The impact of analyzing correct versus incorrect student work samples on students’ mathematical proficiency

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Correct versus Incorrect Student Work Samples
on Students’ Mathematical Proficiency

by

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Abstract

The purpose of this study is to determine if any gains in mathematical understanding differ if calculus learners analyze correct or incorrect student work samples and to investigate students’ perceptions of the effect of analyzing student work samples on their understanding of mathematical concepts. Calculus students will be assigned to two groups: one group analyzing correct student work samples and one group analyzing incorrect student work samples. What difference, if any, exists between groups in determining correct solutions to problems similar to the work samples analyzed? What difference, if any, exists between groups in whether they replicate errors similar to the incorrect work samples? What difference, if any, exists between groups in perceptions of how the analyses of student work samples increase understanding? How do students describe their experiences of analyzing student work samples? Data from enrollees in 10 sections of Basic Calculus at a large university will be analyzed using ANCOVA, independent-samples t-test, and inductive analysis (Hatch, 2002). Results will have implications for teacher practice; for example, revealed group differences will inform the teaching dilemma of what to do about common errors.

Keywords: mathematics, calculus, work samples, errors, mathematical proficiency
Introduction

Many high school graduates in the United States lack mathematical competencies that are considered crucial to success in college-level math courses. Among 2011 high school graduates who took the ACT, 45% met the math college-ready benchmarks; unfortunately, considering that not all students took the ACT, this group represents only 22% of all 2011 graduates (ACT, 2011). This common deficit in math college-readiness is accompanied by inconsistent graduation requirements; for example, currently, only 20 states, plus the District of Columbia, have raised their high school graduation requirements to include four years of mathematics courses that extend through Algebra II (Achieve, 2011). Furthermore, this mathematical deficit appears on college campuses where 50.1% of those seeking an Associate’s degree and 20.7% of those seeking a Bachelor’s degree must take remedial college classes (Complete College America, 2011). Since some studies show that almost one third of United States college freshmen are unprepared for college-level mathematics (Parsad & Lewis, 2003), effective ways to improve students’ mathematical proficiency (NRC, 2001) and to mend students’ mathematical misconceptions are needed.

Problem Statement

Because the transition from high school to college is plagued by mathematical deficits, college professors face the dilemma of deciphering the thinking and misconceptions college students bring to the math classroom. To this end, teachers who have taught the same course multiple times can often predict errors that students make on certain problems. However, this knowledge, by itself, is fruitless; teachers must decide what to do with this knowledge in their classrooms. Predicted common errors could be ignored by the teacher during instruction, mentioned during class in a warning manner, or tackled head-on. One way to confront common
errors and misconceptions is to have students examine, analyze, and reflect on erroneous student work samples in hopes that the underlying misconceptions will be revealed through this type of reflection. This study investigates the effectiveness of this approach. The National Council of Teachers of Mathematics (NCTM) emphasized communication and suggested that students listen to other students’ problem solving strategies in order to learn math concepts (NCTM, 2000); however, while researchers agree that critiquing others work increases understanding, few investigate the dilemma of looking at correct versus incorrect student work samples.

Some conclude that one way to improve mathematical understanding is to have students analyze others’ work, including incorrect work, in hopes of challenging student thinking and pointing out common errors (Kramarski & Zoldan, 2008). Borasi (1994) suggested using student errors as springboards for inquiry and found examination of errors to improve proficiency and confidence in math. However, others, as acknowledged by Borasi (1994) and Kramarski and Zoldan (2008), believe that showing erroneous examples may do more harm than good because students may duplicate the errors. Thus, while teachers of math courses are able to predict the mathematical misconceptions that commonly occur in their classes, they face the dilemma of whether to have students critique correct or incorrect solutions. Research is needed to determine whether having students reflect on correct or incorrect solutions is truly promoting deeper levels of understanding, rather than mere mimicry of procedures.

**Purpose of the Study**

The purpose of this study is to provide evidence that may resolve the dilemma presented above and determine if students gain mathematical understanding by examining samples of other students’ work and if any such gains differ if given correct or incorrect student work samples. In addition to revealing effectiveness of analyzing correct and incorrect student work sample, this
study will also investigate students’ perceptions of the effect of analyzing student work samples on their understanding of mathematical concepts. Calculus students will be assigned to two groups: one group analyzing correct student work samples and one group analyzing incorrect student work samples. The overarching questions for this study is: What difference, if any, exists in students’ mathematical understanding if given correct or incorrect work samples to analyze? More specific questions follow:

- Research Question 1: Does either analysis positively influence students’ overall achievement in the calculus course?
- Research Question 1: Does either analysis positively influence students’ correctly solving problems?
- Research Question 2: Are students more likely to commit common errors if asked to analyze incorrect work samples or correct work samples?
- Research Question 3: Which type of analysis (correct or incorrect) do students perceive as doing more to promote mathematical proficiency?
- Research Question 4: How do students describe their experiences of analyzing student work samples?

**Need for the Study**

This study will contribute to the literature surrounding the impact of having students critiquing peers’ work on mathematical understanding, in particular critiquing correct versus incorrect student work samples. On one side of the debate, many teachers believe that incorrect math should never be shown to students in any context because of students’ inclination to reproduce the incorrect math they have seen. On the other side of the debate, other teachers may believe that if students are only presented with correct solutions, then the students learn to mimic
CORRECT VS. INCORRECT STUDENT WORK SAMPLES

desired results without thinking about concepts, reflecting on meaning, or challenging the ideas of others. The findings of this study will show differences between groups who only analyze incorrect student work samples and those who only analyze correct work samples, provide evidence of group differences in replicating errors they have seen, and reveal how participants perceive gains in various aspects of mathematical reasoning. This study is needed so that instructional decisions and teacher beliefs about instruction may be informed by research. For example, if participants who always analyze correct student work samples are mindlessly mimicking correct solutions, as some believe, then their correct solutions will be more frequent than other groups’ while their perception of gains in conceptual understanding will be lower. On the other hand, if students repeat errors they have seen, as many believe, then this study may reveal a statistical difference between groups’ replication of errors.

Limitations

In this section are descriptions of the factors that could negatively affect the generalizability of this study. These limitations are organized into subsections according to which element of the study they involve: research design, testing, treatments, and groups.

Limitations in Research Design

Limitations include uncontrollable aspects of the course that could weaken the design of this study and thus weaken the generalizability of these findings. For example, because the treatments are a part of an entire calculus class, they cannot be administered in a vacuum-like experiment void of any other modes of instruction. Other limitations follow from the ethical obligation to treat all students in a class fairly.

Since participants’ analyses of student work samples are only one aspect of the course, their associated differences in learning outcomes may not emerge as statistically significant in
this study. In other words, a student’s learning in this class cannot be completely attributed to examining student work samples. Other contributing aspects of the course are class meetings, textbook examples and explanations, and online homework assignments. In an effort to overcome this limitation, data for Research Questions 2 and 3 will be collected about students’ performances on specific types of problems that are most related to the analyses of student work samples.

Because holding different course requirements for different students within the same course would be unethical, this study does not provide a control group (a group that is not analyzing student work samples), which is another limitation. This limitation will prevent us from quantitatively determining if examining student work samples is more effective than not examining student work samples. We will only be able to determine differences in effectiveness between the two groups examining different work samples. To lessen this threat to validity, the survey and the interview protocol will include questions to reveal any gains in understanding that the students may have recognized as resulting from the examination of student work samples.

**Limitations Involving Testing**

Limitations that involve testing include effects caused by giving a pretest, factors that could interfere with testing, and the limitations in scope and nature of the test questions. This section will discuss limitations within each of these aspects.

The existence of a pretest can affect the results of a study if students recognize problems from the pretest and remember how to solve them. Therefore, to limit effect on the results, I will not return the graded pretests to the students. However, the pretest will determine if the groups have similar calculus understanding before the course. A stronger research design that would account for the effect of the pretest is a Solomon’s four-group design, in which a treatment and a
control group are not pretested and are compared to those that are pretested (Solomon, 1949), but requiring assignments for only some of the students in the class would be perceived as unfair.

Another limitation is that the pretest will be administered prior to the drop/add date. As a result, any students who add the class after the first class meeting might be exposed to the online student work analysis prior to taking the pretest. However, the researcher will be able to track individual students’ access of the online material using the options available in Blackboard and will know if the pre-test was taken before or after accessing online material. Because the exams in this class must be consistent with the course description and adopted textbook, the researcher has limited control over the test questions. Some test questions may be a stronger measure of a student’s ability to follow prescribed rules than of a student’s comprehension of underlying mathematical ideas or a student’s creative problem-solving abilities. In an attempt to overcome this limitation, both the survey and the interview protocols will investigate more aspects of the students’ learning.

Limitations Involving Treatments

Ideally, analyses of student work samples arise from real-life situations where students critique each other’s work in study groups or in class discussions. In this study, work samples will be presented electronically and their analysis will be a required part of the course, which may mean that students do not perceive the tasks as relevant and the students’ motivation to reflect may be limited. Lewison (1997) found this negative reaction to the requirement of written reflections in a teacher education course, and Knowles’ study indicated a lack of perception of relevance of the tasks may limit the depth of participants’ responses and their learning (1984; Knowles, Swanson, & Holton, 2011). Information gathered from interviews with another
researcher who audited the Basic Calculus course at this university and who analyzed the student work samples was used in the design of the study to overcome this limitation.

Knowles’ theory of andragogy also implies that a perception of authenticity would aid in the participants’ learning. Although the student work samples that are used in the treatments are not authentic student work, the student work will appear as realistic student work, such as scanned images of handwritten problems on notebook paper. To determine the types of incorrect student work samples to use, the researcher observed common student errors for several years while teaching Basic Calculus. To overcome this limitation of authenticity, the student work samples and questions will be examined for content validity by an expert in mathematics education who is familiar with the Basic Calculus course at this university, and any student work samples deemed inaccurate or confusing will be improved. Some of the chosen errors may be more common than others, and some may be more thought-provoking than others. As a result of such variation, statistically significant differences between groups might be detectable in the case of some learning modules and not others. For this reason, some statistical tests will be performed separately for each error type or for each learning module. The interview data will help reveal qualitative differences between the learning modules to inform the results as well.

Limitations Involving Groups

Student work samples will be provided to the students outside of class meetings, in an online format in an effort to decrease the influence of the lecturer or recitation leader. While there will be no planned discussion of student work samples in class meetings, student-to-student communication about the student work samples is possible. A member of one of the groups might be incidentally exposed to the treatment given to a different group. However, most of the participants will be limited to seeing the content of their own treatment group because the
student work samples are presented online within a password protected learning module on Blackboard. Another limitation is that equal sample sizes for the treatment groups cannot be guaranteed because the researcher cannot foresee which students will consent to participate in the study at the time of the sampling.

**Delimitations**

Instead of using participants from all Basic Calculus courses at the university, this study will only use consenting participants from a large lecture of Basic Calculus for which the researcher lectures. This delimitation is intended to control for the variability that comes with different teachers, such as differences in teaching style and instructional decisions.

So that the students do not feel obligated to consent to participation in this study as a course requirement, the informed consent forms will be collected on the last day of class by Dr. Jo Ann Cady and held by Cady until the researcher has turned in grades for the spring 2012 semester. Because the number of students who will consent to participate in the study is unknown at the beginning of the semester, equal sample sizes for the treatment groups cannot be guaranteed.

Because it would be impractically lengthy to require a pretest that asks participants to do all the calculus tasks covered in the course, the pretest questions are delimited to those that are most related to the student work samples that will be presented. Because of this delimitation, group differences in exam scores will not be as strongly tied to the treatments as the participants’ performances on certain problem types within those exams. For this reason, data analysis will not only include final exam grades, but will also include participants’ performance on certain problem types.

**Assumptions**
Successful math learning is *mathematical proficiency*, as described in the Executive Summary of *Adding It Up* (NRC, 2001). The five strands that are intertwined to form *mathematical proficiency* are (a) *conceptual understanding*, an understanding of mathematical concepts, (b) *procedural fluency*, an ability to carry out procedures successfully, efficiently, and flexibly, (c) *strategic competence*, an ability to represent math problems in such a way that will lead to a solution, (d) *adaptive reasoning*, an ability to use logic and reflection to justify solutions, and (e) *productive disposition*, an inclination to value mathematics and self-efficacy in math (p. 5). This assumption may influence the types of tasks the participants in this study are asked to perform and the types of mathematical responses that are expected. For example, the participants will be asked to explain why a solution makes sense or does not make sense compared to what they know about the mathematical content, which will require them to display *adaptive reasoning* and *conceptual understanding*. This study investigates common errors that are made in calculus classes. Although these errors are in students’ procedures, this study assumes that these procedural errors indicate underlying misconceptions about mathematical ideas. In other words, students make procedural errors due to deficits in *procedural fluency*, *conceptual understanding*, or possibly other strands of *mathematical proficiency* (NRC, 2001).

**Definitions of Terms**

A learning module is an online resource folder that can contain such items as images of student work samples, avatars, fields for participants’ responses, and surveys.

In this study, 10 sections of Basic Calculus meet together in an auditorium two days per week for 50-minute lectures and also meet separately once per week for 75 minutes in smaller classrooms.
The lecture portion of the class will refer to the class meetings in which all sections meet together.

The recitation portion of the class will refer to the class meetings in which the sections meet separately and are taught by graduate students.

**Theoretical Perspective**

This section will outline research and learning theories that provide a framework for the assumptions, methodology, and interpretations in this study. From an ontologically postpositivistic perspective (Guba & Lincoln, 1994), this study seeks to approximate a reality about learning, specifically the differences in learning when participants reflect on and analyze different types of student work samples. No particular result will be anticipated in this study. In a similar manner to Hole (1998) who described education as performing a raindance in which the steps are not known to be effective until the rain comes, this study will have steps done in a systematic way, results recorded, and effectiveness evaluated, but without certainty of cause and effect conclusions.

While much of this research design is quantitative, there is also some qualitative data that will be analyzed. Epistemologically, the researcher will seek to maintain an objective position in relation to the interviewees’ experiences and will attempt to let the qualitative data drive the findings, valuing authenticity. A participant’s experience will be viewed as holistic, as an intertwining of thoughts, skills, observations, occurrences, and understandings. The participants’ awareness of experience is said to have a *collective anatomy* (Marton, 1995, p. 171). No two students may experience the examination of student work samples in the exact same way, just as no two people have the exact same body characteristics, but there still exist some basic rules of anatomy. Similarly, when analyzing students statements of their experiences the examining
student work, some collective anatomy of awareness can still be revealed. In his phenomenographic research, which sought to describe lived experiences, Marton called his data stripped depictions of capability and constraint (1995, p. 171). In the qualitative part of this study, descriptions of experiences will emerge and will be viewed as depictions of capability, such as experiences that promote mathematical understanding, or depictions of constraints, such as experiences that might inhibit mathematical understanding.

Among the early cognitive learning theorists, who seek to characterize how people understand, learn, and think (Atherton, 2011a), Wertheimer, Kohler, and Koffka used Gestalt images, which are nonsensical images that can be identified in different ways by different people, to investigate how the mind finds patterns, gains insights, and solves problems. Gestalt images demonstrate how the mind perceives things in a holistic way and tends to recognize something familiar within nonsensical images in a natural attempt to avoid nonsense (Atherton, 2011b). As students make sense of the work of other students, their minds will be similarly seeking out patterns and comparing it to their own prior mathematical reasoning as a frame of reference. Therefore, this study will take a cognitive approach when considering students’ analysis of work samples.

Under the umbrella of cognitive science, many theorists consider metacognition, one’s ability to monitor their own thinking, to be a valuable part of learning. For example, Mevarech and Kramarski (1997) used a method called IMPROVE, which encouraged students to ask themselves questions as they solve problems, such as “What is the problem?”; “How is this similar or different to prior knowledge or a different problem?”; “What are the strategies appropriate for solving this, and why?”; “When should this strategy by used?”; “What did I do wrong?”; “Does this solution make sense?”; and “How can this be worked a different way?”. The
IMPROVE framework categorizes these metacognitive questions into 4 factors: (a) comprehension of the problem, (b) connections from prior knowledge to new knowledge, (c) use of appropriate strategies, and (d) reflection on the process and solution. Based on this metacognitive approach, this study assumes that using these questions to analyze others’ work will then encourage students to consider such questions when they are learning and solving problems themselves.

In addition to taking a cognitive approach, this study will consider the humanistic qualities underlying Knowles’ (1975) concepts of self-directed learning. Knowles advocated self-directed learning, as opposed to teacher-directed learning, to allow learners to take more initiative in the learning process. Self-directed learning is based on the assumption that as people mature, they are more able to make their own learning choices, that they have an increasing need to make their own learning choices, and that they should develop this increased capacity for self-directed learning as soon as possible. Self-directed learning assumes that the experiences and reflections of the learners are valid learning resources, rather than solely relying on the words of experts in the form of textbooks and papers. Self-directed learning also assumes that learners are naturally task-oriented and learn by problem solving. Finally, self-directed learning assumes that students are intrinsically motivated to learn, meaning factors that are inside the learners are likely to prompt the learners into action. Those intrinsic factors could include curiosity, perception of relevance, or the need to achieve. These assumptions will influence both the design of this study and the interpretation of the results. For example, because the participants are assumed to be natural problem solvers, the participants’ reflections will be organized around specific problems, either correct or incorrect. For an example of how Knowles’ idea of self-directed learning might impact the interpretations in this study, if there is a statistically significant difference in outcome
based on the degree of choice that was given to the participants, then such a difference will be
assumedly a reflection of the participants’ need to make their own learning choices, a need that
Knowles advocated.

In addition to the assumptions underlying self-directed learning, Knowles (1984; Knowles, Swanson, & Holton, 2011) proposed that learners need to perceive that the content has a relevance to their lives. This kind of pragmatism will impact this study; for instance, the interviews may reveal how the perception of relevance might be improved in this study as a way to increase participation and improve the depth of participant reflection. Knowles also asserted that people learn through experiences, and he pointed out that some of these experiences are their own errors. This study assumes Knowles’ assertion that one can learn from one’s own mistakes to be true and seeks to discover the extent to which students are able to learn from others’ mistakes as well.

Review of Literature

Because so many high school graduates and college students in the United States are not ready for college-level math (ACT, 2011; Complete College America, 2011; Parsad & Lewis, 2003), effective ways to improve students’ mathematical proficiency (NRC, 2001) and to mend students’ mathematical misconceptions are needed. Leaders in mathematics education (NCTM 2000, NRC 2001) have called for more emphasis on critical thinking and deeper understanding, warning against over-reliance on purely procedural math instruction. In one study (Schoenfeld, 1988), students who were successful with standardized tests were found to be performing steps without being able to make connections. The students’ only goal was to get the correct answers in the correct form, and they saw themselves as passive consumers of the mathematics that others have created, explored, or discovered. Schoenfeld suggested curricular changes to focus math
instruction on students’ mathematical thinking, rather than following procedures. Some studies have indicated that conceptual understanding and critical thinking can be encouraged by attending to errors (Cherepinsky, 2011; Kasman, 2006; Son & Moseley, 2012; Zerr & Zerr, 2011) or by having students write about mathematics (Green, 2002; Kasman, 2006; Son & Moseley, 2012; Stalder, 2001).

A study from Stephens (2006) suggested that appropriate samples of student work might help mathematical understanding, but specific characteristics of appropriate samples need to be investigated further. Some (Kasman, 2006; Zerr & Zerr, 2011) have discussed advantages of having math students critique proofs; however, proof writing is different from the type of mathematics work from an algebra or introductory calculus course taken by most first-year college students. There are some clues in the current literature about how student work samples might be effectively used in a math content course. For instance, students who first express their ideas in small group settings have more confidence to communicate their ideas in a larger group setting (Reid, Forrestal, & Cook, 1987). Knowles argued that if learners take some initiative in a learning activity, then they are more motivated to learn and are more likely to retain and use what they have learned (1975). He also argued that students should be more self-directed because taking more personal responsibility for learning was a natural part of maturing and because many higher education programs will require students to take initiative in their own learning. Among limitations to this kind of student-centered learning, Sparrow, Sparrow, and Swan (2000) found it to be easier to allow student choice of time and place but more difficult to allow student choice of content.

Because little research has examined the use of student work samples in such math content courses, prior research about preservice teachers’ examination of sample student work
has been reviewed as well. Although analysis of student work samples has been found to increase pedagogical content knowledge (Chamberlin, 2005; Son & Moseley, 2012), little research has examined similar benefits this approach might have on the preservice teachers’ content knowledge. Prior research (Blythe, Allen, & Powell, 1999; Driscoll & Moyer, 2001; NCTM 2001) and projects (Katims & Tolbert, 1998; Kelemanik, Janssen, Miller & Ransick, 1997; Saxe, Gearhart & Nasir, 2001) have shown that in addition to learning about their students’ abilities and thinking, teachers can learn about valid solution methods that may look different from traditional solutions.

Although students’ uses of nontraditional problem-solving strategies are considered “hallmark characteristics of understanding” (Carpenter, 1998), the manner in which nontraditional strategies should be used as student work samples has not been investigated thoroughly. For example, while NCTM (2000) recommends that students should be asked to listen to, justify, and critique the mathematical thinking of others, research has established that deciphering student thinking from written responses is a difficult task even for teachers (Ball, 1990, 1997, 2001; Even & Markovitz, 1995; Even & Tirosh, 1995; Schifter, 2001; Chamberlin, 2005). For this reason, more research should be done to answer questions of effectiveness, such as whether or not to explaining the thinking of other students’ nontraditional solution methods based on work samples is a classroom task that is helpful to conceptual understanding.

Son and Moseley (2012) asked preservice teachers to examine student work samples showing student-invented strategies for multiplying whole numbers. Although the participants were only preservice teachers, the results had some implications for content knowledge, showing more mathematically in-depth responses to the student work samples in which invented strategies were implemented incorrectly than those implemented correctly. Although the results
of that study are not generalizable to content courses in which very few participants are
preservice teachers, the indication that incorrect and correct student work samples elicit a
different quality of response and depth of reflection provides a rationale for more research to be
conducted that compares use of work samples with or without errors in them.

Some perspectives have a developed idea of what role student errors should play in the
classroom. At one end of the spectrum, behaviorism, with views of successful computation as
positive reinforcement, discourages tolerance of errors in the classroom. Behaviorism would
consider attention to errors as a dangerous teaching approach. At the other end of the spectrum,
Borasi (1994) suggests the teaching approach which uses student errors as *springboards for
inquiry*. His study found the following learning opportunities that stemmed from examination of
errors: a) constructive doubt and conflict, b) challenging problem solving, c) open-ended
explorations, d) reflection on the nature of mathematics, e) justification of work, f) initiative and
ownership in learning, g) recognition of humanistic aspects of math, and h) communication of
ideas. Borasi reported four benefits for using student errors as *springboards for inquiry*: a) better
understanding of the nature of math, b) learning significant math content, c) proficiency in doing
math, and d) confidence in math. Borasi asserted that because recognizing something as an error
implies that such a result does not meet one's expectations, errors can be considered an
anomaly and, consequently, “a natural stimulus for reflection and exploration” (p. 168).

Research on conceptual change (Brown & Clement, 1989; Hewson, 1981) and conflict
teaching (Bell, 1983, 1986; Swan, 1983) has suggested that students' errors and misconceptions
in the classroom could generate conflicts that reveal and challenge the students' preexisting
beliefs. If errors are to be used in instruction, many researchers agree that they should be used to
encourage communication of ideas, justification of solutions, questioning, reflection, critical
thinking, inquiry, and flexibility in reasoning (Chi, 2000; Hartman, 2001; Kramarski, 2004; Palincsar & Brown, 1984; Renkl, 1999), but the question of whose errors are most appropriate to use remains to be investigated. Some researchers, like Tirosh (2000), have identified categories of types of errors that can occur in order to better inform teachers of students’ thinking. Cherepinsky (2011) asked students to detect their own errors in incorrect problems, but Stalder (2001) used the teacher’s errors as springboards for discussion. Stalder described a game in which students earn points by detecting their teacher’s mistakes, the errors are then discussed, and the errors then seem less likely to be made by the students; however, the underlying perspective of this game tends toward behaviorism. Rather than solely viewing the errors as undesirable behaviors, more research needs to investigate the underlying mathematical misconceptions associated with the errors.

Kasman’s (2006) study suggests an effective use of errors is possible by using incorrect proofs and asking students to detect the errors. To avoid the vulnerability that students feel when involved in reviewing their peers’ work, the teacher created fictional characters as the originators of the flawed proofs. Similar to the student-invented strategies that were found in teacher education research, some work samples in Kasman’s study used approaches different from those seen in class. In the findings, the students appreciated the puzzle-like quality and the role play aspect, were frustrated when they could not find the error, and gained appreciation for providing written justification of work because the fictional characters could not verbally explain their work. Discovering this appreciation suggests that this type of assignment may encourage students to explain their mathematical thinking, solidifying their ideas, and, thus, improve their abilities to communicate using math.
Communication of mathematical ideas can also be encouraged and assessed by the use of journals and written assignments, which is a common pedagogical tool for reflection on student work samples. Lewison (1997) described some of the problems with using journals to encourage reflection. After teachers responded to journal writing with negativity, while admitting its benefits, she suggested support and authenticity. Discussing audience and purpose, Green (2002) created writing assignments that caused students to use math language in persuasive essays and letters to family. Green suggests that such writing assignments be used to develop mathematical understanding that goes beyond procedural knowledge, but one limitation of writing assignments is difficulty in giving and interpreting teacher feedback. Gao (2003) found that interactive learning situations, such as online assignments, are most effective when the students gain immediate feedback because it emphasizes what was learned and because the students can immediately know answers or acceptable responses. This suggests that in order to determine the effectiveness of student work sample analysis, treatments should provide immediate feedback.

Kramarski and Zoldan (2008) examined effects of 3 interventions, comparing them to the improvements of a control group, on mathematical reasoning, conceptual errors, and metacognitive knowledge. The three approaches were (a) diagnosing errors (DIA), (b) self-questioning (IMPROVE), a framework consisting of comprehension, connection, strategy, and reflection, and (c) a combined approach (DIA+IMPROVE). Participants were 9th-graders (N=115) studying linear functions and graphing. The combined approach was shown the most effective in all outcome variables. While the IMPROVE intervention was shown to be more effective than DIA in problem-solving and metacognition, DIA was shown to be more effective than IMPROVE in reducing student errors. Through these interventions, Kramarski and Zoldan showed the success of reflection and evaluation of student work samples, both correct and
incorrect. However, the study did not look separately at the effectiveness of the correct versus the incorrect student work samples like this study will. Because their study did not have the ability to remove the variance caused by differences in teachers, they asserted a need for more studies about student errors as interventions, especially in situations where the teacher variability can be controlled in some way, such as in this study.

**Procedure**

**Participants**

The sample for this study will consist of all consenting students from among those enrolled in ten sections of a 3-credit-hour, Basic Calculus course. If each of these classes meets their enrollment capacities, this group of potential participants could be as large as 250 students (University of Tennessee Department of Mathematics, 2011). The gender distribution in this sample will likely be similar to the University of Tennessee’s in the 2010 fall semester, in which 47.87% were female, and 52.13% were male. Among these undergraduates, 88.05% were considered residents of Tennessee for tuition purposes; therefore, most of the participants will be residents of Tennessee. The racial make-up of the University of Tennessee undergraduate student body in fall of 2010 is described by percentages in Table 1, and a similar distribution of race among the participants is anticipated for this study.

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As a prerequisite, all enrollees in this course either will have achieved an adequate score on a departmental placement exam or will have successfully completed a College Algebra course with a C or above. Many of the participating students will be undergraduate college students who successfully completed College Algebra in the fall semester of 2011 since students are advised to take College Algebra and Basic Calculus in immediate succession rather than taking a different math course between College Algebra and Basic Calculus (Hagan, 2011a).

Most of the participants who have declared a major will be majoring in business, economics, social science, agriculture, communications, or human ecology because other calculus courses fulfill the degree requirements for mathematics majors, physical science majors, engineering majors, and computer science majors (Hagan, 2011a).

Following approval from the Institutional Review Board of the University of Tennessee, informed consents will be obtained from students in these Calculus classes. Ideally, 10 to 20 interviewees will be chosen after the spring 2012 semester is complete.

Course

In this Basic Calculus Mathematics class comprised of 10 sections, the researcher teaches the lecture portion, and 4 graduate students share the teaching load of leading the recitation portions (University of Tennessee Department of Mathematics, 2011). This course, an introduction to the concepts in differential and integral calculus involving algebraic, logarithmic, and exponential functions, fulfills a Quantitative Reasoning requirement for degree-seeking undergraduates at the University of Tennessee. Topics in this course include rates of change, derivatives, techniques of differentiation, optimization, the definite integral, the Fundamental
Theorem of Calculus, applications of the integral, and techniques of integration, such as integration by substitution. The use of graphing calculators is prohibited in this course. Although small scientific calculators are allowed, the exam items in this course can be successfully completed without a calculator (Hagan, 2011a, 2011b). Course content will align with Larson’s 8th edition of *Brief Calculus, An Applied Approach*, and students will be required to complete corresponding homework questions through an online homework and grading system called WebAssign (Hagan, 2011b; Larson, 2009).

**Sampling**

A lack of control over the teacher variable limited the generalizability of Kramarski and Zoldan’s findings (2008); however, with this dissertation study, stratified random sampling can prevent similar limitations. Because these treatments will be conducted outside of class in an online format, participants who are in the same section can be assigned to different treatment groups. Each section will have two approximately equal-sized groups within it, one analyzing correct student work samples and one analyzing incorrect student work samples. The stratification of sampling will prevent one section from having a heavier proportion of one group than another section. This is important because during the recitation portion of the class, students may discuss different mathematical topics depending on the questions that are asked during those sessions. If different sections were given different types of student work samples, then the differences in class discussions could influence the results and threaten the generalizability of the findings. The strata will be the class in which the students are enrolled, and within each stratum, systematic sampling will select every 2nd student listed on the sampling frame, which will be a list of all those enrolled in each section. This diagram shows how the sampling will be organized. X_A: The participants are asked to analyze correct student work samples.
CORRECT VS. INCORRECT STUDENT WORK SAMPLES

X_B: The participants are asked to analyze incorrect student work samples.

<table>
<thead>
<tr>
<th>Section</th>
<th>Approx. % of class in each group</th>
<th>assignments throughout the semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>2</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>3</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>4</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>5</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>6</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>7</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
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<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
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<tr>
<td>8</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
<tr>
<td>9</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>≈50% ≈50%</td>
<td>X_A X_A X_A X_A X_A X_A X_A X_A</td>
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<tr>
<td></td>
<td></td>
<td>X_B X_B X_B X_B X_B X_B X_B X_B</td>
</tr>
</tbody>
</table>

For the interviews that will be conducted after the semester concludes, I will use purposive sampling to select the interviewees by looking for students who are outliers among those who have consented to follow-up interviews. A participant could be considered an outlier because of his/her degree of improvement or because of an unexpected outcome that is observed about the student. Because there may be a small number of participants who consent to interviews, the purposive sampling criteria cannot be predicted in more detail, but a rationale for participant selection will be provided in the dissertation.

Pilot Study

The pilot study inspired improvements to this study in various aspects, one of which is sampling. In this dissertation study, treatment groups will be randomly selected using stratified,
systematic sampling because there seemed to be bias based on section in the pilot study, which used cluster sampling with each section as a cluster. The diagram below shows how the groups were organized in the pilot study.

$X_A$: The participants were asked to analyze correct student work samples.

$X_B$: The participants were asked to analyze incorrect student work samples.

<table>
<thead>
<tr>
<th>Section</th>
<th>Recitation Leader</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
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</tr>
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<td></td>
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<td>$X_A$</td>
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<tr>
<td>4</td>
<td>Don</td>
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<tr>
<td>6</td>
<td>Alex</td>
<td>$X_B$</td>
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<td>$X_B$</td>
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<td>$X_B$</td>
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<tr>
<td>7</td>
<td>Pat</td>
<td>$X_A$</td>
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<td></td>
<td></td>
<td>$X_B$</td>
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<td></td>
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<td>$X_A$</td>
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<td>$X_B$</td>
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<td>$X_A$</td>
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<td>$X_B$</td>
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<td></td>
<td></td>
<td>$X_A$</td>
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<tr>
<td>8</td>
<td>Bob</td>
<td>$X_B$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_A$</td>
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<tr>
<td></td>
<td></td>
<td>$X_B$</td>
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<td>$X_A$</td>
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<tr>
<td></td>
<td></td>
<td>$X_B$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_A$</td>
</tr>
</tbody>
</table>

Another aspect that was influenced by the pilot study is administration of the pretest. In the pilot study, the pretest questions were given in lecture meetings on different dates throughout the semester; whereas, in this dissertation study, the pretest will be given at the beginning of the semester in the form of a required quiz. This will result in a higher response rate. Also, in this dissertation study, the pretest questions and the similar problems that appear on their exams will be improved based on critique given by an expert validator.

The pilot study influenced some changes to the wording of the prompting questions. For instance, in the pilot study, only one group was asked to summarize what the student did to solve the problem. However, in this dissertation study, this instruction will be included in all student work samples because of its helpfulness to learning which was revealed in the pilot study.

In this dissertation study, the students will not be able to see each other’s responses because the pilot study participants seemed to be uncomfortable posting mathematical
explanations that could be viewed by peers. However, because being able to see other student responses can be beneficial to learning, this dissertation study will simulate this using talking avatars. The words of the avatars will be created by pulling salient phrases from student responses in the pilot study data. Then, to further simulate student-to-student communication, the participants in this dissertation study will have more opportunities to respond to these avatars’ comments online.

Survey instruments in this dissertation study were also influenced by those of the pilot study. In this dissertation study, the students will be asked to rate the treatments’ helpfulness according to each of the five strands of mathematical proficiency (NRC, 2001). This will allow for a more in-depth analysis of the ways in which the student work samples helped the participants’ learning. In the pilot study, an instrument asked the participants to rate the helpfulness of several elements of the course, such as lecture meetings, homework, and recitation meetings, with the student work sample analysis activities were listed as one of those elements, but that instrument did not collect data measuring how the treatments improved learning. In the pilot study, the surveys were provided on the cover page of each exam. In this dissertation study, the surveys will appear on Blackboard immediately following each student work sample activity. By using adaptive release on Blackboard, the survey will not be accessible by a student until after that student has completed the student work analysis.

The choices of methodologies for this study were also influenced by the limitations of the pilot study. Although the pilot study seemed to reveal math ability and thinking, more descriptive data would be telling of other factors. The qualitative interviews of students, which were not included as part of the pilot study, will be added to this study in hopes of revealing more than just mathematical ability, but also attitudinal qualities, such as anxiety or interest level.
Overview of How this Study is Designed

From experience teaching this course, the researcher has identified common errors that students make in calculus and has created learning modules to present those common errors to participants. For each of these common errors, a problem in which the common error is made is presented in a learning module, and that same problem without the error being made is presented in another learning module. The following is a list of those errors.

<table>
<thead>
<tr>
<th>Error Number</th>
<th>Name of Error</th>
<th>Description of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Illegal cancel error</td>
<td>Cancellation of terms instead of factors in the simplification of a rational expression</td>
</tr>
<tr>
<td>2</td>
<td>f(x) + h error</td>
<td>Use of f(x) + h instead of f(x+h) in the limit definition of the derivative</td>
</tr>
<tr>
<td>3</td>
<td>Plus or minus error</td>
<td>Omission of the negative solution when solving a quadratic equation with no linear term</td>
</tr>
<tr>
<td>4</td>
<td>Exponent rule error</td>
<td>Multiplying exponents instead of adding exponents when multiplying factors that have the same base</td>
</tr>
<tr>
<td>5</td>
<td>Quotient rule error</td>
<td>Differentiation of the numerator and denominator separately instead of using the quotient rule for differentiation</td>
</tr>
<tr>
<td>6</td>
<td>Ordered pair error</td>
<td>Using the equation of the first derivative of f(x) to find the f(x) value in a maximum or minimum ordered pair</td>
</tr>
<tr>
<td>7</td>
<td>Quotient integration error</td>
<td>Integration of a rational function’s numerator and denominator separately instead of rewriting as a sum of terms to use the power rule for integration</td>
</tr>
<tr>
<td>8</td>
<td>Plus C error</td>
<td>Using the f(x) value that was given in the initial condition as the constant term C itself instead of using substitution to find the constant term C</td>
</tr>
<tr>
<td>9</td>
<td>Power rule on exponential error</td>
<td>Trying to use the power rule for integration on an exponential function composed with a polynomial function instead of using the chain, exponential, and power rules.</td>
</tr>
</tbody>
</table>

There is both a quantitative part and a qualitative part to this dissertation study. In the quantitative part of the study, the dependent variables will include (a) overall achievement, (b) whether students’ correctly solved problems similar to those presented in the learning modules,
(c) whether students’ made the same type of error that was presented in the learning modules, and (d) students’ perceptions of how much the learning module assignments increased their understanding. In the qualitative part of the study, open-ended interviews will collect data that qualitatively describes the participants’ experiences of analyzing student work samples.

With utilization of charts and research design notation similar to that established by Campbell and Stanley (1963), diagrams will demonstrate the research designs of the quantitative parts of the study. $O_i$, where $i$ is a natural number, will stand for the observation of a variable that was observed $i^{th}$ in order. For example, if there are 3 different times a participant could make a certain type of error, then there might exist $O_1$, $O_2$, and $O_3$, each representing a variable that measures whether the participant makes the error. The order in which the problems appear on the exams and in the class will implicate the number used as $i$; for example, if an error can possibly be made on Pretest’s number 7, Exam 3’s number 10, and the Final Exam’s number 20, then they would be numbered as $O_1$, $O_2$, and $O_3$ respectively. The number of these observations will vary because with some types of errors, there will be more opportunities on exams to make the errors.

Furthermore, $O_{i,j}$, where $j$ is a natural number, will represent an observation that is related to error $j$ listed in the chart above. An example of this notation might be $O_{4,3}$, which might represent a variable that is measured by the fourth opportunity a participant had to make an error of type 3, which is identified as a plus or minus error.

$X_i$, where $i$ is a letter, will stand for the presence of a treatment of a certain type. For example, $X_A$ will represent a learning module that presents a correct student work sample, and $X_B$ will represent a learning module that presents an incorrect student work sample. Furthermore, in the notation $X_{i,j}$, $j$ will indicate the type of error. Consistent with Campbell and Stanley (1963), if the lines are separated by + signs, then subjects will be assigned to the comparisons
groups randomly. The following paragraphs will describe and show one such diagram for each of the 3 research questions associated with a quantitative design.

**Research design for Research Question 1.**

Research Question 1 asks what difference between groups, if any, exists in students’ overall achievement, after accounting for prior knowledge measured by the pretest. $O_1$ in this diagram represents the score on the pretest, $X_A$ represents analysis of correct student work samples, $X_B$ represents analysis of incorrect student work, and $O_2$ represents students’ scores on the final exam.

$O_1$ $X_A$ $O_2$

$++++++++++++$

$O_1$ $X_B$ $O_2$

**Research design for Research Question 2.**

Research Question 2 asks what difference between groups, if any, exists in ability to correctly solve problems similar to those presented in the learning modules, after accounting for prior knowledge measured by the pretest. Instead of measuring solely whether or not the error was made, the variables will measure the correctness of the problem. Coding of the participants’ responses to these problems will be similar to partial credit that would be given when grading an exam. A rubric will be created for each type of problem that is used to measure this variable. The range of the i’s may vary if some problem-types occur more than others on exams; however, a typical range for i would be 3 because each problem that appears in a learning module will have (1) an associated pretest question, (2) a similar problem on a unit exam, and (3) a similar problem on the final exam.
For $X_{i,j}$ in Research Question 2, i represents a treatment of a certain type (A indicating correct student work analysis and B indicating incorrect student work analysis), and j indicates which error type the problem is associated with. These treatments occur after the pretest but before any other observed dependent variables. For each of the 9 types of errors, there will be a diagram that looks similar to this:

\[
O_{1,j} \quad X_{A,j} \quad O_{2,j} \quad O_{3,j}
\]

\[\text{++++++++++++++++++}\]

\[
O_{1,j} \quad X_{B,j} \quad O_{2,j} \quad O_{3,j}
\]

For example, for the illegal cancel error (j=1), if there are 3 times participants see pretest and posttest problems that are similar to the one presented in Learning Module 1, the representative diagram would be as follows:

\[
O_{1,1} \quad X_{A,1} \quad O_{2,1} \quad O_{3,1}
\]

\[\text{++++++++++++++++++}\]

\[
O_{1,1} \quad X_{B,1} \quad O_{2,1} \quad O_{3,1}
\]

**Research design for Research Question 3.**

For Research Question 3, which asks what difference between groups, if any, exists in making errors similar to the incorrect work samples presented in the learning modules, the design diagrams will look the same, except the $O_{i,j}$'s will measure whether the participants make or do not make the error that was demonstrated in the student work. As in Research Question 1, i will indicate the order in which these opportunities to make errors were presented in the course on exams, and j will indicate which type of error is being analyzed. Because there are 9 different types of errors, there also will be 9 of these analyses.
Research design for Research Question 4.

For Research Question 4, which asks what difference between groups, if any, exists in students’ perceptions of how the learning module assignments increase their understanding, each $O_{i,j}$ indicates a variable measured by a rating on a 5-point (1=not helpful, 2=slightly helpful, 3=somewhat helpful, 4=moderately helpful, 5=very helpful) Likert-type scale (1932) of the learning module’s helpfulness to *mathematical proficiency* (NRC, 2001). In this design, $i$ distinguishes the items in the order they appear on the survey ($i = 1$ conceptual understanding, $i = 2$ procedural fluency, $i = 3$ strategic competence, $i = 4$ adaptive reasoning, and $i = 5$ productive disposition), and $j$ indicates which learning module contained the survey, ranging from 1 to 8. In this Research Question, what $j$ represents is slightly different from what it represented in Research Questions 2 and 3. While there are 9 focus errors, there are only 8 learning modules because a few learning modules have two errors demonstrated instead of one. Each of the 8 learning module will have an attached survey. For $X_{i,j}$ in this research question, $i$ will represent a treatment of a certain type (A indicating correct student work sample and B indicating incorrect student work sample).

\[ X_{A,j} \quad O_{1,j} \quad O_{2,j} \quad O_{3,j} \quad O_{4,j} \quad O_{5,j} \]

++++++++++++++++++++++++++++++

\[ X_{B,j} \quad O_{1,j} \quad O_{2,j} \quad O_{3,j} \quad O_{4,j} \quad O_{5,i} \]

This diagram can also be shown in the form of a table, such as the one that follows:
### Likert-type Scores on Survey of Helpfulness to *Mathematical Proficiency* (NRC, 2001)

<table>
<thead>
<tr>
<th>Treatment Group</th>
<th>Learning Module</th>
<th>Conceptual Understanding</th>
<th>Procedural Fluency</th>
<th>Strategic Competence</th>
<th>Adaptive Reasoning</th>
<th>Productive Disposition</th>
<th>Overall Mathematical Proficiency</th>
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</thead>
<tbody>
<tr>
<td>Correct</td>
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<td></td>
<td></td>
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<tr>
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<td>4</td>
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**Research design for Research Question 5.**

Research Question 5 asks about descriptions of the experiences of calculus learners as they are asked to analyze student work samples. The interview protocol for these follow-up interviews will be similar to the protocol found in Appendix A, but may have more specific questions that relate to their individual responses.

**Pretest**

Nine pretest questions will be contained in one quiz that will be administered on the first
recitation meeting, since attendance is typically high during the first week of classes. In the cases of students who add the class after the administration of the pretest, they are expected to take the pretest once they are enrolled in the class. This pretest is found in Appendix C. The pretest has been deemed valid in content by an expert in mathematics education who is familiar with the Basic Calculus course.

Treatments

In descriptions of the learning modules, a $X_A$ learning module will refer to one which focuses on correct student work, and a $X_B$ learning module will refer to one which focuses on a sample of incorrect student work, where one of the errors from the list is made. In the presentation of student work samples and student responses in this study, the researcher will use Voki (2011), to create speaking avatars. Each of the first 6 $X_A$ and $X_B$ learning modules consists of 5 pages described below:

1. The first page contains one student work sample, typed questions, and a talking avatar.
2. The second page contains the same typed questions that were on the first page and an input field for the participants to type their responses to the QUESTIONS.
3. The third page has 3 talking avatars that speak various responses to the student work sample. The script for these 3 avatars was developed by pulling examples from the student responses given in the pilot study. This page also shows the student work sample.
4. The fourth page of one of these learning modules is an input field where the participants can add additional comments.
5. The fifth page is the survey for rating the helpfulness to mathematical proficiency (NRC, 2001).
In the pilot study, there seemed to be trends in the engagement levels of the students. For example, near the beginning of the semester, there was a hesitance toward participating in the assignment, demonstrated by student questions about the nature of the assignment and some assignments left incomplete. However, after responding to several work samples, they seemed to be more comfortable with the assignments. Then, when the students seemed to be very accustomed to the framework of questioning, demonstrated by answering the questions with slightly more haste and with phrases similar to those they had used in prior responses, the researcher decided that the seventh and eighth learning modules should be formatted differently, to include different questions and to include 2 student work samples instead of 1. To encourage engagement and to further stretch thinking, the seventh and eighth learning modules were created according to the format described below:

1. The first page will have a talking avatar, a student work sample, and 3 questions. If it is a $X_A$ learning module, the solution will be correct and the strategy shown will be traditional. In other words, the student work sample will look similar to the examples found in the course textbook. If it is a $X_B$ learning module, the solution will be incorrect because one of the identified common errors will have occurred.

2. The second page will show the questions and will have in input field for the participants’ responses.

3. The third page has 3 talking avatars that speak various responses to the student work sample. The script for these 3 avatars was developed by pulling examples from the student responses given in the pilot study. This page also shows the student work sample again.
4. The fourth page will have the same math problem as was used on the first page. If it is an X_A learning module, it will be correct but will be solved using a method that is not traditional. If it is an X_B learning module, the answer will be incorrect because of a common error different from the one shown on the first page. The avatar and 3 accompanying questions here will be different from those on the first page.

5. The fifth page is an input field for the participants’ responses.

6. The sixth page has 3 talking avatars that speak various responses to the student work sample. The script for these 3 avatars was developed by pulling examples from the student responses given in the pilot study. This page also shows the student work sample.

7. The seventh page will be the survey measuring perception of helpfulness to mathematical proficiency (NRC, 2001).

For the groups who analyze correct student work samples, the seventh and eighth learning modules will present the same problem solved in two different ways. One student work sample will present a traditional solution that follows the solution patterns in the textbook, and the other student work sample will present a nontraditional strategy that has not been presented in the textbook or in class. More specifically, the nontraditional method in these samples will use knowledge of differentiation rules to figure out an integral rather than use of traditional integration rules. For the groups who analyze incorrect student work samples, the seventh and eighth learning modules will feature 2 different common errors.

Below is a screenshot of the first page of an intervention of type X_A, in which the avatar introduces herself as a calculus student named Melissa who needs help. She then asks questions similar to those that accompany the work sample.
The questions that accompany this correct student work sample include the following:

1. “Briefly describe the steps Melissa took. Do you think this solution is correct? Explain.”

   This question corresponds to the comprehension component of the IMPROVE framework for self-questioning.

2. “From your understanding of what a derivative is, does Melissa’s method make sense? Explain.” This question corresponds to the connection component of the IMPROVE framework for self-questioning.

3. “Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.” This question corresponds to the strategy component of the IMPROVE framework for self-questioning.

4. “Can you share another way of finding this derivative?” This question corresponds to the reflection component of the IMPROVE framework for self-questioning (Mevarech & Kramarski, 1997).
The second page of the learning module shows these questions and an input field for participants to submit their responses. On Blackboard, this input page was made by creating a test with one short answer question. Using adaptive release, each participant will be required to submit at least one response here before the following pages of the learning module will be accessible to that particular student. Below is a screenshot of an example of a third page of a learning module.

After the participant hears these avatars explain their reasoning, he/she may want to comment on what they learned from others’ explanations, and the fourth page will allow participants to make additional commentary.

The questions that accompany the student work samples vary depending on whether the error was or was not made. If the error was made, the questions would be similar to the following:
1. “Briefly describe the steps Lisa took. Why do you think this solution is incorrect?” This question corresponds to the comprehension component of the IMPROVE framework for self-questioning.

2. “What would you say to Lisa to help with the problem?” This question corresponds to the reflection component of the IMPROVE framework for self-questioning.

3. “From your understanding of what a derivative is, does Lisa’s answer seem reasonable or unreasonable? Explain.” This question corresponds to the connection component of the IMPROVE framework for self-questioning.

4. “What specific steps or strategies could Lisa use to avoid making this type of error?” This question corresponds to the strategy component of the IMPROVE framework for self-questioning (Mevarech & Kramarski, 1997).

In the 7th and 8th learning modules, the patterns of accompanying questions are different. For the XA learning modules, which show correct work samples, the traditional problem-solving strategy is presented first, and the accompanying questions are similar to the following:

1. Explain the steps taken in Allison’s solution. Do you think this solution is correct?

2. From your understanding of what an integral is, does this method in Allison’s solution make sense? Explain.

3. Do you think the method shown here will work every time, or could there be special circumstances that would prevent Allison’s method from working? Explain.

After the participant submits at least one response, he/she will be able to view the avatars speaking other students’ responses followed by a second student work sample solved in a non-traditional manner. The questions accompanying it will be as follows:

1. Explain the steps taken in Julie’s solution. Do you think this solution is correct?
2. From your understanding of what an integral is, does Julie’s method make sense? Explain.

3. Compare Allison’s solution and Julie’s solution. Describe how the two methods differ and how they are similar. Is one method "better"? Explain.

After the participant submits at least one response to this one, he/she will be able to view the avatars speaking other students’ responses followed by the 5-item survey.

In the seventh and eighth learning modules assignments, the \( X_B \) learning modules, which show incorrect student work samples, will have questions similar to the following to accompany the first incorrect work sample:

1. First explain what Bruce did. Did Bruce just make a misstep or is there some overarching concept that Bruce doesn't understand?

2. Explain the correct procedures. What would be your answer? What specific steps or strategies within this process could Bruce use to avoid this error?

3. From your understanding of what an integral is, does your answer seem more reasonable than Bruce’s? Explain.

In these \( X_B \) learning modules, the questions that accompany the second incorrect work sample will be as follows:

1. First explain what Lewis did. Did Lewis just make a misstep or is there some overarching concept that Lewis doesn't understand?

2. From your understanding of what an integral is, does your answer seem more reasonable than Lewis’s? Explain.

3. Compare Bruce’s work to Lewis’s work. Would you say that one error is "worse" than the other? Explain.
After responding to each of the student work samples, avatars that provide other students’ responses will be accessible, and at the very end of each learning module, the 5-item survey will be presented.

**Data Collection**

This section includes descriptions of the instruments used and methods for collecting data in this study. The comprehensive final exam scores will be used to measure overall achievement of the participants. Problems on exams that are similar to problems presented in learning modules will be used to measure whether students can solve these problems correctly. Also, all exam problems that present opportunities to make errors similar to those in the learning modules will be used to collect data. The coding framework might vary with the problem type that is being examined; however, with most multi-step problems, one column will include values for a dichotomous variable measuring whether or not the participant reached the step in which the error of interest commonly occurs. A number 1 would indicate that the student reached that step, and a number 0 would indicate the student did not reach that step, either by leaving it blank or by making an earlier misstep that prevented the error of interest from being a possibility. The second column would correspond to a dichotomous variable indicating whether the error of interest is committed, with a number 1 indicative of making the error and 0 indicative of not making the error. The diagram below shows the possible categories that were described:
Another column will indicate the total number of points each participant earned on each exam problem when the exams are graded. Partial credit will be given based on rubrics, which will be included in the dissertation.

For the 5-item 5-point Likert-type scale (1932) survey that is administered online at the end of the learning module, each of the 5 items corresponds to one of the 5 strands of mathematical proficiency (NRC, 2001). In the screenshot below, Question 1 corresponds to conceptual understanding, Question 2 corresponds to procedural fluency, Question 3 corresponds to strategic competence, Question 4 corresponds to adaptive reasoning, and Question 5 corresponds to productive disposition (NRC, 2001).
CORRECT VS. INCORRECT STUDENT WORK SAMPLES

Preview Survey: Feedback about Learning Module

Description: This is where you are asked to rate the helpfulness of this learning module.

Instructions: Please rate this learning module (1=not helpful, 5=very helpful) in its helpfulness to these different parts of learning math.

Multiple Attempts: This Survey allows multiple attempts.

Force Completion: This Survey can be saved and resumed later.

Question Completion Status:

Question 1
Rate the helpfulness of this learning module as it relates to your understanding of mathematical ideas.

- 1. Not helpful
- 2. Slightly helpful
- 3. Somewhat helpful
- 4. Moderately helpful
- 5. Very helpful

Save Answer

Question 2
Rate the helpfulness of this learning module as it relates to your ability to follow rules of math.

- 1. Not helpful
- 2. Slightly helpful
- 3. Somewhat helpful
- 4. Moderately helpful
- 5. Very helpful

Save Answer

Question 3
Rate the helpfulness of this learning module as it relates to your ability to develop a problem-solving strategy.

- 1. Not helpful
- 2. Slightly helpful
- 3. Somewhat helpful
- 4. Moderately helpful
- 5. Very helpful

Save Answer

Question 4
Rate the helpfulness of this learning module as it relates to your ability to think about and explain your answers.

- 1. Not helpful
- 2. Slightly helpful
- 3. Somewhat helpful
- 4. Moderately helpful
- 5. Very helpful

Save Answer

Question 5
Rate the helpfulness of this learning module as it relates to your attitudes about math and about your abilities.

- 1. Not helpful
- 2. Slightly helpful
- 3. Somewhat helpful
- 4. Moderately helpful
- 5. Very helpful

Save and Submit
The questions in the survey shown above are

1. Rate the helpfulness of the learning module as it relates to your understanding of mathematical ideas.

2. Rate the helpfulness of the learning module as it relates to your ability to follow rules of math.

3. Rate the helpfulness of the learning module as it relates to your ability to develop a problem-solving strategy.

4. Rate the helpfulness of the learning module as it relates to your ability to think about and explain your answers.

5. Rate the helpfulness of the learning module as it relates to your attitudes about math and about your abilities.

Possible responses for each are (1) not helpful, (2) slightly helpful, (3) somewhat helpful, (4) moderately helpful, and (5) very helpful. Participants’ responses to each survey question within each module will be recorded in a separate column.

For Research Question 4, the interview protocols used will be similar in content to the protocol found in Appendix A, which was piloted to interview a researcher who audited Basic Calculus in the fall 2011 semester with a different instructor and then completed these learning modules. The interview questions will be open-ended questions that invite participants to describe their unique experiences. The protocol will include questions about the participants’ background in math, about their experiences of responding to the different formats of learning modules, about their experiences of responding to each of the questions that appeared in learning modules, about their experiences of hearing other students’ responses, and how each of the 5
strands of *mathematical proficiency* (NRC, 2001) related to their experiences of completing the learning modules. These interviews will be audio-recorded and later transcribed.

**Data Analysis**

For Research Questions 1, 2, and 3, an Analysis of Covariance (ANCOVA) statistical test will be appropriate for determining group differences while adjusting for any group’s initial advantage indicated in the pretest scores. ANCOVA will increase the power of the comparison by removing the variance caused by the pretest-associated variables. The assumptions of normality and homoscedasticity will be tested. F values and p-values will be reported.

For Research Question 2, inter-rater reliability will be determined for the variables that are based on partial credit given. For Research Questions 4, internal consistency reliability will be estimated by a Cronbach’s Alpha test, which tests how all items on a test relate to all other items on the test. On Research Questions 4, independent-samples t-test would be an appropriate statistical test to determine differences between groups’ mean ratings on the mathematical proficiency survey.

For the qualitative part of this study, Research Question 5, the interview data will analyzed in a manner similar to what Hatch described as inductive analysis (2002), which borrows heavily from Spradley’s ideas (1979, 1980) of detecting salient domains by paying attention to the relationships between these domains. Using X and Y as symbols for salient domains, the categories of relationships include “X is a kind of Y,” “X is a place in Y,” “X is a result of Y,” “X is a reason for doing Y,” “X is a place for doing Y,” “X is used for Y,” “X is a way to do Y,” “X is a step in Y,” “X is a characteristic of Y.” Hatch’s inductive analysis (2002) includes analysis within domains as well as an analysis of themes across domains. Because this investigation inquires about the qualitative differences in participants’ experiences,
phenomenography is an appropriate methodology to employ in this data analysis. Therefore, in addition to using the relationships suggested in the above list, Marton’s (1995) framework of capabilities and constraints will be considered when seeking qualitative differences in the lived experiences of the participants.
CORRECT VS. INCORRECT STUDENT WORK SAMPLES

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Appendix A

Interview Protocol

Introduction: The purpose of this interview is to get descriptions of what it is like to respond to student work samples of differing types. This information will be presented anonymously and the names will be changed in the transcription of this interview. This is an interview with ____________________________. The date and time of this interview is ____________________________.

Question 1. Tell me about your background in math and in education.

Question 2. As an auditor in a Basic Calculus class, you were not a typical student taking this course for credit. Tell me about how you feel this difference might have impacted your analysis or responses.

Question 3. On several occasions in your responses, you used language like, “My first thought was …; however, …” or “I had to examine it further to realize….?” Tell me about your experiences at those times. (Prompting questions: Were you experiencing a change? A challenge to your beliefs? An AHA moment?)

Question 4. Did those experiences (refer to the ones described in Question 3) differ depending on the type of student work sample?
Question 5. At one point, there was a transition in the format of the student work sample prompts. Instead of presenting only 1 correct solution to analyze, there were 2 correct solutions to the same problem to analyze. One of them used a traditional approach, and one of them used a non-traditional approach. Tell me about this transition. How did that change in format cause your analysis experiences to be different? (Prompting questions: What was your first thought when you noticed the change in format?)

I am going to read the text questions that accompanied the student work samples without showing you the specific student work samples. I’d like you to describe your experiences when you had to respond to these different types of accompanying questions.

Question 6. “Do you think this solution is correct?” (Prompting questions: Did you work the problems on your own? Did you catch any errors in your own work?)

Question 7. “From your understanding, does this method make sense? Explain.” which is a question that was asked about correct student work samples. (Prompting questions: What criteria did you use? Did you look at the method or the solution?)

Question 8. “Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.” (Prompting question: How did you think of those types of problems where it wouldn’t work?)
Question 9. “Can you share another way of finding this?” (Prompting: Sometimes you responded with a simple “no.” Describe that experience.)

Question 10. “Why do you think this solution is incorrect?” (Prompting question: Did your experience involve thinking about the procedures that were taken, or the math concepts behind the procedures, or some other way of thinking about the math?)

Question 11. “What would you say to this student to help with the problem?” (Prompting question: How did your experience differ when the recipient of your comments changed?)

Question 12. “From your understanding, does this student’s answer seem reasonable or unreasonable? Explain.” which was used with incorrect work samples. (Prompting questions: Because you knew the answer was incorrect, did you anticipate responding with an answer of “unreasonable”? What was it like for you when you responded that an incorrect answer seemed reasonable?)

Question 13. “What specific steps or strategies could this student use to avoid this type of error?” (Prompting question: After responding to that question, do you think you were likely to take your own advice?)

Question 14. “Compare work sample A and work sample B. Describe how the two methods differ and how they are similar. Is one method ‘better’? Explain.” (Prompting questions: How was your experience different when you made this value judgment? You chose generalizeability
of method as one of your judgment criteria. How did you make that decision? You were complimentary of the nontraditional method. Describe why.)

Question 15. “Compare work sample A and work sample B. Would you say that one error is ‘worse’ then the other? Explain.” (Prompting questions: How was your experience different when you made this value judgment? How did you choose the criteria for this judgment?)

I would like you to read your responses to the first student work sample. (Provide a copy of it, and wait for participant to read it.)

I would like you to complete this learning module on Blackboard. After engaging with the learning module, typing in your response as a student would, observing the feedback, I will ask you about the different parts of this experience. (Present the learning module that corresponds to the first prompt. Leave the table to allow the participant to do this.)

Question 16. Describe your experience of hearing and seeing Allison, the avatar, and her work.

Question 17. Describe how you experienced typing in your response.

Question 18. Describe your experience of hearing other students’ responses.

Question 19. Describe your experience of having the opportunity to add other comments at the end of the module.
According to the Executive Summary of *Adding It Up* (NRC, 2001), the 5 strands that are intertwined to form *mathematical proficiency*:

- **conceptual understanding**, an understanding of mathematical concepts,
- **procedural fluency**, an ability to carry out procedures successfully, efficiently, and flexibly,
- **strategic competence**, an ability to represent math problems in such a way that will lead to a solution,
- **adaptive reasoning**, an ability to use logic and reflection to justify solutions, and
- **productive disposition**, an inclination to value mathematics and self-efficacy in math (p. 5).

Question 20. I’d like you to read this brief explanation of *mathematical proficiency* (NRC, 2001). (Provide a copy to participant, and allow time for reading.) How did these experiences relate to each of these 5 strands?
Appendix B

Learning Module #1

Correct Work Sample

Examine the student work that follows.

\[
\lim_{x \to 2} \frac{2x^3 - 4x^2 + 3}{x^2}
\]

\[
= \frac{2(2)^3 - 4(2)^2 + 3}{(2)^2}
\]

\[
= \frac{16 - 16 + 3}{4}
\]

\[
= \frac{3}{4}
\]

Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what a limit is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding this limit?

Incorrect Work Sample

Examine the student work that follows.
Answer the following questions in your reflective response:

Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what a limit is, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module #2**

**Correct Work Sample**

Examine the student work that follows.
Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what a derivative is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding this derivative?

**Incorrect Work Sample**

Examine the student work that follows.

Find the derivative of \( f(x) = x^2 + 3x \) using the limit definition.

\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
\lim_{h \to 0} \frac{(x^2 + 3x + h) - (x^2 + 3x)}{h}
\]

\[
\lim_{h \to 0} \frac{x^2 + 3x + h - x^2 - 3x}{h}
\]

\[
\lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1
\]

Uh oh! The answer should be \( 2x + 3 \).

Answer the following questions in your reflective response:

Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what a derivative is, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module #3**

**Correct Work Sample**
Examine the student work that follows.

$$\frac{3}{x^2-4}$$ is undefined when $$x^2-4=0$$.

$$x^2-4=0$$

$$(x+2)(x-2)=0$$

$$x=-2, x=2$$

$$f(x)$$ is continuous over $$(-\infty, -2)$$, $$(-2, 2)$$, and $$(2, \infty)$$.

Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what continuity is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding these intervals?

**Incorrect Work Sample**

Examine the student work that follows.

$$\frac{3}{x^2-4}$$ is undefined when $$x^2-4=0$$.

$$x^2-4=0$$

$$(x+2)(x-2)=0$$

$$x=-2, x=2$$

$$f(x)$$ is continuous over $$(-\infty, 2)$$ and $$(2, \infty)$$.

Uh oh! The answer should have said that the intervals where $$f(x)$$ is continuous are $$(-\infty, -2)$$, $$(-2, 2)$$, and $$(2, \infty)$$. 
Answer the following questions in your reflective response:

Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what continuity is, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module # 4**

**Correct Work Sample**

Examine the student work that follows.

\[
\text{Use the product rule to find the derivative of } \ f(x) = \sqrt[3]{x} \cdot (3\sqrt{x} - 4).
\]

\[
f(x) = x^{\frac{3}{2}} \cdot (3x^{\frac{3}{2}} - 4) \left\text{ rewritten} \right.
\]

\[
f'(x) = \frac{\partial}{\partial x} \left( \frac{3}{2} x^{\frac{3}{2}} - 4 \right) = \left( \frac{3}{2} \cdot x^{\frac{3}{2}} \right)(x^{\frac{3}{2}})
\]

\[
f'(x) = \frac{\partial}{\partial x} \left( \frac{3}{2} x^{\frac{3}{2}} \right) - 6x^{\frac{3}{2}} + \frac{3}{2} x^{\frac{3}{2}}
\]

\[
f'(x) = 6x^{\frac{3}{2}} - 6x^{\frac{3}{2}}
\]

Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what a derivative is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding this derivative?

**Incorrect Work Sample**
Examine the student work that follows.

Use the product rule to find the derivative of \( f(x) = \sqrt{x}(3\sqrt{x} - 4) \).

\[
f'(x) = \frac{2}{x^{3/2}} \left(3x^{1/2} - 4\right) \quad \text{product rule: } f'g + g'f
\]

\[
f'(x) = \left(\frac{3}{2} x^{1/2}\right)(3x^{1/2} - 4) + \left(\frac{3}{2} \cdot \frac{1}{2}\right)(x^{3/2})
\]

\[
f'(x) = \frac{9}{2} x^{1/2} - 6x^{1/2} + 3 \cdot \frac{3}{2} x^{-3/4}
\]

\[
f'(x) = 6\sqrt{x} \left(\sqrt{x} - 1\right)
\]

Uh oh.

The answer should have been

\[
6\sqrt{x} \left(\sqrt{x} - 1\right)
\]

Answer the following questions in your reflective response:

Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what a derivative is, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module # 5**

**Correct Work Sample**

Examine the student work that follows.
Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what a derivative is, does this method make sense? Explain.

Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding this derivative?

**Incorrect Work Sample**

Examine the student work that follows.

Answer the following questions in your reflective response:
Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what a derivative is, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module # 6**

**Correct Work Sample**

Examine the student work that follows.

![Correct Work Sample Image]

Answer the following questions in your reflective response:

Do you think this solution is correct?

From your understanding of what a derivative is and what extrema are, does this method make sense? Explain how.
Do you think this method will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Can you share another way of finding these extrema?

**Incorrect Work Sample**

Examine the student work that follows.

Answer the following questions in your reflective response:

Why do you think this solution is incorrect?

What would you say to this student to help with the problem?

From your understanding of what a derivative is and what extrema are, does this student’s answer seem reasonable or unreasonable? Explain.

What specific steps or strategies could this student use to avoid this type of error?

**Learning Module # 7**
Correct Work Sample

Work sample A.

\[
\int \frac{3x^2 + 4}{x^2} \, dx \quad \text{Find the indefinite integral.}
\]

\[
\int \left( \frac{3x^2}{x^2} + \frac{4}{x^2} \right) \, dx = \int \left( 3 + \frac{4}{x^2} \right) \, dx
\]

\[
3x + \frac{4x^{2-1}}{2-1} + C = 3x + \frac{4x}{1} + C
\]

\[
= 3x - \frac{4}{x} + C
\]

1. Explain the steps taken in work sample A. Do you think this solution is correct?

2. From your understanding of what an integral is, does this method in work sample A make sense? Explain.

3. Do you think the method shown in work sample A will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Work sample B.

\[
\int \frac{3x^3 + 4}{x^2} \, dx \quad \text{Find the indefinite integral.}
\]

\[
\frac{f \cdot \frac{g'}{g} = \frac{3x^3 + 4}{x}} {x^3 - 3} \Rightarrow x = g
\]

\[
3x^3 + 4 = (\text{antiderivative of } x) - (1)(\text{antiderivative of } x^3)
\]

\[3x^3 + 4 = (\text{antiderivative of } x^3) - 1(x^4)
\]

\[3x^3 + 4 = 2x^2 - x^2 - 4
\]

\[3x^3 + 4 = 2x^2 - x^2
\]

\[3 = 2n - n
\]

\[3 = n
\]

So \( f' = \frac{6x}{x^2 + 4} \) and \( f = 3x^2 - 4 \)

\[
= \frac{3x^2 - 4}{x} + C
\]

4. Explain the steps taken in work sample B. Do you think this solution is correct?
5. From your understanding of what an integral is, does this method in work sample B make sense? Explain.

6. Compare work sample A and work sample B. Describe how the two methods differ and how they are similar. Is one method "better"? Explain.

**Incorrect Work Sample**

**Work sample A**

\[
\int \frac{3x^2+4}{x^2} \, dx \quad \text{Find the indefinite integral.}
\]

\[
= \int \left(\frac{3}{x^2} + \frac{4}{x^2}\right) \, dx = \int \left(3 + 4\right) \, dx = \int 7 \, dx
\]

\[
= 7x + C \quad \text{Uh oh! That's not correct!}
\]

1. First explain what the student did in work sample A. Did this student just make a misstep or is there some overarching concept that this student doesn't understand?

2. Explain the correct procedures. What would be your answer? What specific steps or strategies within this process could this student use to avoid this error?

3. From your understanding of what an integral is, does your answer seem more reasonable than the one found in work sample A? Explain.

**Work sample B**
4. First explain what the student did in work sample B. Did this student just make a misstep or is there some overarching concept that this student doesn't understand?

5. From your understanding of what an integral is, does your answer seem more reasonable than the one found in work sample B? Explain.

6. Compare work sample A to work sample B. Would you say that one error is "worse" than the other? Explain.

Learning Module # 8

Correct Work Sample

Work sample A.
When you created your rooftop garden, you added three and three fourths ounces of earth worms. Your worm population has been increasing at a rate of \(2e^{\frac{8}{7}t}\) ounces per year, where \(t\) represents the age of your garden in years. Find a function that tells how many ounces of worms there are in your garden at time \(t\). Also find how many ounces of worms there would be at exactly three years.

\[
\int 2e^{\frac{8}{7}t} \, dt = 2 \int e^{\frac{8}{7}t} \, dt = \frac{7}{8} \int e^{\frac{8}{7}t} \cdot \frac{8}{7} \, dt \\
= \frac{7}{4} \int e^u \, du \\
= \frac{7}{4} e^u + C \\
= \frac{7}{4} e^{\frac{8}{7}t} + C \\
\]

\[(0, \frac{3}{4}) \quad (0, \frac{15}{4})\]

\[
\frac{15}{4} = \frac{7}{4} e^{\frac{8}{7}(0)} + C \\
\frac{15}{4} = \frac{7}{4} [1] + C \\
\frac{15}{4} - \frac{7}{4} = C \\
C = 2 \\
\]

\[
f(t) = \frac{7}{4} e^{\frac{8}{7}t} + 2 \\
f(3) = \frac{7}{4} e^{\frac{24}{4}} + 2 \\
\approx 56 \text{ ounces of worms at } t = 3 \text{ yrs.}
\]

1. Explain the steps taken in work sample A. Do you think this solution is correct?
2. From your understanding of what an integral is, does this method in work sample A make sense? Explain.

3. Do you think the method shown in work sample A will work every time, or could there be special circumstances that would prevent this method from working? Explain.

Work sample B.

When you created your rooftop garden, you added three and three fourths ounces of earth worms. Your worm population has been increasing at a rate of \(2e^{\frac{8}{7}t}\) ounces per year, where \(t\) represents the age of your garden in years. Find a function that tells how many ounces of worms there are in your garden at time \(t\). Also find how many ounces of worms there would be at exactly three years.

\[
2e^{\frac{8}{7}t} = m \cdot e^{\frac{8}{7}t} \cdot n \quad \text{as long as} \quad m \cdot n = 2
\]

Let

\[
n = \frac{8}{7}, \quad \Rightarrow \quad m \cdot \frac{8}{7} = 2
\]

\[
m = 2 \cdot \frac{7}{8}
\]

\[
m = \frac{7}{4}
\]

So \(2e^{\frac{8}{7}t}\) can be written as

\[
\frac{7}{4} \cdot e^{\frac{8}{7}t} \cdot \frac{8}{7}
\]

Looks like chain rule derivative

\[
f'(t) = \frac{7}{4} \left[ e^{\frac{8}{7}t} \cdot \frac{8}{7} \right] + 0
\]

\[
f(t) = \frac{7}{4} \left[ e^{\frac{8}{7}t} \right] + C
\]

\[
f(0) = \frac{15}{4} \quad \rightarrow \quad \frac{15}{4} = \frac{7}{4} \left[ e^{\frac{8}{7}(0)} \right] + C
\]

\[
\frac{15}{4} = \frac{7}{4}[1] + C
\]

\[
\frac{15}{4} - \frac{7}{4} = 2
\]

\[
f(t) = \frac{7}{4} \left[ e^{\frac{8}{7}t} \right] + 2
\]

\[
f(3) = \frac{7}{4} \left[ e^{\frac{24}{7}} \right] + 2 \approx 56 \text{ oz in year 3}
\]
4. Explain the steps taken in work sample B. Do you think this solution is correct?

5. From your understanding of what an integral is, does this method in work sample B make sense? Explain.

6. Compare work sample A and work sample B. Describe how the two methods differ and how they are similar. Is one method "better"? Explain.

**Incorrect Work Sample**

**Work sample A.**
1. First explain what the student did in work sample A. Did this student just make a misstep or is there some overarching concept that this student doesn't understand?

2. Explain the correct procedures. What would be your answer? What specific steps or strategies within this process could this student use to avoid this error?

3. From your understanding of what an integral is, does your answer seem more reasonable than the one found in work sample A? Explain.
Work sample B.

When you created your rooftop garden, you added three and three fourths ounces of earth worms. Your worm population has been increasing at a rate of $2e^{\frac{8}{7}}$ ounces per year, where $t$ represents the age of your garden in years. Find a function that tells how many ounces of worms there are in your garden at time $t$. Also find how many ounces of worms there would be at exactly three years.

\[\int 2e^{\frac{8}{7}t} \, dt = 2\int e^{\frac{8}{7}t} \, du = \frac{7}{8} \int e^{\frac{8}{7}t} \cdot \frac{8}{7} \, dt\]

\[\quad = \frac{7}{4} \int e^u \, du\]

\[\quad = \frac{7}{4} e^{u+1} + C\]

\[\quad = \frac{7}{4} e^{\frac{8}{7}t+1} + C\]

\[\quad = \frac{7e^{\frac{8}{7}t+1}}{4(\frac{8}{7}t+1)} + C\]

\[\quad = \frac{15}{4} = \frac{7e^{0+1}}{4(0+1)} + C\]

\[\quad = \frac{15}{4} = \frac{7e^{\frac{3}{4}+1}}{4(\frac{3}{4}+1)} + C\]

\[\quad = C = \frac{15}{4} - \frac{7e}{4} \approx -1\]

\[f(t) = \frac{7e^{\frac{8}{7}t+1}}{4(\frac{8}{7}t+1)} - 1\]

\[f(3) = \frac{7e^{\frac{24}{7}+1}}{4(\frac{24}{7}+1)} - 1 \approx 33.12\]

Uh oh! This is not correct!

4. First explain what the student did in work sample B. Did this student just make a misstep or is there some overarching concept that this student doesn't understand?
5. From your understanding of what an integral is, does your answer seem more reasonable than the one found in work sample B? Explain.

6. Compare work sample A to work sample B. Would you say that one error is "worse" than the other? Explain.
CORRECT VS. INCORRECT STUDENT WORK SAMPLES

Appendix C

QUIZ

NAME: ________________________________

Math 125 students come from various mathematical backgrounds. The purpose of this quiz is to see what you might already know about algebra. Give each problem a try.

1. Rewrite the expression \( \frac{2x^3 - 4x^2}{x^2} \) as an expression with two terms.

2. If \( f(x) = 3x^2 - 2x \), find \( f(x + a) \).

3. Solve \( x^2 - 9 = 0 \).

4. Circle the function(s) that are quotient(s).

\[
f(x) = \frac{x^4 - x^2 + 5}{x^2 + x} \quad f(x) = e^x \quad f(x) = \sqrt{x + 4}
\]

\[
f(x) = x^4 + 7x^3 \quad f(x) = \frac{e^x - x}{x^2} \quad f(x) = \ln |x|
\]

5. On the graph of this function \( f(x) = 4x^2 - 3x + 2 \), a point exists at \( x = 1 \). State this ordered pair (___,____).

6. Multiply \( \sqrt{x}(2x^2 + \sqrt{x}) \).

7. Rewrite \( \frac{5x^2 + 7}{x^2} \) so that it is a sum of terms.

8. If \( mn = 2 \) and \( m = \frac{8}{7} \), then what is \( m \)?

9. If \( f(x) = 4e^x + C \) and \( f(0) = 8 \), then what is \( C \)?