# Forthcoming: Journal of Socio-Economics

How Realistic is the Supply/Demand Equilibrium Story?
A Simple Demonstration of False Trading and its
Implications for Market Equilibrium

Neil H. Buchanan Rutgers School of Law – Newark 123 Washington Street Newark, New Jersey 07102 neilb@andromeda.rutgers.edu March 29, 2006 ABSTRACT:

Transactions at non-equilibrium prices are "false trades." Under

standard assumptions, markets without false trading produce Pareto-

efficient outputs. This paper demonstrates graphically the complications

created when false trades occur, showing that quantities produced deviate

from Pareto-efficient quantities except under unique conditions. In a general

equilibrium framework, this spills over to cause Pareto-inefficient results in

other markets as well. These observations call into question the use of

standard supply-and-demand equilibrium theory as a starting point for

policy analysis.

JEL Codes: B41, D50

Keywords: Tâtonnement, False Trading, Pareto Efficiency, Convergence,

Disequilibrium

This paper uses very simple graphing techniques to re-examine one of the most basic issues in neoclassical economics: convergence to market equilibrium. As has long been noted (see especially Arrow and Hahn, 1971), the absence of the so-called "Walrasian Auctioneer" in a market creates a difficult theoretical problem. Specifically, even if one could divine the excess demand functions in every market, and even if there were a set of prices that would create simultaneous general equilibrium in all markets, how could we be confident that any individual market will actually reach its equilibrium—much less that an entire market economy will actually find its way into its general equilibrium? The process known as *tâtonnement* (a French word meaning "groping" or "trial and error") is the mechanism through which neoclassical economists have typically attempted to demonstrate that markets move toward their (partial or general) equilibrium values.

An arguably plausible description of the real-world process of market clearing shows, however, that even under the usual assumptions of perfect competition, the neoclassical equilibrium output in any particular market will only be produced under a unique set of conditions—conditions that do not presumptively hold in real markets.

(This is the case for all but a very special class of markets, i.e., futures markets.)

Moreover, this problem—the failure to reach Pareto-efficient outcomes—exists in both partial equilibrium and general equilibrium frameworks.

The standard story motivating our understanding of market equilibrium is therefore potentially quite misleading. It requires that there be no "false trading," that is, trading at prices other than general equilibrium values. A graphical analysis that carefully depicts the

nuances of the groping process is significantly more complicated than the simple supply and demand story. This complication, moreover, is not due to transaction costs or other supposed market imperfections. It is, rather, fundamental to the process of trading itself.

I argue below that the standard textbook *explanation* of market adjustment is generally accurate and, therefore, that the standard *graph* is, at best, optimistic. This is contrary to the standard story. If pressed, most economists would most likely claim that the graph (and, more fundamentally, the underlying algebra) is correct and that the explanation is merely "fudged" to make students' lives easier. This difference is not merely unfortunate; it highlights an underappreciated, central difficulty with standard economic theory.

After demonstrating how false trading changes the results of the standard analysis, I will draw out further implications and discuss some extensions of these conclusions. Most importantly, attempts to extend the analysis to explain dynamic rather than static processes do not obviate the problem. Rather, they make the problem even more challenging to the standard analysis.

### I. Theoretical Background

The basic result demonstrated later in this paper—that standard market-clearing analysis ignores false trades and thus does not capture potential sources of disequilibrium—is neither new nor seriously disputed. Economists who work in this area have long since concluded that convergence to equilibrium is, at best, an open theoretical issue. For example, Franklin Fisher (1987, p. 26) points out:

"... [W]e have no rigorous basis for believing that equilibria can be achieved or maintained if disturbed. Unless one robs words of their meaning and defines every state of the world as an 'equilibrium' in the sense that agents do what they do instead of

doing something else, there is no disguising the fact that this is a major lacuna in economic analysis."

It is always possible, of course, that even a "major lacuna" ultimately might have no impact on the validity or usefulness of a theory, if the theory's central elements do not in some important sense rise or fall on the missing piece of the puzzle. Unfortunately, neoclassical theory does rely crucially on the achievement and maintenance of equilibria. Fisher (1987, p. 27) goes on to note:

"Tâtonnement stability requires extremely strong special assumptions. This has extremely important implications. Indeed it is not too strong to say that the entire theory of value is at stake. ... Moreover, comparative static analysis, that major tool of theory, will miscompute the effects of a displacement of equilibrium ..."

Indeed, long before Fisher wrote those words, F.Y. Edgeworth (1934) noted: "[T]he equations of exchange enable us at best to determine the final position, not the steps by which it is reached."

In one sense, therefore, this result is "well known." That is, there is a literature that has concluded that supply and demand analysis is, at best, overdrawn. (See, for example, Fisher (1976), Hahn (1961), Hahn (1962), Hahn and Negishi (1962), Scarf (1960), and Uzawa (1962). Hahn and Negishi show that most work to that point relied on tâtonnement equilibrium. More recently, Goeree, Hommes, and Weddepohl (1998) extend the tâtonnement analysis into multi-dimensional systems, finding that the results are bounded but not unique.) While this paper is not designed to review the historical literature on this subject, it is important to recognize that the concept of convergence to equilibrium has been the subject of much discussion.

In a more practical sense, however, this important logical gap in standard market theory might not be so well known at all. The tâtonnement concept is apparently absent from most curricula. Graduate students, to say nothing of undergraduates, are rarely if ever taught that there is no theoretical certainty that a perfectly flexible market will clear at Pareto-efficient values. In fact, the consistent message in most economics courses appears to be that when there are no artificial imperfections (such as monopolies, externalities, etc.), markets will reach an efficient, desirable equilibrium set of outputs.

The absence of the subject from economics courses is not—or need not be—the result of some kind of conspiracy. Instead, the problem is much more nuanced. As Schiffman (2004, p. 1083) describes the process generally:

> [E]xclusion is subtle; perhaps the term 'strong deemphasis' is more accurate. Often, a deemphazised idea has been articulated by a well-known mainstream economist (perhaps even a Nobel Prize winner), in a prestigious journal that is universally read. It might also have appeared in a popular textbook. Yet, there may be an impression that the idea has gone out of style. The idea may be far from the consciousness of most mainstream economists, motivate very little research, and be discussed only within a small circle. It is not mentioned to graduate students (let alone undergraduates), who are unaware of its existence.

The practical importance of this gap in the theory can most easily be found in the applied fields of economics, which have increasingly come to emphasize market-clearing analysis. For example, Mankiw (1988, p. 23) argued that the major advance in macroeconomics in the previous two decades was its newfound reliance on "rigorous" microeconomic foundations—foundations that very much rely upon the notion that

Recent conversations with graduate students at one of the top Ph.D. programs in Economics in the United States confirm that tâtonnement is either not taught or not emphasized. Students who had passed their theory exams, when asked if they had been asked to learn about tâtonnement, replied, "What's that?"

markets clear at their general equilibrium values. Under this view, market-clearing models are superior to older macroeconomic models, which relied on *ad hoc* assumptions that lacked such supposedly rigorous microeconomic building blocks.

Similarly, adherents of the so-called Law and Economics movement frequently argue that legal analysis should be dedicated to designing a legal framework that guarantees that Pareto-efficient outcomes will be achieved in every market. "How can the legal system set efficient prices...? In a law and economics perspective, customary law can be viewed as a process for generating legal rules that is analogous to a price mechanism in a partial equilibrium framework." (Parisi, 2001, abstract). (Note that this emphasizes partial rather than general equilibrium. As demonstrated below, neither the partial nor the general equilibrium framework is immune to the problems raised by false trading.) Efficiency is thus achieved either by removing legal structures that inhibit market clearing or by attempting to mimic the hypothetical result that a market would reach in supply-and-demand equilibrium.<sup>2</sup>

The analysis below, then, is designed to introduce a new (and very simple) way to visualize why the concept of false trading matters, as a way of reminding some readers, and perhaps informing others, that we cannot safely assume that markets always clear at

The law and economics field faces the additional difficulty that many of the people studying it have not undergone graduate training in economics—or, at least, have not been required to do so—further increasing the likelihood that they have not come across the issues highlighted here. As noted above, however, having graduate training in economics is no guarantee that a scholar has been exposed to these issues.

their equilibrium (supply-equals-demand) values. This is, therefore, not a paper about the history of economic thought on convergence to equilibrium but is, rather, an attempt to explore and illuminate the implications of this concept for the use of supply-and-demand analysis in analyzing real-world markets.

# II. The Starting Point—Simple Supply and Demand Analysis

One of the first concepts presented to students in a "Principles of Economics" class is market equilibrium, that is, the clearing of product and factor markets through price adjustment. The description is usually a variant on the following explanation, taken from one of the market-leading introductory textbooks:

"Could a price [above equilibrium] persist *over a period of time*? Certainly not. The very large surplus... would prompt competing sellers to bid down the price to encourage buyers to take this surplus off their hands.

"Can [a price below equilibrium] persist as a market price? No. Competition among buyers will bid up the price...

"By trial and error we have eliminated every price but [the equilibrium price]. At [this] price ..., and only at this price, ... there is neither a shortage nor a surplus..."

(McConnell and Brue, 1990; first emphasis added; second emphasis in original)

That explanation, or a close variant of it, is offered in most Principles of Economics textbooks currently available. Professors and teaching assistants hammer away at this "simple" concept at the beginning of both micro- and macroeconomic principles courses. Whether in words, algebra, or on a graph, students quickly learn that the price charged for any good "tends toward" (or "is forced to equal" or "moves naturally toward," etc.) its equilibrium price—the price that equates quantity supplied with quantity demanded. Unfortunately, this seemingly innocuous recitation of the standard supply and demand

lecture glosses over a fundamentally important logical gap. The clue is in the assertion that agents will bid the price toward equilibrium "over time."

As an example, one can imagine observing that the price in a particular market was \$25 at 2:00pm, \$35 at 3:00, \$38 at 4:00 and \$40 at 5:00. Faced with these data, one might conclude that "the price tended toward \$40 over time." At the same time, the quantity would also be "tending" toward its equilibrium value. But, as Joan Robinson asked (1978, 144), how do quantities *tend*? Quantities either are sold, or they are not sold. Continuing with the preceding example, if one also observed that 5 units were bought between 1:00 and 2:00, 9 units between 2:00 and 3:00, 14 units between 3:00 and 4:00, and 20 units between 4:00 and 5:00, what can one say about the "tendency" of the quantity in this market? Even with the convenient data assumed here—an upward progression in quantities as the afternoon proceeds—it is still unclear whether the quantity was tending toward 20 units (the total sold in the final hour) or 48 units (the total quantity sold during the afternoon). A time frame must be specified to know which quantity matters. It is the logical difficulty presented by the notion of quantities "tending"—and the typically unspoken issue of the passage of time while market participants make trades—that is the basis of the following analysis.

### <u>Illustrating Discontinuities in Demand and Supply Curves</u>

The analysis below is built upon almost all of the standard features of a simple microeconomic, supply-and-demand analysis. Economic actors are assumed to have reservation prices distributed along a continuum (which is equivalent to saying that there is no assumption of a representative agent). Standard assumptions about continuity, etc. are preserved. The fundamental difference between the analysis below and the standard textbook analysis is the suspension of the assumption that the Walrasian Auctioneer guarantees immediate clearing of all markets, which is functionally equivalent to assuming

that there is no re-contracting and that goods are not resold. This difference forces the analysis to focus more carefully on the passage of time.

Why is it necessary to introduce the assumption that goods are not resold, or that there is no recontracting? A clear explanation of the assumptions underlying the standard equilibrium story is found in Henderson and Quandt's classic graduate-level mathematical microeconomics textbook:

"Imagine that buyers and sellers arrive in the market without any fore-knowledge as to what will become the going price...

"Assume that production is instantaneous and producers arrive in the market without any actual output... Whenever a buyer and a seller enter into a contract, they both reserve the right to recontract with any person who makes a more favorable offer. Assume that some consumer makes an initial bid and offers a price of p<sup>0</sup> dollars for the commodity. This price is recorded and made public by an auctioneer who is an impartial observer of the trading process. Buyers and sellers will attempt to enter into contracts with each other at the price  $p^0$ ... Some of the consumers who have not been able to satisfy their demand will be induced to raise their bids... As soon as this higher price p(1) is recorded and made public by the auctioneer, sellers break their old contracts and recontract at the higher price... The process of recontracting continues as long as the price announced by the auctioneer is below the equilibrium price, i.e., as long as the quantity demanded exceeds the quantity supplied. When the equilibrium price is reached, neither consumers nor producers have an incentive to recontract any further. Recontracting is discontinued, entrepreneurs instantaneously produce and deliver the output for which they have contracted, and the exchange is completed." (Henderson and Quandt, 1980, 145-146, emphasis in original)

This stylized description of the market process is arguably a close description of trading for commodities that have futures markets. By definition, such markets are characterized by actors having the luxury of long time periods during which they can contract and recontract, and the markets are also populated by actors who are well-informed and familiar with the bidding and recontracting processes. Futures markets also rely upon goods that do not lose value (depreciate) over time. The very specific and

limited nature of such markets, however, highlights the importance of the false trading problem for all other markets.

The important welfare result that is guaranteed by reaching market equilibrium is, of course, that the quantity produced is Pareto-efficient. That is true because economic actors will voluntarily trade every fractional unit of the good that is more valuable (according to the willingness-to-pay criterion) to the buyer than to the seller, whereas they will not trade units for which the opposite is true. Social welfare is maximized in this story, it is worth repeating, because the *quantity* traded is the equilibrium quantity. The price is merely a lever that adjusts to guarantee that efficient outputs are produced.

The key to understanding the model below is to pay especially close attention to the (entirely standard) definitions of demand and supply curves: the demand (supply) curve shows, for every price, the quantity that households (firms) would be willing to buy (sell) at that price. Each point on a curve corresponds, by definition, to a consumer or firm who values that marginal unit of the good at exactly the price depicted on the relevant curve. Continuity is achieved by assuming infinite divisibility of units. Thus, in Figure 1, consumers would be willing to purchase 50 units of bread at a price of \$1.00 per loaf. The demand curve tells us that there is only one infinitesimal fraction of a loaf for which a consumer is willing to pay a price of exactly \$1.00 per loaf. Every quantity less than 50 units could be sold for more than \$1.00 per loaf.

If trading begins in this market (on any given "day") at a price of \$1.00, people will learn of the price and start to arrive at the bakeries to buy loaves of bread. Of course, people have no way of knowing whether the price that they are paying or receiving is "false." It is simply the current price. If, after an arbitrary period of time, the price of bread changes, what can we say about the demand for the product? Imagine that 10 loaves of bread were purchased while the price was \$1.00. Those consumers who

purchased their desired quantities will now have a demand of zero, meaning that they must be removed from the demand curve.

In Figure 1, if we assume that the consumers who were willing to buy bread for prices between \$1.25 and \$1.50 per unit were the ones who actually bought the ten loaves before the price was changed (who, as it were, queued up first), we now must "break" the demand curve to exclude those now-satisfied demanders. The consumers who were willing to purchase up to 30 units at prices of \$1.50 or higher are still in the market, as are the consumers who were willing to make purchases at prices less than \$1.25. Shifting the below-\$1.25 demanders ten units to the left and erasing the 10 units of demand between \$1.25 and \$1.50 depicts the disappearance of the ten now-satisfied units of demand. This is shown by the bold, discontinuous curve, D'.

A similar story can be told for any given supply curve. Consider the supply curve depicted in Figure 2. If trade opens at \$15 per unit, and 20 units are sold by suppliers who would have been willing to receive between \$9 and \$12 per unit, the supply curve breaks, and the discontinuous supply curve is as shown in bold, S'. One might object that the usual assumption in microeconomics is that marginal costs increase only as output increases; so it would seem that any satisfied units of supply would have to come from the origin, since they are the first units produced. However, if the market includes heterogeneous firms, some of which are higher-cost producers than others, there is nothing to guarantee that the lowest-cost producers will happen to be the ones who actually sell their output during the period of time when the price is \$15. As with demanders, anyone who is willing to transact at the current price might be able to complete a transaction during the period of time when the price is \$15, depending on whether customers happen to show up at their door.

Note that two of the standard, unrealistic assumptions of perfect competition theory are preserved in this analysis. First, the Law of One Price still governs, which is to deny the reality in many markets that different firms—and even different outlets of the same

firm—charge different prices at the same point in time. Second, this model assumes that the groups of demanders and suppliers who "disappear" from their relevant curves (because they have made false trades) are grouped together. It is certainly possible that, for example, the ten units in the demand example above were sold to consumers whose maximum acceptable prices were \$3.72, \$3.56, \$3.50, \$3.29, \$3.00, \$2.90, \$2.84, \$2.77, \$2.60, and \$2.53. For the sake of graphing, however, it is useful to assume that the satisfied units are compact, allowing us to limit ourselves to only one discontinuity per time period.

Were either of these assumptions to be relaxed, moreover, the standard result would be even *less* likely to occur than in the analysis below. By maintaining the assumptions of the standard story, with the exception of false trading, we can be certain that the new result is solely due to this new assumption.

# III. A Graphical Analysis of False Trading

We can now examine what happens in an actual market as the process of "tending toward equilibrium" progresses. Start with Figure 3, a run-of-the-mill supply and demand graph. The equilibrium values of price and quantity are  $P^*$  and  $Q^*$ , respectively. If the price is at  $P_H$ , then the quantity supplied  $(Q_{S,H})$  exceeds the quantity demanded  $(Q_{D,H})$ , creating a surplus.

According to the textbooks, the market participants will "notice" that there is a surplus. How? Presumably, the firms start to sell their product at price  $P_H$  and gradually find that they are not making as many sales as they had hoped. When they notice this, they lower their prices and wait again. If the new price clears the market, then they have no reason to change the price; otherwise, they will continually change their price in a ceaseless search for profits. While it requires additional special assumptions to guarantee that this progression converges on equilibrium, the analysis below will proceed as if that is true.

Suppose that we start with a market that had actually achieved its equilibrium price at the close of business on Tuesday. On Wednesday morning, consumers might wake up with different preferences, reflected in the demand curve D<sup>#</sup> on Figure 4. Therefore, the old equilibrium point is no longer the equilibrium point for this market. The textbooks would say that the next step is simple: the market adjusts to the new equilibrium: P\*\* and Q\*\*. But will it? And if so, how?

If we assume that, before the sellers notice that they are no longer in equilibrium, a random number  $\alpha$  of false trades occurs at the old price, P\*, then we need to know who made the false purchases and who made the false sales.<sup>3</sup> If the consumers who valued those units most highly happen to buy those  $\alpha$  units, then the demand curve will simply shift to the left by  $\alpha$  units. This is shown in Figure 5 (which does not display the original demand curve, D, since it is no longer relevant). The  $\alpha$  units that are highest on the demand curve have been satisfied, so the remaining unsatisfied demand is depicted by the left-shifted curve D'. Similarly, if the falsely-sold units happen to be the  $\alpha$  units that are produced by the lowest-cost producers, then the unsatisfied supply will be represented by the left-shifted curve S'.

At this point, there would seem to be no problem. The intersection of the S' and D' curves lies at  $P^{**}$ , and the quantity at that equilibrium is exactly  $\alpha$  units less than  $Q^{**}$ . If  $(Q^{**}-\alpha)$  units are now sold, in addition to the  $\alpha$  units "falsely" sold at price  $P^{*}$ , the total quantity will equal  $Q^{**}$ , which is the Pareto-efficient quantity with the new demand curve, D#, and the old supply curve, S. It is as if the vertical axis had simply been shifted to the right by  $\alpha$  units. The standard result is still in sight—at least, for a partial equilibrium story—even though some units have traded at non-equilibrium prices.

The range within which  $\alpha$  may fall is from zero up to the quantity that is demanded on D# at P\*, because every unit in that range is more highly valued by the potential buyers than P\*.

Alternatively, let us continue to assume that the highest valued demanders made the false trades, but now the suppliers who were lucky enough to sell at the too-high price were those who valued their production most highly. In Figure 6, this is shown by the disappearance from the supply curve of the  $\alpha$  units to the left of Q\*—a result certainly consistent with the definition of the supply curve, since those producers received a price (P\*) at or above their subjective valuation of their production. This new, discontinuous supply curve, along with the left-shifted demand curve, now defines a new equilibrium with some rather important properties.

When they realize that they are in a surplus situation, the firms (and, possibly, the households) will bid down the price. There is a vanishingly-small probability that they will immediately bid the price to  $P^{\wedge}$ , the price at the intersection of the now-relevant, broken supply and demand curves, S' and D'. This would clear what remains of the market. Once the market has cleared, though, what are the equilibrium values of P and Q? The price has settled at  $P^{\wedge}$ , *not* the equilibrium price that the textbooks would have predicted (in this case,  $P^{**}$ ); and the <u>total</u> quantity sold is  $(Q^{\wedge} + \alpha)$ , which is unequivocally greater than  $Q^{**}$ .

If the firms are not so lucky in choosing their second price, the result will be a second round of false trades. Since the firms can, in principle, choose any price below P\*, there is no way to predict whether they will choose a price above or below P^, which is the price necessary to clear what remains of the supply and demand in the market. Even more important, there is no way to predict the order in which buyers and sellers will line up to satisfy their preferences at the potentially endless series of false prices. Each succeeding day could, therefore, start in a situation that was a disequilibrium during the previous day; and the process would be analytically identical to the process above, ending in a cleared market on any given day only if the price chosen were the unique price at which the (repeatedly-broken) demand and supply curves intersect.

It is perhaps tempting to surmise that the number of false trades might diminish with time as the market experienced repeated failures to reach equilibrium and, therefore, that the process would ultimately come to rest. If that were true, then at least the theory could stand on the claim that while the standard equilibrium is not reached immediately, at least we will observe a *tendency* toward some kind of equilibrium. Unfortunately, this is precisely the conclusion that Fisher described as being a "major lacuna" in neoclassical theory, as discussed above. That is, we have no theory that demonstrates that it will actually happen. While there is certainly a non-zero probability of the market clearing on any given day of trading, that is the most that can be known from the theory. We do not know how large that probability is. And even if the market ultimately settles on a point that is an "equilibrium" in the sense of clearing what remains of the market, the problem remains that the total output produced and sold is not generally the Pareto-efficient output that is generally assumed.<sup>4</sup>

The important implication of this analysis is that there is only one way for the Pareto-efficient result to hold: the demanders and suppliers must line up for their false transactions such that no unsatisfied demander who values her marginal unit of the good

Marshall (1920, p. 151 footnote 1) concedes the point that there are some situations in which the market can reach an equilibrium that is not a true equilibrium. Describing a situation where a merchant is "in urgent need of a certain amount of ready money," he concludes that the "price at which the market closed would be an equilibrium price: and though not properly described as the equilibrium price, it would be very unlikely to diverge widely from that price." (emphasis in original) Thus, while he considered it a matter of lesser importance, he was clearly aware of the possibility that the market could clear at a point that was not efficient in his sense of the term.

less than any other unsatisfied demander—and no unsatisfied supplier who values her marginal unit more than any other unsatisfied supplier—ever makes a false transaction. It is, at best, an open question whether the Invisible Hand has the dexterity to coordinate a sequence of transactions of such exquisite complexity.

Put differently, this is not a "best case/worst case" story. It might better be described as a "best case/all other cases" story. There *is* a possibility that every market will clear in the way that the textbooks describe, but the assumptions necessary to support such an outcome seem highly unlikely. This is, of course, ultimately an empirical issue, and it could indeed turn out that life resembles the best case; but even the acknowledgement that reaching the Pareto-efficient equilibrium (or tending toward that equilibrium) is an empirical matter is an important qualification to the usual presumption. If the "usual story" is not known to be reliably likely, that uncertainty should inform our analysis.

### Other Cases and Extreme Curves

Clearly, one could draw graphs starting with a price that is below the marketclearing price. The same arduous graphical analysis would show that the range of possible results is limitless, and that the probability of reaching the Pareto-efficient result is again unknown.

This result does not hold, however, when either curve is perfectly vertical. This is easily understood and straightforward, so we can thankfully omit the graphs. In the case of a vertical D or S curve, the immutable fact is that the total quantity sold will always equal the quantity defined by the perfectly inelastic curve. It does not matter how many different false prices are tried, because either the demanders or suppliers (whichever group is inelastic) are going to achieve their desired total quantity. As each inelastic actor is satisfied, they leave the picture. In terms of equilibrium quantities, therefore, the standard story is correct for zero-elasticity cases—though only for the partial equilibrium analysis.

Similarly, a horizontal supply or demand curve will produce a Pareto-efficient result. In Figure 7, we see a horizontal supply curve and a price from "yesterday" below today's equilibrium. Since  $P^*$  is below the marginal valuation of every produced unit, no seller will enter a false trade. If buyers bid up the price to  $P^{**}$ , the story is over. Even if the price jumps higher, say to  $P^{\wedge}$ , some false trades will be made (again, assume a random quantity  $\alpha$ ), the demand curve will become discontinuous, and the price will change. The remaining demand, however, has shifted exactly  $\alpha$  units to the left, because  $\alpha$  units were sold at  $P^{\wedge}$ . Even if the price reaches  $P^{**}$  only for the marginal demander who values the good at exactly  $P^{**}$ , the total quantity sold will still add up to  $Q^{**}$ . Similar logic will prevail for a horizontal demand curve.

# General Equilibrium

In a general equilibrium framework, the above story becomes even more complicated, as the factors that might cause quantities to deviate from their general equilibrium values multiply. Even if the underlying preferences or technology in any given market do not change overnight, changes in other markets can affect the equilibrium in the original market. Thus, even assuming that a general equilibrium has been established at some point, a change in *any* market can lead to groping processes in *all* markets. For example, recall that Figure 5 displayed a situation in which transactions at the false price did not lead to an inefficient quantity in that market, due to the fortuitous order in which actors lined up at the different prices. In a general equilibrium framework, however, the price paid does matter, because different prices paid in one market generate income effects in other markets.<sup>5</sup>

16

As a technical clarification, the income effect will exist where there are systematic differences in the preferences of the traders on either side of the supply-demand interaction in the relevant product market. Although such differences seem likely (indeed,

That is, if a general equilibrium solution was based upon the requirement that people were going to buy a total of 50 computers at \$1000 each, but some of them actually pay more than \$1000, then the total spent on computers will exceed \$50,000. This money has to come from somewhere and go somewhere, which means that demand and supply curves in other markets will shift. Again, this sets in motion a new groping process in markets for which nothing fundamental has changed, i.e., for which the underlying preferences or technology have not shifted. Even the seemingly benign cases discussed above thus become problematic. If false trades are possible, therefore, all markets are opened up to the possibility of non-Pareto-efficient equilibria.

The implication of this analysis, therefore, is that we cannot be confident in any rigorous way that prices will adjust to create either partial or general equilibrium, which means that freely-adjusting markets may not reliably provide Pareto-efficient results. One can choose to call the end result of such processes equilibrium; but this is not an equilibrium with the welfare properties normally associated with market equilibria, as discussed further below.<sup>6</sup>

an assertion that there are *no* differences would be an assertion in need of proof), it is true that this is also ultimately an empirical matter, as discussed above.

A colleague argued that the term "equilibrium" implies that everyone is content with their choices, given the choices made by others. Clearly, that is not the case if those who made false trades at high prices see others buying at lower prices (and, I would add, if they believe that their trades are otherwise identical to the lower-priced trades). Moreover, a non-Pareto efficient equilibrium would be an oxymoron by this definition, such that the situation depicted in Figure 6 at (P^, Q^) could not be called an equilibrium

#### IV. Criticisms and Extensions

At first blush, some readers might suspect that the graphical analysis above is a variation on the familiar "Cobweb Theorem." While the graphical technique shown here might appear mechanically similar to the Cobweb analysis, however, the underlying analysis is quite distinct. The Cobweb Theorem postulates a set of difference equations, where the supply curve is dependent on a one-period lagged value of price. The model then displays a series of values of P and Q, where the series converges or diverges depending upon the relative slopes of the supply and demand curves. Each of those values is, however, an *equilibrium* between the demand curve and the current-period supply curve, the latter of which is assumed to be vertical (reflecting the inelastic quantity of the good that exists at any given moment). The model, therefore, displays a dynamic process caused by timing differences between supply and demand decisions. It does not challenge the standard assumptions of market-clearing analysis. The analysis presented in this paper, on the contrary, assumes that both supply and demand decisions are made in response to contemporaneous price information. The unexpected result is due entirely to trades made *out of equilibrium*.

A more telling response to this analysis might be that it inappropriately relies on a strictly static analytical framework. It is certainly true that the satisfaction of a supplier or a demander at a particular moment does not *permanently* remove them from a market. The unit of time on the horizontal axis is an essential piece of information. For example, does the demand curve for bread in Figure 1 represent "loaves per day" or "loaves per week"? Even if a person is lucky enough to make a false trade on Wednesday at a low

at all. In the current context, however, I am using equilibrium in the more traditional sense of a situation in which no further trades would be made. It is simply a point of rest.

18

non-equilibrium price, they might reappear on Thursday's demand curve. Therefore, even if the market has achieved some kind of (non-Pareto-efficient) equilibrium by the close of business on Wednesday, the whole game starts anew on Thursday, anyway. (Moreover, the game would start on Thursday at an arbitrary price.)

Since any time period—in this case, a day—is arbitrary, there is no way of knowing when the satisfied demanders (or suppliers) will re-enter the market, seeking to purchase (sell) further units at their possibly-new reservation prices. Therefore, one might argue, the analysis must be carried out using "continuous equilibrium," in which every instant of time is theoretically a separate static moment of trading.

Unfortunately, this step—which, as Eatwell and Milgate (1983) suggest, is the direction in which most modern work in economics has moved—actually makes the problem worse. Continuous equilibrium necessarily assumes that the trades that take place at every moment are Pareto-efficient equilibrium trades (i.e., without false trading). This is not proved; it is assumed. As the time period involved shrinks, though, this assumption seems less and less tenable. While one could imagine at least the possibility of convergence to some sort of equilibrium if the time period were a year, the supposition that economic actors can avoid false trades on a moment-by-moment basis is open to dispute, to say the least. Whatever the time period, however, the analysis above shows that any real-time process with false trades will result in the production of non-Pareto-efficient quantities.

It might also be possible to re-cast this discussion into a "repeated game" framework. That is, market participants might not be able to adjust prices quickly and perfectly the first time trading begins; but continuous opportunities to do so (with the opening of each new trading day—however long one defines a day) will allow them to perfect their abilities to move more quickly toward equilibrium prices while avoiding false trades.

Whether this can be proved mathematically has been addressed elsewhere (for example, in Kalai and Lehrer (1993) and Jordan (1992)) and is not central to the present analysis. That result, moreover, would simply mean that Walrasian equilibria can be reached, but only after some learning process has been completed. Thus, markets might fail to clear simply because the participants are still learning how to make them clear.

It is important to emphasize, however, that this is a possibility, not a certainty. Under appropriate assumptions, a repeated game framework might well demonstrate that at some point a day would come when the price and quantity start at equilibrium levels and stay there. In such a case, the question then becomes one of timing, with the definition of a "trading day" no longer an abstraction but the central question in determining the usefulness of a supply-and-demand model. The point here, though, is not to compare specific markets to see which have longer or shorter trading periods. Instead, the point is to suggest that the standard supply-and-demand result is derived from an implicit assumption that the trading day is short enough to make any deviations from the Pareto-efficient quantity mere curiosities.

At a minimum, therefore, this analysis implies that false trading is another potential explanation for sluggish price (and wage) adjustment—one which is not created by "imperfections" such as minimum wages, regulation, rent controls, etc. If the trading day is long enough, and if prices are therefore moving slowly enough to interact with production decisions, then the deviations of quantity from its equilibrium level will affect the use of scarce resources.

If new markets and new market participants are being created all the time, moreover, this would mean that there will be a continuously replenished source of disequilibrium. In other words, the world might not wait for the game to repeat itself enough times to reach the point where a day starts and ends with a price of P\* and a quantity of Q\*. On the way to that equilibrium, the S and D curves might shift due to

exogenous factors, requiring the learning process to discover and incorporate new information and to discard old information.

Moreover, since general equilibrium means that all markets clear simultaneously, even mature markets and market participants would not be immune to the spillover effects inherent in this process, because a market that is in the process of finding its equilibrium (or even one that is already there) can be thrown off by shocks to other markets that can change the prices of complements and substitutes, etc.

This analysis is thus not intended to make the unsustainable claim that there is no theoretical mechanism by which markets could reach equilibrium, all else constant, if enough time were allowed to elapse. Instead, even without challenging the assumptions underlying any such theory, the point is that any path that takes time to traverse can be disrupted. The discussion in this paper focuses on one of the less-discussed reasons why the path to equilibrium might take more time than we might initially assume, and it shows how the standard story is at least incomplete in explaining how markets could ultimately clear without any participants having engaged in false trading.

Finally, it is worth remembering that the contrast here is between any real-time picture of the process of markets groping toward equilibrium and the instantaneous equilibrium associated with the Walrasian Auctioneer. Although it is sometimes helpful to think of the auctioneer as if he were acting in real time—announcing prices, taking bids, calculating excess demands, announcing new prices, etc.—the point of imagining the auctioneer is precisely to imagine that disequilibrium is discovered and corrected instantly. The real world always takes longer than the auctioneer would take (by definition), because the auctioneer is a metaphor for perfect market clearing without false trades or the passage of time. The more reasons we have to believe that real-world market clearing is sluggish, uncertain, and subject to corrections, the more we should bear those imperfections in mind when applying the standard model.

# Is Being "At Rest" Enough?

The analysis in this paper demonstrates that the possibility of false trading significantly complicates the process by which prices change and outputs are determined. Yet this process can still end with the market in a situation that one could plausibly call equilibrium, i.e., the market has cleared, and there are no unconsummated trades. Is this not ultimately what matters? From the standpoint of welfare economics, it is not.

First, even if the market does clear at the end of a trading day, this is not an equilibrium in the traditional sense because it will not persist. When trading reopens on the next day, the market will almost certainly not be at rest—even if there have been no exogenous changes after the market closed the night before. An equilibrium, on the other hand, should continue to hold once it has been reached, *ceteris paribus*. In the situation depicted above, however, the "equilibrium" that exists at the end of one day is generally a disequilibrium the next day, even when there are no changes in preferences or technology.

Second, the welfare properties of the equilibria in this model are suboptimal in the Pareto sense, in that more units of the good are produced than should be produced. That is, the market produces an arbitrary number of units of the good whose marginal social costs, *ex ante*, exceed their marginal social benefits. This is true even though prices adjust to eliminate shortages and surpluses.

Finally, it is important to remember just how unique were the examples provided above. The groping process depicted in Figure 6 ended after only two rounds of price changes. Although any attempt to depict an alternative scenario would be extremely difficult to graph in a comprehensible fashion, false trades could actually continue for any arbitrary number of "gropes," with no mechanism to guarantee market clearing even at the end of any given day.

#### V. Conclusion

The assumption of simple supply-and-demand equilibrium can mask a fundamental problem inherent in the standard approach to economic equilibrium. At any given moment, we cannot assume that trades are made at equilibrium prices nor that markets "tend toward" any useful conception of equilibrium. Efficiency is not guaranteed, and the process of convergence to equilibrium is inherently open-ended.

Of course, while it could turn out as an empirical matter that all of the conditions necessary to achieve the Pareto-efficient equilibrium are actually achieved on a consistent basis in most (or all) markets, there is no principled reason to assume that that is so.

Unless and until such a demonstration is available, belief in the standard story of market equilibrium is a matter of faith. As Robert Chirinko and Robert Eisner (1983, p. 163) once noted in a somewhat different context: "Faith is indeed sometimes rewarded. But for our part, in this instance, we remain agnostic." Those who wish to continue to believe will find nothing in the analysis here to *prove* that we do not regularly experience Pareto-efficient equilibria. At the very least, though, the conditions required to turn such belief into reality are more complicated than we have typically assumed.

### References

- Arrow, K., Hahn, F., 1971. General Competitive Analysis. Holden-Day, San Francisco.
- Chirinko, R.S., Eisner, R., 1983. Tax Policy and Investment in Major U.S.

  Macroeconomic Econometric Models. Journal of Public Economics, 139--166.
- Eatwell, J., Milgate, M., 1983. Introduction, in: Eatwell, J., Milgate, M. (eds.), Keynes's Economics and the Theories of Value and Distribution. Oxford University Press, New York, pp. 1--17.
- Edgeworth, Francis Y., cited in a letter from R. L. Hall to Roy Harrod, 17 February 1934.
- Fisher, F.M., 1976. A Non-Tatonnement Model with Production and Consumption. Econometrica, 907--38.
- Fisher, F.M., 1987. Adjustment Processes and Stability, in: Eatwell, J., Milgate, M., Newman, P. (eds.), The New Palgrave: A Dictionary of Modern Economics, Vol. 1. Macmillan, London, pp. 26--29.
- Goeree, J.K., Hommes, C., Weddepohl, C., 1998. Stability and Complex Dynamics in a Discrete Tatonnement Model. Journal of Economic Behavior and Organization, 395--410.
- Hahn, F.H., 1961. A Stable Adjustment Process for a Competitive Economy. Review of Economic Studies, 62--65.
- Hahn, F.H., 1962. On the Stability of Pure Exchange Equilibrium. International

- Economic Review, 206--14.
- Hahn, F.H., Negishi, T., 1962. A Theorem on Non-Tatonnement Stability. Econometrica, 463--69.
- Henderson, J.M., Quandt, R.E., 1980. Microeconomic Theory: A Mathematical Approach, 3rd edition. McGraw-Hill, New York.
- Jordan, J.S., 1992. Convergence to Rational Expectations in a Stationary Linear Game.

  Review of Economic Studies, 109--23.
- Kalai, E., Lehrer, E., 1993. Rational Learning Leads to Nash Equilibrium. Econometrica, 1019--45.
- Mankiw, N.G., 1988. Recent Developments in Macroeconomics: A Very Quick Refresher Course. Journal of Money, Credit, and Banking, 22--44.
- Marshall, A., 1920. Principles of Economics, 8th ed. (abridged) Prometheus Books, Amherst NY, 1997.
- McConnell, C., Brue, S.L., 1990. Economics, 11th ed. McGraw-Hill, New York.
- Parisi, F., 2001. The Formation of Customary Law. George Mason Law & Economics Research, Paper No. 01-06.
- Robinson, J.V., 1978. Contributions to Modern Economics. Basil Blackwell, Oxford.
- Scarf, H., 1960. Some Examples of Global Instability of the Competitive Equilibrium.

International Economic Review, 157--72.

- Schiffman, D.A., 2004. Mainstream economics, heterodoxy and academic exclusion: a review essay. European Journal of Political Economy 20, 1079-95.
- Uzawa, H., 1962. On the Stability of Edgeworth's Barter Process. International Economic Review, 218-32.













