma)(2-x_1-x_2)(1+\sigma)x_i^{\sigma}}{\left(\left(1-\gamma\right)\left(x_1^{1+\sigma}+x_2^{1+\sigma}\right)\right)^2}$$

$$= -p\left(\frac{1}{(2-x_1-x_2)} + \frac{(1+\sigma)x_i^{\sigma}}{x_1^{1+\sigma}+x_2^{1+\sigma}}\right) \tag{21}$$

Using (6) and (16), the ratio of real per capita income is given by

$$\frac{\omega_i}{\omega_j} = \frac{x_i^{1+\sigma}p + 1 - x_i}{x_j^{1+\sigma}p + 1 - x_j}$$

Differentiating this yields after simplification and substitution from (21):

$$\frac{\partial(\omega_i/\omega_j)}{\partial x_i} = w_j^{-1}\left(\frac{\omega_i}{\omega_j}x_j^{1+\sigma} - x_i^{1+\sigma}\right)\left(-p\left(\frac{1}{(2-x_1-x_2)} + \frac{(1+\sigma)x_i^{\sigma}}{x_1^{1+\sigma}+x_2^{1+\sigma}}\right) + p(1+\sigma)x_i^{\sigma} - 1\right) \tag{22}$$

Consider a symmetric equilibrium, with $x = x_1 = x_2$ and $\omega_1 = \omega_2$. Since $t_i = 0$ in equilibrium, the above equation must equal zero for both countries. Using (13), we have

$$p = \frac{\gamma(1-x)}{(1-\gamma)x^{1+\sigma}}$$

Substituting in (22) and setting equal to zero yields after cancellation:

$$\frac{\gamma(1-x)}{(1-\gamma)x}(1+\sigma) - 1 = 0$$

which has a unique solution

$$x = \frac{\gamma+\gamma\sigma}{1+\gamma\sigma}$$

At an interior equilibrium, intersectoral equality of wages prevails. At a symmetric equilibrium, setting $t = t_1 = t_2$ this implies

$$1 - t = x^{\sigma}p + t\frac{(1-x)}{x}$$

Using the equilibrium expressions for p and x from above, this yields a unique solution

$$t = \frac{\gamma\sigma}{1+\gamma\sigma}$$

Since x and t are both in the interval $(0;1)$ for all admissible parameter values, there exists a unique symmetric interior equilibrium. Stability depends on the properties of the Jacobian

$$J = \begin{pmatrix} a & b & -(\gamma-1) & 0 \\ b & a & 0 & -(\gamma-1) \\ c & d & 0 & 0 \\ d & c & 0 & 0 \end{pmatrix}$$

where, due to the fact that it is a symmetric equilibrium,

$$
\begin{aligned}
a &= \frac{\partial}{\partial x_1}\left(\gamma x_1 \left(\frac{w_1}{\hat{w}_1} - 1\right)\right) = \frac{\partial}{\partial x_2}\left(\gamma x_2 \left(\frac{w_2}{\hat{w}_2} - 1\right)\right); \\
b &= \frac{\partial}{\partial x_2}\left(\gamma x_1 \left(\frac{w_1}{\hat{w}_1} - 1\right)\right) = \frac{\partial}{\partial x_1}\left(\gamma x_2 \left(\frac{w_2}{\hat{w}_2} - 1\right)\right) \\
c &= f'(0)\frac{\partial^2(\omega_1/\omega_2)}{\partial x_1^2} = f'(0)\frac{\partial^2(\omega_2/\omega_1)}{\partial x_2^2}; \\
d &= f'(0)\frac{\partial^2(\omega_1/\omega_2)}{\partial x_1 x_2} = f'(0)\frac{\partial^2(\omega_2/\omega_1)}{\partial x_1 x_2}
\end{aligned}
$$

Evaluation of a and b yields $a = b = -\frac{1}{2}\gamma$, and since $\frac{\partial^2(\omega_1/\omega_2)}{\partial x_1 x_2} = -\left(\frac{\omega_2}{\omega_1}\right)^2 \frac{\partial^2(\omega_2/\omega_1)}{\partial x_1 x_2}$ a symmetric equilibrium will have $d = 0$. Furthermore, symmetry implies that $c = -(1+\theta)x^{\theta}\frac{p}{x(1-x)} < 0$. Hence the Jacobian can be rewritten

$$
J = \begin{pmatrix} -\frac{1}{2}\gamma & -\frac{1}{2}\gamma & -(\bar{\ } - 1)\gamma & 0 \\ -\frac{1}{2}\gamma & -\frac{1}{2}\gamma & 0 & -(\bar{\ } - 1)\gamma \\ c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \end{pmatrix}
$$

which has eigenvalues $\pm\sqrt{(c\gamma(1-\bar{\ }))}$, $-\frac{1}{2}\gamma \pm \frac{1}{2}\sqrt{\gamma^2 + 4c\gamma(1-\bar{\ })}$. Since c is negative, the real parts of the last two roots will be negative. The first two roots, however, are purely imaginary and the stability properties of the equilibrium cannot be determined from the Jacobian alone.

In order to show that the symmetric solution represents the unique interior equilibrium it is sufficient (by symmetry) to show that there are no interior equilibria with $0 < x_2 < x_1 < 1$. The proof is in five steps.

Step 1: When $t_1 > 0; t_2 > 0; x_1 > x_2$; and $\dot{x}_1 = 0$ then $t_1 = 0$ implies that $(1+\theta)px_1^{\theta} > 1 > px_1^{\theta}$:

The first inequality, $(1+\theta)px_1^{\theta} > 1$, follows from the expression for t_1 by observing that $\left(\frac{\omega_1}{\omega_2}x_2^{1+\theta} - x_1^{1+\theta}\right)$ can be written

$$
\begin{aligned}
\left(\frac{\omega_1}{\omega_2}x_2^{1+\theta} - x_1^{1+\theta}\right) &= \frac{px_1^{1+\theta} + 1 - x_1}{px_2^{1+\theta} + 1 - x_2}x_2^{1+\theta} - x_1^{1+\theta} \\
&= \left[\frac{p + \frac{1-x_1}{x_1^{1+\theta}}}{p + \frac{1-x_2}{x_2^{1+\theta}}} - 1\right]x_1^{1+\theta}
\end{aligned}
$$

Thus, $\left(\frac{\omega_1}{\omega_2}x_2^{1+\theta} - x_1^{1+\theta}\right)$ is negative for $\frac{1-x_1}{x_1^{1+\theta}} < \frac{1-x_2}{x_2^{1+\theta}}$ or, equivalently, for $x_1 > x_2$.

The second inequality, $1 > px_1^{\theta}$, is a straightforward implication of $\dot{x}_1 = 0$ and $t_1 > 0$:

Step 2: If $\theta \cdot 1$ and $(1+\theta)px_1^{\theta} > 1 > px_1^{\theta}$ then $\left(\frac{\omega_2}{\omega_1}x_1^{1+\theta} - x_2^{1+\theta}\right) > (1-z)x_1^{1+\theta}$ where $z = \frac{x_2}{x_1}$. The expression $\left(\frac{\omega_2}{\omega_1}x_1^{1+\theta} - x_2^{1+\theta}\right)$ can be rewritten

$$\left(\frac{\omega_2}{\omega_1}x_1^{1+\gamma} - x_2^{1+\gamma}\right) = \left(\frac{w_2}{w_1} - z^{1+\gamma}\right)x_1^{1+\gamma}$$
$$= \left(\frac{1 + zx_1(px_1^{\gamma}z^{\gamma} - 1)}{1 + x_1(px_1^{\gamma} - 1)} - z^{1+\gamma}\right)x_1^{1+\gamma}$$
$$= \frac{1 - z + (1 - x_1)(z - z^{1+\gamma})}{1 - x_1(1 - px_1^{\gamma})}x_1^{1+\gamma}$$
$$> (1 - z)x_1^{1+\gamma}$$

Step 3: $(1+\gamma)px_1^{\gamma} > 1$ implies that $(1+\gamma)px_2^{\gamma} > z^{\gamma}$:

Step 4: If $\gamma \cdot 1; 1+\gamma \cdot \frac{\bar{\ }}{1-\ }$ and $(1+\gamma)px_1^{\gamma} > 1 > px_1^{\gamma}$ then

$$p\left(\frac{1}{2 - x_1 - x_2} + \frac{(1+\gamma)x_2^{\gamma}}{x_1^{1+\gamma} + x_2^{1+\gamma}}\right) > x_1^{-(1+\gamma)}$$

To see this, first use the definition of p and $px_1^{\gamma} > \frac{1}{1+\gamma}$ to get

$$p\left(\frac{1}{2 - x_1 - x_2} + \frac{(1+\gamma)x_2^{\gamma}}{x_1^{1+\gamma} + x_2^{1+\gamma}}\right) > \frac{1}{1+\gamma}\left(\frac{x_1^{-\gamma}}{2 - x_1 - x_2} + \frac{z^{\gamma}}{x_1^{1+\gamma} + x_2^{1+\gamma}}\right)$$
$$= x_1^{-(1+\gamma)}\left(\frac{1}{1+\gamma}\frac{x_1}{2 - x_1 - x_2} + \frac{z^{\gamma}}{1 + z^{1+\gamma}}\right)$$

and then – using $px_1^{\gamma} < 1$ (and the definition of p) –

$$x_1^{-(1+\gamma)}\left(\frac{1}{1+\gamma}\frac{x_1}{2 - x_1 - x_2} + \frac{z^{\gamma}}{1 + z^{1+\gamma}}\right) > x_1^{-(1+\gamma)}\left(\frac{\frac{1}{1+\gamma}\frac{\bar{\ }}{1-\ } + z^{\gamma}}{1 + z^{1+\gamma}}\right)$$
$$> x_1^{-(1+\gamma)}$$

Step 5: The value of t_2 is determined by

$$\frac{\partial(\omega_2/\omega_1)}{\partial x_2} = w_1^{-1}\left(\frac{\omega_2}{\omega_1}x_1^{1+\gamma} - x_2^{1+\gamma}\right)\left(p\left(\frac{1}{(2 - x_1 - x_2)} + \frac{(1+\gamma)x_2^{\gamma}}{x_1^{1+\gamma} + x_2^{1+\gamma}}\right) + p(1+\gamma)x_2^{\gamma} - 1\right)$$

and using the results from steps 2-4 it follows immediately that

$$\frac{\partial(\omega_2/\omega_1)}{\partial x_2} > w_1^{-1}\left[(1 - z)x_1^{1+\gamma}x_1^{-(1+\gamma)} + z^{\gamma} - 1\right] = z^{\gamma} - z \geq 0 \text{ for } z < 1; \gamma \cdot 1$$

which completes the proof. ■

Proof of Proposition 9. Assume $x_1 = 0$. We then have $x_2 = 1$ and $t_2 = 0$. To see this, note first that $x_2 < 1$ would imply that

$$1 - t_2 = px_2^{1+\gamma} + t_2\frac{1 - x_2}{x_2} = \frac{\bar{\ }}{1-\ }\frac{2 - x_2}{x_2} + t_2\frac{1 - x_2}{x_2}$$

Since by assumption t_2 cannot be negative and $\bar{\ } > \frac{1}{2}$, it follows that the LHS of this equation is less than or equal to one while the RHS is strictly greater than one for $x_2 \cdot 1$: Hence $x_2 = 1$ and $w_2 = p = \frac{\bar{\ }}{1 \text{ ¡ } \bar{\ }} > 1$. Using 22 straightforward calculation shows that $\frac{@(!_2=!_1)}{@x_2} < 0$ for $x_1 = 0$ and $x_2 = 1$, and equation 17 then implies that in equilibrium $t_2 = 0$. The strict inequalities $\frac{@(!_2=!_1)}{@x_2} < 0$ and $w_2 = p = \frac{\bar{\ }}{1 \text{ ¡ } \bar{\ }} > 1$ imply that in the neighborhood of an equilibrium with $x_1 = t_1 = 0$ the dynamics of $(x_1; t_1)$ are determined by the two-dimensional subsystem obtained from equations 17 and 10 by ...xing the values of x_2 and t_2 at $x_2 = 1$ and $t_2 = 0$ (and leaving out the dynamic equations for x_2 and t_2): The Jacobian for this subsystem is given by

$$J = \begin{pmatrix} 0 & 1 \\ \text{¡}\, , & 0 \end{pmatrix}$$

which implies saddlepoint instability. Hence there can be no stable equilibria with $x_1 = 0$.

We now consider boundary solutions with $x_1 = 1$ and $0 < x_2 < 1$. In this case we get

$$\begin{aligned}
\frac{@(!_2=!_1)}{@x_2} &= w_1^{\text{¡}1}\left(\frac{!_2}{!_1}x_1^{1+°} \text{ ¡ } x_2^{1+°}\left(p\left(\frac{1}{(2 \text{ ¡ } x_1 \text{ ¡ } x_2)} + \frac{(1+°)x_2^{°}}{x_1^{1+°} + x_2^{1+°}}\right) + p(1+°)x_2^{°} \text{ ¡ } 1\right)\right) \\
&= p^{\text{¡}1}\left(\frac{w_2}{p} \text{ ¡ } x_2^{1+°}\left(p\left(\frac{1}{(1 \text{ ¡ } x_2)} + \frac{(1+°)x_2^{°}}{1 + x_2^{1+°}}\right) + p(1+°)x_2^{°} \text{ ¡ } 1\right)\right) \\
&= p^{\text{¡}1}\left((1 \text{ ¡ } x_2)\left(\frac{1}{(1 \text{ ¡ } x_2)} + \frac{(1+°)x_2^{°}}{1 + x_2^{1+°}}\right) + \frac{1}{x_2}\left(p(1+°)x_2^{1+°} \text{ ¡ } x_2\right)\right) \\
&= p^{\text{¡}1}\left(1 + (1 \text{ ¡ } x_2)\frac{(1+°)x_2^{°}}{1 + x_2^{1+°}} + °px_2^{°} + \frac{1}{x_2}\left(px_2^{1+°} \text{ ¡ } x_2\right)\right) \\
&= p^{\text{¡}1}\left(1 + (1 \text{ ¡ } x_2)\frac{(1+°)x_2^{°}}{1 + x_2^{1+°}} + °px_2^{°} \text{ ¡ } \frac{t_2}{x_2}\right) \\
&> 0
\end{aligned}$$

where the inequality follows from the fact that $\frac{t_2}{x_2}$ must be less than one at an equilibrium with $0 < x_2 < 1$ (use equation 10). Since $\frac{@(!_2=!_1)}{@x_2}$ is strictly positive equation 17 implies that $t_2 = 1$ at an equilibrium. With $t_2 = 1$; however, we must have $x_2 = 1$; which contradicts the assumption that $0 < x_2 < 1$. In other words, there can be no stable equilibria with $x_1 = 1$: This proves the ...rst claim.

Now assume $t_1 = 0; \underline{x}_1 = 0; 0 < x_1 < 1$ and $t_2 > 0; \underline{x}_2 = 0; 0 < x_2 < 1$. We then have

$$w_1 = px_1^{°} = 1$$

and

$$w_2 = px_2^{°} + t_2\frac{1 \text{ ¡ } x_2}{x_2} = 1 \text{ ¡ } t_2:$$

It follows that $x_1 > x_2$:

We now show that if $x_1 > x_2$ and $\underline{x}_1 = 0$ then we must have $t_2 > 0$: Using 17 and 22 the sign of t_2 is determined by

$$\frac{@(!_2=!_1)}{@x_2} = w_1^{¡1}\left(\frac{!_2}{!_1}x_1^{1+°} ¡ x_2^{1+°}\right)\left(p\left(\frac{1}{(2 ¡ x_1 ¡ x_2)} + \frac{(1+°)x_2^{°}}{x_1^{1+°}+x_2^{1+°}}\right) + p(1+°)x_2^{°} ¡ 1\right) \tag{23}$$

Using the expressions for $w_1; w_2$ and p , 6 can be rewritten

$$\frac{@(!_2=!_1)}{@x_2} = w_1^{¡1}\left(w_2 ¡ \left(\frac{x_2}{x_1}\right)^{1+°}\left(x_1\left(\frac{\bar{}}{1¡\bar{}}\frac{x_1^{°}}{x_1^{1+°}+x_2^{1+°}} + \frac{(1+°)\left(\frac{x_2}{x_1}\right)^{°}x_1^{°}}{x_1^{1+°}+x_2^{1+°}}\right) + (1+°)\left(\frac{x_2}{x_1}\right)^{°} ¡ 1\right)\right)$$

or, de...ning $z = \frac{x_2}{x_1}$ and $w_2 = x_2z^{°} + 1 ¡ x_2$,

$$\begin{aligned}\frac{@(!_2=!_1)}{@x_2} &= w_1^{¡1}\left(1 ¡ x_1z + x_2z^{°} ¡ z^{1+°}\left(\frac{\bar{}}{1¡\bar{}} + (1+°)z^{°}\right) + (1+°)z^{°} ¡ 1\right)\\ &= w_1^{¡1}\left(1 ¡ z + (1¡x_1)z ¡ z^{1+°}\left(\frac{\bar{}}{1¡\bar{}} + (1+°)z^{°}\right) + (1+°)z^{°} ¡ 1\right)\end{aligned}$$

We have $0 < z < 1; 0 < x_1 < 1$ and, by assumption, $1 > ° > 0$ and $\bar{} > :5$ (and hence $z^{°} > z > z^{1+°}$ and $\frac{\bar{}}{1¡\bar{}} > 1$). Using these results, we get

$$\begin{aligned}\frac{@(!_2=!_1)}{@x_2} &> w_1^{¡1}(1 ¡ z + (1+°)z^{°} ¡ 1)\\ &> w_1^{¡1}°z^{°} > 0\end{aligned}$$

and it follows that there is no equilibrium with $\underline{x}_1 = \underline{x}_2 = t_2 = 0$ and $0 < t_2 < 1$:

In order to prove that there is no equilibrium with $t_i = 1$ we show that if $\underline{x}_1 = \underline{x}_2 = 0$ and $t_2 = 1$ then $t_2 < 0$. Observe ...rst that $t_2 = 1$ implies $\underline{x}_2 > 0$ for $x_2 < 1$ and hence that in equilibrium we must have $x_2 = 1$. Substituting $x_2 = 1$ into the equations for $p; w_1; w_2$ we get

$$\begin{aligned}p &= \frac{\bar{}}{1¡\bar{}}\frac{1¡x_1}{1+x_1^{1+°}}\\ w_1 &= px_1^{1+°} + (1¡x_1)\\ w_2 &= p\\ \frac{w_1}{w_2} &= x_1^{1+°} + \frac{(1¡x_1)}{\frac{\bar{}}{1¡\bar{}}\frac{1¡x_1}{1+x_1^{1+°}}} = x_1^{1+°} + \left(1 + x_1^{1+°}\right)\frac{1¡\bar{}}{\bar{}} = \frac{1¡\bar{} + x_1^{1+°}}{\bar{}}\end{aligned}$$

and $\frac{@(!_2=!_1)}{@x_2}$ can be written

$$\begin{aligned}
\frac{@(!_2=!_1)}{@x_2} &= w_1^{i\,1}\left(\frac{!_2}{!_1}x_1^{1+°} - 1\right)\left(p\left(\frac{1}{(1-x_1)} + \frac{(1+°)}{x_1^{1+°}+1}\right) + p(1+°) - 1\right) \\
&= w_1^{i\,1}\left(\frac{-}{1--+x_1^{1+°}}(1-x_1)\left(\frac{1}{(1-x_1)} + \frac{1+°}{x_1^{1+°}+1}\right) + \frac{-}{1--}\frac{1-x_1}{1+x_1^{1+°}}(1+°) - 1\right) \\
&= w_1^{i\,1}\left(\frac{-}{1--+x_1^{1+°}}\left(1+(1+°)\frac{(1-x_1)}{x_1^{1+°}+1}\right) + \frac{-}{1--}\frac{1-x_1}{1+x_1^{1+°}}(1+°) - 1\right)
\end{aligned}$$

The equilibrium condition $\underline{x}_1 = 0$ implies that either $x_1 = 1$ or

$$px_1^{°} = \frac{-}{1--}\frac{1-x_1}{1+x_1^{1+°}}x_1^{°} \cdot 1 - \frac{t_1}{x_1} \cdot 1 \text{ and } x_1 < 1$$

Substituting $x_1 = 1$ into the expression for $\frac{@(!_2=!_1)}{@x_2}$ it is readily seen that in this case $t_2 < 0$. In the case with $x_1 < 1$ we get

$$\begin{aligned}
\frac{@(!_2=!_1)}{@x_2} &= w_1^{i\,1}\left(-\left(1+\frac{-}{1--+x_1^{1+°}}\right) + (1+°)\frac{(1-x_1)}{x_1^{1+°}+1}\left(\frac{-}{1--} - \frac{-}{1--+x_1^{1+°}}\right)\right) \\
&\cdot w_1^{i\,1}\left(-\left(1+\frac{-}{1--+x_1^{1+°}}\right) + (1+°)x_1^{i\,°}\frac{1--}{-}\left(\frac{-}{1--} - \frac{-}{1--+x_1^{1+°}}\right)\right) \\
&= w_1^{i\,1}\left(-\left(1+\frac{-}{1--+x_1^{1+°}}\right) + (1+°)\frac{x_1}{1--+x_1^{1+°}}\right) \\
&= \frac{w_1^{i\,1}}{1--+x_1^{1+°}}\left[-\left(1+x_1^{1+°}\right) + (1+°)x_1\right]
\end{aligned}$$

The expression in square brackets is increasing in x_1 for $x_1 < 1$. Hence,

$$\begin{aligned}
\frac{@(!_2=!_1)}{@x_2} &= \frac{w_1^{i\,1}}{1--+x_1^{1+°}}\left[-\left(1+x_1^{1+°}\right) + (1+°)x_1\right] \\
&< \frac{w_1^{i\,1}}{1--+x_1^{1+°}}[-1+°] \text{ for } x_1 < 1
\end{aligned}$$

It follows that if $° \cdot 1$ then $t_2 < 0$ for $t_2 = 1$; $\underline{x}_1 = \underline{x}_2 = 0$: This proves the second claim. ■

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