Capacitor Voltage Control Techniques of the Z-source Inverter: A Comparative Study

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Keywords: Z-source inverter, maximum constant boost control, small signal modeling, single-loop capacitor voltage control, dual-loop capacitor voltage control

Abstract

The Z-source inverter (ZSI) is a recently proposed single-stage power conversion topology. It adds voltage boost capability for complementing the usual voltage buck operation of a traditional voltage source inverter (VSI) with improved reliability. In this paper, a single-loop and dual-loop capacitor voltage control techniques for the ZSI are digitally designed based on a third order small signal model of the ZSI, implemented using a digital signal processor (DSP) and compared. Simulation and experimental results of a 30 kW ZSI during input voltage changes, load disturbances and steady state operations are presented and compared. The results show that the dual-loop capacitor voltage control technique achieves better steady state and transient performance and enlarge the stability margins of the ZSI compared to the single-loop capacitor voltage control technique.

Introduction

The Z-source inverter (ZSI), as shown in Fig. 1, is an emerging topology for power electronics DC-AC converters. It can utilize the shoot-through (ST) state to boost the input voltage, which improves the inverter reliability and enlarge its application field. In comparison with other power electronics converters, it provides an attractive single stage DC-AC conversion with buck-boost capability, reduced cost, reduced volume and higher efficiency.
due to a lower component number. For emerging power generation technologies, such as fuel cells, photovoltaic arrays and wind turbine, the ZSI is a very promising and competitive topology [1-3].

Since the ZSI was proposed in 2003 [4], lots of work had been done in the ST control methods. Four different ST control methods have been proposed in the literature, which are: simple boost control (SBC) [4], maximum boost control (MBC) [5], maximum constant boost control (MCBC) [6] and modified space vector modulation boost control (MSVMBC) [7] methods. In [8], the authors present a review of these four ST boost control methods and present a comparison between them based on simulation and experimental results. They concluded that the MCBC method seems to be the most suitable boost control method for inserting the ST state within the switching states of the ZSI. Therefore, the MCBC method will be used in this paper.

The control strategy of the ZSI is an important issue and several feedback control strategies have been investigated in recent publications [9-24]. There are four methods for controlling the ZSI DC-link voltage, which are: capacitor voltage control [9-17], indirect DC-link voltage control [18, 19], direct DC-link control [20-22] and unified control [23, 24]. In [9-11], the capacitor voltage is controlled by regulating the ST duty ratio using different control methods. Where, in [9] a PI controller is used to control the ST duty ratio and the modulation index is set to be $M = 1 - D_o$ using the SBC method. While, in [10] a PI controller with saturation is used to control the ST duty ratio and a PI controller tuned by a neural network for wide range control is used in [11], where the MSVMBC method is used in [10-11]. In [12-14], nonlinear control methods are used to control the capacitor voltage using the SBC method, where the gain scheduling combined with the state feedback control was used in [12, 13] and sliding mode control method was used in [14]. In [15-17], the capacitor voltage is controlled by regulating the ST duty ratio and the output voltage is controlled by regulating the modulation index using the MSVMBC method with two separate control loops with PI controllers as in [15, 16] and a neural network controllers as in [17]. In [18], a PID-like fuzzy controller is used to indirectly control the average DC-link voltage using the SBC method, where the average DC-link voltage is calculated by measuring the capacitor voltage using the following relation $V_i = V_o/(1 - D_o)$. Moreover, in [19], the peak DC-link voltage is indirectly controlled by controlling the peak AC output voltage using a PI controller to regulate the modulation index and the ST duty ratio is calculated by measuring the input voltage and comparing it by the required peak DC-link voltage using the MSVMBC method. In [20, 21], the peak DC link voltage is directly controlled by regulating the ST duty ratio, the peak DC-link voltage is measured using an additional circuit, because of its pulsating nature, which makes the control algorithm more complex, the SBC method was used in [20] and the MSVMBC was used in [21]. In [22], the peak DC-link voltage is directly controlled by regulating the ST duty ratio using the SBC method, where the peak of the DC-link voltage was estimated by measuring both the input and the capacitor voltages, as $V_{ip} = 2V_c - V_{in}$. In [23, 24], the unified control method is used to regulate the modulation index and the ST simultaneously by controlling the AC output voltage using a single PI controller using the MSVMBC method. Table 1, presents a review of the above mentioned control methods with their drawbacks.

In all the above mentioned control methods, a single-loop voltage control technique was used. However, in high power converters, a single-loop voltage control has two problems. The first problem is that, the inductor current is not regulated and can be overloaded during transient events and the limited stability limits is the second problem [25]. Therefore, a dual-loop voltage control is preferred over a single-loop voltage control in high power converters to overcome the above mentioned problems [26].

This paper presents, a comparative study between two capacitor voltage control techniques for controlling the DC-link voltage of the ZSI. The table [1] shows a review of previous ZSI control strategies.

<table>
<thead>
<tr>
<th>Reference No.</th>
<th>Control method</th>
<th>Controller type</th>
<th>Controller variable</th>
<th>Drawbacks</th>
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<tbody>
<tr>
<td>9</td>
<td>$V_c$ control</td>
<td>Single loop control with a PID controller</td>
<td>$D_0$</td>
<td>The controller is designed based on a second order model</td>
</tr>
<tr>
<td>10</td>
<td>$V_c$ control</td>
<td>Single loop control with a PI controller with saturation</td>
<td>$D_0$</td>
<td>The controller is not designed</td>
</tr>
<tr>
<td>11</td>
<td>$V_c$ control</td>
<td>Single loop control with a PI controller</td>
<td>$D_0$</td>
<td>The controller parameters is tuned by neural network (NN)</td>
</tr>
<tr>
<td>12, 13</td>
<td>$V_c$ control</td>
<td>Single loop nonlinear control</td>
<td>$D_0$ and $M$</td>
<td>Complex control algorithm</td>
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<tr>
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<td>Complex control algorithm</td>
</tr>
<tr>
<td>15, 16, 17</td>
<td>$V_c$ control</td>
<td>Two control loops with a PI controller</td>
<td>$D_0$ and $M$</td>
<td>The controller is not designed or tuned by NN</td>
</tr>
<tr>
<td>18</td>
<td>Indirect $V_i$ control by controlling $V_{ac}$</td>
<td>PID fuzzy controller</td>
<td>$D_0$</td>
<td>The controller is not accurate, because the relation between is not linear and the controller is not designed</td>
</tr>
<tr>
<td>19</td>
<td>Indirect $V_i$ control by controlling $V_{ac}$</td>
<td>PI controller with saturation</td>
<td>$M$</td>
<td>The ST is not controlled and the controller is not designed</td>
</tr>
<tr>
<td>20</td>
<td>Direct $V_i$ control</td>
<td>Single loop PID controller</td>
<td>$D_0$</td>
<td>External complex sensing circuit and the controller is not designed</td>
</tr>
<tr>
<td>21</td>
<td>Direct $V_i$ control</td>
<td>Two control loops with a PI controller</td>
<td>$D_0$ and $M$</td>
<td>External complex sensing circuit and the controller is not designed</td>
</tr>
<tr>
<td>22</td>
<td>Direct $V_i$ control</td>
<td>Dual loop $V_i$ control, the outer voltage loop has a PI controller and the inner current loop has a P controller</td>
<td>$D_0$</td>
<td>A simplified representation of the 3-phase load of the ZSI</td>
</tr>
<tr>
<td>23, 24</td>
<td>Unified control</td>
<td>Single loop with a PI controller</td>
<td>$V_{ref}$ for the MSVM</td>
<td>Suitable for isolated operations and the controller is not designed</td>
</tr>
</tbody>
</table>
the ZSI, which are single-loop and dual-loop control techniques. A digital PI controller with anti-windup correction is used for both control techniques. The performance of both control techniques is verified and compared by simulation and experimental results of a 30 kW ZSI with inductive load during input voltage changes, load disturbances and steady state operations.

A small signal model of the ZSI

Many methods for modeling power converters have been reported in the literature [27, 28]. Among these methods, the state space averaged small-signal modeling, which is the most widely used to model power converters. Therefore, an accurate small-signal model of the ZSI is needed, which gives not only a global but also a detailed view of system dynamics and provides guidelines for the system controllers design since the required transfer functions could be derived accordingly.

General operation of a ZSI can be illustrated by simplifying the AC system, with state variables: capacitor voltages and inductor currents in the network, and load current) as a vector.

Let us define the state variables (capacitor voltages and inductor currents) for the simplified circuit presented in Fig. 2. Where, \( R_c \) is calculated based on output power balance of the two circuits (Fig. 1 and Fig. 2) as \( R_s = \frac{8 | Z_{s1} | \cos \varphi}{L_1} \) and \( L_1 \) is determined so that the time constant of the DC load is the same as the AC load [12]. Two operation modes involving two different circuit topologies can be identified in the ZSI operation as shown in Fig. 3. In Mode 1, Fig. 3-a, the energy transferred from source to load is zero because the load side and source side are decoupled by the ST state and the open side circuit by an equivalent RL load in parallel with a switch.

As shown in Fig. 3. In Mode 1, Fig. 3-a, the energy transferred from source to load is zero because the load side and source side are decoupled by the ST state and the open side circuit by an equivalent RL load in parallel with a switch.

Due to Z-network symmetry, \( i_{L1}(t) = i_{L2}(t) = i(t) \) and \( v_{C1}(t) = v_{C2}(t) = v(t) \), therefore the system order reduced to a third order system, with state variables: \( i(t) = v(t) \) and \( i(t) \) [29, 30]. Using state space averaging method, and performing small signal perturbation for a given operating point, one gets:

\[
\left[ \begin{array}{c} \tilde{L}(t) \\ \tilde{V}(t) \\ \tilde{I}(t) \end{array} \right] = \left[ \begin{array}{ccc} 0 & \frac{2D_0 - 1}{L} & 0 \\ - \frac{2D_0}{C} & 0 & - \frac{(1 - D_0)}{C} \\ 0 & \frac{-R_s}{L_1} & 0 \end{array} \right] \left[ \begin{array}{c} \tilde{L}(t) \\ \tilde{V}(t) \\ \tilde{I}(t) \end{array} \right] + \left[ \begin{array}{c} \frac{2D_0 - V_m}{L} \\ \frac{-2L_1 + I_l}{C} \\ \frac{-2V_m + V_m}{L_1} \end{array} \right] \cdot \tilde{d}(t)
\]

The steady state values can be calculated by \( Ax + Bu = 0 \), as:

\[
V_c = \frac{1 - D_0}{1 - 2D_0} V_m
\]

\[
I_L = \frac{1 - D_0 - I_l}{1 - 2D_0}
\]

\[
I_l = \frac{V_c}{R_l}
\]

In small-signal modeling and transient analysis, the response of one state variable to multiple small-signal perturbations can be expressed as a linear combination of the variable response to each individual perturbation. The capacitor voltage and the inductor current can be expressed as a linear combination of the variable response to each individual perturbation as:

\[
\tilde{v}(s) = G_{vd}(s)\tilde{d}(s) + G_{vi}(s)\tilde{\nu}(s)
\]

\[
\tilde{i}(s) = G_{id}(s)\tilde{d}(s) + G_{ii}(s)\tilde{\nu}(s)
\]

where \( G_{vd}(s), G_{vi}(s), G_{id}(s) \) and \( G_{ii}(s) \) are given by:

\[
\tilde{L}(s) = 2(1 - D_0)\tilde{v}(s) - \mu_0(s) - (1 - D_0)\tilde{\nu}(s) + (-2V_c + V_m)\tilde{d}(s)
\]

\[
\tilde{I}(s) = 2(1 - D_0)\tilde{v}(s) - \mu_0(s) - (1 - D_0)\tilde{\nu}(s) + (-2V_c + V_m)\tilde{d}(s)
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\]
Predicting the right half plane (RHP) zeros of the related transfer functions is one of the major advantages of the small-signal modeling. By considering the control-to-capacitor voltage transfer function, given by Eq. 6 as an example, the numerator is a quadratic function. As shown in Eq. 7, it can be acquired that if the discriminant $4 \alpha^2 - 4c = 0$, there will be two different poles $\alpha$ and $\beta$, if the discriminant $b^2 - 4ac > 0$. Regarding this case, it can be acquired that $a = (-2I_L + I_I)lC_L < 0$ and $c = (1 - 2D_0)(2V_C - V_m)R_I > 0$, therefore, this transfer function has two zeros; one is negative while the other is a positive one, which is called RHP zero. This identifies a non-minimum phase characteristic in the capacitor voltage response that is known to potentially introduce stability issues in the closed loop regulated system. The design of a feedback controller with an adequate phase margin becomes more difficult when RHP zeros appear in the transfer function, since it tends to destabilize the wide-bandwidth feedback loops, implying high-gain instability and imposing control limitations [29, 30].

**Capacitor voltage control strategies**

In order to conduct a fair comparison between the single-loop and dual-loop capacitor voltage control strategies of the ZSI, they must be designed using the same procedure. The direct digital design method is used to design the single and the dual-loop controllers, using the following steps: first, the continuous time control to outputs transfer functions ($G_{cl}(s)$ and $G_{cl}(s)$) of the ZSI are discretized using the zero order hold (ZOH) method. After that, once the discrete transfer functions of the system are available, the digital controllers are designed directly in the z-domain using methods similar to the continuous time frequency response methods [31]. This has the advantage that the poles and zeros of the digital controllers are located directly in the z-domain, resulting in a better load transient response, as well as better phase margin and bandwidth for the closed loop power converter [32].

**Single-loop capacitor voltage control**

Fig. 4 shows the entire digital single-loop capacitor voltage control technique of the ZSI containing the voltage loop controller, $G_{cl}(z)$, the zero order hold, $(1 - e^{-Ts}(s))$, the computational delay, $e^{-\tau_s}$, the control to capacitor voltage continuous time transfer function $G_{cd}(s)$ and the modified modulation to ST transfer function $G_{md}(s)$. Where $G_{md}(s)$ is expressed by [9]:

\[
G_{md}(s) = \frac{D_o(s)}{V_m'(s)} = \frac{2}{V_m}
\]

In this digital implementation the chosen sampling scheme results in a computation delay of half the sampling period ($T_d = T/2$). The loop gain for the single loop capacitor voltage control can be expressed by [31]:

\[
G_{cl}(z) = \sum \left( 1 - e^{-\tau_s} \cdot e^{-T_s} \cdot G_m(z) \cdot G_{cl}(s) \right)
\]

In this paper, a digital PI controller with anti-windup will be designed based on the required phase margin, and critical frequency, using the bode diagram of the system in the z-domain, the transfer function of the digital PI controller in z-domain is given by:

\[
G_c(z) = K_p + \frac{K_I T_c}{z - 1}
\]

where

\[
K_p = \cos \theta \frac{G_p(z)}{G_P(z)}
\]

\[
K_I = \sin \theta \cdot \frac{f_{ce}}{G_P(z)}
\]
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Fig. 5 shows the entire dual-loop capacitor voltage control technique of the ZSI containing the voltage loop and current loop controllers $G_{cv}(s)$, $G_{ci}(s)$, the zero order hold, the computational delay, the control to outputs transfer functions $G_{vd}(s)$, $G_{id}(s)$ and the modified modulation to ST transfer function $G_{M}(s)$, respectively. The loop gains for inner current loop and outer voltage loop can be expressed as:

$$T_i(z) = G_{cv}(z) \cdot G_{id}(z)$$  \hspace{1cm} (16)

$$T_v(z) = \frac{G_{cv}(z) \cdot G_{di}(z) \cdot G_{id}(z)}{1 + T_i(z)}$$  \hspace{1cm} (17)

where the discretized inductor current to control transfer function is given by:

$$G_{id}(z) = Z \left\{ \frac{1 - e^{-Ts}}{s} \cdot e^{-Ts} \cdot G_{M}(s) \cdot G_{id}(s) \right\}$$  \hspace{1cm} (18)

As an example, Fig. 6 shows the bode plots for the current loop gain and voltage loop gain for the dual-loop control technique, respectively, with the system parameters listed in Table 2. The plots indicate that the current loop gain has a crossover frequency as high as 1 kHz, with a phase margin of 65° and a gain margin of 10 dB. To avoid interaction between the sub-systems, low control bandwidth is used for the voltage loop. The resulting outer voltage loop has a crossover frequency of 100 Hz and a phase margin of 59° and a gain margin of 25 dB.

**Simulation and experimental results**

The dynamic performance of the ZSI with single-loop and dual-loop capacitor voltage control techniques has been tested using MATLAB simulation and experimental verification using the system parameters in Table 2. A 30 kW ZSI prototype, as shown in Fig. 7, has been designed, implemented and tested up to 10 kW. The eZdsp™ F2808 evaluation board is used for the realization of the capacitor voltage control techniques and the real time workshop (RTW) is used for automatic code generation.

**Table 2: Experimental and simulation parameters of the ZSI**

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<thead>
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</thead>
<tbody>
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<td>200 V</td>
</tr>
<tr>
<td>Capacitor reference voltage</td>
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</tr>
<tr>
<td>Inductance</td>
<td>650 µH</td>
</tr>
<tr>
<td>Inductance internal resistance</td>
<td>0.22 Ω</td>
</tr>
<tr>
<td>Capacitance</td>
<td>320 µF</td>
</tr>
<tr>
<td>Capacitance internal resistance</td>
<td>0.9 mΩ</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>AC load inductance</td>
<td>340 µH</td>
</tr>
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<td>AC load resistance</td>
<td>12.5 Ω</td>
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</table>

and

$$\theta = 180^\circ + \phi_m - \angle G_p(z)$$  \hspace{1cm} (15)

**Dual-loop capacitor voltage control**

Fig. 5 shows the entire dual-loop capacitor voltage control technique of the ZSI containing the voltage loop and current loop controllers $G_{cv}(s)$, $G_{ci}(s)$, the zero order hold, the computational delay, the control to outputs transfer functions $G_{vd}(s)$, $G_{id}(s)$ and the modified modulation to ST transfer function $G_{M}(s)$, respectively. The loop gains for inner current loop and outer voltage loop can be expressed as:

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The dynamic performance of the ZSI with single-loop and dual-loop capacitor voltage control techniques has been tested using MATLAB simulation and experimental verification using the system parameters in Table 2. A 30 kW ZSI prototype, as shown in Fig. 7, has been designed, implemented and tested up to 10 kW. The eZdsp™ F2808 evaluation board is used for the realization of the capacitor voltage control techniques and the real time workshop (RTW) is used for automatic code generation.
Figs. 8-10 show the simulation and experimental results of the single-loop capacitor voltage control technique and Figs. 11-13 show the simulation and experimental results of the dual-loop capacitor voltage control technique during input voltage step down by 7.5% with the same load, load increasing and decreasing by 50% and steady state operations. It is noticeable that the experimental results in both cases match the simulation results very well.

By comparing Fig. 8 with Fig. 11, in case of input voltage step down, one can conclude that there is a noticeable oscillations in the inductor current and the output line voltage in case of the single-loop control technique. Furthermore, during load disturbances, Fig. 9 and Fig. 12, the single-loop control strategy, compared to the dual-loop control strategy, suffers from large oscillations in the inductor current, capacitor voltage, DC-link voltage and output line voltage during load increasing by 50%. In addition, by comparing Fig. 10 and Fig. 13, the dual-loop control strategy gives better steady state performance compared with the single-loop control strategy. Therefore, the dual loop capacitor voltage control technique achieves better steady state and transient performances.

As well-known, the zeros in the RHP of the s-domain will be mapped outside the unity circle in the z-domain. Also, as the gain of a closed loop controlled system is increased, the root locus position of the open loop system poles track towards the open loop system zeros. Since the control to capacitor voltage transfer function, Eq. 6, has a zero in the RHP, this means that attempting to control the capacitor voltage with a single PI voltage controller will cause the open loop poles to track across the unity circle in the z-domain into the RHP as the controller gain is increased, thus making the system unstable, as indicated in Fig. 14-a. In contrast, the control to inductor current transfer function given by Eq. 8 contain only a left hand plane (LHP) zeros and consequently do not has a non-minimum phase characteristic. Controlling the ZSI inductor current with a closed loop PI controller makes the system closed loop poles track to the open loop system zeros, but in this case their path is away from the unity circle and towards the origin of the z-domain. When the outer voltage controller is wrapped around the current controlled system, the RHP zero reappears in the overall system transfer function, Eq. 17, and will cause the current controlled system poles to once again track to the RHP as the outer loop PI gain is increased. However, the pole locations of the cascaded system at the system final design gains are still inside the unity circle, as shown in Fig. 14-b. Therefore, the insertion of the inner PI current control loop has modified the primary system response to make it over damped and more stable. This has allowed the gains of the outer PI voltage control loop to then be increased to achieve a good overall system dynamic response, while still avoiding the stability limits of the single-loop voltage control system.
Conclusion

This paper presents two techniques for digitally control the capacitor voltage of the ZSI. Both control techniques are designed based on a small-signal mode of the ZSI using a bode diagram of the discrete time transfer functions of the system to achieve the required phase margin and critical frequency. Experimental and simulation results of a 30 kW ZSI during input voltage changes, load disturbances and steady state for both single and dual-loop control techniques are presented and compared. In addition, the stability of both control techniques is discussed. Finally, the dual-loop capacitor voltage control technique achieves better steady state and transient performance and more stable behavior compared to the single-loop capacitor voltage control technique.

References


Fig. 10: Steady state response of the ZSI: (a, c and e) simulation results and (b, d and f) experimental result using single-loop control technique.
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Fig. 11: ZSI response during input voltage step down by 7.5 %: (a and c) simulation results and (b and d) experimental results using dual-loop control technique.


Fig. 12: ZSI response during load increasing and decreasing by 50%: (a and c) simulation results and (b and d) experimental results using dual-loop control technique.
Fig. 13: Steady state response of the ZSI: (a, c and e) simulation results and (b, d and f) experimental result using dual-loop control technique.
Fig. 14: Z-domain root locus for single-loop (a) and dual-loop (b) capacitor voltage controlled ZSI

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