

# Allotment in First-Price Auctions: An Experimental Investigation

Luca Corazzini\*, Stefano Galavotti†, Rupert Sausgruber‡, Paola Valbonesi§

## Abstract

We experimentally study the effects of allotment – the division of an item into homogeneous units – in independent private value auctions. We compare a single-item, first-price auction with two equivalent treatments with allotment: a two-unit discriminatory auction and two simultaneous single-unit first-price auctions. We find that allotment mitigates overbidding, with this effect being stronger in the discriminatory auction. In the allotment treatments, we observe large and persistent bid spreading. Across treatments, the discriminatory auction is the least efficient and generates the lowest revenue.

**JEL classification:** H57, D44.

**Keywords:** Allotment, multi-unit auction, discriminatory auction, first-price auction, laboratory experiment.

---

\*Department of Economics and Management, University of Padova, Via del Santo 33, 35123 Padova, Italy.  
Email: luca.corazzini@unipd.it.

†Department of Economics and Management, University of Padova, Via del Santo 33, 35123 Padova, Italy.  
Email: stefano.galavotti@unipd.it.

‡**Corresponding author.** Department of Public Economics, University of Innsbruck, Universitätsstraße 15, 6020 Innsbruck, Austria. Phone: +43 512 507 7176; Fax: +43 512 507 2970. Email: rupert.sausgruber@uibk.ac.at.

§Department of Economics and Management, University of Padova, Via del Santo 33, 35123 Padova, Italy.  
Email: paola.valbonesi@unipd.it.

# 1 Introduction

In settings where an auctioneer wants to allocate a certain quantity of a divisible good to a number of bidders, a fundamental question is whether to sell the entire quantity as a single bundle or to allot it into several distinct units. The existing economic literature dealing with multi-unit auctions has highlighted the importance of this issue as bidders may strategically respond to the presence of multiple units. For instance, the value a bidder attaches to a specific unit (and thus her bid) might depend on the number of units already acquired (or expected to be acquired) in the auction (see, among others, Engelbrecht-Wiggans and Kahn, 1998a and 1998b; Ausubel and Cramton, 2002). Also, the presence of multiple units might foster entry by other (small) bidders and thus alter the degree of competition and affect behavior by incumbents (see, e.g., Goeree et al., 2013). In deriving these results, the literature takes as given that, when the number of bidders is fixed, units are homogenous and do not display any complementarity or substitutability in value, the allotment of a divisible good would not affect bids, efficiency and revenue.

In this paper, we propose a novel experimental design to illustrate that the decision to bundle or allot the quantity of a divisible good may play a greater than hitherto expected role. We show that moving from a single-unit to an “equivalent” multiple-unit auction may cause a reaction by bidders not predicted by standard theory. In particular, we shed light on how allotment affects bidding (and overbidding), efficiency and revenue.

To isolate these effects, we experimentally compare “pay-as-bid” auctions<sup>1</sup> with and without allotment, adopting the simplest possible setting in which allotment should have no impact on the strategic incentives of bidders. The units traded in the auctions are homogenous and do not display any complementarity or substitutability, as then comparing single-unit against multi-unit auctions would have to account for complex effects on bidders’ incentives. We also keep constant the number of bidders and the distribution of private valuations to rule out the possibility that allotment affects the degree of competition or the thickness of the demand.

The experimental design includes three treatments. The first treatment consists of a standard single-unit, first-price, independent private value auction and represents our benchmark. In the second treatment, bidders compete in a two-unit discriminatory auction: each bidder places two bids, the two highest bids are deemed winning and the bidder(s) who placed the winning bids is (are) assigned the units and pays (pay) the amount of her (their) winning bid(s). Finally, in the third treatment, bidders participate in two simultaneous first-price auctions, each involving a single unit.<sup>2</sup>

Our experimental results can be summarized as follows. First, in all treatments we observe overbidding – bids above the theoretical risk neutral Nash equilibrium –, with bids being more aggressive in the benchmark than in the allotment treatments. Second, we detect bid spreading – different bids for the two units – in the allotment formats. Third, in all treatments, but especially in the discriminatory auction, a considerable number of units are inefficiently

---

<sup>1</sup>Pay-as-bid auctions are auctions in which the winner(s) pays (pay) what she (they) bid. In a single-unit context, the pay-as-bid auction corresponds to the first-price auction, while in a multi-unit context it corresponds to the discriminatory auction.

<sup>2</sup>Although our experimental design is highly stylized and not meant to reproduce particular real-world auctions, it is easy to refer it to adopted practices. For instance, the allotment design in the second treatment is commonly endorsed in auctions involving financial assets, tradeable (emission) permits, electricity quotas, and raw materials. On the other hand, the allotment design in our third treatment is frequently observed on Ebay and other online platforms where private or public operators manage simultaneous auctions for identical products.

assigned to the subjects with the lowest valuation. Fourth, while no significant differences in bidders' surplus are detected across treatments, we find that the auctioneer's revenue is lower in the discriminatory auction than in the other two treatments.

Overall, our results complement previous findings which question the validity of the standard assumption of risk neutrality to account for bidders' behavior. In line with recent advancements in behavioral economics, we propose risk aversion and "joy of winning" as possible interpretations of our experimental evidence. We show that these behavioral hypotheses successfully account for bid spreading, as well as for the observed differences across treatments.

The rest of the paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 presents the experimental design and the theoretical predictions under the standard assumption of risk neutral bidders. Section 4 reports the results of our experiment. Section 5 discusses risk aversion and joy of winning as potential explanations for our results. Finally, Section 6 concludes with some policy implications from our experimental findings.

## 2 Literature review

Although to date the theoretical and experimental literature on auctions is vast, a thorough understanding of the behavioral effects of allotment is still missing. That selling many objects together rather than separately can generate relevant effects has been clear since the work of Palfrey (1983). The author theoretically shows that, with two bidders and under a second-price rule, allocating different goods as a single bundle is revenue superior (but inferior in terms of total welfare) to selling them through parallel auctions.<sup>3</sup>

Recently, Popkowski Leszczyc and Häubl (2010) have empirically investigated into the research question advanced by Palfrey (1983). They report results from a field experiment based on a second-price rule: they show that when there is complementarity between goods, a bundle auction generates higher revenue than two parallel auctions for the components of the bundle; however, the opposite occurs when the goods are substitute. There are three main differences between these contributions and our paper. First, we do not introduce any complementarity/substitutability between goods; rather, we assume that the two goods are identical and that each bidder's valuation is the same for both units. This allows us to study the effects that allotment exerts on bidders' behavior. Second, these papers do not consider multi-unit auctions in which bidders place multiple bids, one for each item involved. Finally, despite its practical relevance, these two papers do not consider the pay-as-bid pricing rule.

Most of the economic interest in selling mechanisms involving multiple goods has concentrated on multi-unit auctions, investigating the properties of the standard formats (uniform-price, discriminatory and Vickrey and their open outcry counterparts). In particular, the theory on discriminatory auctions (see Engelbrecht-Wiggans and Kahn, 1998a) has shown that, when the units have decreasing marginal valuations, (i) bidders shade all bids and (ii) the difference between two bids placed by a bidder tends to be smaller than the difference between the corresponding valuations.

From the experimental point of view, economists have mainly focused on uniform-price and Vickrey auctions, while less attention has been devoted to the discriminatory format.<sup>4</sup>

---

<sup>3</sup>The work of Palfrey has been subsequently extended by Chakraborty (1999).

<sup>4</sup>For a detailed survey on single-unit and multi-unit experimental auctions, see Kagel (1995) and Kagel and

Some relevant exceptions are: Goeree et al. (2013), who study preemptive bidding in discriminatory and ascending auctions in which entry has a negative externality on incumbent bidders; Goswami et al. (1996) and Sade et al. (2006), who investigate cooperative behavior in discriminatory and uniform-price common value auctions with pre-play communication; Engelmann and Grimm (2009), who analyze bidders' behavior and efficiency in five different multi-unit auction formats: discriminatory, uniform-price sealed-bid, uniform-price open, Vickrey and Ausubel auctions. This last paper is the most related to ours, as it considers the same setting: two symmetric bidders having flat demand for two homogeneous units, with independent private values. In their discriminatory auction, in contrast to theoretical predictions under the (traditional) assumption of risk neutral bidders, Engelmann and Grimm (2009) find a strong evidence of overbidding and bid spreading. In a companion paper (Grimm and Engelmann, 2005), they advance risk aversion and joy of winning as possible explanations for this puzzling evidence, finding empirical arguments that suggest the superiority of the latter behavioral artifact.<sup>5</sup>

We depart from the study by Engelmann and Grimm (2009) as we are interested in assessing the effects of allotment on bids and efficiency in alternative pay-as-bid auction formats. To this end, we compare results from the discriminatory auction with an equivalent (in value) single-unit first-price auction. Moreover, in order to check whether results depend on the specific form of allotment, we also consider an additional treatment in which bidders participate in two identical and simultaneous single-unit first-price auctions.

Finally, our paper also relates to the literature on combinatorial auctions in which buyers can submit bids on either single units or packages. Combinatorial auctions are useful allocation mechanisms when units are characterized by the presence of synergies or when, because of their budget capacity and dimension, there are bidders interested in subsets of goods only (Cramton et al., 2006). In these settings, if the items are allocated separately, while there is an incentive to place high bids to get the extra value associated with the synergy, a bidder may refrain from aggressive bidding in order to avoid exposure to losses in case she wins only a limited number of units. This phenomenon, called the exposure problem, may negatively affect the efficiency and revenue of the auction (Kagel and Levin, 2005; Katok and Roth, 2004). A combinatorial auction reduces the risk of exposure and allows for more efficient allocations, as recently shown by Chernomaz and Levin (2012).<sup>6</sup>

We share with the literature on combinatorial auctions the attention to the effects of introducing the possibility of allotting items. However, the two approaches cannot be directly compared as they present relevant differences in the strategy profiles they endorse. In particular, in combinatorial auctions, package bids compete with single-item bids in the same auction; in our study, instead, we compare an auction in which only package bids are allowed with two formats in which bidders can exclusively place separate bids.

---

Levin (2011).

<sup>5</sup>The hypothesis of joy of winning is discussed in Section 5 below.

<sup>6</sup>In the setting considered by Chernomaz and Levin (2012), two local bidders (i.e., bidders interested in only one good) compete with one global bidder (i.e., a bidder interested in both goods and who can benefit from synergies) in two simultaneous (first-price) auctions and a combinatorial auction; their results highlight that if the value of the synergy is sufficiently high, the latter format can improve efficiency and revenue.

### 3 Experimental design and predictions

In order to assess how allotment affects bids and efficiency, we compare results from a benchmark treatment in which bidders compete for a single item with those observed in two equivalent (in terms of experimental features and potential payoff) auction formats with multiple and identical units. For each treatment, we ran three sessions. Each session included 15 periods and involved 18 participants, for a total of 162 subjects.

#### 3.1 Treatments

The benchmark treatment, *1A1U*, consists of a single-unit, first-price, independent private value auction with two bidders. Once pairs are formed, each bidder privately observes the value of an indivisible item. In each period, values are randomly and independently drawn from the same uniform distribution with support  $[0, 200]$ . All the characteristics of the distribution used to generate private values are common knowledge. After observing their values, bidders simultaneously place their bids. The bidder who places the highest bid wins the item and her earnings are given by the difference between her value and her bid. The other bidder earns nothing.

In the second treatment, *1A2U*, in every period the two bidders compete in a two-unit discriminatory auction. To each bidder, the two units have the same value, which, in each period, is randomly and independently drawn from a uniform distribution with support  $[0, 100]$ . This implies that the distribution (in terms of mean and upper and lower bounds) of the sum of the private values in *1A2U* coincides with that used in the benchmark, *1A1U*. The rules of the auction are as follows: given her value, each bidder places two bids. Once the four bids are collected, the two units are assigned to the bidder(s) who placed the two highest bids. For any unit acquired, the bidder earns an amount given by the difference between her value and the corresponding winning bid. As in *1A1U*, if a bidder does not obtain any unit, she earns nothing.

In the third treatment, *2A1U*, allotment is introduced by letting the two bidders participate in two identical and simultaneous first-price auctions, each involving a single unit. Apart from this aspect, all other experimental features are identical to those adopted in *1A2U*. Given her value, each bidder places two bids, one in each auction. In each of the two auctions, the highest bid wins the corresponding unit. Earnings are then computed as in *1A2U*.

#### 3.2 Procedures

Upon their arrival, subjects were randomly assigned to a computer terminal. In all sessions, instructions were distributed at the beginning of the experiment and read aloud.<sup>7</sup> Before the experiment started, subjects were asked to answer a number of control questions to make sure that they understood the instructions as well as the consequences of their choices. When necessary, answers to the questions were privately checked and explained. At the beginning of the experiment, the computer randomly formed three rematching groups of six subjects each. The composition of the rematching groups was kept constant throughout the session. At the beginning of every period, subjects were randomly and anonymously divided into pairs. Pairs were randomly formed in every period within rematching groups. Subjects were told

---

<sup>7</sup>Experimental instructions are included in Appendix C.

that pairs were randomly formed in such a way that they would never interact with the same opponent in two consecutive periods.<sup>8</sup>

In every period, subjects were presented with two consecutive screens. On the first screen, subjects were informed about their private value and were required to place their bids (one in *1A1U*, two in *1A2U* and *2A1U*). On the second screen, each subject was informed about the winning bid(s) as well as her payoff in that period. Payoffs were expressed in points and accumulated over periods. Subjects started the experiment with a balance of 300 points to cover the possibility of losses. At the end of the experiment, the number of points obtained by a subject during the experiment was converted at an exchange rate of 3 euro per 100 points and monetary earnings were paid in cash privately.

The experiment took place at the Experimental Laboratory of the University of Innsbruck in November 2011. Participants were mainly undergraduate students, recruited by using *ORSEE* (Greiner, 2004). The experiment was computerized using the *z-Tree* software (Fischbacher, 2007). On average, subjects earned 12.00 Euro in sessions lasting 45 minutes (including the time for instructions and payment). Before leaving the laboratory, subjects completed a short questionnaire containing questions on their socio-demographics and their perception of the experimental task.

### 3.3 Testable predictions: bids, spread and efficiency

In order to derive the testable predictions on bidding behavior and efficiency in the three treatments, we develop a simple theoretical framework based on the assumption of risk neutral bidders. Although a systematic violation of risk neutrality has been extensively documented by the economic literature, it still constitutes the standard assumption in models studying the equilibrium properties of different auction formats. Moreover, as discussed in the next pages, the assumption of risk neutrality provides two very intuitive predictions in our experiment, namely equivalence (in terms of bids, total efficiency and auctioneer's revenue) across auction formats, and identical bids in the allotment treatments, *1A2U* and *2A1U*.

In *1A1U*, the item auctioned off has a value  $V_i$  for bidder  $i$ . Values are private information, but it is commonly known that they are *i.i.d.* random variables with uniform distribution over the interval  $[0, 200]$ . After observing her own value, bidder  $i$  places her bid,  $a_i$ . The item is assigned to the bidder who places the highest bid and the winner pays her bid. Bidder  $i$ 's payoff is  $V_i - a_i$  if she wins the auction and zero otherwise.

In the allotment treatments, *1A2U* and *2A1U*, the two bidders compete for two units: in *1A2U*, the two units are sold in the same auction; in *2A1U*, they are sold separately, in two identical and simultaneous auctions. In both cases, the two units have the same value,  $v_i$ , to bidder  $i$ . Values  $v_1$  and  $v_2$  are *i.i.d.* random variables with uniform distribution over  $[0, 100]$ . Therefore, the sum of the values of the two units,  $2v_i$ , is uniformly distributed over the interval  $[0, 200]$ , just like  $V_i$  in *1A1U*.

In *1A2U*, bidder  $i$  places two bids,  $b_{i,1}$  and  $b_{i,2}$ , with  $b_{i,1} \geq b_{i,2}$  (i.e.  $b_{i,1}$  and  $b_{i,2}$  are the highest and lowest bids of bidder  $i$ , respectively). Four bids are thus collected within a pair, the two highest bids win and are paid by the corresponding bidder(s). Bidder  $i$ 's payoff is  $2v_i - b_{i,1} - b_{i,2}$  if she wins both units,  $v_i - b_{i,1}$  if she wins only one unit, and zero otherwise.

In *2A1U*, bidder  $i$  places one bid,  $c_{i,1}$ , in the first auction and one bid,  $c_{i,2}$ , in the second. In each auction, the highest bid wins the unit and the corresponding bidder pays it. If bidder

---

<sup>8</sup>The rematching protocol implemented in the experiment was intended to increase the number of independent observations per treatment.

$i$  wins both auctions, her payoff is thus  $2v_i - c_{i,1} - c_{i,2}$ ; if she wins only the first (respectively, second) auction, her payoff is  $v_i - c_{i,1}$  (respectively,  $v_i - c_{i,2}$ ); if she wins neither auction, her payoff is null.

Under the assumption of risk neutral bidders, each of the three treatments admit a unique symmetric (Bayes-)Nash equilibrium in pure strategies. Given the equilibria in the three treatments, we can state the following testable predictions (see Appendix B for all the proofs):

**H1. Bid equivalence.** *In all treatments, bids are equal to half the value assigned to that unit. Thus, for given private value, bids in 1A1U are equal to the sum of the two bids in 1A2U and 2A1U.*

**H2. Zero-spread.** *In 1A2U and 2A1U, bidders place two identical bids for the two units.*

**H3. Efficiency and revenue equivalence.**<sup>9</sup> *In all treatments, the units are allocated to the bidder with the highest private value. Thus, on average, the overall surplus is the same across treatments. Finally, in every treatment, the overall surplus is split equally between bidders and the auctioneer.*

## 4 Experimental results

We organize our results as follows. First, we compare treatments with respect to the extent of overbidding by looking at both the (sum of the) bids and the proportion of subjects that bid above the risk neutral ( $RN$ ) equilibrium level. Second, focusing on the two treatments with allotment,  $1A2U$  and  $2A1U$ , we study bid spreading. Third, we investigate differences across treatments in overall efficiency, bidders' surplus and auctioneer's revenue.

The non parametric tests discussed below are based on 27 independent observations (9 rematching groups per treatment). Similarly, the parametric models presented in the next subsections properly control for dependency of observations over repetitions by either clustering standard errors or introducing random effects at the rematching group level.

### 4.1 (Over-)bidding behavior

Figure 1 shows the bids over all periods in the three treatments.

[Figure 1 about here]

We observe overbidding in all our three treatments. In particular, the proportion of bids above the  $RN$  level is 89.14% in  $1A1U$ , 79.38% in  $1A2U$ , and 83.46% in  $2A1U$ . A (two-sided) Mann-Whitney rank-sum test rejects the null hypothesis that the average number of periods in which a subject overbids in  $1A1U$  is the same as that in the two treatments with allotments ( $z = 2.160$ ,  $p < 0.05$ ). The previous finding is mainly driven by the comparison between  $1A1U$

---

<sup>9</sup>Given that in the experiment bidders' earnings are observable while their true surplus is not, throughout the paper we will adopt the convention common in this literature of measuring a bidders' surplus with her monetary earnings (for each unit, the difference between the valuation and the price paid). The allocation will thus be considered efficient (i.e., it maximizes the sum of revenue and bidders' earnings) if and only if both units are allocated to the bidder with the highest valuation. Clearly, this definition of welfare is correct if bidders are risk-neutral, but it is not under different specifications of the utility function.

and  $1A2U$  ( $z = 2.075$ ,  $p < 0.05$ ), whereas the difference between the benchmark and  $2A1U$  is not significant ( $z = 1.634$ ,  $p = 0.102$ ).

*Table 1* reports the Probit marginal effect estimates for the probability of overbidding in the three treatments.

[*Table 1* about here]

As shown in column (4), allotment reduces the probability of overbidding. Indeed, the coefficient of  $1A2U \& 2A1U$  is negative and highly significant. Column (5) reveals that, compared with  $1A1U$ , the probability of overbidding in  $1A2U$  is significantly lower. The treatment effect remains negative, although non significant (according to a two-sided test) in  $2A1U$ . Finally, the difference between the coefficients of  $1A2U$  and  $2A1U$  is not significant ( $\chi^2(1) = 1.57$ ,  $p = 0.210$ ).

Moving to the magnitude of overbidding, *Table 2* reports results from parametric panel regressions to study the determinants of the (sum of the) bids in the three treatments.

[*Table 2* about here]

By looking at the first three columns of *Table 2*, the coefficient of *Value* is significantly greater than 0.5 in  $1A1U$  ( $\chi^2(1) = 422.73$ ,  $p < 0.01$ ), in  $1A2U$  ( $\chi^2(1) = 166.89$ ,  $p < 0.01$ ) and in  $2A1U$  ( $\chi^2(1) = 317.41$ ,  $p < 0.01$ ).<sup>10</sup> As indicated by the coefficient of  $1A2U \& 2A1U$  in column (4), after controlling for the private value and the linear time trend, allotment significantly decreases the (sum of the) bids. Moreover, when we replace  $1A2U \& 2A1U$  with two distinct treatment dummies, we find that both  $1A2U$  and  $2A1U$  have the same sign, although (according to a two-sided test) it is significant in  $1A2U$  only. The difference between the coefficients of  $1A2U$  and  $2A1U$  in column (5) is not significant ( $\chi^2(1) = 0.38$ ,  $p = 0.539$ ).<sup>11</sup> As a final observation, the (sum of the) bids significantly decrease(s) over periods as indicated by the coefficient of *Period*.<sup>12</sup> We summarize the previous findings as follows.

**R0. Overbidding.** *In all treatments, subjects bid above the RN equilibrium level.*

**R1. Less overbidding in the allotment treatments.** *Bids in  $1A1U$  are greater than the sum of the two bids in the allotment treatments. This effect is stronger when  $1A1U$  is compared with  $1A2U$ .*

## 4.2 Bid spread in $1A2U$ and $2A1U$

In contrast with what predicted under risk neutrality, we find that 84.32% and 75.31% of bids in  $1A2U$  and  $2A1U$  respectively associated with bid spread. *Figure 2* illustrates the difference between bids per value in  $1A2U$  and  $2A1U$ .

<sup>10</sup>This result is confirmed by a (two-sided) Wilcoxon signed-rank test, which rejects the null hypothesis that the (sum of the) bid(s) is equal to the *RN* level in the three treatments ( $z = 2.666$ ,  $p < 0.01$ ).

<sup>11</sup>In line with the previous results, a (two-sided) Mann-Whitney rank-sum test strongly rejects the null hypothesis that the mean of the bid(s) in  $1A1U$  is the same of that in the treatments with allotment ( $z = 2.160$ ,  $p = 0.031$ ). Again, the negative effect of allotment on bids is stronger when  $1A1U$  is compared to  $1A2U$  ( $z = 2.075$ ,  $p = 0.038$ ), rather than when it is compared to  $2A1U$  ( $z = 1.634$ ,  $p = 0.102$  for the two-sided test). We do not find any significant difference between  $1A2U$  and  $2A1U$  ( $z = -0.574$ ,  $p = 0.566$ ).

<sup>12</sup>The linear trend included in the regressions starts from 0 in the first period. This facilitates the interpretation of the intercepts included in the regressions: the coefficient of the constant term expresses the mean of the dependent variable in the first period; the coefficient of the treatment dummies records changes of the mean in the first period relative to the benchmark,  $1A1U$ .



[Figure 2 about here]

Over all periods, relative to the *RN* level, the bid spread is 30.33% in *1A2U* and 29.57% in *2A1U*. In both treatments, the relative spread is significantly greater than 0 according to a (two-sided) Wilcoxon signed-rank test (in both treatments,  $z = 2.667$ ,  $p < 0.01$ ). Moreover, in 29.88% and 30.00% of offers made in *1A2U* and *2A1U*, respectively, the relative spread is greater than 40%. Finally, as highlighted by the previous figure, the size of the bid spread is increasing in the assigned value.

In Table 3, we parametrically investigate the determinants of both the size and the probability of bid spreading in the two allotment treatments.

[Table 3 about here]

There is a positive and highly significant correlation between the size of bid spread and the private value in that period. Interestingly, as shown by the coefficient of *Period* in columns (1), (3) and (5), the size of the spread either remains stable or increases over periods. This confirms that rather than converging to the level predicted under the assumption of risk neutrality, subjects persistently and intentionally choose to place different bids. As indicated by the coefficient of *2A1U* in column (5), we do not observe significant differences in the size of the spread between *1A2U* and *2A1U*.<sup>13</sup> By looking at the probit marginal effect estimates in columns (2), (4) and (6), we find that the probability of bid spreading positively depends on the assigned value and increases over periods. Finally, column (6) shows that bid spreading is more likely to occur in *1A2U* than in *2A1U*.

**R2. Bid spread.** *In the allotment treatments, 1A2U and 2A1U, we observe a persistent and large-in-size bid spread. The size of bid spread is the same in the two treatments. The probability of bid spread is higher in 1A2U than in 2A1U.*

Given the results about overbidding and bid spread, in Table 4 we look at the size of both the highest and the lowest bids in the allotment treatments.

[Table 4 about here]

Both the highest and the lowest bids observed in *1A2U* and *2A1U* are associated with overbidding. The coefficient of *Value* in columns (1)-(4) is always significantly greater than 0.5 (for the highest bid:  $\chi^2(1) = 372.00$ ,  $p < 0.01$  in *1A2U*;  $\chi^2(1) = 610.64$ ,  $p < 0.01$  in *2A1U*; for the lowest bid:  $\chi^2(1) = 30.39$ ,  $p < 0.01$  in *1A2U*;  $\chi^2(1) = 77.52$ ,  $p < 0.01$  in *2A1U*). A (two-sided) Wilcoxon signed-rank test rejects the null hypothesis that the highest and lowest bid in both treatments are equal to the *RN* level (for the highest and lowest bids in *1A2U* as well the highest bid in *2A1U*:  $z = 2.667$ ,  $p < 0.01$ ; for the lowest bid in *2A1U*:  $z = 2.547$ ,  $p = 0.011$ ). Moreover, by looking at columns (5) and (6), we do not find any significant difference between treatments in both the highest and the lowest bids.

**R0'. Overbidding.** *In the allotment treatments, both bids are associated with overbidding.*

---

<sup>13</sup>This result is confirmed by non parametric tests: a (two-sided) Mann-Whitney rank-sum test does not reject the null hypothesis that the spread in *1A2U* is the same that in *2A1U* ( $z = 0.751$ ,  $p = 453$ ).

By looking at *Table 1* and *Table 3*, we find that the difference in overbidding across auction formats is mainly driven by the lowest (rather the highest) bids in the allotment treatments. Indeed, the coefficients attached to *Value* in the two regressions based on the highest bids in *1A2U* and *2A1U* (columns (1) and (3) of *Table 4*) are similar to those observed in *1A1U* (column (1) of *Table 2*). On the other hand, the coefficients of *Value* in the regressions based on the lowest bids in *1A2U* and *2A1U* (columns (2) and (4) of *Table 4*) are substantially smaller than those observed in *1A1U*.<sup>14</sup>

The finding that subjects in *1A2U* overbid with both their lowest and highest bids represents a remarkable difference with the results of Engelmann and Grimm (2009): in a discriminatory auction similar to our *1A2U*, they find that the lowest bid is, on average, below the *RN* level (i.e. underbidding). This discrepancy in results can be due to the different (re)matching protocol used in the two experiments. While the partner matching used in Engelmann and Grimm (2009) can induce subjects to collude over periods and thus to lower bids, the random rematching protocol implemented in our experiment makes collusion (virtually) impossible.

### 4.3 Efficiency and revenue

We now turn our attention to the level of efficiency achieved in the three treatments. The assumption of risk neutrality implies full allocative efficiency in all treatments: the unit(s) is (are) assigned to the subject with the highest private value.

To investigate this issue, for each pair and in every period, we construct three measures: (i) the *relative efficiency*, defined as the ratio between the achieved total welfare and the maximum possible welfare; (ii) the *relative auctioneer's revenue*, given by the ratio between the winning bid(s) and the maximum possible welfare; (iii) the *relative bidders' surplus*, corresponding to the ratio between the monetary payoff of the winning bidders and the maximum possible welfare. *Table 5* reports the estimates from GLS random effects models to compare relative efficiency, relative auctioneer's revenue, and relative bidders' surplus in the three treatments, computed by averaging at the rematching group level.

[*Table 5* about here]

As shown by the first two columns of *Table 5*, relative efficiency is significantly higher in *1A1U* than in the two allotment treatments. This result seems to be mainly driven by the larger efficiency loss recorded in *1A2U* relative to the other two treatments. By looking at column (2) of *Table 5*, we find a significant welfare loss in all treatments. The relative efficiency in *1A1U* (as measured by the constant term) is significantly smaller than 1 ( $\chi^2(1) = 28.43$ ,  $p < 0.01$ ). Similarly, the measures of relative efficiency in *1A2U* and *2A1U* – expressed by the linear combination of the constant term with the corresponding treatment dummy – are significantly lower than 1 (in *1A2U*:  $\chi^2(1) = 79.58$ ,  $p < 0.01$ ; in *2A1U*:  $\chi^2(1) = 35.95$ ,  $p < 0.01$ ). The loss of relative efficiency in all treatments is associated with allocative inefficiency: the percentage of winning bids placed by subjects with the lowest private value is 10.67% in

<sup>14</sup>We run two additional panel regressions (with clustered standard errors). The dependent variable in the first (second) regression is equal to the unique bid in *1A1U* and to the highest (lowest) bid in the two allotment treatments, *1A2U* and *2A1U*. As controls, both regressions include the allotment dummy, *1A2U&2A1U*, the value of one item and the time trend. The estimate of *1A2U&2A1U* is not significant in the first regression while it is negative and significant at the 1% level in the second regression.

1A1U, 29.85% in 1A2U, and 18.04% in 2A1U. Finally, the difference in the relative efficiency between 1A2U and 2A1U is highly significant ( $\chi^2(1) = 8.19, p < 0.01$ ).<sup>15</sup>

Moving to the relative auctioneer's revenue and focusing on columns (3) and (4) of Table 5, we do not find significant differences between 1A1U and the two allotment treatments. However, when we replace the allotment dummy, 1A2U&2A1U, with two separate treatment dummies, we find that 1A2U is associated with the lowest relative auctioneer's revenue. Indeed, the coefficient of 1A2U in column (4) is negative and highly significant and the difference between the estimates of 1A2U and 2A1U is highly significant ( $\chi^2(1) = 79.58, p < 0.01$ ).<sup>16</sup>

Focusing on columns (5) and (6) of Table 4, we find no significant differences in the relative bidders' surplus between treatments, either in the regressions (both the coefficients of 1A2U and 2A1U in column (6) as well as the estimate of 1A2U&2A1U are not significant; the difference between the coefficients of 1A2U and 2A1U in column (6) is not significant:  $\chi^2(1) = 0.00, p = 0.992$ ), or by using non parametric tests (according to a two-sided Mann-Whitney rank-sum test, the difference in the relative bidders' surplus between 1A1U and the treatments with allotment is not significant:  $z = -0.463, p = 0.643$ ; similarly, the difference between 1A2U and 2A1U is not significant:  $z = 0.044, p = 0.965$ ).

Finally, because of overbidding, in all treatments the relative auctioneer's revenue is greater than the relative bidders' surplus. Indeed, for each treatment, a (two-sided) Wilcoxon signed-rank test rejects the null hypothesis that the relative auctioneer's revenue is equal to the relative bidders' surplus ( $z = 2.667, p < 0.01$ ).

**R3. Differences in efficiency and revenue.** *In all treatments, we observe allocative inefficiency. Efficiency and revenue are higher in 1A1U and 2A1U than in 1A2U. Treatments do not significantly differ in terms of bidders' surplus. Finally, in all treatments the auctioneer's revenue is higher than bidders' surplus.*

In order to check for robustness of our main findings to repetition effects, we have replicated the parametric analysis on different subsets of 5 periods. Both the sign and the magnitude of the treatment dummies remain stable across subsets of periods.<sup>17</sup>

## 5 Discussion

Our findings question the validity of the assumption of risk neutrality to describe bidders' behavior at least for the following four considerations. First, in line with the existing experimental literature, we detect overbidding in all treatments. Second, allotment reduces overall bids, with this effect being more pronounced in the discriminatory auction, 1A2U. Third, relative to the other two treatments, 1A2U is associated with the lowest allocative efficiency and auctioneer's revenue. Finally, in the allotment treatments, 1A2U and 2A1U, we observe

<sup>15</sup>In line with these findings, a (two-sided) Mann-Whitney rank-sum test rejects the null hypothesis that the relative efficiency in 1A1U is the same as that in the two allotment treatments ( $z = 2.057, p < 0.05$ ). Similarly, the test detects a significant difference in relative efficiency between 1A2U and 2A1U ( $z = -2.693, p < 0.01$ ) as well as between 1A1U and 1A2U ( $z = 2.517, p < 0.05$ ). No significant differences are observed between 1A1U and 2A1U ( $z = 1.015, p = 0.310$ ).

<sup>16</sup>A (two-sided) Mann-Whitney rank-sum test confirms the previous result as it rejects the null hypothesis that the relative auctioneer's revenues in 1A1U and 1A2U are equal ( $z = 1.810, p = 0.070$ ).

<sup>17</sup>Results of these robustness check are available upon request.

large and persistent bid spreading. In order to provide an interpretation of our experimental results, we develop in this section two simple theoretical frameworks: one is based on the assumption of risk aversion, the other on joy of winning.

Bidders' risk aversion is probably the most invoked argument to account for the common phenomenon of overbidding in experimental auctions, and it is beyond question that it may constitute an important factor in explaining deviations from the theoretical  $RN$  equilibrium. However, the large body of experimental findings that has accumulated so far suggests that risk aversion alone is not sufficient.<sup>18</sup> Accordingly, several other models that rationalize observed behaviors in auctions have been proposed.<sup>19</sup>

One of the most popular of these alternative explanations is the joy of winning hypothesis, according to which bidders get a positive benefit from the mere fact of winning. As long as this extra-benefit is anticipated by bidders, it has an impact on their optimal bids. Joy of winning was first introduced by Cox et al. (1988) as an element that, added to risk aversion, may help improve the fitness to experimental data. Goeree et al. (2002), in their quantal response equilibrium model, and Grimm and Engelmann (2005), in the context of discriminatory auctions, have shown that joy of winning can be a valid alternative to risk aversion to track actual behavior of bidders. Recently, Roider and Schmitz (2012) show that joy of winning and its counterpart – disutility of losing – together are able to rationalize much of the existing experimental evidence on single-unit auctions. Joy of winning has also found increasing support from recent studies in neuroscience and psychology, according to which placing subjects into competitive environments can trigger a desire to win (see Delgado et al., 2008, and Malhotra, 2010, both adopting an auction setup).

That a desire to defeat opponents may affect individuals' behavior is indeed credible in small-stake competitions, as typically are laboratory auctions; it is less convincing for professionals participating in large-stake auctions, like those for the sale of high-valued assets (licenses, treasury bills) or for the procurement of works. However, once we realize that the essential feature of the joy of winning hypothesis is that there is a latent factor which, upon winning the auction, increases the actual payoff of a bidder beyond the simple difference between value and price, then we can imagine several latent factors that may have the above effect also in large-stake real world auctions: for example, winning a license or obtaining an important procurement contract may have a positive reputation effect on the firm, that can translate into higher profits in the future or in other markets in which the firm is operating. Also, it can increase the bargaining power of the firm or generating a learning by doing effect or increase the probability of accessing to future business opportunities or avoid the cost of excess capacity.

Similarly, while it is commonly accepted and widely documented that most individuals are indeed averse to risk in auction and non-auction settings, the economic literature has typically assumed that firms are risk neutral agents. However, there is evidence that even firms may display a risk averse behavior. This has been shown empirically by Athey and

---

<sup>18</sup>In the words of Kagel (1995, p. 525), "It is probably safe to say that risk aversion is one element, but far from the only element, generating bidding above the risk neutral Nash equilibrium". Kirchkamp et al. (2010), using a novel experimental design that manipulates bidders' risk preferences through the number of income-relevant auctions, show that risk preferences explain overbidding by about 50%.

<sup>19</sup>For example, Engelbrecht-Wiggans and Katok (2008, 2009) and Filiz-Ozbay and Ozbay (2007) argue that bidders overbid to prevent regret in case they do not win the auction. Armantier and Treich (2009) consider misperception of probabilities. Others have suggested different notions of equilibrium (e.g. Goeree et al., 2002) or non-equilibrium models (e.g. Crawford and Iriberri, 2007). For an overview of the competing explanations, see Kagel (1995) and Kagel and Levin (2011).

Levin (2001) in an auction setting and by Kawasaki and McMillan (1987), Asanuma and Kikutani (1992), Yun (1999) in subcontracting relations. In a recent experimental paper, List and Mason (2011) show that CEOs exhibit risk-aversion.

The two models are formalized as follows:

**Risk Aversion.** When bidders are risk averse, their utility function  $u(\cdot)$  is strictly concave.<sup>20</sup> In *1A1U*, bidder  $i$ 's payoff is  $u(v_i - a_i)$  if she wins the auction and zero otherwise. In *1A2U*, bidder  $i$ 's payoff is  $u(2v_i - b_{i,1} - b_{i,2})$  if she wins both units,  $u(v_i - b_{i,1})$  if she wins one unit only and zero otherwise. Finally, in *2A1U*, bidder  $i$ 's payoff is  $u(2v_i - c_{i,1} - c_{i,2})$  if she wins both auctions,  $u(v_i - c_{i,1})$  if she wins only the first auction,  $u(v_i - c_{i,2})$  if she wins only the second auction and zero otherwise.

**Joy of winning.** Following Grimm and Engelmann (2005), we assume that the joy of winning is captured by a multiplicative parameter,  $w > 1$ . In *1A1U*, while bidder  $i$ 's value for the item is  $V_i$ , her perceived value conditional on winning the auction is  $wV_i$ . Therefore, bidder  $i$ 's (perceived) payoff is  $wV_i - a_i$  if she wins the auction, and zero otherwise. In *1A2U* and *2A1U*, a bidder can win one or two units. We assume that, if a bidder wins both units, the second unit does not provide any additional source of utility.<sup>21</sup> Formally, in *1A2U*, bidder  $i$ 's (perceived) payoff is  $wv_i + v_i - b_{i,1} - b_{i,2}$  if she wins both units,  $wv_i - b_{i,1}$  if she wins (only) one unit, and zero otherwise. Similarly, in *2A1U* bidder  $i$ 's (perceived) payoff is  $wv_i + v_i - c_{i,1} - c_{i,2}$  if she wins both auctions,  $wv_i - c_{i,1}$  if she wins only the first auction,  $wv_i - c_{i,2}$  if she wins only the second auction, and zero otherwise.

The (Bayes-)Nash equilibria of the three auction formats under the hypotheses of risk aversion and joy of winning give rise to the following predictions (all the proofs can be found in Appendix B).

*h0. Overbidding. In all treatments, bidders bid above the RN equilibrium level. In the allotment treatments, both bids are associated with overbidding.*

Under risk aversion, bidders tend to bid more aggressively than under risk neutrality as they are willing to pay a higher price in order to increase the probability of winning.

Under joy of winning, bidders, anticipating that winning will provide an extra-utility, are willing to pay more for the item and thus bid more aggressively. Notice that, in the allotment treatments, overbidding involves also the second unit (even though no extra-utility is associated to it) because, since a bidder's second unit bid competes with the other bidder's first unit bid, and because first unit bids are now more aggressive, it is optimal to increase also the second unit bid.

*h1. Less overbidding in the allotment treatments. Under joy of winning, bids in 1A1U are greater than the sum of the two bids in the allotment treatments.*

By assumption, in *1A2U* and *2A1U*, the extra utility from winning is associated only with the first unit won. Therefore, in equilibrium, a bidder places one aggressive bid whose

<sup>20</sup>Apart from differentiability, we do not make any assumption on the particular form of the utility function. Hence, our results apply to any specific form of risk aversion. Moreover, they are also independent from the probability distribution of values.

<sup>21</sup>This assumption can be relaxed by attaching an extra utility also to the second unit won; as long as this extra utility is lower than the one associated with the first unit, the theoretical implications do not qualitatively change.

purpose is to obtain the (highly valued) first unit (which also incorporates joy of winning) and one moderate bid that is addressed to obtain the second unit (see *h2* below). Of course, bid spreading is not possible in *1A1U* and bidders have a unique opportunity to obtain the value of the item as well as the extra utility from winning. Thus, the unique bid in *1A1U* is greater than the sum of the two bids in the allotment treatments. A similar comparison is not possible under risk aversion as equilibrium bidding functions cannot be explicitly derived.

*h2. Bid spread. In the allotment treatments, 1A2U and 2A1U, bidders place two different bids for the two units. Bidding functions (and thus bid spread) are identical in 1A2U and 2A1U.*

Under risk aversion, bid spreading is due to the concavity of the utility function. Suppose a bidder makes two identical bids. Then it is always profitable to (slightly) increase the first bid and decrease the second: in so doing, the probability of winning the first unit will increase and the probability of winning the second unit will decrease, which is profitable as the marginal utility of the first unit is higher than that of the second. Intuitively, bid spreading increases the probability of winning only one unit; for a risk averse agent, this is preferable to making two identical bids which would lead to a high probability of either winning both units or none.

The joy of winning hypothesis implies that, in the allotment treatments, bidders perceive the first unit to be more valuable than the second. Therefore, they are willing to pay more for the first unit and thus bid more aggressively than on the second.

*h3. Differences in efficiency and revenue. In 1A1U, the item is efficiently allocated to the bidder with the highest private value. In the allotment treatments, instead, allocative efficiency is not guaranteed. In all treatments, the auctioneer's revenue is higher than bidders' surplus. Moreover, under joy of winning, the auctioneer's revenue is higher in 1A1U than in the allotment treatments.*

The difference in allocative efficiency between *1A1U* and the allotment treatments is due to bid spreading. Indeed, in *1A2U* and *2A1U*, it is possible that the highest bid placed by the bidder with the lowest private value exceeds the lowest bid of the bidder with the highest private value. Of course, this cannot occur in *1A1U* where the single item is always allocated to the bidder with the highest evaluation. The fact that the revenue is higher than bidders' surplus is implied by *h0*. Finally, the fact that the auctioneer's revenue is higher in *1A1U* than in the two treatments with allotment, *1A2U* and *2A1U*, is a direct consequence of *h1*. This last result holds under joy of winning, whereas no revenue ranking can be assessed under risk aversion.

The theoretical predictions *h0-h3* described above match most of the experimental results *R0-R3* presented in Section 4. There are only two results that are not aligned with the theoretical predictions: first, we observe higher allocative efficiency in *2A1U* than in *1A2U*; second, we observe a higher probability of bid spreading in *1A2U* than in *2A1U*. However, these two anomalies can be reasonably accomplished using behavioral arguments.

The difference in allocative efficiency could be a consequence of the structure of the equilibria in the two treatments. As shown in the Appendix B, *2A1U* admits two equilibria in pure strategies in which bidders must properly coordinate their bids in a way that, in each auction, the highest bid of one bidder competes with the lowest bid of the other bidder. Suppose that bidders have opposite expectations about which equilibrium will be played, so that

they mis-coordinate and both place their highest bid in the first auction and their lowest bid in the second. In this case, the bidder with the highest private value wins both auctions and allocative efficiency is achieved. Clearly, this coordination problem is not present in *1A2U*. Given the absence of communication among bidders and the random re-matching protocol implemented in our experiment, allocative efficiency is (expected to be) higher in *2A1U* than in *1A2U*.

The difference in the probability of bid spreading in the allotment treatments could be the result of the response of bidders to the coordination problem inherent in *2A1U* mentioned above. Consider a bidder in *2A1U* and suppose she is uncertain on whether her opponent will place a high bid in the first auction and a low bid in the second or viceversa. This uncertainty could then lead her to believe that the opponent is going to play a mixed strategy, in which the pure strategies (high bid in the first auction, low bid in the second) and (low bid in the first auction, high bid in the second) are played with equal probability. It can be shown that it might be optimal to respond to such belief by making two identical bids in the two auctions. Evidently, this cannot occur in *1A2U*. Hence, the difference in the probability of bid spreading could be due to the fact that, in *2A1U*, some bidders respond to the coordination problem by making two identical bids. Notice also that, when this happens, the probability of an efficient allocation increases with respect to the equilibrium case, so this is an additional argument to support the difference in allocative efficiency between the two treatments.

As a final step, we investigate the empirical relevance of the assumptions of risk aversion and joy of winning by combining subjects' bids with the information from the post-experimental questionnaire. In particular, we draw from two questions: asking subjects to self-report their risk attitude and inquiring into their perception of the joy of winning during the experiment.<sup>22</sup>

*Table 6* presents the proportions of subjects reporting either to be risk averse or to enjoy winning for different levels of overbidding in the different treatments. In particular, for each subject, we first run a regression with the (sum of the) bid(s) as the dependent variable and the (sum of the) private value(s) and the constant term as controls. Given the coefficient of *Value*, each subject is classified according to the extent of overbidding. Then, for each of the four categories, *Table 6* shows the proportions of subjects who reported to be risk averse or to enjoy winning.

[*Table 6* about here]

In *Table 6*, when we look at the pooled data, the proportions of subjects that reported either to be risk averse or to enjoy winning increase with overbidding. Next, in *Table 7*, we re-run the panel regressions in *Table 2* by adding the two dummies from the questionnaire, *RA* (for risk aversion) and *JoW* (for joy of winning).

[*Table 7* about here]

By looking at the regressions with pooled data (columns (4) and (5)), *RA* and *JoW* are both significant and have the expected sign. Robust evidence (although with different levels

---

<sup>22</sup>The first question focuses on risk aversion. Subjects are asked to report on a 7-point scale (with 1 indicating risk aversion and 7 risk seeking) whether, in general terms, they are willing to take or to avoid risks. The second question refers to the joy of winning. Subjects are asked whether they agree (on the basis of a 7-point scale, with 1 indicating strong disagreement and 7 strong agreement) with the statement that in a generic period of the experiment, winning (at least one of) the unit(s) was very important to them, regardless of the corresponding monetary payoff.

of significance) is found when we analyze each treatment separately (columns (1), (2) and (3)). Hence, subjects reporting to be either risk averse or to enjoy winning tend to place, on average, higher bids. Interestingly, relative to the results in *Table 2*, adding *RA* and *JoW* weakens the effect of allotment, as indicated by the lower significance of *1A2U* & *2A1U*.

## 6 Conclusion

Although economists and practitioners have long recognized the pro-competitive effects of allotment in auctions,<sup>23</sup> how bidders actually respond to the presence of multiple units remains an open question.

This paper reports results from a laboratory experiment designed to empirically identify the effects of allotment – moving from a single-unit to multiple-unit auctions – on bidders’ behavior, efficiency and revenue. To this end, we compare three treatments: a first-price auction in which two units are sold as a single bundle; a two-unit discriminatory auction; two simultaneous first-price single-unit auctions. The experiment is designed so that the standard theory of risk-neutral bidders gives the same equilibrium predictions in the three treatments: allotment (and the allotment format chosen) should play no role.

Our main result is, however, that allotment significantly mitigates overbidding, with this effect being more evident in the discriminatory auction. Furthermore, in both allotment treatments, we find a persistent tendency of subjects to place different bids for the two identical units auctioned. Bid spread and differences in overbidding across treatments have important consequences on efficiency and revenue associated with the auction formats. In particular, we detect a substantial loss in efficiency and revenue in the discriminatory auction relative to the other two treatments.

Our experimental results strongly reject the prediction of perfect equivalence of the three treatments implied by the standard assumption of risk neutral bidders. Although discriminating between alternative explanations is beyond the scope of the present study, we have leaned on two popular behavioral hypothesis proposed in experimental auctions: risk aversion and joy of winning. Overall, the predictions derived under these hypothesis successfully rationalize our experimental evidence.

With the usual caveats about generalizing results from laboratory experiments to complex real world situations, our study offers two important policy recommendations. First, in choosing the auction format to implement, the auctioneer should seriously take into account that this choice may have a greater than expected impact on bidders’ behavior and, consequently, on his revenue. As our experimental results reveal, allotment either reduces or leaves unchanged the auctioneer’s revenue. Combined with the additional direct (administrative, organizational and managerial) costs generally associated with allotment, the previous consideration questions whether allotment is an optimal strategy for a revenue maximizing

---

<sup>23</sup>For instance, the recent “Green Paper on the modernization of EU public procurement policy” (European Commission, 2011) points out that allotment can encourage small and medium enterprises (SMEs) to participate in public procurement auctions, whereby SMEs are considered of crucial importance for stimulating job creation, economic growth and innovation; for these reasons, since 2006, the French Public Procurement Code contains the mandatory principle that all the public contracts should be awarded in separate lots, unless the allotment shows a technical, economic or financial disadvantage. In a similar vein, in designing auctions to allocate *CO*<sub>2</sub> emission allowances, allotment is recommended to avoid limits in participation (see, e.g., the NYSERDA Report, 2007, p.35).



seller. Second, in those cases in which allotment is required to pursue general and important social goals, our study highlights the superiority of parallel auctions with respect to equivalent discriminatory auctions in enhancing efficiency and revenue.

## References

- [1] Armantier, O., Treich, N., 2009. Subjective probabilities in games: an application to the overbidding puzzle. *International Economic Review* 50, 1079-1102.
- [2] Asanuma, B., Kikutani, T., 1992. Risk absorption in Japanese subcontracting: a microeconomic study of the automobile industry. *Journal of the Japanese and International Economies* 6, 1-29.
- [3] Athey, S., Levin, J., 2001. Information and competition in U.S. forest service timber auctions. *Journal of Political Economy* 109, 375-417.
- [4] Ausbel, L.M., Cramton, P., 2002. Demand reduction and inefficiency in multi-unit auctions. University of Maryland, Working Paper No. 96-07.
- [5] Chakraborty, I., 1999. Bundling decisions for selling multiple objects. *Economic Theory* 13, 723-733.
- [6] Chernomaz, K., Levin, D., 2012. Efficiency and synergy in a multi-unit auction with and without package bidding: an experimental study. *Games and Economic Behavior* 76, 611-635.
- [7] Cox, J.C., Smith, V.L., Walker, J.M., 1988. Theory and individual behavior of first-price auctions. *Journal of Risk and Uncertainty* 1, 61-99.
- [8] Cramton, P., Shoham, Y., Steinberg, R., 2006. *Combinatorial Auctions*. Cambridge: MIT Press.
- [9] Crawford, V.P., Iriberri, N., 2007. Level-k auctions: can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions? *Econometrica* 75, 1721-1770.
- [10] Delgado, M.R., Schotter, A., Ozbay, E.Y., Phelps, E.A., 2008. Understanding overbidding: using the neural circuitry of reward to design economic auctions. *Science* 321, 1849-1852.
- [11] Engelbrecht-Wiggans, R., Kahn, C.M., 1998a. Multi-unit pay-your-bid auctions with variable awards. *Games and Economic Behavior* 23, 25-42.
- [12] Engelbrecht-Wiggans, R., Kahn, C.M., 1998b. Multi-unit auctions with uniform prices. *Economic Theory* 12, 227-258.
- [13] Engelbrecht-Wiggans, R., Katok, E., 2008. Regret and feedback information in first-price sealed-bid auctions. *Management Science* 54, 808-819.
- [14] Engelbrecht-Wiggans, R., Katok, E., 2009. A direct test of risk aversion and regret in first-price sealed-bid auctions. *Decision Analysis* 6, 75-86.

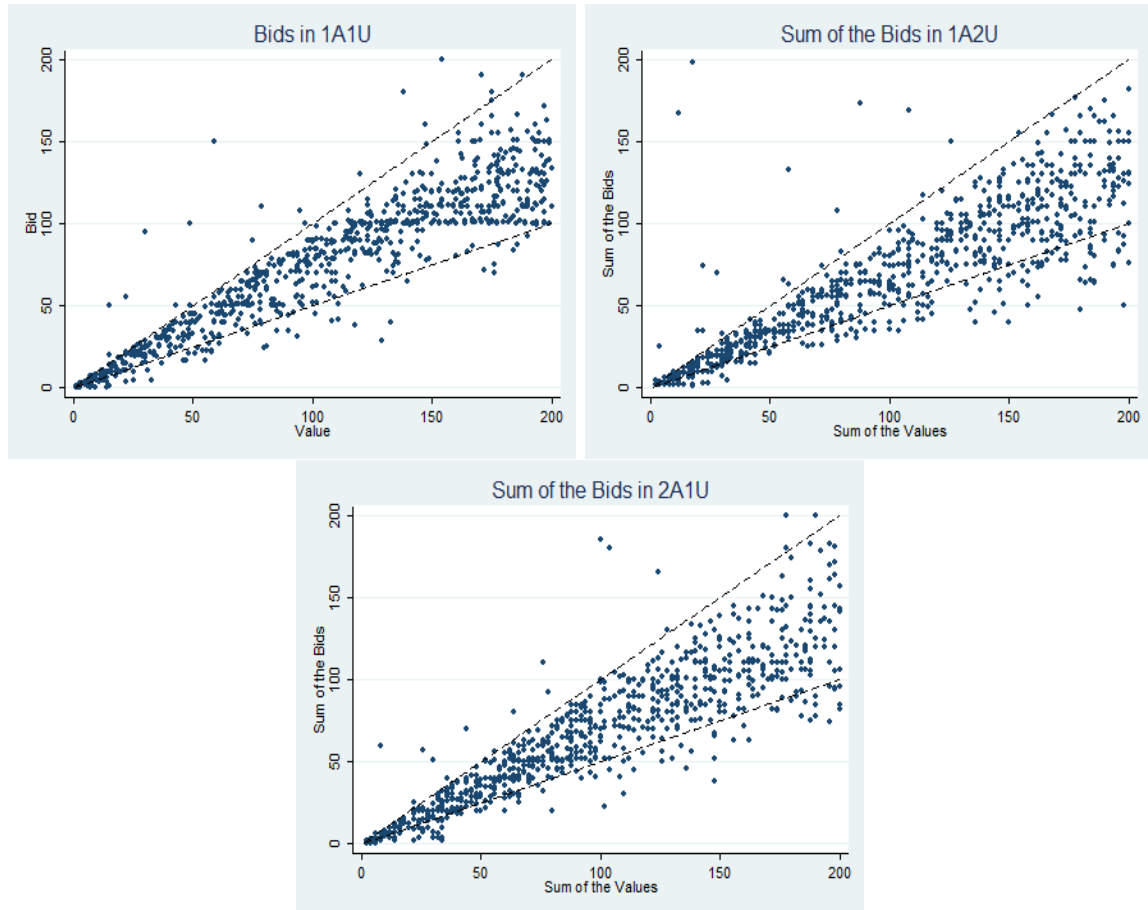
- [15] Engelmann, D., Grimm, V., 2009. Bidding behaviour in multi-unit auctions – an experimental investigation. *The Economic Journal* 119, 855-882.
- [16] European Commission, 2011. Green Paper on the modernisation of EU public procurement policy. Available at: <http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2011:0015:FIN:EN:PDF>.
- [17] Filiz-Ozbay, E., Ozbay, E.Y., 2007. Auctions with anticipated regret: theory and experiment. *The American Economic Review* 97, 1407-1418.
- [18] Fischbacher, U., 2007. Z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics* 10, 171-178.
- [19] Goeree, J.K., Holt, C.A., Palfrey, T.R., 2002. Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory* 104, 247-272.
- [20] Goeree, J.K., Offerman, T., Sloof, R., 2013. Demand reduction and preemptive bidding in multi-unit license auctions. *Experimental Economics* 16, 52-87.
- [21] Goswami, G., Noe, T.H., Rebello, M.J., 1996. Collusion in uniform-price auctions: experimental evidence and implication for treasury auctions. *Review of Financial Studies* 9, 757-785.
- [22] Greiner, B., 2004. An online recruitment system for economic experiments. In: Kremer, K., Macho, V. (Eds.). *Forschung und Wissenschaftliches Rechnen. GWDG Bericht 63*. Göttingen: Gesellschaft für Wissenschaftliche Datenverarbeitung, 79-93.
- [23] Grimm, V., Engelmann, D., 2005. Overbidding in first price private value auctions revisited: implications of a multi-unit auctions experiment. In: Schmidt, U., Traub, S. (Eds.). *Advances in Public Economics: Utility, Choice, and Welfare*. Dordrecht: Springer, 235-54.
- [24] Kagel, J.H., 1995. Auctions: a survey of experimental research. In: Kagel, J.H., Roth, A.E. (Eds.). *The Handbook of Experimental Economics*. Princeton, NJ: Princeton University Press, 501-585.
- [25] Kagel, J.H., Levin, D., 2005. Multi-unit demand auctions with synergies: behavior in sealed-bid versus ascending-bid uniform-price auctions. *Games and Economic Behavior* 53, 170-207.
- [26] Kagel, J.H., Levin, D., 2011. Auctions: a survey of experimental research, 1995-2010. to appear in *The Handbook of Experimental Economics*.
- [27] Katok, E., Roth, A.E., 2004. Auctions of homogeneous goods with increasing returns: experimental comparison of alternative “Dutch” auctions. *Management Science* 50, 1044-1063.
- [28] Kawasaki, S., McMillan, J., 1987. The design of contracts: evidence from Japanese subcontracting. *Journal of the Japanese and International Economies* 1, 327-349.
- [29] Kirchkamp, O., Reiss, J.P., Sadrieh, A., 2010. A pure variation of risk in private-value auctions. Mimeo. Available at: [http://www.kirchkamp.de/pdf/Risk2010\\_01-c.pdf](http://www.kirchkamp.de/pdf/Risk2010_01-c.pdf)

- [30] List, J.A., Mason, C.F., 2011. Are CEOs expected utility maximizers? *Journal of Econometrics* 162, 114-123.
- [31] Malhotra, D., 2010. The desire to win: the effects of competitive arousal on motivation and behavior. *Organizational Behavior and Human Decision Processes* 111, 139-146.
- [32] NYSERDA Report, 2007. Auction design for selling  $CO_2$  emission allowances under the regional greenhouse gas initiative. Available at: [http://www.rggi.org/docs/rggi\\_auction\\_final.pdf](http://www.rggi.org/docs/rggi_auction_final.pdf)
- [33] Palfrey, T.R., 1983. Bundling decisions by a multiproduct monopolist with incomplete information. *Econometrica* 51, 463-483.
- [34] Popkowski Leszczyc, P.T.L., Häubl, G., 2010. To bundle or not to bundle: determinants of the profitability of multi-item auctions. *Journal of Marketing* 74, 110-124.
- [35] Roider, A., Schmitz, P.W., 2012. Auctions with anticipated emotions: overbidding, underbidding, and optimal reserve prices. *The Scandinavian Journal of Economics* 114, 808-830.
- [36] Sade, O., Schnitzlein, C., Zender, J.F., 2006. Competition and cooperation in divisible good auctions: an experimental examination. *Review of Financial Studies* 19, 195-235.
- [37] Yun, M., 1999. Subcontracting relations in the Korean automotive industry: risk sharing and technological capability. *International Journal of Industrial Organization* 17, 81-108.

**Acknowledgements.** We thank Paolo Bertoletti, Maria Bigoni, Ottorino Chillemi, Christopher Cotton, Dirk Engelmann, Jose-Antonio Espin-Sanchez, Carlos Alós-Ferrer, Oscar Mitnik, Marco Pagnozzi, Alexander Rasch, Achim Wambach and participants at the PRIN Workshop in Capri, the XXVII Jornadas de Economía Industrial in Murcia, the Economics Department Seminar in Pavia, the 2012 ASSET Conference in Cyprus and the seminars at the Chaire EPPP- Paris 1, Pantheon Sorbonne, the "Design and Behavior" group of the University of Cologne, the PET 13 - International meeting of the Association for Public Economic Theory, for their comments. We are indebted to Giancarlo Spagnolo for very helpful discussions. Financial support from the University of Padova (University Project CPDA084881/08), the Italian Ministry of Education (PRIN No.20089PYFHY\_002) and the Austrian Science Fund (FWF - Project S 103070-G14) is gratefully acknowledged. All remaining errors are our own.

## Appendix

### A Figures and Tables



**Figure 1** – (Sum of the) bids in  $1A1U$ ,  $1A2U$  and  $2A1U$ .

Table 1. Probability of overbidding in 1A1U, 1A2U and 2A1U

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.001*** ( $1.8 \cdot 10^{-4}$ )	0.001 (0.001)	0.001*** ( $1.8 \cdot 10^{-4}$ )	0.001*** ( $2.2 \cdot 10^{-4}$ )	0.001*** ( $2.2 \cdot 10^{-4}$ )
<i>Period</i>	-0.002 (0.003)	$1.7 \cdot 10^{-4}$ (0.004)	-0.003 (0.002)	-0.002 (0.002)	-0.002
<i>1A2U&amp;2A1U</i>				-0.076*** (0.028)	
<i>1A2U</i>					-0.109*** (0.032)
<i>2A1U</i>					-0.062 (0.046)
<i>lpl</i>	-245.882	-407.405	-350.385	-1016.305	-1013.195
<i>Wald</i> – $\chi^2$	41.42	61.14	36.22	46.69	
<i>p</i> > $\chi^2$	0.000	0.232	0.000	0.000	0.000
<i>Obs.</i>	810	810	810	2430	2430

Notes. This table reports Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over all periods. Columns (1)-(3) consider the three treatments, separately. Regressions in columns (4) and (5) are based on pooled data. The dependent variable is a dummy that takes a value of 1 if the (sum of the) bid(s) of the subject in the period is associated with over-bidding. *Value* is the (sum of the) private value(s) assigned to the subject in the period. *Period* is a linear time trend that starts from 0 in the first period of the experiment. *1A2U* and *2A1U* are treatment dummies. *1A2U&2A1U* is a dummy that takes value of 1 if the treatment is either *1A2U* or *2A1U*. Estimates remain unchanged when *Period* is excluded from the regressions. Significance levels are denoted as follows: \* :  $p < 0.1$ ; \*\* :  $p < 0.05$ ; \*\*\* :  $p < 0.01$ .

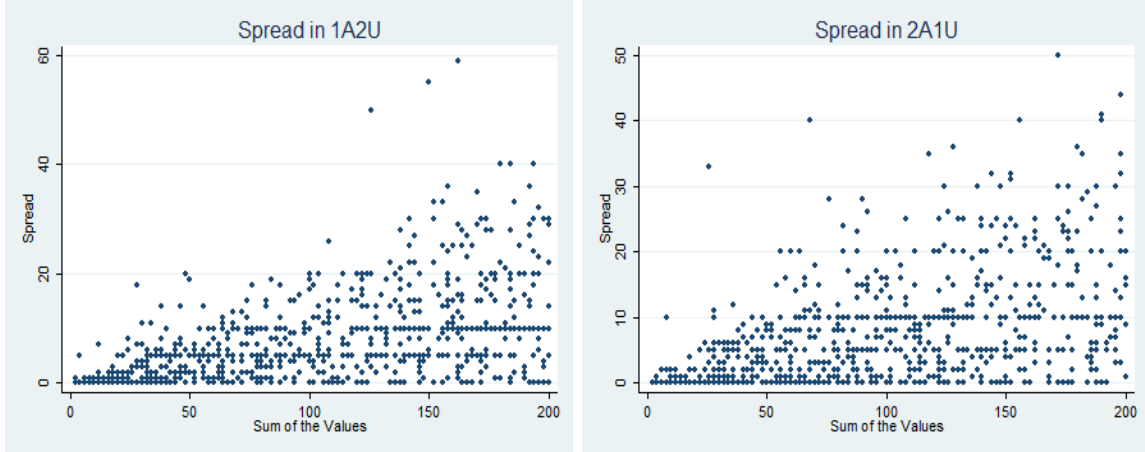


Figure 2 – Bid spread in 1A2U and 2A1U.

Table 2. (Sum of the) Bids in 1A1U, 1A2U and 2A1U

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.681*** (0.009)	0.633*** (0.010)	0.659*** (0.009)	0.658*** (0.005)	0.658*** (0.005)
<i>Period</i>	-0.494*** (0.117)	-0.640*** (0.135)	-0.800*** (0.111)	-0.649*** (0.070)	-0.649*** (0.070)
<i>1A2U &amp; 2A1U</i>				-4.367*** (2.120)	
<i>1A2U</i>					-5.128** (2.478)
<i>2A1U</i>					-3.606 (2.479)
<i>Constant</i>	7.689*** (2.248)	8.496*** (2.316)	8.423*** (2.180)	11.145*** (1.884)	11.141*** (1.904)
<i>lrl</i>	-3361.717	-3488.945	-3333.645	-10208.017	-10206.006
<i>Wald - <math>\chi^2</math></i>	6014.09	3784.23	5430.88	14722.16	14721.77
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	810	810	810	2430	2430

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the (the sum of the) bid(s) placed by the subject in the period. The other remarks of Table 1 apply.

Table 3. Bid spread in 1A2U and 2A1U

	1A2U		2A1U		Pooled	
	Size	Prob.	Size	Prob.	Size	Prob.
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Value</i>	0.071*** (0.004)	0.002*** ( $2.6 \cdot 10^{-4}$ )	0.068*** (0.004)	0.003*** ( $3.5 \cdot 10^{-4}$ )	0.070*** (0.003)	0.002*** ( $2.1 \cdot 10^{-4}$ )
<i>Period</i>	0.055*** (0.049)	0.007*** (0.003)	0.117*** (0.050)	0.008*** (0.003)	0.086*** (0.035)	0.007*** (0.002)
<i>2A1U</i>					−0.101 (0.794)	−0.084** (0.036)
<i>Constant</i>	0.096 (0.726)		−0.215 (0.853)		−0.012 (0.663)	
<i>lrl (lpl)</i>	−2653.633	−314.564	−2678.307	−401.947	−5327.147	−716.958
<i>Wald</i> − $\chi^2$	367.03	27.59	294.99	109.54	659.62	96.19
<i>p</i> > $\chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	810	810	810	810	2430	2430

Notes. Columns (1), (3) and (5) report coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in these regression is the absolute value of the difference of the two bids placed by the subject in the period. Columns (2), (4) and (5) report Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over all periods. The dependent variable is a dummy that takes a value of 1 if the subject places two different bids in the period. The other remarks of Table 1 apply.



Table 4. Highest and lowest bids in 1A2U and 2A1U

	1A2U		2A1U		Pooled	
	Highest	Lowest	Highest	Lowest	Highest	Lowest
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Value (1 unit)</i>	0.704*** (0.011)	0.562*** (0.011)	0.728*** (0.009)	0.592*** (0.010)	0.715*** (0.007)	0.576*** (0.008)
<i>Period</i>	-0.292*** (0.069)	-0.348*** (0.074)	-0.342*** (0.057)	-0.459*** (0.064)	-0.317*** (0.045)	-0.403*** (0.049)
<i>2A1U</i>					0.683 (1.206)	0.785 (1.401)
<i>Constant</i>	4.304*** (1.133)	4.195*** (1.289)	4.110*** (1.071)	4.311*** (1.309)	3.885*** (0.973)	3.887*** (1.116)
<i>lrl</i>	-2945.333	-3006.303	-2795.9459	-2891.461	-5752.116	-5903.973
<i>Wald - <math>\chi^2</math></i>	4444.91	2475.37	6213.57	3261.75	10314.65	5610.32
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	810	810	810	810	2430	2430

Notes. This table reports report coefficient estimates (standard errors in parentheses) from both two-way linear random effects model over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in columns (1), (3) and (5) is the highest bid of the subject in the period. The dependent variable in columns (2), (4) and (6) is the lowest bid of the subject in the period. *Value (1 unit)* refers to the private value assigned to one unit, only. All the other remarks of *Table 1* apply.

Table 5. Relative efficiency in 1A2U and 2A1U

	RE		RAR		RBS	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>1A2U&amp;2A1U</i>	-0.013** (0.006)		-0.038 (0.024)		0.015 (0.023)	
<i>1A2U</i>		-0.022*** (0.006)		-0.055** (0.027)		0.015 (0.027)
<i>2A1U</i>		-0.004 (0.006)		-0.021 (0.027)		0.015 (0.027)
<i>Period</i>	0.002*** (0.001)	0.002*** (0.001)	-0.008*** (0.001)	-0.008*** (0.001)	-0.009*** (0.001)	-0.009*** (0.001)
<i>Constant</i>	0.968*** (0.006)	0.968*** (0.006)	0.790*** (0.021)	0.790*** (0.021)	0.180*** (0.020)	0.180*** (0.021)
<i>Wald - <math>\chi^2</math></i>	12.70	22.17	56.83	58.45	60.13	60.11
<i>p &gt; <math>\chi^2</math></i>	0.002	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	405	405	405	405	405	405

Notes. This table reports coefficient estimates from GLS models that include random effects at the rematching group level. The dependent variable in columns (1) and (2), is the mean of the relative efficiency of the rematching group in the period. The dependent variable in columns (3) and (4), is the mean of the relative auctioneer's revenue of the rematching group in the period. The dependent variable in columns (5) and (6), is the mean of the relative bidders' surplus of the rematching group in the period. All the other remarks of *Table 1* apply.

Table 6. Risk aversion and joy of winning in explaining overbidding

	1A2U			1A2U			2A1U			Pooled		
	Obs.	RA	JoW	Obs.	RA	JoW	Obs.	RA	JoW	Obs.	RA	JoW
$bid/value \leq 0.5$	0	0	0	11	0.182	0.364	7	0.286	0	18	0.222	0.222
$0.5 < bid/value \leq 0.625$	16	0.375	0.313	13	0.385	0.154	13	0.385	0.077	42	0.381	0.191
$0.625 < bid/value \leq 0.75$	29	0.379	0.276	18	0.444	0.278	19	0.526	0.211	66	0.439	0.258
$0.75 < bid/value$	9	0.777	0.222	12	0.5	0.333	15	0.333	0.267	36	0.5	0.278

Notes. This table shows the proportions of subjects that report either to be risk averse or to agree with the joy of winning statement for different levels of overbidding. *RA* is a dummy that takes value 1 if the subject reports to be risk averse (1, 2, 3 in the first question). *JoW* takes value 1 if the subject agrees with the joy of winning statement (5, 6, 7 in the second question).

Table 7. Bids, risk aversion and joy of winning in 1A1U, 1A2U and 2A1U

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.681*** (0.009)	0.633*** (0.010)	0.660*** (0.009)	0.658*** (0.005)	0.658*** (0.005)
<i>RA</i>	4.835** (2.353)	6.217* (3.758)	1.055 (3.264)	4.695** (1.840)	4.664** (1.841)
<i>JoW</i>	2.467 (2.589)	4.530 (4.090)	10.847** (4.184)	4.825** (2.115)	5.015** (2.126)
<i>Period</i>	-0.493*** (0.117)	-0.640*** (0.135)	-0.800*** (0.111)	-0.649*** (0.070)	-0.649*** (0.070)
<i>1A2U&amp;2A1U</i>				-3.883* (2.050)	
<i>1A2U</i>					-4.870** (2.382)
<i>2A1U</i>					-2.878 (2.394)
<i>Constant</i>	4.870* (2.503)	4.857 (2.975)	6.214** (2.677)	7.747*** (2.987)	7.703*** (2.109)
<i>lrl</i>	-3355.838	-3482.530	-3325.905	-10199.444	-10197.308
<i>Wald - <math>\chi^2</math></i>	6020.51	3788.57	5442.95	14739.69	14739.89
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	810	810	810	2430	2430

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group. The other remarks of Table 1 and Table 5 apply.

## B Proofs of the predictions under risk neutrality, joy of winning and risk aversion

### B.1 Risk neutrality

Under risk neutrality, the symmetric equilibrium of  $1A1U$  is well known: each bidder bids according to  $a^{RN}(V) = \frac{1}{2}V$ .<sup>24</sup>

It is trivial to show that, in  $1A2U$  and in  $2A1U$ , every bidder makes two identical bids according to the same equilibrium bidding function used in a single-unit first-price auction, namely:<sup>25</sup>

$$b_1^{RN}(v) = b_2^{RN}(v) = c_1^{RN}(v) = c_2^{RN}(v) = \frac{1}{2}v.$$

### B.2 Joy of winning

We characterize equilibrium bids in the allotment treatments and their implication assuming that, if a bidder obtains both units, only the first provides the joy of winning  $w$ . It is rather straightforward to obtain equilibrium bids in the more general case in which also the second unit provides an extra-utility, say  $w_2$ . As long as  $w_2 < w$  (i.e., the joy of winning provided by the second unit won is lower than that associated to the first), all the predictions that follow still hold.

#### B.2.1 Equilibrium bids in $1A2U$

Consider one bidder with value  $v$  drawn from a uniform distribution over the interval  $[0, \bar{v}]$ . Let us denote by  $b_1$  and  $b_2$ , with  $b_1 \geq b_2$ , her highest and lowest bids, respectively. The expected payoff of this bidder is

$$\Pi(b_1, b_2; v) = (wv - b_1)F(b_2^{-1}(b_1)) + (v - b_2)F(b_1^{-1}(b_2))$$

where  $b_1(\cdot)$  and  $b_2(\cdot)$  are, respectively, the highest and the lowest bids of the opponent and  $F(z) = z/\bar{v}$ , for  $z \in [0, \bar{v}]$ .

If the optimal bids  $(b_1^*, b_2^*)$  are such that  $b_1^* > b_2^*$  (we will verify this condition in the sequel), then they must satisfy the following first order (necessary) conditions:

$$\begin{cases} (wv - b_1^*)(b_2^{-1}(b_1^*))' = b_2^{-1}(b_1^*), \\ (v - b_2^*)(b_1^{-1}(b_2^*))' = b_1^{-1}(b_2^*). \end{cases} \quad (\text{B.1})$$

In a symmetric equilibrium, it must be that  $b_1^* = b_1(v)$  and  $b_2^* = b_2(v)$ . The first order conditions for a symmetric equilibrium  $(b_1(v), b_2(v))$  are thus:

$$\begin{cases} (wv - b_1(v))(b_2^{-1}(b_1(v)))' = b_2^{-1}(b_1(v)), \\ (v - b_2(v))(b_1^{-1}(b_2(v)))' = b_1^{-1}(b_2(v)). \end{cases}$$

This system of differential equations, together with the boundary conditions<sup>26</sup>  $b_1(0) = b_2(0) = 0$  and  $b_1(\bar{v}) = b_2(\bar{v}) = \bar{b}$ , defines a symmetric equilibrium  $(b_1^{JoW}(v), b_2^{JoW}(v))$ .<sup>27</sup> It is

<sup>24</sup>For convenience, in the rest of this Appendix bidders' indexes are omitted, unless necessary.

<sup>25</sup>We do not provide a formal proof of this result. However, it can be easily derived from the model under the joy of winning hypothesis, by letting  $w$  tend to 1.

<sup>26</sup>The boundary conditions follow from a simple weak dominance argument.

<sup>27</sup>Provided that,  $b_1^{JoW}(v) > b_2^{JoW}(v)$ , for all  $v \in (0, \bar{v})$ .

convenient to write the system above in terms of inverse bidding functions. To this end, let us define  $\phi_1 = b_1^{-1}$  and  $\phi_2 = b_2^{-1}$ . The system of differential equations becomes:

$$\begin{cases} (w\phi_1(b) - b)\phi_2'(b) = \phi_2(b), \\ (\phi_2(b) - b)\phi_1'(b) = \phi_1(b). \end{cases}$$

Together with the boundary conditions  $\phi_1(0) = \phi_2(0) = 0$  and  $\phi_1(\bar{b}) = \phi_2(\bar{b}) = \bar{v}$ , the previous system defines a symmetric equilibrium. Using standard techniques, one obtains the following solutions in terms of inverse bidding functions:

$$\phi_1^{JoW}(b) = \frac{2b}{w(1 - Cb^2)}, \quad \phi_2^{JoW}(b) = \frac{2b}{1 + Cb^2},$$

where  $C = (w^2 - 1)/w^2\bar{v}^2$ . Going back to the direct bidding functions, the solution is

$$b_1^{JoW}(v) = \frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv}, \quad b_2^{JoW}(v) = \frac{1 - \sqrt{1 - Cv^2}}{Cv}.$$

Below, we will show that  $b_1^{JoW}(v) > b_2^{JoW}(v)$ , for all  $v \in (0, \bar{v})$ .

### B.2.2 Equilibrium bids in 2A1U

The expected payoff of a bidder with value  $v$  who bids  $c_1$  in the first auction and  $c_2$  in the second is:

$$\begin{aligned} \Pi(c_1, c_2; v) = & (wv - c_1)F(c_1^{-1}(c_1)) + (wv - c_2)F(c_2^{-1}(c_2)) \\ & - (w - 1)vF(\min(c_1^{-1}(c_1); c_2^{-1}(c_2))), \end{aligned}$$

where  $c_1(\cdot)$  and  $c_2(\cdot)$  are her opponent's bidding function in the first and the second auction, respectively.

We first show that a symmetric equilibrium in which both bidders bid  $c_1(v)$  in the first auction and  $c_2(v)$  in the second auction does not exist. More generally, we show that for a bidder it is never optimal to place two bids  $c_1$  and  $c_2$  such that  $c_1^{-1}(c_1) = c_2^{-1}(c_2)$ . Now, let  $c_1^*$  and  $c_2^*$  be the optimal bids for a bidder and suppose that  $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$ . The expected payoff is not differentiable at  $(c_1^*, c_2^*)$ . However, at that point, left and right partial derivatives are well defined. For  $(c_1^*, c_2^*)$  to be optimal, the left partial derivatives must be nondecreasing and the right partial derivatives must be non-increasing (otherwise, a small increase or decrease in bids would be profitable). This corresponds to require that

$$\begin{cases} (wv - c_1^*)(c_1^{-1}(c_1^*))' - c_1^{-1}(c_1^*) - (w - 1)v(c_1^{-1}(c_1^*))' \geq 0, \\ (wv - c_1^*)(c_1^{-1}(c_1^*))' - c_1^{-1}(c_1^*) \leq 0, \end{cases}$$

which cannot be satisfied simultaneously since the left hand side of the first equation is strictly lower than the left hand side of the second.

As a consequence, the optimal bids  $(c_1^*, c_2^*)$  must be such that  $c_1^{-1}(c_1^*) \neq c_2^{-1}(c_2^*)$ . Suppose, without loss of generality, that  $c_1^{-1}(c_1^*) > c_2^{-1}(c_2^*)$ . The expected payoff is now differentiable at  $(c_1^*, c_2^*)$ . Thus, the first order conditions are

$$\begin{cases} (wv - c_1^*)(c_1^{-1}(c_1^*))' = c_1^{-1}(c_1^*), \\ (v - c_2^*)(c_2^{-1}(c_2^*))' = c_2^{-1}(c_2^*). \end{cases}$$

Notice the perfect analogy with the first order conditions for optimal bids in  $1A2U$  - see system (B.1). If  $(b_1^*, b_2^*)$  are the highest and the lowest equilibrium bids in  $1A2U$  when the opponent's highest and lowest bidding functions are  $(b_1(\cdot), b_2(\cdot))$ , then, in  $2A1U$ ,  $b_1^*$  is the optimal bid in the first auction and  $b_2^*$  is the optimal bid in the second auction when the opponent bids according to  $b_2(\cdot)$  in the first auction and to  $b_1(\cdot)$  in the second auction. Formally, there is an equilibrium of  $2A1U$  in which bidding functions are:<sup>28</sup>

$$\begin{cases} c_{1,1}^{JoW}(v_1) = b_1^{JoW}(v_1), & c_{1,2}^{JoW}(v_1) = b_2^{JoW}(v_1), \\ c_{2,1}^{JoW}(v_2) = b_2^{JoW}(v_2), & c_{2,2}^{JoW}(v_2) = b_1^{JoW}(v_2). \end{cases}$$

In this equilibrium, bidder 1 respectively places the highest bid in the first auction and the lowest bid in the second, while bidder 2 makes the opposite. There is also a specular equilibrium in which bidder 1 respectively places the lowest bid in the first auction and the highest bid in the second, while bidder 2 makes the opposite:

$$\begin{cases} c_{1,1}^{JoW}(v_1) = b_2^{JoW}(v_1), & c_{1,2}^{JoW}(v_1) = b_1^{JoW}(v_1), \\ c_{2,1}^{JoW}(v_2) = b_1^{JoW}(v_2), & c_{2,2}^{JoW}(v_2) = b_2^{JoW}(v_2). \end{cases}$$

### B.2.3 Bid spread in $1A2U$ and $2A1U$

Given that the equilibrium bidding functions are the same in  $1A2U$  and  $2A1U$ , we only need to show bid spreading in  $1A2U$ , that is,  $b_1^{JoW}(v) > b_2^{JoW}(v)$ , for all  $v \in (0, \bar{v})$ , or

$$\frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv} > \frac{1 - \sqrt{1 - Cv^2}}{Cv}, \forall v \in (0, \bar{v}).$$

By multiplying both sides by  $Cwv$  and rearranging, we obtain  $\sqrt{1 + Cw^2v^2} + w\sqrt{1 - Cv^2} > w + 1$ . By simple algebra, this condition reduces to  $w^2 - 1 - Cw^2v^2 > 0$ , or  $(w^2 - 1)(1 - v^2/\bar{v}^2) > 0$ , which is satisfied for all  $v \in (0, \bar{v})$ .

### B.2.4 Overbidding

Showing that, in  $1A1U$ ,  $a^{JoW}(V) > a^{RN}(V)$  for all  $V \in (0, \bar{V}]$  is trivial.

To have overbidding in  $1A2U$  (and in  $2A1U$ ), it must be that  $b_2^{JoW}(v) > b_2^{RN}(v)$ , namely:

$$\frac{1 - \sqrt{1 - Cv^2}}{Cv} > \frac{1}{2}v, \forall v \in (0, \bar{v}].$$

By simple algebra, the previous condition reduces to  $C^2v^4 > 0$ , which is satisfied for all  $v \in (0, \bar{v}]$ .

### B.2.5 Efficiency in $1A1U$ , inefficiency in $1A2U$ and $2A1U$

Allocative efficiency in  $1A1U$  is implied by the symmetric nature of the equilibrium and the fact that equilibrium bidding functions are strictly increasing. As a consequence, the bidder with the highest private value places the highest bid and wins the auction. In both  $1A2U$  and  $2A1U$ , the bidder with the highest private values wins for sure one unit, as her highest bid is

<sup>28</sup>In  $2A1U$ , since there is an asymmetry in bids in the two auctions, we have to re-introduce bidders' indexes. Thus,  $c_{i,1}^{JoW}(v_i)$  and  $c_{i,2}^{JoW}(v_i)$  indicate the equilibrium bidding functions of bidder  $i$  in  $2A1U$ .

greater than both opponents' bids. However, there is a strictly positive probability that the bidder with the lowest private values wins one unit with her highest bid. Indeed, notice that an inefficient allocation of the second unit occurs if either  $v_1 > v_2$  but  $b_2^{JoW}(v_1) < b_1^{JoW}(v_2)$ , or  $v_1 < v_2$  but  $b_1^{JoW}(v_1) > b_2^{JoW}(v_2)$ . The ex-ante probability of an inefficient allocation of the second unit is the probability that one of these events occurs, namely:

$$\int_0^{\bar{v}} [F((b_2^{JoW})^{-1}(b_1^{JoW}(v_2))) - F((b_1^{JoW})^{-1}(b_2^{JoW}(v_2)))] f(v_2) dv_2.$$

The previous expression is strictly positive. Indeed, since  $b_2^{JoW}(\cdot) < b_1^{JoW}(\cdot)$ , we have  $(b_2^{JoW})^{-1}(\cdot) > (b_1^{JoW})^{-1}(\cdot)$  and  $(b_2^{JoW})^{-1}(b_1^{JoW}(\cdot)) > (b_1^{JoW})^{-1}(b_2^{JoW}(\cdot))$ .

### B.2.6 Bidders bid more in 1A1U than in 1A2U and 2A1U

We show that, for  $V = 2v$ ,  $a^{JoW}(V) > b_1^{JoW}(v) + b_2^{JoW}(v)$ , for all  $v \in (0, \bar{v}]$ . It is sufficient to show that  $a^{JoW}(V) > 2b_1^{JoW}(v)$ , namely:

$$\frac{1}{2}wV = wv > 2 \frac{\sqrt{1 + Cw^2v^2} - 1}{Cwv}.$$

By simple algebra, the previous condition reduces to  $C^2w^4v^4 > 0$ , which holds for all  $v \in (0, \bar{v}]$ .

### B.2.7 Auctioneer's revenue is higher in 1A1U than in 1A2U and 2A1U

This implication follows from the previous result. Indeed, suppose that bidders' private values are  $v'$  and  $v''$ , with  $v' \geq v''$ , in the allotment treatments, while they are  $V' = 2v'$  and  $V'' = 2v''$  in 1A1U. The auctioneer's revenue in 1A1U is simply  $a^{JoW}(V')$ . The auctioneer's revenue in 1A2U and 2A1U are either  $b_1^{JoW}(v') + b_2^{JoW}(v')$  if  $b_2^{JoW}(v') > b_1^{JoW}(v'')$  or  $b_1^{JoW}(v') + b_1^{JoW}(v'')$  if  $b_2^{JoW}(v') < b_1^{JoW}(v'')$ . In the former case, we have already shown that  $a^{JoW}(v') > b_1^{JoW}(v') + b_2^{JoW}(v')$ . In the latter case, since  $v' \geq v''$ , we have that  $2b_1^{JoW}(v') \geq b_1^{JoW}(v') + b_1^{JoW}(v'')$ . However, by previous results,  $a^{JoW}(V') > 2b_1^{JoW}(v')$ , which implies that  $a^{JoW}(V') > b_1^{JoW}(v') + b_1^{JoW}(v'')$ .

### B.2.8 0-spread can be rational in 2A1U but not in 1A2U

Suppose that, in 2A1U, bidder  $i$  believes that her opponent is going to play the following mixed strategy: with probability 1/2, she will play  $c_H(v)$  in the first auction and  $c_L(v)$  in the second; with probability 1/2, she will play  $c_L(v)$  in the first auction and  $c_H(v)$  in the second, where  $c_H(v) > c_L(v)$ , for all  $v \in (0, \bar{v})$ . Bidder  $i$ 's expected payoff would thus be:

$$\begin{aligned} \Pi(c_1, c_2; v) = & \frac{1}{2} \{ (wv - c_1) [F(c_H^{-1}(c_1)) + F(c_L^{-1}(c_1))] \\ & + (wv - c_2) [F(c_L^{-1}(c_2)) + F(c_H^{-1}(c_2))] \\ & - (w - 1)v [F(\min(c_H^{-1}(c_1); c_L^{-1}(c_2))) + F(\min(c_L^{-1}(c_1); c_H^{-1}(c_2)))] \} \end{aligned}$$

Suppose that the optimal bids  $(c_1^*, c_2^*)$  are such that  $c_H^{-1}(c_1^*) < c_L^{-1}(c_2^*)$ . Then, they must satisfy the following first order conditions:

$$\begin{cases} (wv - c_1^*)(c_L^{-1}(c_1^*))' + (v - c_1^*)(c_H^{-1}(c_1^*))' = c_H^{-1}(c_1^*) + c_L^{-1}(c_1^*), \\ (wv - c_2^*)(c_L^{-1}(c_2^*))' + (v - c_2^*)(c_H^{-1}(c_2^*))' = c_H^{-1}(c_2^*) + c_L^{-1}(c_2^*). \end{cases}$$

The two conditions above are clearly satisfied when  $c_1^* = c_2^*$ .

This argument shows that, if the opponent plays the above mixed strategy, then it is locally optimal to place two identical bids in the two auctions. Whether this is also globally optimal depends on the exact shape of  $c_H(\cdot)$  and  $c_L(\cdot)$ . It can be shown that, for reasonable beliefs on  $c_H(\cdot)$  and  $c_L(\cdot)$ , this is indeed the case.

Clearly, in  $1A2U$ , such a mixed strategy is meaningless: a mixed strategy in which the two bids are  $c_H(v)$  and  $c_L(v)$  with probability  $1/2$  and  $c_L(v)$  and  $c_H(v)$  with probability  $1/2$  is simply a pure strategy in which the two bids with  $c_H(v)$  and  $c_L(v)$  are played with probability one, so in this case placing two identical bids cannot be optimal.

### B.3 Risk Aversion

#### B.3.1 Equilibrium bids in $1A2U$

We characterize a symmetric equilibrium in differentiable, strictly increasing strategies. Using the same notation as before, in  $1A2U$ , the expected payoff of a bidder with value  $v$  is:

$$\Pi(b_1, b_2; v) = u(2v - b_1 - b_2)F(b_1^{-1}(b_2)) + u(v - b_1)[F(b_2^{-1}(b_1)) - F(b_1^{-1}(b_2))],$$

where  $u(\cdot)$  is a strictly increasing and strictly concave function, with  $u(0) = 0$ .

If the optimal bids  $(b_1^*, b_2^*)$  are such that  $b_1^* > b_2^*$  (we will verify this condition in the sequel), then they must satisfy the following first order necessary conditions:

$$\begin{cases} -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*)) - u'(v - b_1^*)[F(b_2^{-1}(b_1^*)) - F(b_1^{-1}(b_2^*))] \\ + u(v - b_1^*)f(b_2^{-1}(b_1^*))(b_2^{-1}(b_1^*))' = 0, \\ -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*)) + \\ + [u(2v - b_1^* - b_2^*) - u(v - b_1^*)]f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))' = 0. \end{cases} \quad (\text{B.2})$$

In a symmetric equilibrium, it must be that  $b_1^* = b_1(v)$  and  $b_2^* = b_2(v)$ . Thus, the first order conditions for a symmetric equilibrium are:

$$\begin{cases} -u'(2v - b_1(v) - b_2(v))F(b_1^{-1}(b_2(v))) - u'(v - b_1(v))[F(b_2^{-1}(b_1(v))) \\ - F(b_1^{-1}(b_2(v)))] + u(v - b_1(v))f(b_2^{-1}(b_1(v)))(b_2^{-1}(b_1(v)))' = 0, \\ -u'(2v - b_1(v) - b_2(v))F(b_1^{-1}(b_2(v))) \\ + [u(2v - b_1(v) - b_2(v)) - u(v - b_1(v))]f(b_1^{-1}(b_2(v)))(b_1^{-1}(b_2(v)))' = 0. \end{cases}$$

This system of differential equations, together with the boundary conditions  $b_1(0) = b_2(0) = 0$  and  $b_1(\bar{v}) = b_2(\bar{v}) = \bar{b}$  defines a symmetric equilibrium  $(b_1^{RA}(v), b_2^{RA}(v))$ .<sup>29</sup>

#### B.3.2 Equilibrium bids in $2A1U$

The expected payoff of a bidder with value  $v$  in  $2A1U$  is:

$$\begin{aligned} \Pi(c_1, c_2; v) = & u(v - c_1)F(c_1^{-1}(c_1)) + u(v - c_2)F(c_2^{-1}(c_2)) \\ & + [u(2v - c_1 - c_2) - u(v - c_1) - u(v - c_2)]F(\min(c_1^{-1}(c_1); c_2^{-1}(c_2))). \end{aligned}$$

Suppose that, as in a symmetric equilibrium, the optimal bids  $c_1^*, c_2^*$  are such that  $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$ . Notice that, at this point, the expected payoff is not differentiable. However, it

---

<sup>29</sup>Provided that,  $b_1^{RA}(v) > b_2^{RA}(v)$ , for all  $v \in (0, \bar{v})$ .



admits right and left partial derivatives. In particular, the right partial derivatives must be non-positive and the left partial derivatives must be nonnegative (otherwise, any increase or decrease in bids would be profitable):

$$\begin{aligned}\frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^+} &\leq 0, & \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^-} &\geq 0; \\ \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^+} &\leq 0, & \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^-} &\geq 0.\end{aligned}$$

Let us focus on the first two conditions (by symmetry, the same holds for the other two conditions), which are:

$$-u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' \leq 0, \quad (\text{B.3})$$

$$-u'(2v - c_1^* - c_2^*)F(c_1^{-1}(c_1^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_2^*)]f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' \geq 0. \quad (\text{B.4})$$

Since  $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$ , the first term of (B.3) coincides with that of (B.4). Let us concentrate on the second term. Since  $u(\cdot)$  is strictly concave, the incremental ratio  $[u(z + h) - u(z)]/h$  is strictly decreasing in  $z$ . Therefore, we have

$$u(2v - c_1^* - c_2^*) - u(v - c_2^*) < u(v - c_1^*).$$

But then, (B.3) and (B.4) cannot simultaneously hold. This means that the optimal bids  $(c_1^*, c_2^*)$  are such that  $c_1^{-1}(c_1^*) \neq c_2^{-1}(c_2^*)$ . In particular, they imply that there are no symmetric equilibria.

Now, suppose that the optimal bids  $(c_1^*, c_2^*)$  are such that  $c_1^{-1}(c_1^*) > c_2^{-1}(c_2^*)$ . The expected payoff is now differentiable at  $(c_1^*, c_2^*)$  and the first order conditions are:

$$\begin{cases} -u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) - u'(v - c_1^*) [F(c_1^{-1}(c_1^*)) - F(c_2^{-1}(c_2^*))] \\ + u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))' = 0 \\ -u'(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_1^*)] \times \\ \times f(c_2^{-1}(c_2^*))(c_2^{-1}(c_2^*))' = 0 \end{cases} \quad (\text{B.5})$$

Notice the perfect analogy with the first order conditions for optimal bids in *1A2U* - see system (B.2). Therefore, in *2A1U* there is an equilibrium in which:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_1^{RA}(v_1), & c_{1,2}^{RA}(v_1) = b_2^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_2^{RA}(v_2), & c_{2,2}^{RA}(v_2) = b_1^{RA}(v_2). \end{cases}$$

In this equilibrium, bidder 1 respectively places the highest bid in the first auction and the lowest bid in the second, while bidder 2 makes the opposite. As under joy of winning, there is also a specular equilibrium in which bidder 1 respectively places the lowest bid in the first auction and the highest bid in the second, while bidder 2 makes the opposite:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_2^{RA}(v_1), & c_{1,2}^{RA}(v_1) = b_1^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_1^{RA}(v_2), & c_{2,2}^{RA}(v_2) = b_2^{RA}(v_2). \end{cases}$$

### B.3.3 Bid spread in 1A2U and 2A1U

Given the analogy between 1A2U and 2A1U in terms of equilibrium bids, we focus on 1A2U and show that, if  $(b_1^{RA}(v), b_2^{RA}(v))$  constitutes a symmetric equilibrium in differentiable strategies, then it must be that  $b_1^{RA}(v) > b_2^{RA}(v)$ , for all  $v \in (0, \bar{v})$ .

Consider a bidder with value  $v \in (0, \bar{v})$  who finds optimal to bid  $b_1^* = b_2^* = b^*$ . Then, it must be the case that deviations are non-profitable, namely:

$$\frac{\partial \Pi(b^*, b^*)}{\partial b_1} \leq 0, \quad \frac{\partial \Pi(b^*, b^*)}{\partial b_2} \geq 0.$$

In a symmetric equilibrium,  $b_1^* = b_1(v)$ ,  $b_2^* = b_2(v)$ , and since  $b_1^* = b_2^* = b^*$ , then  $b_1(v) = b_2(v) = b^*$ . The conditions above reduce to

$$b_2'(x) \geq \frac{u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)}$$

and

$$b_1'(x) \leq \frac{u(2v - 2b^*) - u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)}.$$

Since  $u(\cdot)$  is strictly concave and  $u(0) = 0$ , we have that, for  $y > 0$ ,  $2u(y) > u(2y)$ . Therefore,

$$\frac{u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)} > \frac{u(2v - 2b^*) - u(v - b^*)}{u'(2v - 2b^*)} \frac{f(v)}{F(v)},$$

which implies that  $b_2'(v) > b_1'(v)$ . Thus, whenever the two bids are equal, the bidding function corresponding to the lowest bid is steeper than that of the highest bid. However, this is not possible under the assumption that  $b_1(\cdot)$  and  $b_2(\cdot)$  are differentiable.

### B.3.4 Overbidding

It is well known that  $a^{RA}(V) > a^{RN}(V)$ . Let us prove overbidding in 1A2U and 2A1U. Starting from 1A2U, suppose that the opponent bids according to generic (strictly increasing) bidding strategies,  $b_1(\cdot)$  and  $b_2(\cdot)$ . Under risk neutrality, the first order conditions that define the optimal lowest bid  $b_2^*$  of a bidder with value  $v$  is

$$(v - b_2^*)f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))' = F(b_1^{-1}(b_2^*)). \quad (\text{B.6})$$

Now, suppose that under risk aversion, the bidder bids according to  $b_2^*$  defined by (B.6). The partial derivative of her expected payoff with respect to  $b_2$  evaluated at  $b_2^*$  is

$$-u'(2v - b_1 - b_2^*)F(b_1^{-1}(b_2^*)) + [u(2v - b_1 - b_2^*) - u(v - b_1)]f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))'.$$

By using (B.6), the last expression can be written as

$$\frac{u(2v - b_1 - b_2^*) - u(v - b_1)}{v - b_2^*} - u'(2v - b_1 - b_2^*),$$

which is strictly positive. Indeed, notice that the first addend is the incremental ratio of  $u(\cdot)$  from  $(v - b_1)$  to  $(2v - b_1 - b_2^*)$ . The second term is the derivative of  $u(\cdot)$  evaluated at  $(2v - b_1 - b_2^*)$ . Since  $u(\cdot)$  is strictly concave, the first term is strictly greater than the second. This shows that, under risk aversion, for any possible bidding strategy adopted by the opponent, a bidder has an incentive to increase her lowest bid with respect to the corresponding equilibrium level under risk neutrality.

### B.3.5 Efficiency in $1A1U$ , inefficiency in $1A2U$ and in $2A1U$

The argument is identical to the one used under joy of winning (see section B.2.5). Inefficiency in  $1A2U$  and in  $2A1U$  follows directly from bid spread.

### B.3.6 0-spread can be rational in $2A1U$ but not in $1A2U$

The argument is identical to the one used under joy of winning (see section B.2.8). In this case, if the other bidder, with probability  $1/2$ , bids  $c_H(v)$  in the first auction and  $c_L(v)$  in the second and, with probability  $1/2$ , bids  $c_L(v)$  in the first auction and  $c_H(\cdot)$  in the second (where  $c_H(v) > c_L(v)$ , for all  $v \in (0, \bar{v})$ ), then bidder  $i$ 's expected payoff is:

$$\begin{aligned} \Pi(c_1, c_2; v) = & \frac{1}{2} \{ u(v - c_1) [F(c_H^{-1}(c_1)) + F(c_L^{-1}(c_1))] \\ & + u(v - c_2) [F(c_L^{-1}(c_2)) + F(c_H^{-1}(c_2))] \\ & + [u(2v - c_1 - c_2) - u(v - c_1) - u(v - c_2)] \\ & \times [F(\min(c_H^{-1}(c_1); c_L^{-1}(c_2))) + F(\min(c_L^{-1}(c_1); c_H^{-1}(c_2)))] \} \end{aligned}$$

If the optimal bids  $(c_1^*, c_2^*)$  are such that  $c_H^{-1}(c_1^*) < c_L^{-1}(c_2^*)$ , they must satisfy the following first order conditions:

$$\begin{cases} u(v - c_1^*)(c_L^{-1}(c_1^*))' - u'(v - c_1^*)[c_L^{-1}(c_1^*) - c_H^{-1}(c_2^*)] - u(v - c_2^*)(c_H^{-1}(c_1^*))' \\ + u(2v - c_1^* - c_2^*)(c_H^{-1}(c_1^*))' - u'(2v - c_1^* - c_2^*)[c_H^{-1}(c_1^*) + c_H^{-1}(c_2^*)] = 0, \\ u(v - c_2^*)(c_L^{-1}(c_2^*))' - u'(v - c_2^*)[c_L^{-1}(c_2^*) - c_H^{-1}(c_1^*)] - u(v - c_1^*)(c_H^{-1}(c_2^*))' \\ + u(2v - c_1^* - c_2^*)(c_H^{-1}(c_2^*))' - u'(2v - c_1^* - c_2^*)[c_H^{-1}(c_1^*) + c_H^{-1}(c_2^*)] = 0. \end{cases}$$

The two conditions above are clearly satisfied when  $c_1^* = c_2^*$ .

## C Experimental instructions

In what follows, we present the instructions given to participants in the three treatments. Instructions were originally written in German. We first present the part of *Instructions* that is common to all treatments (*[All treatments]*) and, then, the part of *Detailed Instructions* which is specific of each to the three treatments (*[1A1U]*, *[1A2U]*, *[2A1U]*).

### **Instructions - *[All treatments]***

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully you can earn an amount of money that will be paid in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with the other participants. If you have questions raise your hand and one of the assistants will come and answer it. The rules that you are reading are the same for all participants.

*General rules.* For showing up on time, you will receive 4 euro. During the experiment you will receive points. At the end of the experiment, the total number of points you have accumulated will be converted into Euro at the rate of 100 points = 3 euro. Your final payment will be composed of the show-up fee of 4 euro plus the amounts of money that you will earn during the experiment. Your final payment will be paid to you in cash immediately at the end of the experiment.

### **Detailed Instructions - *[1A1U]***

In this experiment you will participate in one auction that involves one unit of a hypothetical good. If you acquire the unit, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn.

In total, two persons will participate in the auction. Thus, in addition to you, there is another participant who will also want to acquire the unit involved in the auction. Exactly like you, the other participant will be given a resale value before the beginning of the auction. This value will be, again, randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore drawn independently of each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale value of the other participant, nor will the other participant be informed of your resale value.

*Rules of the auction.* The unit of the good involved in the auction will be auctioned off according to the following rules. You, as well as the other participant, will place one bid for the unit. The bid will be equivalent to the number of points that you are willing to pay to acquire the unit. Given the choices of the two participants, the highest bid wins the unit involved in the auction. This means that you will acquire the unit if you place the highest bid in the auction. In the case of identical bids, the winning bid will be randomly determined. If you acquire the unit, your earnings in points will be given by the difference between the resale value and your winning bid. If you do not acquire the unit, you will earn nothing. Note that you may also generate losses if you acquire the unit by bidding more than the resale value. Eventual losses will be subtracted from your total earnings in points.

*Example.* The following table reports the two hypothetical bids placed by the two participants in the auction. The two bids are ranked in order from the highest to the lowest. The participant placing the highest bid wins the unit and pays 87 points to acquire that unit.

Auction	
<i>Rank</i>	<i>Bid</i>
1	87
2	53

*Repetitions of the experimental task.* The experiment consists of 15 periods. In each period you will participate in an auction involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bid, the winning bid and how many points you have obtained in that auction.

#### **Detailed Instructions - [1A2U]**

In this experiment you will participate in one auction that involves two units of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the auction have the same resale value.

In total, two persons will participate in the auction. Thus, in addition to you, there is another additional participant who will also want to acquire the two units involved in the auction. Exactly like you, the other participant will be given a resale value for each of the two units before the beginning of the auction. Also for the other participant, the two units are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

*Rules of the auction.* The two units of the good involved in the auction will be auctioned off according to the following rules. You and the other participant will each place two bids, one for each unit. Each bid will be equivalent to the number of points that you are willing to pay to acquire the corresponding unit. Given the choices of the two participants, the two highest bids win the two units involved in the auction. This means that you will acquire one unit if you place one of the two highest bids in the auction. Similarly, you will acquire both units if you place the two highest bids in the auction. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

*Example.* The following table reports the four hypothetical bids placed by the two participants in the auction. The four bids are ranked in order from highest to lowest. The participant placing the highest bid wins the first unit and pays 87 points to acquire that unit.

The participant placing the second highest bid wins the second unit and pays 77 points to acquire that unit.

Auction	
<i>Rank</i>	<i>Bid</i>
1	87
2	77
3	66
4	53

*Repetitions of the experimental task.* The experiment consists of 15 periods. In each period, you will participate in an auction involving two units. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the two winning bids in the auction and how many points you have obtained in the auction.

**Detailed Instructions - [2A1U]** In this experiment you will participate in two simultaneous auctions, each involving one unit of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the two auctions have the same resale value.

In total, two persons will participate in the two auctions. Thus, in addition to you, there is another participant who will also want to acquire the two units involved in the two auctions. Exactly like you, the other participant will be given the resale value of each of the two units before the beginning of the two auctions. Also for the other participant, the units in the two auctions are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the two auctions are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the two auctions, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

*Rules of the auctions.* The two units of the good involved in the two auctions will be auctioned off according to the following rules. You and the other participant will each place two bids, one in each of the two auctions. Each bid will be equivalent to the number of points that you are willing to pay to acquire the unit in the corresponding auction. Given the choices of the two participants, in each auction, the highest bid wins the corresponding unit. This means that you will acquire one unit if you place the highest bid in one of the two auctions. Similarly, you will acquire two units if you place the highest bids in both auctions. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate

losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

*Example.* The following two tables report the bids placed by the two participants in the two auctions. In each auction, the two bids are ranked in order from highest to lowest. The participant placing the highest bid in the first auction wins the first unit and pays 87 points to acquire that unit. The participant placing the highest bid in the second auction wins the second unit and pays 77 points to acquire that unit.

Auction A		Auction B	
<i>Rank</i>	<i>Bid</i>	<i>Rank</i>	<i>Bid</i>
1	87	1	77
2	53	2	66

*Repetitions of the experimental task.* The experiment consists of 15 periods. In each period you will participate in two simultaneous auctions, each involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the winning bids in the two auctions and how many points you have obtained in each of the two auctions.

## D Robustness checks to control for the effects of repetition (not intended for publication)

In what follows, we replicate the main parametric analysis presented in Section 4 on different subsets of periods to control for the effects of repetition. All the regressions include a linear time trend that starts from 0 in the first period of the subset. Results remain almost unchanged if the linear time trend is excluded from the regressions.

Table 2a. (Sum of the) Bids in 1A1U, 1A2U and 2A1U in periods 1-5

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.695***	0.659***	0.701***	0.685***	0.685***
	(0.018)	(0.023)	(0.018)	(0.012)	(0.012)
<i>Period</i>	0.092	-2.357**	-1.100*	-1.140**	-1.140**
	(0.724)	(0.911)	(0.648)	(0.444)	(0.444)
<i>1A2U&amp;2A1U</i>				-3.161	
				(2.910)	
<i>1A2U</i>					-4.459
					(3.387)
<i>2A1U</i>					-1.861
					(3.388)
<i>Constant</i>	5.589	9.713**	5.631*	9.105***	9.087***
	(3.667)	(3.946)	(3.022)	(2.814)	(2.831)
<i>lrl</i>	-1176.500	-1242.133	-1158.312	-3596.702	-3594.271
<i>Wald - <math>\chi^2</math></i>	1471.69	707.99	1572.87	3396.22	3396.49
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	270	270	270	810	810

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 1-5 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the (the sum of the) bid(s) placed by the subject in the period. The other remarks of Table 1 apply.



Table 2b. (Sum of the) Bids in 1A1U, 1A2U and 2A1U in periods 6-10

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.665***	0.651***	0.665***	0.660***	0.660***
	(0.013)	(0.013)	(0.015)	(0.008)	(0.008)
<i>Period</i>	0.469	-1.079**	-0.953*	-0.512*	-0.512*
	(0.514)	(0.511)	(0.518)	(0.297)	(0.297)
<i>1A2U&amp;2A1U</i>				-4.967**	
				(2.173)	
<i>1A2U</i>					-5.790**
					(2.538)
<i>2A1U</i>					-4.142
					(2.539)
<i>Constant</i>	4.391*	3.166	3.129	6.872***	6.868***
	(2.469)	(2.519)	(2.486)	(2.044)	(2.062)
<i>lrl</i>	-1092.234	-1103.514	-1104.0787	-3301.786	-3299.728
<i>Wald - <math>\chi^2</math></i>	2606.59	2446.75	2020.18	7057.93	7056.12
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	270	270	270	810	810

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 6-10 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the (the sum of the) bid(s) placed by the subject in the period. The other remarks of Table 1 apply.

Table 2c. (Sum of the) Bids in 1A1U, 1A2U and 2A1U in periods 11-15

	1A1U	1A2U	2A1U	Pooled	
	(1)	(2)	(3)	(4)	(5)
<i>Value</i>	0.677*** (0.012)	0.614*** (0.015)	0.625*** (0.014)	0.640*** (0.008)	0.640*** (0.008)
<i>Period</i>	-0.774* (0.465)	0.099 (0.541)	-0.562 (0.502)	-0.362 (0.292)	-0.362 (0.292)
<i>1A2U&amp;2A1U</i>				-4.790* (2.465)	
<i>1A2U</i>					-4.973* (2.904)
<i>2A1U</i>					-4.606 (2.904)
<i>Constant</i>	3.786 (2.446)	3.559 (2.712)	4.032 (2.962)	6.726*** (2.252)	6.724*** (2.289)
<i>lrl</i>	-1059.598	-1115.521	-1089.300	-3276.262	-3274.279
<i>Wald - <math>\chi^2</math></i>	3028.38	1741.76	1925.36	6382.57	6381.51
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	270	270	270	810	810

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 11-15 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the (the sum of the) bid(s) placed by the subject in the period. The other remarks of Table 1 apply.

Table 3a. Bid spread in 1A2U and 2A1U in periods 1-5

	1A2U		2A1U		Pooled	
	Size	Prob.	Size	Prob.	Size	Prob.
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Value</i>	0.065*** (0.008)	0.002*** ( $3.8 \cdot 10^{-4}$ )	0.050*** (0.007)	0.002*** ( $4.7 \cdot 10^{-4}$ )	0.057*** (0.005)	0.002*** ( $2.9 \cdot 10^{-4}$ )
<i>Period</i>	-0.119 (0.280)	0.008 (0.019)	0.335 (0.263)	0.015 (0.018)	0.112 (0.192)	0.011 (0.013)
<i>2A1U</i>					-0.382 (1.071)	-0.093* (0.048)
<i>Constant</i>	1.030 (1.197)		1.234 (1.157)		1.349 (0.993)	
<i>lrl(lpl)</i>	-923.998	-124.468	-913.465	-156.219	-1835.979	-280.885
<i>Wald - <math>\chi^2</math></i>	73.00	20.72	50.70	14.58	121.34	41.13
<i>p &gt; <math>\chi^2</math></i>	0.000	0.000	0.000	0.001	0.000	0.000
<i>Obs.</i>	270	270	270	270	540	540

Notes. Columns (1), (3) and (5) report coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 1-5 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in these regression is the absolute value of the difference of the two bids placed by the subject in the period. Columns (2), (4) and (5) report Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over periods 1-5. The dependent variable is a dummy that takes a value of 1 if the subject places two different bids in the period. The other remarks of Table 1 apply.

Table 3b. Bid spread in 1A2U and 2A1U in periods 6-10

	1A2U		2A1U		Pooled	
	Size	Prob.	Size	Prob.	Size	Prob.
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Value</i>	0.690*** (0.006)	0.002*** ( $3.8 \cdot 10^{-4}$ )	0.074*** (0.006)	0.003*** ( $3.9 \cdot 10^{-4}$ )	0.071*** (0.004)	0.003*** ( $2.7 \cdot 10^{-4}$ )
<i>Period</i>	0.071 (0.227)	-0.002 (0.017)	0.059 (0.225)	0.030 (0.018)	0.069 (0.159)	0.038 (0.0127)
<i>2A1U</i>					-0.038 (0.906)	-0.068* (0.038)
<i>Constant</i>	0.434 (0.919)		-0.060 (1.073)		0.208 (0.824)	
<i>lrl(lpl)</i>	-865.204	-102.886	-865.834	-119.585	-1728.080	-223.821
<i>Wald</i> - $\chi^2$	145.22	37.14	137.50	38.31	284.40	68.56
<i>p</i> > $\chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
<i>Obs.</i>	270	270	270	270	540	540

Notes. Columns (1), (3) and (5) report coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 6-10 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in these regression is the absolute value of the difference of the two bids placed by the subject in the period. Columns (2), (4) and (5) report Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over periods 6-10. The dependent variable is a dummy that takes a value of 1 if the subject places two different bids in the period. The other remarks of Table 1 apply.

Table 3c. Bid spread in 1A2U and 2A1U in periods 11-15

	1A2U		2A1U		Pooled	
	Size	Prob.	Size	Prob.	Size	Prob.
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Value</i>	0.074*** (0.006)	0.001*** ( $2.0 \cdot 10^{-4}$ )	0.078*** (0.007)	0.003*** ( $6.2 \cdot 10^{-4}$ )	0.075*** (0.005)	0.002*** ( $2.8 \cdot 10^{-4}$ )
<i>Period</i>	0.005 (0.216)	-0.011 (0.014)	0.333 (0.258)	-0.034** (0.016)	0.163 (0.168)	-0.022** (0.011)
<i>2A1U</i>					0.091 (0.976)	-0.085** (0.042)
<i>Constant</i>	0.526 (1.004)		-0.480 (1.288)		0.051 (0.920)	
<i>lrl(lpl)</i>	-864.068	-85.157	-903.066	-116.021	-1767.387	-202.283
<i>Wald</i> - $\chi^2$	158.54	10.87	113.89	16.27	261.72	26.44
<i>p</i> > $\chi^2$	0.000	0.004	0.000	0.000	0.000	0.000
<i>Obs.</i>	270	270	270	270	540	540

Notes. Columns (1), (3) and (5) report coefficient estimates (standard errors in parentheses) from two-way linear random effects models over periods 11-15 accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable in these regression is the absolute value of the difference of the two bids placed by the subject in the period. Columns (2), (4) and (5) report Probit marginal effect estimates (robust standard errors clustered at the rematching group level in parentheses) over periods 11-15. The dependent variable is a dummy that takes a value of 1 if the subject places two different bids in the period. The other remarks of Table 1 apply.

Table 5a. Relative efficiency in 1A2U and 2A1U in periods 1-5

	<i>RE</i>		<i>RAR</i>		<i>RBS</i>	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>1A2U&amp;2A1U</i>	-0.016 (0.015)		-0.029 (0.037)		0.004 (0.044)	
<i>1A2U</i>		-0.034** (0.015)		-0.050 (0.043)		0.001 (0.052)
<i>2A1U</i>		0.002 (0.015)		-0.008 (0.043)		0.007 (0.052)
<i>Period</i>	0.003 (0.003)	0.003 (0.003)	-0.015** (0.007)	-0.015** (0.007)	0.018** (0.008)	0.017** (0.008)
<i>Constant</i>	0.965*** (0.014)	0.965*** (0.013)	0.803*** (0.033)	0.803*** (0.033)	0.162*** (0.039)	0.162*** (0.040)
<i>Wald - <math>\chi^2</math></i>	2.28	7.81	4.64	5.58	5.02	5.04
<i>p &gt; <math>\chi^2</math></i>	0.320	0.050	0.098	0.134	0.081	0.169
<i>Obs.</i>	135	135	135	135	135	135

Notes. This table reports report coefficient estimates from GLS random effects models (robust standard errors in parentheses clustered at the rematching group level). The dependent variable in columns (1) and (2), is the mean of the relative efficiency of the rematching group in the period. The dependent variable in columns (3) and (4), is the mean of the relative auctioneer's revenue of the rematching group in the period. The dependent variable in columns (5) and (6), is the mean of the relative bidders' surplus of the rematching group in the period. All the other remarks of Table 1 apply.

Table 5b. Relative efficiency in 1A2U and 2A1U in periods 6-10

	<i>RE</i>		<i>RAR</i>		<i>RBS</i>	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>1A2U&amp;2A1U</i>	−0.013*		−0.040*		0.019	
	(0.008)		(0.023)		(0.021)	
<i>1A2U</i>		−0.014		−0.061**		0.033
		(0.009)		(0.025)		(0.024)
<i>2A1U</i>		−0.012		−0.019		0.006
		(0.009)		(0.025)		(0.024)
<i>Period</i>	0.006**	0.006**	−0.007**	−0.007**	0.013***	0.013***
	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)
<i>Constant</i>	0.972***	0.972***	0.743***	0.743***	0.230***	0.230***
	(0.008)	(0.008)	(0.020)	(0.019)	(0.018)	(0.018)
<i>Wald</i> − $\chi^2$	7.39	7.43	7.06	10.05	13.09	14.41
<i>p</i> > $\chi^2$	0.023	0.059	0.029	0.018	0.001	0.002
<i>Obs.</i>	135	135	135	135	135	135

Notes. This table reports report coefficient estimates from GLS random effects models (robust standard errors in parentheses clustered at the rematching group level). The dependent variable in columns (1) and (2), is the mean of the relative efficiency of the rematching group in the period. The dependent variable in columns (3) and (4), is the mean of the relative auctioneer's revenue of the rematching group in the period. The dependent variable in columns (5) and (6), is the mean of the relative bidders' surplus of the rematching group in the period. All the other remarks of Table 1 apply.

Table 5c. Relative efficiency in 1A2U and 2A1U in periods 11-15

	<i>RE</i>		<i>RAR</i>		<i>RBS</i>	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>1A2U&amp;2A1U</i>	-0.011 (0.008)		-0.045* (0.027)		0.022 (0.024)	
<i>1A2U</i>		-0.019** (0.008)		-0.054* (0.031)		0.012 (0.028)
<i>2A1U</i>		-0.002 (0.008)		-0.037 (0.031)		0.032 (0.028)
<i>Period</i>	-0.002 (0.002)	-0.002 (0.002)	-0.004 (0.004)	-0.004 (0.004)	$4.0 * 10^{-4}$ (0.004)	$4.0 * 10^{-4}$ (0.004)
<i>Constant</i>	0.986*** (0.008)	0.986*** (0.007)	0.718*** (0.023)	0.718*** (0.023)	0.270*** (0.021)	0.270*** (0.021)
<i>Wald - <math>\chi^2</math></i>	2.68	6.78	4.13	4.36	0.88	1.41
<i>p &gt; <math>\chi^2</math></i>	0.262	0.079	0.127	0.225	0.644	0.7028
<i>Obs.</i>	135	135	135	135	135	135

Notes. This table reports report coefficient estimates from GLS random effects models (robust standard errors in parentheses clustered at the rematching group level). The dependent variable in columns (1) and (2), is the mean of the relative efficiency of the rematching group in the period. The dependent variable in columns (3) and (4), is the mean of the relative auctioneer's revenue of the rematching group in the period. The dependent variable in columns (5) and (6), is the mean of the relative bidders' surplus of the rematching group in the period. All the other remarks of Table 1 apply.