Stability Analysis of a Unified Power Flow Controller

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Abstract—The paper concerns with stability analysis of a Unified Power Flow Controller (UPFC). A nonlinear model is described in the abc-reference frame and then transformed into the dq0-reference frame. The model includes shunt and series compensators of the UPFC. Stability analysis is presented first in a three-dimensional space of system parameters. A linearized model is obtained in state-space form to be used in small-signal stability studies. Effects of system parameters on stability are investigated using eigenvalue analysis. These include resistance, reactance, and modulation indices of the shunt and series components of the UPFC. Simulation results show that the stability of the UPFC can be affected by these parameters. The results are useful for UPFC designers and engineers who set technical specifications of these control devices.

I. INTRODUCTION

The continuous growth of demand in electric power systems requires establishing new generating plants and transmission lines. These requirements are becoming more restrictive due to economic and environmental impacts. Conventional control devices with fixed or mechanically switchable components have low speed, high wearing and require frequent maintenance. Therefore, new fast control devices need to be developed to increase transmission capacity by utilizing existing generating units and loading transmission lines closer to their thermal limits even in contingency conditions. New Flexible AC Transmission System (FACTS) devices are used in practical power systems to improve performance [1], [2].

The Unified Power Flow Controller (UPFC) is one of the most effective FACTS devices [3], [4]. The concept of the UPFC was proposed by Gyugyi in 1991 [5]. The UPFC is used for the real time control and dynamic compensation for AC transmission systems, providing more flexibility in power system today. Moreover, the UPFC has the unique capability to control simultaneously or independently, all the parameters affecting power flow in the transmission line which are voltage, impedance, and phase angle. Also, it can independently control both the real and reactive power flows in the transmission line [6], [7].

This paper demonstrates the UPFC main configuration and basic operation for each branch, including the series and shunt branch control. A nonlinear model is used to describe the UPEC system. Stability is investigated in the three-dimensional parameter space [8], [9]. A linearized state-space model is obtained from the nonlinear system equations. It is used to study the small-signal stability of the UPFC by eigenvalue analysis. Effects of system parameters, e.g. resistances and inducances, on stability are investigated. Simulation results can help system designers and engineers in selecting proper parameter values for maintaining stability and acceptable technical specifications.

II. UPFC DESCRIPTION

A. UPFC Circuit

Figure 1 shows the UPFC circuit which consists of a shunt and a series branches. Each branch is considered as a voltage source converter using semiconductor devices of fully controlled type, such as the IGBTs (Insulated-Gate Bipolar Transistors) and the GTOs (Gate Turn-Off) thyristors. The shunt and series converters are connected to the transmission line through step down transformers. They are operated from a common DC link provided by a DC storage capacitor which has a capacitance value selected to improve dynamic performance with cost-benefit consideration [1], [6]. These back to back converters can independently generate (or absorb) reactive power at its own AC output terminal.

B. Operation of UPFC

The basic function of the shunt converter is to provide the real power required to the series converter from the AC power system [7], [10]. It can generate or absorb reactive power at the UPFC connected busbar, which is independent of the active power transfer to (or from) the DC circuit [7].
Therefore it can provide reactive power compensation for the transmission line and thus provide indirect voltage regulation at the input terminal of the UPFC [3], [6].

The main function of the UPFC is provided through the series converter by injecting an AC voltage in series with controllable magnitude $V_{SE}$ ($0 \leq V_{SE} \leq V_{SE_{max}}$) and phase angle $\alpha_{SE}$ ($0 \leq \alpha_{SE} \leq 360^\circ$). This is achieved at the power frequency through a series connected transformer with the transmission line. The converter output voltage injected in series with the line can be used for direct voltage control, series compensation, phase shifter and their combinations. The real power exchanged at AC terminal of the insertion transformer is converted by the series converter into DC power which appears at the DC link as positive real power demanded by the shunt converter. The reactive power exchange at the AC terminal is generated internally by the inverter. Fig. 2 shows the equivalent circuit of the Pulse Width Modulation (PWM) inverter.

\[ V_o = \left( \frac{m}{2} \cdot \cos \alpha \right) V_{dc} \]  

III. NONLINEAR EQUATIONS OF THE UPFC

A. ABC-Frame

Fig. 3 shows a simplified equivalent circuit for the parallel and series branches. Neglecting harmonics for simplicity, we can write the equations of this equivalent circuit as follows:

\[ V_S = L_{SH} \frac{di_{SH}}{dt} + R_{SH} i_{SH} + V_{SH} \]  
\[ V_S = L_{SE} \frac{di_{SE}}{dt} + R_{SE} i_{SE} - V_{SE} + V_R \]  
\[ V_{SH} = \frac{m_{SH}}{2} \cdot V_{dc} \cdot \cos \alpha_{SH} \]  
\[ V_{SE} = \frac{m_{SE}}{2} \cdot V_{dc} \cdot \cos \alpha_{SE} \]

Where, $V_S$ is the AC voltage and $V_{dc}$ is the DC voltage, $m_{SH}$ and $m_{SE}$ are the modulation indices of the shunt and series inverter respectively, $\alpha_{SH}$ and $\alpha_{SE}$ are the phase displacement of the shunt and series inverter voltage respectively, $R_{SH}$ and $R_{SE}$ are the equivalent resistance of the shunt and series branch respectively, and $R_{SH}$ and $R_{SE}$ are the equivalent inductance of the shunt and series branch respectively.

C. DQ0-Frame:

Equations (6) to (10) are:

\[ \frac{d}{dt} i_{SHd} = -R_{SH} \frac{\alpha_{SH}}{L_{SH}} i_{SHd} + \omega_{SH} i_{SHq} + \frac{\alpha_{H}}{L_{SH}} V_{SH} - \frac{m_{SH} \cdot \alpha_{H}}{2L_{SH}} V_{dc} \cdot \cos \alpha_{SH} \]  
\[ \frac{d}{dt} i_{SHq} = -R_{SH} \frac{\alpha_{SH}}{L_{SH}} i_{SHq} - \omega_{SH} i_{SHd} + \frac{\alpha_{H}}{L_{SH}} V_{SH} - \frac{m_{SH} \cdot \alpha_{H}}{2L_{SH}} V_{dc} \cdot \sin \alpha_{SH} \]  
\[ \frac{d}{dt} i_{SEd} = -R_{SE} \frac{\alpha_{SE}}{L_{SE}} i_{SEd} + \omega_{SE} i_{SEq} + \frac{\alpha_{H}}{L_{SE}} (V_{Sd} - V_{Rd}) + \frac{m_{SE} \cdot \alpha_{H}}{2L_{SE}} V_{dc} \cdot \cos \alpha_{SE} \]  
\[ \frac{d}{dt} i_{SEQ} = -R_{SE} \frac{\alpha_{SE}}{L_{SE}} i_{SEQ} - \omega_{SE} i_{SEd} + \frac{\alpha_{H}}{L_{SE}} (V_{Sq} - V_{Rq}) + \frac{m_{SE} \cdot \alpha_{H}}{2L_{SE}} V_{dc} \cdot \sin \alpha_{SE} \]  
\[ \frac{dV_{dc}}{dt} = \frac{3}{4} \cdot C \cdot \omega_{SH} \cdot \cos \alpha_{SH} \cdot i_{SHd} + \frac{3}{4} \cdot C \cdot \omega_{SH} \cdot \sin \alpha_{SH} \cdot i_{SHq} \]  
\[ + \frac{3}{4} \cdot C \cdot \omega_{SE} \cdot \cos \alpha_{SE} \cdot i_{SEQ} + \frac{3}{4} \cdot C \cdot \omega_{SE} \cdot \sin \alpha_{SE} \cdot i_{SEQ} \]

For performing steady-state analysis of the UPFC, we use the following per unit values for shunt and series system parameters and for the dc-circuit [11]:

\[ R_{SH} = 0.015, \quad L_{SH} = 0.15, \]  
\[ R_{SE} = 0.01, \quad L_{SE} = 0.1, \]  
\[ C = 0.5, \quad f = 50 \text{ Hz}, \]  
\[ \omega = 2\pi f, \quad \omega_a = 2\pi f, \]  
\[ V_{Sd} = 1, \quad V_{Sq} = 0, \]  
\[ V_{Rd} = 0.94, \quad V_{Rq} = 0.34 \]
The steady state analysis can provide behavior indication of the model to choose the preferable setting range of the series and shunt inverter to achieve the required response.

By changing the value of the series inverter modulation index \( m_{SE} \) to 0.1 and the phase angle displacement of the series inverter voltage \( \alpha_{SE} \) to 40 degrees the system becomes unstable at a certain value of the shunt inverter modulation index \( m_{SH} \) as shown in Fig. 5.

### IV. LINEARIZED MODEL OF UPFC

Equations (6) to (10) of Section III. B., describe the dynamics of the UPFC in dq0 frame. These equations are nonlinear and may be expressed in general form as:

\[
X = f(x, u)
\]

Where;

\[
X = \begin{bmatrix}
i_{SHd} \\
i_{SHq} \\
i_{SEd} \\
i_{SEQ} \\
V_{dc}
\end{bmatrix}^T, \\
U = \begin{bmatrix}
m_{SH} \\
\alpha_{SH} \\
m_{SE} \\
\alpha_{SE}
\end{bmatrix}^T
\]

To linearize the above equation, the jacobians defined below are used to find the elements of the system matrices:

\[
A_{ij} = \frac{\partial f_i}{\partial x_j}, \quad B_{ik} = \frac{\partial f_i}{\partial u_k}
\]

where; \( i = 1 \) to 5, \( j = 1 \) to 5, and \( k = 1 \) to 4.

The linearized system is written in the form;

\[
\Delta X = A \Delta x + B \Delta u
\]

Where, \( \Delta x \) and \( \Delta u \) are the state and input signal deviations from their steady state values. The state coefficient matrix \( A \) and the input coefficient \( B \) are:

\[
A = \begin{bmatrix}
-R_{q,SH} & \omega & 0 & 0 & -m_{SE} V_{dc} \cos \alpha_{SH}^0 \\
0 & -R_{q,SH} & \omega & 0 & -m_{SE} V_{dc} \sin \alpha_{SH}^0 \\
0 & 0 & -R_{q,SH} & \omega & m_{SE} V_{dc} \cos \alpha_{SE}^0 \\
0 & 0 & 0 & -R_{q,SH} & m_{SE} V_{dc} \sin \alpha_{SE}^0 \\
k m_{SH} \cos \alpha_{SH} & k m_{SH} \sin \alpha_{SH} & k m_{SE} \cos \alpha_{SE} & k m_{SE} \sin \alpha_{SE} & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-V_{dc} \omega \cos \alpha_{SH}^0 & m_{SH} \omega V_{dc} \sin \alpha_{SH}^0 & 0 & 0 \\
0 & -m_{SH} \omega V_{dc} \cos \alpha_{SH}^0 & 0 & 0 \\
-k \alpha_a & k m_{SH} \alpha & 0 & 0 \\
0 & 0 & -k \alpha_c & k m_{SE} \alpha
\end{bmatrix}
\]

Where, \( k = \frac{3}{4} C \omega_b \)

\[
a = \left( \cos \alpha_{SH} \cdot i_{SHd} + \sin \alpha_{SH} \cdot i_{SHq} \right) \\
b = \left( -\sin \alpha_{SH} \cdot i_{SHd} + \cos \alpha_{SH} \cdot i_{SHq} \right) \\
c = \left( \cos \alpha_{SE} \cdot i_{SEd} + \sin \alpha_{SE} \cdot i_{SEq} \right) \\
d = \left( -\sin \alpha_{SE} \cdot i_{SEd} + \cos \alpha_{SE} \cdot i_{SEq} \right)
\]
The eigenvalues of the matrix $A$ contain information on the small signal stability of the system. They can be calculated by using MATLAB software. In general, the eigenvalue has the following complex form:

$$
\lambda = \sigma \pm j \omega_d
$$

The reciprocal of the real part $|1/\lambda|$ defines the time constant in seconds by which the oscillations decay. The imaginary part $(\omega_d)$ gives the frequency of oscillations in rad/s. The damped natural frequency $f_d = \omega_d/2\pi$, Hz. The damping ratio $\zeta = \sigma/|\lambda|$.

If all eigenvalues of the system are on the left hand side of the s-plane, the system is stable. This means that, for stability, the real parts of all eigenvalues should be negative, otherwise the system is unstable.

V. SMALL SIGNAL STABILITY ANALYSIS

Table I shows the system eigenvalues for the base case; in which the parameter values of the resistances, inductances, capacitance, modulations indices, and phase displacements of the voltages of shunt and series inverters, and the dc voltage are as listed in the same table. The system has three modes; two complex conjugate $\lambda_{12}, \lambda_{34}$ and one real $\lambda_5$. All real parts are negative and therefore the system is stable. The first mode has a damping ratio $\zeta = 0.0665$, and a damped natural frequency $f_d = 61.94$ Hz. The damping ratio and the damped natural frequency of the second mode are $\zeta = 0.0995$ and $f_d = 50$ Hz, respectively. With the parameter values indicated in the table, although the system is stable the damping is poor.

Further analysis shown in Table II indicates that the first mode $\lambda_{12}$ which oscillates with 61.94 Hz is related to the shunt compensator; whilst mode two $\lambda_{34}$ which oscillates with 50 Hz is related to the series compensator of the UPFC. The real eigenvalue $\lambda_5$ is related to the capacitor. For $R_{SH} = 0$, the system becomes unstable as the real part of $\lambda_{12}$ is positive; the real eigenvalue $\lambda_5$ is also positive. If $R_{SH}$ is zero, the real part of mode two $\lambda_{34}$ becomes positive and therefore the system is unstable. If the capacitance $C$ is zero, $\lambda_5$ becomes zero, which indicates that the order of the system is reduced to 4.

Tables III, IV, V, and VI show the effects of the resistances and the inductances of the shunt and series parts of the UPFC. Increasing the resistance values leads to an increase in the magnitude of the real part of the corresponding eigenvalues, thus improving stability. Higher values of the shunt or the series inductances ($L_{SH}$ or $L_{SE}$) deteriorate stability.

Table VII shows the effect of the capacitance $C$ on the system stability. The value of $C$ influences mainly the real eigenvalue $\lambda_5$. If the value of $C$ is increased, the magnitude of $\lambda_5$ increases. Mode one $\lambda_{12}$ is also affected by the value of $C$. As $C$ increases, the magnitude of the real part decreases, whilst the imaginary part increases indicating higher frequency of oscillation in the system response.

Tables VIII and IX show the effects of the modulation indices $m_{SH}$ and $m_{SE}$ on the system stability. The two oscillatory modes $\lambda_{12}$ and $\lambda_{34}$ are not much sensitive to variations in $m_{SH}$ or $m_{SE}$. The modulation indices have direct effect on the real eigenvalue $\lambda_5$. For $m_{SE} = 0.1$ the system is unstable because $\lambda_5$ becomes positive.
TABLE VIII
EFFECT OF SHUNT MODULATION INDEX

<table>
<thead>
<tr>
<th>m_sh = 0.1</th>
<th>m_sh = 0.3</th>
<th>m_sh = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,2} = -31.42 \pm 314.16i )</td>
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<td>( \lambda_{1,2} = -31.42 \pm 314.16i )</td>
</tr>
<tr>
<td>( \lambda_{3,4} = -32.45 \pm 304.30i )</td>
<td>( \lambda_{3,4} = -32.45 \pm 304.30i )</td>
<td>( \lambda_{3,4} = -32.45 \pm 304.30i )</td>
</tr>
<tr>
<td>( \lambda_5 = 2.07 )</td>
<td>( \lambda_5 = 1.13 )</td>
<td>( \lambda_5 = -6.00 )</td>
</tr>
</tbody>
</table>

TABLE IX
EFFECT OF SERIES MODULATION INDEX

<table>
<thead>
<tr>
<th>m_se = 0.1</th>
<th>m_se = 0.3</th>
<th>m_se = 0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{1,2} = -25.58 \pm 396.27i )</td>
<td>( \lambda_{1,2} = -25.58 \pm 396.27i )</td>
<td>( \lambda_{1,2} = -25.58 \pm 396.27i )</td>
</tr>
<tr>
<td>( \lambda_{3,4} = -31.42 \pm 314.16i )</td>
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<td>( \lambda_{3,4} = -31.42 \pm 314.16i )</td>
</tr>
<tr>
<td>( \lambda_5 = 11.68 )</td>
<td>( \lambda_5 = 9.62 )</td>
<td>( \lambda_5 = 7.45 )</td>
</tr>
</tbody>
</table>

VI. DISCUSSIONS AND CONCLUSIONS

Stability analyses of the UPFC have been performed using a mathematical model describing system dynamics. The UPFC is based on the pulse width modulation (PWM) technique assuming an approximate inverter output voltage and neglecting the harmonics for simplifying the analysis.

The non-linear mathematical model is based on the transformation of the three-phase system equations into an orthogonal synchronously rotating coordinate system (dq0).

The steady state analysis has been applied first to demonstrate the system stability in a 3-dimensional parameter space. This is to illustrate the effect of the modulation ratio and phase displacement on the stability of the UPFC device. When the modulation index of the series inverter is changed from 0.5 to 0.1 at the same phase displacement \( \alpha_{SE} = 40^\circ \), the system becomes unstable at a certain value of the shunt inverter modulation index \( m_{SH} \). Therefore with the steady state analysis the performance of the system for specified operating data can be investigated.

A linearized model of the UPFC has been obtained in a state-space form to study the stability of the system for certain operating point. The model has been used to investigate the impacts of system parameters on small-signal stability. The stability has been evaluated by using eigenvalue analysis. The results have shown that the system may become unstable for certain values of system parameters. The UPEC system has two oscillatory modes and one monotonic. The frequencies of oscillations of these modes range from about 50 to 60 Hz. In general, the damping ratios of the oscillatory modes have found to be low. This conclusion suggests the need for stabilizing control signals to improve the dynamic performance of UPFC devices.

The simulation results are useful for system designers and planning engineers in selecting proper parameter values for maintaining stability and setting acceptable technical specifications.

REFERENCES