Voltage Unbalance Emission Assessment in Interconnected Power Systems

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Abstract—The International Electrotechnical Commission (IEC) Technical Report IEC/TR 61000-3-13:2008 considers voltage unbalance (VU) emission assessment as a key aspect of the VU management of power systems. Compliance assessment of unbalanced installations at the post connection stage is essential to ensure that the limits set by the IEC VU emission allocation methodology in the pre-connection stage are met. Although, VU is known to be caused by load asymmetries and inherent network asymmetries, locating all VU emission sources is not a straightforward process, especially in a network with interconnections. Such assessment methodologies should ensure that the contributions from various sources of unbalance to the total VU emission are determined using data which is not overly demanding. This paper presents deterministic methodologies which can be used to assess constituent components of post-connection VU level at the point of evaluation in an interconnected network utilising post connection voltage/current measurements and known system parameters. The theoretical bases are developed to cover different load types including induction motors. Emission assessment outcomes of different study systems obtained by employing the proposed methodologies are verified using unbalanced load flow analysis.

Index Terms—power quality, voltage unbalance, current unbalance, voltage unbalance emission allocation, voltage unbalance emission assessment, network asymmetry, load asymmetry

I. INTRODUCTION

Voltage unbalance (VU) is a power quality problem that can lead to adverse effects on both supply utilities and customer installations. The primary causes of VU are untransposed transmission lines and asymmetrical distribution of loads [1]. VU management requires carefully developed approaches for identification of different unbalance sources and evaluation of their individual contributions on the total emission level to ensure the utilisation of total VU absorption capacity of the power system. In this regard, the IEC technical report IEC/TR 61000-3-13:2008 [1] prescribes a VU emission allocation methodology enabling evaluation of individual emission limits to unbalanced installations. The emission allowance determined for a particular busbar is then apportioned to loads and lines using the \( k_{UE} \) factor approach which has seen to follow a rudimentary direction as it disregards its sensitivity to power system diversities [2], [3]. In addition, limited knowledge exists on the location of VU sources and their influence on total unbalance emission at various busbars [4], [5] and [6]. Nonetheless, a considerable collection of literature exists on definitions of VU [7], causes, effects [8] and related standards and mitigation techniques [9].

With the release of [1], there is another requirement to quantify the post-connection VU emission of a load against the imposed emission allocation. In this regard, the recent work covered in [4] presents a new deterministic approach on compliance assessment at the post connection stage of installations for radial power systems thus significantly extending the preliminary work covered in the CIGRE/CIRED C4.109 working group report on emission assessment techniques [11]. The new approach is based on the use of complex VU factors and utilises pre-connection and post-connection voltage/current measurements together with known system parameters. Further, it is sufficiently generalised to disaggregate the total VU emission at the point of evaluation\(^1\) (POE) into its constituent components (i.e. due to load asymmetry, line asymmetry and upstream source contributions).

Compared to the case of radial power systems, identification of sources of VU and their level of contribution at each busbar is a much more involved task in interconnected networks as a result of the complexity of interactions that occur between the many sources of unbalance [12]. Even in the well established IEC emission allocation methodology, VU emission in interconnected networks is not dealt with an exact manner taking multiple interactions into account. The approaches followed in some case studies undertaken [5], [6], are not sufficiently general and theoretically sound for VU emission assessment.

The main objective of this paper is to present a new, deterministic methodology on VU emission assessment in interconnected networks providing further improvements to the IEC work on VU management. The proposed study extends the theoretical modelling presented in [4], giving a generalised classification on constituent components of net VU emission at individual busbars. The models established are validated using 3-phase unbalanced load flow analyses.

The structure of this paper is as follows: the existing approaches on VU emission assessment giving further emphasis to the methodologies in [4] are reviewed in Section II. The proposed theoretical bases for the evaluation of various VU emission contributions in an interconnected network are given in Section III. Section IV presents the results associated with the verification process related to the proposed methodologies. Conclusions are given in Section V.

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\(^1\)The point in the public power system where the emission level of a given installation is to be assessed against the given emission limit.
II. REVIEW OF THE EXISTING APPROACHES ON VU EMISSION ASSESSMENT

The approach given in [11] evaluates the VU emission caused by the connection of a load \(U_{2,i}\) at the POE using the difference of post-connection and pre-connection VU emission measurements \(U_{2,i} = U_{2,\text{post-connection}} - U_{2,\text{pre-connection}}\). Although the connection of a load changes the VU emission level at the POE, the load is not solely responsible for the change in the emission as it contains a contribution made by line asymmetry which is a measure of the negative-positive sequence coupling impedance of the line \(Z_{21,t}\). Therefore, \(U_{2,i}\) consists of contributions made by load asymmetry \((U_{2,i}(\text{load}))\) as well as line asymmetry \((U_{2,i}(\text{line}))\). Accordingly, the post-connection VU emission (in terms of the negative sequence voltage) at the POE can be decomposed, identifying its constituent components as given in (1).

\[
U_{2,\text{post-connection}} = KU_{2,\text{pre-connection}} + U_{2,i}(\text{load}) + U_{2,i}(\text{line})
\]

where \(K\) is a complex scaling factor.

\(U_{2,\text{pre-connection}}\) represents the emission contribution made by the upstream unbalanced voltage source as the pre-connection measurement is made under open circuit condition at the POE. Hence, the generalised outcome given in (1) can be identified as the basis for evaluating the contributions made by the load, line and upstream source asymmetries on the overall post-connection VU emission at the POE. Accordingly, the work presented in [4] has established a post-connection emission assessment methodology for radial power systems (referring to Fig. 1) using complex VU factors \(^4\) (obtained by normalising all terms in (1) using positive sequence voltage at the POE) to decompose post-connection VU emission at the POE into individual components as shown in (2)\(^5\).

\[
VUF_{\text{POE}} = VUF_{\text{load}} + VUF_{\text{line}} + VUF_{\text{source}}
\]

where \(VUF_{\text{load}}\) is the contribution made by the unbalanced load connected; \(VUF_{\text{line}}\) is the contribution made by asymmetrical line; \(VUF_{\text{source}}\) is the contribution made by upstream unbalanced voltage source (i.e. the unbalance which propagates from the upstream source to the POE). The load can be either passive or an induction motor or a mix of the same. The mathematical formulation presented in [4] evaluates these decoupled components in a generalised manner as given in Table\(^6\) I in such a way that asymmetries associated with individual power system components are reflected by the respective decoupled formulation. Accordingly, the contribution made by line asymmetry \((VUF_{\text{line}})\) is governed by a term; the negative-positive sequence coupling impedance of the line \((Z_{21,t})\) which is a measure of line asymmetry. The contribution made by upstream voltage source is established using the pre-connection emission measurement at the POE. For passive loads (Table I, column 2), total upstream unbalance is seen to propagate to the POE as the source contribution which is equal to \(VUF_{\text{source}}\). Further, the generalised expression for the load contribution is governed by negative-positive sequence coupling impedance of the load \((Z_{21,\text{rec}})\) which is later modified using the current unbalance factor \(^7\) \((CUF)\) and the VU factor at the POE \((VUF_{\text{POE}})\) to facilitate the emission assessment related to passive loads where the details of the impedances may not be known.

In the case of induction motor loads (which do not contain any intrinsic unbalance), it is well known that they are affected by unbalance of the supply source while acting as VU compensators. Thus, the post-connection emission assessment at the POE is composed of two components as shown in Table I (column 3) which include the influence made by upstream source by propagation \((Z_{22,\text{m}} Z_{21,\text{t}} (\frac{Z_{21,m}}{Z_{21,t}}) VUF_{\text{source}}\) represents a portion of the upstream source VU factor) and the contribution made by an asymmetrical line \((-\frac{Z_{22,m}}{Z_{21,m}} (\frac{Z_{21,t}}{Z_{22,m}})\) on the total VU emission. Here, the scaling factor of upstream source unbalance has a magnitude less than unity, showing the improvement in VU emission made as a result of the connection of an induction motor at the POE. The absence of a term corresponding to load contribution is a consequence of the symmetrical nature of the induction motor.

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\(^5\) In terms of negative sequence voltages [11].

\(^6\) When transmission lines are not completely transposed, unequal mutual impedances which arise as a result of the asymmetrical electromagnetic coupling between phase conductors cause unbalanced voltages across three phases. Presence of these unequal mutual impedances result in \(Z_{21,t}\) in the sequence domain which is identified as the negative-positive sequence coupling impedance.

\(^4\) The VU factor \((VUF)\) is defined as the ratio of negative sequence voltage to positive sequence voltage [1].

\(^3\) All terms that appear in (2) are expressed in terms of VU factors.

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**TABLE I**

**VU EMISSION ASSESSMENT IN RADIAL POWER SYSTEMS: DECOUPLED CONTRIBUTIONS [4]**

<table>
<thead>
<tr>
<th>Individual contributor</th>
<th>Passive loads</th>
<th>Induction motor loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load asymmetry VUF</td>
<td>(Z_{21,\text{rec}})</td>
<td>(Z_{21,\text{rec}})</td>
</tr>
<tr>
<td>Line asymmetry VUF</td>
<td>(Z_{21,t})</td>
<td>(Z_{21,t})</td>
</tr>
<tr>
<td>Source asymmetry VUF</td>
<td>(Z_{22,\text{m}})</td>
<td>(Z_{22,\text{m}})</td>
</tr>
</tbody>
</table>

\(^7\) \(Z_{x,y,t}\) is the sequence impedance of the transmission line. \(x\) and \(y\) can be replaced with 1 and 2 which stand for positive and negative sequence respectively, \(V_{\text{reg-line}}\) is the voltage regulation of the line defined as the ratio of positive sequence voltage drop in the line to positive sequence voltage at the load end. \(Z_{x,m}\) is the sequence impedance of the motor \((x=1\text{ or }2)\).

\(^8\) Defined as the ratio of negative sequence current to positive sequence current [1].
III. ASSESSMENT OF INDIVIDUAL VU EMISSION CONTRIBUTIONS IN AN INTERCONNECTED NETWORK: THEORETICAL BASES

The proposed deterministic methodology for interconnected power systems extends the concepts and approaches summarised in Section II with regard to radial power systems. Concurrently existing sources of VU while taking their interactions at the POE into account are analysed in a generalised manner.

A. Classification of VU Emission Contributors at the POE

The outcome of the post-connection VU emission at the POE for a radial network given in (2) evaluates the source contribution \( (VUF_{source}) \) using pre-connection emission measurement at the POE. This statement is strictly valid for radial systems where an equivalent fixed unbalanced voltage source can be assumed to exist at an upstream point which is not disturbed by the connection of the downstream load. Use of such an argument is not valid for an interconnected network considering a POE (i.e. the busbar under observation) since the pre-existing busbar unbalance levels in the entire network are affected by the unbalanced load once connected at the POE. For this reason, the proposed methodology related to interconnected networks utilises only the post-connection snapshot based VU measurements at busbar levels to quantify individual contributions from various unbalanced sources reflected at the POE.

![Fig. 2. Interconnected power system](image)

As shown in Fig. 2, identification of different emission contributors at the POE is carried out by defining a hypothetical boundary that encircles all surrounding busbars (identified as local busbars) which are connected to the busbar under assessment (POE) via lines (identified as local lines). Accordingly, post-connection VU level at the POE is decomposed by considering (a) the influence made by local load at the POE (identified as local load contribution), (b) the influence made by all transmission lines connected to the POE (identified as local line contributions) and (c) influence made by VU levels at local busbars to address the VU that propagates to POE from local busbars (identified as local busbar contributions).

The current drawn by the load at the POE affects the pre-existing busbar VU levels (except for voltage controlled busbars) due to the additional negative sequence voltage drops on the lines. Therefore, local busbars are identified as dependent VU sources which include pre-existing (pre-connection) unbalance as well as the influence made by the local load on the rest of the system outside the hypothetical boundary (due to the added current to the network). In effect, this decomposition process can be applied to any busbar in the power system to determine the influence made by the local load, local lines and local busbars on the net unbalance measurement made at the POE.

B. Location of Different Sources of Unbalance in an Interconnected Power System

The interconnected power system in Fig. 3 is used to demonstrate the location of unbalance contributors at busbar 3 (POE). The total VU measurement (emission) at busbar 3 can be decomposed by identifying three factors: (a) influence made by the local load (b) influence made by the local lines and (c) influence made by the local busbars that represent dependent voltage sources which are influenced by the connection of the load.

- Busbar 3 is connected to busbars 1 and 2 through transmission lines 3-1 and 3-2 respectively. Thus, inherent asymmetries associated with these two lines can have an influence on the total VUF at bus 3. Hence, these two lines are identified as local lines and the total line contribution at busbar 3 \( (VUF_{line,3}) \) is made up of two components namely \( VUF_{line,3-1} \) and \( VUF_{line,3-2} \).
- Since busbars 1 and 2 are directly connected to busbar 3 through local lines, voltage asymmetries of these local busbars influence the net emission level at busbar 3 as dependent unbalanced sources. Hence, the total local busbar contribution \( (VUF_{d,source,3}) \) can be decomposed as \( VUF_{d,source,3-1} \) and \( VUF_{d,source,3-2} \).
- The influence made by the local load on the total VUF at busbar 3 is identified as \( VUF_{load,3} \).

Based on the listed classification of various influencing sources of VU, a generalised model is developed to assess the individual emission contributions. The hypothetical 3-bus power system (enclosed with dotted lines) shown in Fig. 3 is used to demonstrate the derivation of the theoretical bases giving due consideration to the following aspects:

- Power system is assumed to operate under sinusoidal steady state conditions.
- Zero sequence VU is ignored assuming 3-wire systems.

![Fig. 3. Interconnected power system](image)
\[ Y_{21:L3}U_{1,3} + Y_{22:L3}U_{2,3} = Y_{21:31}U_{1,1} + Y_{22:31}U_{2,1} + Y_{21:32}U_{1,2} + Y_{22:32}U_{2,2} + Y_{21:33}U_{1,3} + Y_{22:33}U_{2,3} \]  
\[ (Y_{22:L3} - Y_{22:33}) VUF_3 = Y_{21:31} - Y_{21:L3} + Y_{21:31}(1 + V_{\text{drop-t31}}) + Y_{22:31}VUF_1 (1 + V_{\text{drop-t31}}) + Y_{21:32}(1 + V_{\text{drop-t32}}) + Y_{22:32}VUF_2 (1 + V_{\text{drop-t32}}) \]

- Loads are modelled as a coupled sequence-impedance matrix in the case of passive loads and as a decoupled impedance matrix in the case of induction motors.

Passive loads and induction motor loads are separately considered in developing the theoretical basis. Theoretical outcomes are verified initially using a 3-bus test network which is followed by applying the same methodologies to the IEEE 14 bus test system.

C. Separation of VU Emission Contributors - Passive Loads

The mathematical model presented in this section is sufficiently general to assess the VU emission caused by any type of passive load (i.e. constant impedance, current or power or any mix) since load contribution is derived using voltage/current measurements at the POE in addition to the known network parameters. Total VU emission at busbar 3 is decomposed by extending nodal analysis to 3-phase systems. Referring to the hypothetical 3-bus system shown in Fig. 3, nodal current at busbar 3 \((I_{3,3})\) where \(x=a,b,c\) in the 3-phase domain can be expressed as in (3) [13].

\[
\begin{pmatrix}
I_{a,3} \\
I_{b,3} \\
I_{c,3}
\end{pmatrix} = 3 \sum_{i=1} \begin{pmatrix}
Y_{aa:3i} & Y_{ab:3i} & Y_{ac:3i} \\
Y_{ba:3i} & Y_{bb:3i} & Y_{bc:3i} \\
Y_{ca:3i} & Y_{cb:3i} & Y_{cc:3i}
\end{pmatrix} \begin{pmatrix}
U_{a,i} \\
U_{b,i} \\
U_{c,i}
\end{pmatrix}
\]  
\[(3)\]

where \(a,b,c\) refer to the three phases; \(U_{x,i}\) is the bus voltage vector for \(i^{th}\) bus and \(Y_{x:y:3i}\) refers to the elements of nodal bus admittance matrix associated with the 3-bus system. Further, \(i=1,2,3\) is any bus in the hypothetical 3-bus power system.

The corresponding nodal equation in the sequence domain can then be derived (employing the a-b-c to 0-1-2 transformation matrix) as follows:

\[
\begin{pmatrix}
I_{0,3} \\
I_{1,3} \\
I_{2,3}
\end{pmatrix} = 3 \sum_{i=1} \begin{pmatrix}
Y_{00:3i} & Y_{01:3i} & Y_{02:3i} \\
Y_{10:3i} & Y_{11:3i} & Y_{12:3i} \\
Y_{20:3i} & Y_{21:3i} & Y_{22:3i}
\end{pmatrix} \begin{pmatrix}
U_{0,i} \\
U_{1,i} \\
U_{2,i}
\end{pmatrix}
\]  
\[(4)\]

where the subscripts 0, 1 and 2 refer to the zero, positive and negative sequences respectively and the subscript 3 refers to busbar 3.

Positive and negative sequence currents at busbar 3 can be extracted from (4) as given by (5) and (6) neglecting the influence made by zero sequence terms.\(^3\)

\[ I_{1,3} = \sum_{i=1} Y_{11:3i} U_{1,i} \]  
\[(5)\]

\(^3\)The contribution made by the sum of products \(\sum_{i=1} Y_{12:3i}U_{2,i}\) on \(I_{1,3}\) is assumed to be negligible in deriving \(I_{1,3}\) [4].

\(^4\)Practically speaking, the zero sequence unbalance can be ignored, considering its minor contribution compared to negative sequence unbalance from an equipment impact perspective and/or because of three-wire situations [1].

Further, positive and negative sequence currents at busbar 3 \((I_{1,3} \text{ and } I_{2,3})\) can be rewritten in terms of load admittance and load voltage (bus 3 voltage) in sequence domain as in (7) and (8)

\[ I_{1,3} = Y_{11:L3} U_{1,3} \]  
\[(7)\]

\[ I_{2,3} = Y_{21:L3} U_{1,3} + Y_{22:L3} U_{2,3} \]  
\[(8)\]

where \(Y_{x:y:L3}\) represents load 3 admittance in the sequence domain.

Substitution of \(I_{2,3}\) given in (8) simplifies (6) as shown in (9). VU factor at busbar 3 \((VUF_3 = \frac{I_{2,3}}{I_{1,3}}})\) can be now obtained by further simplifying (9) as shown in (10). Here, positive sequence voltages at busbars 1 and 2 are rearranged using the voltage at busbar 3 and the voltage drops through respective lines (i.e. \(U_{1,1} = U_{1,3} + U_{1,3-1}\) and \(U_{1,2} = U_{1,3} + U_{1,3-2}\)). Accordingly, normalized positive sequence voltage drop of line 3-1 \((V_{\text{drop-t31}} - \text{normalised using bus 3 voltage})\) is defined as \(\frac{U_{1,1} - U_{1,3}}{U_{1,3}}\) and \(V_{\text{drop-t32}}\) is the voltage drop of line 3-2 \((\text{normalised using bus 3 voltage})\) defined as \(\frac{U_{1,2} - U_{1,3}}{U_{1,3}}\). In the analysis of the radial power system (in Section II) this voltage drop was identified as voltage regulation because of the unidirectional power flow from source to load. \(VUF_3\) is the VU factor at busbar 1 as given by \(\frac{I_{2,3}}{I_{1,3}}\) and similarly \(VUF_2 = \frac{I_{2,2}}{I_{1,2}}\) is the VU factor at busbar 2. The load impedances are comparatively large compared to transmission line impedances for all practical needs. Thus, line admittance compared to the load admittance is large and hence the difference between the negative sequence load admittance and negative sequence line admittance can be approximated as (11):

\[ (Y_{22:L3} - Y_{22:33}) \approx -Y_{22:33} \]  
\[(11)\]

Further, total positive-negative sequence coupling admittance seen at a busbar is equal to the negative of the sum of individual positive-negative sequence coupling admittances of all connected lines to busbar 3 (considering the properties of nodal Y bus admittance matrix in sequence domain).

\[ Y_{21:33} = -(Y_{21:31} + Y_{21:32}) \]  
\[(12)\]
can be analysed to identify the individual emission contributions made by different sources of unbalance at busbar 3.

- The factor \( \frac{Y_{21:31}}{Y_{22:33}} \) can be identified as the influence made by load asymmetry at busbar 3 as \( Y_{21:31} \) represents the positive-negative sequence coupling admittance which is a measure of the load asymmetry. For a balanced load, \( Y_{21:31} = 0 \) and hence the term \( \frac{Y_{21:31}}{Y_{22:33}} = 0 \) thus verifying the influence made by the load (\( VUF_{load,3} \)).

- \( Y_{21:31} \) is the positive-negative sequence coupling admittance of line 3-1 which represents the respective line asymmetry. Hence, if the line is symmetrical, the factor \( \frac{Y_{21:31}}{Y_{22:33}} \) indicates it as the influence made by the positive-negative sequence coupling admittance of line 3-1 (\( VUF_{load,3} \)). Similarly, the factor \( \frac{Y_{22:31}}{Y_{22:33}} \) is the influence made by line 3-2 (\( VUF_{line,3-2} \)). Thus, the total line contribution at busbar 3 (\( VUF_{line,3} \)) is given by \( VUF_{line,3} = VUF_{load,3} + VUF_{line,3-1} + VUF_{line,3-2} \).

- If the voltages of neighbourhood busbars which are connected to bus 3 (i.e. bus 1 and 2) are balanced, respective VU factors (i.e. \( VUF_1 \) and \( VUF_2 \)) do not exist. Therefore, the factor \( \frac{Y_{21:31}}{Y_{22:33}} \) (1 + \( VUF_{load,3} \)) gives the influence made by busbar 1 emission on the total unbalance at busbar 3 (\( VUF_{source,3-1} \)) as a propagated quantity since all the other terms associated with it are fixed for the line connecting the two busbars. Similarly, \( \frac{Y_{22:31}}{Y_{22:33}} \) (1 + \( VUF_{load,3} \)) gives the influence made by busbar 2 (\( VUF_{source,3-2} \)) - propagated unbalance from busbar 2 to busbar 3) on the total \( VUF_3 \). Hence, the total influence made by local busbars as dependent unbalance sources can be established as

\[
VUF_{source,3} = VUF_{source,3-1} + VUF_{source,3-2}
\]

The emission contribution made by load asymmetry \( \frac{Y_{21:31}}{Y_{22:33}} \) can be further modified as shown in (14) which is independent of load admittances to cater for other passive load types (for example constant current or constant power loads) of which load impedances are not known. The details of the derivation of (14) is given in Appendix A.

\[
VUF_{load,k} = (CUF_k - VUF_k) \sum_{i \neq k} n Y_{1:i:k} Y_{2:i:k} V_{drop-t(i-k)} (1 + V_{drop-t(i-k)}) VUF_1 (1 + V_{drop-t(i-k)}) VUF_{source,k} \tag{16}
\]

\[
VUF_{line,k} = n \sum_{i \neq k} VUF_{source,k-i} = -n \sum_{i \neq k} Y_{2:i:k} Y_{2:i:k} V_{drop-t(i-k)} \tag{17}
\]

\[
VUF_{source,k} = n \sum_{i \neq k} \left( \frac{Y_{2:i:k}}{Y_{2:k:k}} \right) (1 + V_{drop-t(i-k)}) VUF_i \tag{18}
\]

Local load contribution \( VUF_{load,k} \) represents the influence made by load asymmetry only on the POE. The contributions made by the local load on the rest of the system outside the boundary which is embedded in the local busbar asymmetries is dependent on the level of local load unbalance and network unbalance and is seen to be essentially the responsibility of the network owner.

D. Separation of VU Emission Contributors - Induction Motor Loads

Although 3-phase induction motors are inherently symmetrical devices, they are affected by the supply source unbalance while exhibiting their ability to compensate for pre-existing unbalance levels in general. Hence, it is important to examine the level of compensation provided by induction motors.

A VU emission assessment study in the case of 3-phase induction motors is carried out by replacing the passive load at bus 3 in the interconnected network discussed in Section III-C with a 3-phase induction motor, represented by three decoupled admittances in sequence domain. Therefore, expressions for positive and negative sequence motor currents \( I_{1,m} \) and \( I_{2,m} \) respectively can be established as:

\[
I_{1,m} = U_{1,3} Y_{1,m} \tag{19}
\]

\[
I_{2,m} = U_{2,3} Y_{2,m} \tag{20}
\]

where \( Y_{1,m} \) and \( Y_{2,m} \) are the positive and negative sequence motor currents respectively. Negative sequence current given by (20) is substituted in (6) and can be simplified using the same approach used in the case of passive loads to evaluate the total VU at busbar 3 as given in (21). Similar to the case of the passive load, individual emissions from different influencing sources of unbalance can be identified by analysing (21).

\[
Y_{21:31} \text{ is the positive-negative sequence coupling admittance of line 3-1 which represents the respective line asymmetry. Hence, if the line is symmetrical, the factor } \frac{Y_{21:31}}{Y_{22:33}} = 0 \text{ indicating that it as the influence made by line 3-1 (} VUF_{line,3-1} \text{) on total VU emission at busbar 3. Similarly, the factor } \frac{Y_{22:31}}{Y_{22:33}} \text{ is the influence made by line 3-2}
\]
\[ VUF_3 = \frac{Y_{21:31} V_{\text{drop-t31}}}{Y_{2:m} - Y_{22:33}} + \frac{Y_{21:32} V_{\text{drop-t32}}}{Y_{2:m} - Y_{22:33}} + \frac{Y_{22:31} (1 + V_{\text{drop-t31}})}{Y_{2:m} - Y_{22:33}} VUF_1 + \frac{Y_{22:32} (1 + V_{\text{drop-t32}})}{Y_{2:m} - Y_{22:33}} VUF_2 \]  
(21)

(VUF\text{line},3\rightarrow 2). Thus, the total line contribution at busbar 3 VUF\text{line},3\rightarrow 1 + VUF\text{line},3\rightarrow 2.

If the voltages of neighbourhod busbars (i.e. busbars 1 and 2) which are connected to busbar 3 are balanced, the respective VU factors (i.e. VUF_3 and VUF_2) are equal to zero. Therefore, the term \( \frac{Y_{22:31} (1 + V_{\text{drop-t31}})}{Y_{2:m} - Y_{22:33}} VUF_1 \) represents the influence made by busbar 1 VU asymmetry on the total unbalance at busbar 3 (labeled VUF\text{dsource},3\rightarrow 1). Similarly, the term \( \frac{Y_{22:32} (1 + V_{\text{drop-t32}})}{Y_{2:m} - Y_{22:33}} VUF_2 \) is the influence made by busbar 2 VU asymmetry on the total unbalance at busbar 3 (labeled VUF\text{dsource},3\rightarrow 2). Hence, the total influence made by local busbars as dependent unbalance sources can be established as VUF\text{dsource},3 = VUF\text{dsource},3\rightarrow 1 + VUF\text{dsource},3\rightarrow 2.

The magnitude of these scaling factors (\( \frac{Y_{22:31} (1 + V_{\text{drop-t31}})}{Y_{2:m} - Y_{22:33}} \) or \( \frac{Y_{22:32} (1 + V_{\text{drop-t32}})}{Y_{2:m} - Y_{22:33}} \)) can be shown to be less than unity demonstrating the fact that 3-phase induction motors help to improve pre-existing VU levels at busbar 3.

Accordingly, a generalised model can be established to determine emission contributions made by individual sources to the overall VU factor at a busbar with an induction motor as follows. For an n bus power system, the total VU emission at busbar k (VUF_k) where an induction motor load is connected, can be written as the summation of the influences made by the lines (VUF\text{line},k) and the influences made by local busbars (VUF\text{dsource},k) as shown in (22), (23) and (24). There is no term corresponding to load contribution resulting from the symmetrical nature of the induction motors.

\[ VUF_k = VUF_{\text{line},k} + VUF_{\text{dsource},k} \]  
(22)

\[ VUF_{\text{line},k} = \sum_{i \neq k}^{n} VUF_{\text{line},k-i} = \sum_{i \neq k}^{n} \frac{Y_{21:ki} V_{\text{drop-t(k-i)}}}{Y_{2:m} - Y_{22:kk}} \]  
(23)

\[ VUF_{\text{dsource},k} = \sum_{i \neq k}^{n} VUF_{\text{dsource},k-i} \]  
(24)

This methodology can be used when a load comprises multiple induction motors at the POE using the approach presented in [14] which evaluates an equivalent (effective) set of motor parameters.

IV. VERIFICATION OF THE METHODOLOGY

The proposed methodology was verified using an unbalanced load-flow program written in MATLAB in combination with analyses employing PSCAD/EMTDC and DiGSIILENT PowerFactory simulation platforms. Results related to two test systems are presented in the following sections.

A. Verification of the Methodology: Three bus MV Test System

The 3-bus MV system [15] shown in Fig. 4 is considered where two cases are considered:(a) constant power loads at busbars 1, 2 and 3 and (b) constant power loads at busbars 1 and 2 and an induction motor at busbar 3. The details of this test system and the loads are as follows:

- HV: 66 kV, MV: 12.47 kV, 60 Hz, three-wire
- Transmission lines: all lines are considered as identical in construction and untransposed (line lengths are shown in Fig. 4).  
  - Positive sequence series line admittance = (0.9743 – j3.6568) pu/km
  - Positive-negative sequence series line coupling admittance = (0.1618 + j0.2766) pu/km
- Loads:
  - Passive load: 12 MVA, 6 MVA and 3 MVA 3-phase constant power loads (at bus 1, 2 and 3 respectively) with lagging power factors of 0.95, 0.75 and 0.85 in a, b and c phases respectively.
  - Induction motor load: A 2250 HP, 2.3 kV, 3-phase induction motor of which the equivalent circuit parameters are given in Appendix B.

![Fig. 4. 3-bus MV test system](image-url)

1) Case (a): Passive Loads Only: The 3-bus test system as shown in Fig. 4, supplying constant power loads is simulated using an unbalanced load flow program in MATLAB. Busbar VU factors and current unbalance factors of loads\(^{10}\) obtained using unbalanced load flow analysis were used to evaluate the VU emission decomposition outcomes in terms of complex unbalance factors for different busbars.

Table II - Case (a) presents total VU factors at different busbars obtained using both load flow analysis and the proposed methodology together with constituent emission contribu-
Simulation results | Proposed formulation results
---|---
Positive sequence voltage (U1) pu | VUF at kth bus (VUF_{k,LF})% | VUF at kth bus (VUF_{k,calc})% | Local load contribution (VUF_{load,k})% | Local line contribution (VUF_{line,k})% | Local busbar contribution (VUF_{d,source,k})% |
**Case (a) Unbalanced 3-phase constant power loads at buses 1, 2 and 3**
1 | 1.01∠−2.1 | 0.54∠−57.2 | 0.56∠−57.4 | 0.26∠−64.7 | 0.12∠−8.2 | 0.26∠−75.2 |
2 | 0.96∠−4.3 | 1.10∠−85.9 | 1.12∠−84.2 | 0.46∠−99.6 | 0.34∠179.2 | 0.76∠−73.1 |
3 | 0.97∠−3.7 | 0.93∠−92.3 | 0.94∠−81.4 | 0.13∠−63.8 | 0.09∠179.6 | 0.83∠−77.6 |
**Case (b) Unbalanced 3-phase constant power loads at buses 1, 2 and induction motor load at busbar 3**
1 | 1.02∠−92.1 | 0.41∠−57.5 | 0.47∠−57.5 | 0.24∠−64.1 | 0.12∠10.0 | 0.21∠−79.9 |
2 | 0.96∠−94.1 | 0.92∠−89.9 | 0.92∠−89.8 | 0.48∠−63.5 | 0.36∠179.2 | 0.51∠−73.3 |
3 | 0.98∠−93.4 | 0.60∠−89.3 | 0.65∠−89.3 | 0.00 | 0.10∠−173.4 | 0.64∠−80.4 |

The vital entries of Table II are:
- **Column 3 - VUF_{k,LF}** - total VUF at busbar 'k' obtained using the load flow
- **Column 4 - VUF_{k,calc}** - total VUF at busbar 'k' obtained using the proposed methodology
- **Column 5 - VUF_{load,k}** - component of VUF contributed by load asymmetry at busbar 'k'
- **Column 6 - VUF_{line,k}** - component of VUF contributed by line asymmetry at busbar 'k'
- **Column 7 - VUF_{d,source,k}** - component of VUF contributed by local busbar VU asymmetries at busbar 'k'

It can be seen that for a selected busbar, vector addition of VUF_{load,k}, VUF_{line,k} and VUF_{d,source,k} forms the resultant VU factor - VUF_{k,calc} which is approximately equal to the corresponding VU factor obtained using load flow (VUF_{k,LF}) thus verifying the proposed methodology.

The phasor representation of all VU emission outcomes for busbar 3 is further illustrated in Fig. 5-(a) where individual VU emission levels are indicated as a percentage next to the individual phasors. In the phasor diagram:
- VUF_{k,LF} - post-connection resultant VU emission at the POE (busbar 3) obtained using load flow analysis
- VUF_{k,calc} - post-connection resultant VU emission at the POE evaluated using the theoretical formulation
- VUF_{load,3} - VU emission at the POE caused by the load asymmetry evaluated using the theoretical formulation (application of (16))
- VUF_{line,3} (\(= \sum \text{VUF}_{\text{line,3-1}}\)) - total VU emission influence made by all asymmetrical lines connected to the POE (application of (17)) which is further expanded as VUF_{line,3-1} + VUF_{line,3-2} to represent the contributions made by single lines 3-1 (VUF_{line,3-1} = \(12.47\angle179^\circ\)) and 3-2 (VUF_{line,3-2} = 0.09∠−2^\circ).
- VUF_{d,source,3} (\(= \sum \text{VUF}_{\text{d,source,3-1}}\)) - total VU emission influence made by local busbars (application of (18)) on the POE which is also expanded in the phasor diagram as VUF_{d,source,3-1} + VUF_{d,source,3-2} to represent the individual contributions made by local busbars 1 (VUF_{d,source,3-1} = 0.25∠−56^\circ) and 2 (VUF_{d,source,3-1} = 0.6∠−87^\circ).

It is to be noted that the VUF_{k,calc} represents the reference phasor with a phase angle of zero degrees. Therefore, all other phasors are represented with respect to that reference phasor.

The phasor summation of all individual emission phasors (VUF_{load,3} + \(\sum \text{VUF}_{\text{line,3-1}}\) + \(\sum \text{VUF}_{\text{d,source,3-1}}\)) makes the resultant VUF obtained by the proposed methodology and the nearly coincident resultant VU emission phasors established through load flow and theoretical formulation demonstrate the validity of the mathematical formulation.

2) **Case (b): Induction Motor Load at Busbar 3:** The 3-bus test system was modelled in DigSILENT PowerFactory and in PSCAD/EMTDC (as a verification) simulation platforms with a 2.3 kV, 2250 HP 3-phase induction motor replacing the constant power load (3 MVA) at busbar 3. The induction motor was connected to the network via a 12.47/2.3 kV transformer (leakage reactance of 5% pu). The VU emission separation outcomes which were evaluated (application of (23) and (24)) using known network parameters, measured VU factors at busbars and measured current unbalance factors of passive loads at different busbars are given in the Table II - Case (b).

Similar to case (a), for busbars 1 and 2, individual emission contributions made by load, line and local busbar asymmetries (Columns 5, 6 and 7 respectively) constitute the resultant VUF as given in Column 4 which is approximately equal to the VUF obtained from simulations (in Column 3).

At busbar 3 where the 3-phase induction motor is connected, VUF_{load,3} is zero as a result of the symmetrical nature of the induction motor where the total emission is made up of VUF_{line,3} and VUF_{d,source,3}. All VU phasors associated with busbar 3 are shown in Fig. 5-(b). VUF_{line,3} is expanded in to two vectors (VUF_{line,3-1} and VUF_{line,3-2}) to illustrate the influence made by individual lines 3-1 (VUF_{line,3-1} = 0.19∠−179^\circ) and 3-2 (VUF_{line,3-2} = 0.09∠−2^\circ). Similarly vector addition of VUF_{d,source,3-1} + VUF_{d,source,3-2} makes the total local busbar contribution at busbar 3 (VUF_{d,source,3-1} = 0.25∠−58^\circ, VUF_{d,source,3-2} = 0.4∠−93^\circ). The resultant VU emission phasors obtained through the proposed formulation and through simulation are closely aligned, thus illustrating the validity of the proposed formulation. Furthermore, connection
of the induction motor at busbar 3 has attenuated the prevailing unbalance levels in the system compared to the case where a only passive load was connected at the same busbar.

Fig. 5. Phasor diagrams for Bus 3 VU emission vectors: (a). Bus 3 contains a 3 MVA constant power load, (b). Bus 3 contains a 2250 HP, 3 ph. induction motor load

B. Verification of the Methodology: IEEE 14 Bus Test System

The proposed methodology was applied to the IEEE 14 bus test system shown in Fig. 6 of which balanced voltage controlled busbars (1, 2, 3, 6 and 8) supply constant power loads. The 60 Hz, three wire system data used are as per [15] and [16]. All transmission lines are considered to be identical in configuration and of different lengths and untransposed.

- Positive sequence series line admittance = (0.2729 - j1.0244) × 10^2 pu/km
- Positive-negative sequence series line coupling admittance = (0.0453 + j0.0775) × 10^2 pu/km

The IEEE 14 bus test system was simulated using the unbalanced load flow program in MATLAB where two cases were considered: (a) balanced 3-phase loads, and (b) unbalanced 3-phase loads in which the load unbalance was set to approximately 10% in terms of current unbalance factor.

Similar to the case of the 3-bus MV test system, Table III presents the VU emission outcomes obtained through the proposed methodology and the load flow analysis for both cases. For a selected busbar, vector addition of individual emission contributions \( VUF_{load,k}, VUF_{line,k} \) and \( VUF_{d\text{source},k} \) forms the resultant VU factor \( VUF_{k,cal} \) (Column 4) which is approximately equal to the corresponding VU factor obtained using load flow \( VUF_{k,LF} \) (column 3) thus verifying the proposed methodology. For voltage controlled busbars (1, 2, 3, 6 and 8), resultant VUF is equal to zero.

Further, Fig. 7 shows the graphical representation of distribution of resultant VU factors (magnitudes and angles) at different busbars obtained from both the proposed methodology (labeled as “VUF-cal”) and the load flow analysis (labeled as “VUF-LF”). The resultant VU emission factors obtained through theoretical formulation and load flow analyses are in close agreement thus demonstrating the validity of the formulation. In Case (a), the influence made by the load on total unbalance is negligible since loads are balanced. But, the formulation gives a minute contribution for \( VUF_{load,k} \) due to the negative sequence current flowing in the network that contains asymmetrical lines. Resultant VU emission is mainly governed by the asymmetrical line contributions and hence \( VUF_{d\text{source},k} \) at different busbars reflect the effects of emissions associated with asymmetrical lines in the entire system. In Case (b), the increase in the net unbalance emission at busbars is due to the added effect of load asymmetries.

Fig. 6. IEEE 14 bus test system

Fig. 7. Magnitudes and angles of VU factors at different busbars: IEEE 14 bus test system, Case (a) - 3-phase balanced loads, Case (b) - 3-phase unbalanced loads
This paper has extended the concept of complex VU factor based formulation of post connection VU emission assessment developed for radial systems to interconnected power systems. The methodology utilises snapshot based post-connection steady state measurements together with known system parameters to determine the net unbalance at the POE. The resultant VU emission at a given busbar was established as a summation of decoupled emission contributions as recognised by line asymmetries, load asymmetries and local (neighbourhood) busbar voltage asymmetries. The contribution made by line asymmetries was characterised by the positive-negative sequence coupling admittance of a line. Load contribution was evaluated employing the current unbalance factor in the case of passive loads whereas in the case of 3-phase induction motors, the use of decoupled negative sequence admittances demonstrated the VU compensation effect.

The deterministic methodology of VU emission assessment as described in this paper can be applied in practical environments by using modern power quality instrumentation. GPS synchronised power quality instruments that are programmed with the proposed algorithm can take snapshot based measurements (voltage, current) and by using system data and system status that are being continuously updated (e.g. changes in network configurations), VU emission assessment can be carried out in real time. Such a scheme will allow network service providers to automate the VU emission assessment process. Further, the work presented in this paper can be considered as a basis for enhancing the VU management process.

V. CONCLUSIONS

TABLE III

<table>
<thead>
<tr>
<th>Bus</th>
<th>Simulation results</th>
<th>Proposed formulation results</th>
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<tbody>
<tr>
<td></td>
<td>VUF at kth bus</td>
<td>VUF at kth bus</td>
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<tr>
<td>#</td>
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<td>$V_{U_{k}}$</td>
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<td>13</td>
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APPENDIX A

VU EMISSION CONTRIBUTION DUE TO LOAD ASYMMETRIES: PROOF (16)

Based on the expressions for load currents at busbar 3 (i.e. (7) and (8)), the current unbalance factor at busbar 3 ($C_{UF3} = \frac{I_{2,3}}{I_{1,3}}$) can be derived as:

$$I_{2,3} = \frac{Y_{21,1,3} + Y_{22,1,3}}{Y_{11,1,3}} U_{2,3}$$

(A.1)
For passive loads, positive sequence load admittance and negative sequence load admittance are equal (i.e., \(Y_{11:L3} = Y_{22:L3}\)). Therefore, (A.1) can be simplified as:

\[
\frac{Y_{21:L3}}{Y_{11:L3}} = CUF_3 - VUF_3 \quad (A.2)
\]

Thus the VU emission due to load asymmetries can be modified as:

\[
\frac{Y_{21:L3}}{Y_{22:33}} = \left( CUF_3 - VUF_3 \right) \frac{Y_{11:L3}}{Y_{22:33}} \quad (A.3)
\]

In (A.3), the term \(Y_{11:L3}\) can be expressed in terms of nodal bus admittances and normalised positive sequence voltage drop of lines connected to the busbar 3 by equating positive sequence currents of busbar 3 given in (5) and (7).

\[
Y_{11:L3} = Y_{11:31}V_{\text{drop-31}} + Y_{11:32}V_{\text{drop-32}} \quad (A.4)
\]

Thus, the VU emission due to load asymmetries is given by;

\[
\frac{Y_{21:L3}}{Y_{22:33}} = \left( CUF_3 - VUF_3 \right) \frac{Y_{11:L3}}{Y_{22:33}} \quad (A.5)
\]

**APPENDIX B**

**INDUCTION MOTOR PARAMETERS**

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>60 Hz, 4-Pole Induction Motor Parameters Adopted from [17]</th>
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<tbody>
<tr>
<td>Power rating</td>
<td>2250 (HP)</td>
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<tr>
<td>Line Voltage</td>
<td>2300 (V)</td>
</tr>
<tr>
<td>Motor speed</td>
<td>1786 (rpm)</td>
</tr>
<tr>
<td>(r_s)</td>
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<td>(X_{ls})</td>
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<td>(X_M)</td>
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<td>(r_L)</td>
<td>0.022(Ω)</td>
</tr>
<tr>
<td>(J)</td>
<td>63.87 (kg m²)</td>
</tr>
</tbody>
</table>

**REFERENCES**


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