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Long-term environmental problems and strategic intergenerational transfers

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Abstract

The impacts of long-lived stock pollutants and the measures supposed to address them link current and future generations. Altruism towards successor generations is a prerequisite for resolving the resulting intergenerational equity issues. Preference asymmetry and imperfect altruism introduce strategic conflicts between generations. Here, a current generation decides on a combination of abatement and whether to provide an imperfect backstop. The future generation decides whether to use the backstop or not. We identify three outcomes: (1) Technology denial, in which the current generation deliberately rejects the imperfect backstop to avoid misuse by the future generation. (2) Underabatement, in which the current generation provides the backstop but reduces abatement activities; and (3) Overabatement, in which the current generation provides the backstop but increases abatement activities. The outcome depends non-trivially on the assessment of collateral damages of the future generation. Uncertainty over future preferences renders technology denial more likely.

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1 Introduction

Many environmental problems facing the world today involve long-lived stock pollutants. The long-term persistence of these pollutants confronts today's generation with policy challenges that have a significant intergenerational equity dimension. Two responses to these challenges have received most attention. One is the abatement of polluting activities by today's generation, thus reducing the stock of pollution passed on to the future. The other is the development of a technological backstop, thus enabling future generations to manage more successfully the impacts of the stock pollution problem generated by their predecessors. Both responses impose costs now and yield benefits only in the future. For any of these activities to be undertaken, therefore, today's generation needs to exhibit a significant degree of altruism vis-à-vis its successors.

As the general economics literature reminds us, however, altruism offers no protection against strategic conflicts between providers of altruistically motivated transfers and their recipients: The 'rotten kid' theorem (Becker 1974, Bergstrom 1989), the 'Samaritan's dilemma' (Buchanan 1975), and the 'strategic bequest motive' (Bernheim et al. 1985) are only the most prominent examples of a wide literature on how preference asymmetries between donors and recipients can lead to inefficiencies due to strategic behavior (see Laferrere and Wolff 2006 for a survey). In addition, although benevolent, altruism typically suffers from well-known imperfections. Prime among them is the failure to accept consumer sovereignty of a recipient. Instead, donors attach merit connotations to consumption or production bundles, giving rise to 'paternalistic altruism' (Bergstrom 1982, Pollak 1988, Jacobson et al. 2007). Therefore, while altruism is a prerequisite for resolving the intergenerational equity issues concerning long-term environmental problems, preference asymmetries between generations and imperfections of altruism jointly give rise to the possibility of important strategic distortions.

This paper studies the presence and nature of strategic distortions that arise when a generation decides the level of abatement and whether to provide a technological backstop to a successor generation. Both preference asymmetry and paternalistic altruism matter: Asymmetric preferences may lead to the future generation failing to use the backstop as intended. Paternalistic altruism may lead to the current generation reconsidering its abatement and technological transfer decision. Jointly, both properties may lead to significant deviations from the combined abatement and technology transfer that would be optimal if the strategic dimensions were absent.

To provide a tractable and instructive setting, we assume a society consisting of two non-overlapping generations. Before the true marginal damage of pollution stock is revealed, the current generation decides on a combination of abatement and technology-enabling investment (TEI) leading to an imperfect backstop technology: While the backstop fully eliminates the damages by the stock pollutant, it also causes other specific damages. The future generation, having learned the true marginal damage, decides whether to use the technology. In this set-up, paternalistic altruism arises out of the combination of uncertainty over the true marginal damage at the time of decision and the imperfection of the backstop: The current generation develops the backstop with the intention of it being used discriminately, i.e. only in the case of marginal damages being high. Preference asymmetry, on the other hand, arises out of the difference in the valuation of the backstop-induced damages: The successor generation could be more or less concerned than the current generation about these damages. To make the analysis of the strategic considerations concrete, we envision a welfare-maximizing outcome that involves a specific combination of abatement and the provision of the backstop and to which the strategic distortions can be compared and contrasted. We provide a specific illustration by drawing on the economic literature on so-called 'geoengineering' technologies (Barrett 2008; Victor 2008; Schelling 1996). Referring to 'deliberate large-scale interventions in the climate system with the aim of reversing the effects of anthropogenic climate change' (Keith 2000), the climate change context conveniently captures the spirit of the stock pollution problem and the damage uncertainty while the unintended side effects of geoengineering exemplify the imperfect backstop.

Using this framework, we show that intergenerational transfers to resolve long-term environmental problems can involve strategic distortions, leading to a rich set of possible outcomes, some of them with troubling characteristics. The current generation may deliberately deny the imperfect backstop in anticipation that the future generation will use the technology either indiscriminately or not at all. The current generation may also decide that it is rational to accompany the development of the backstop with a reduction in abatement efforts. This decision can arise for two distinct reasons: One is that abatement becomes less productive because of an anticipation that future generation will use the backstop regardless of the level of damages. The other reason is that the backstop becomes more productive because the current generation strategically abates less in anticipation of a successor generation not using the backstop regardless of the realization of marginal damages: By decreasing abatement levels, the current generation can return its successor to a discriminate use regime. The current generation may also decide that it is rational to increase abatement levels in anticipation of the future generation using the backstop indiscriminately. With higher levels of abatement, the backstop becomes less productive and the successor generation can be returned to a discriminate use regime. Uncertainty over the future generation's preferences leads to important changes in the current generation's strategy: As the variance of the preference distribution function increases, technology denial quickly becomes the preferred option. For those cases in which the backstop is provided, the relationship between the variance and abatement levels is typically non-monotonic. The reason is that the abatement level needs to maximize benefits across a probabilistic distribution of strategic equilibria.

In terms of focus and results, this paper touches on three literatures: One is the general literature on strategic conflicts between donors and recipients as covered above. While sharing the focus on strategic distortions and time inconsistency with Lindbeck and Weibull (1988) and considering an enlarged strategy space (Bergstrom 1989), the model developed here differs from this literature in two respects. The first is the fact the transfers studied here are not *inter vivos*. As a result, the current generation's strategy cannot involve transfers contingent on observable recipient behavior (Bruce and Waldman 1991, Cremer and Pestieau 1996). The second aspect concerns the number and types of intergenerational transfers. Not only does the model feature a combination of two transfers (as opposed to one in the case of the 'rotten kid theorem'; see Bergstrom 1989), but it also introduces a novel type of transfer alongside transfers in-kind, namely a *transfer of technology*. While transfers in-kind can limit the strategic distortions by restricting the choice set (Bruce and Waldman 1991), technology transfer can be shown to have the opposite effect.

The second literature to which the paper relates studies the question of environmental preference changes from generation to generation. There is a small set of recent papers that study from a normative perspective how the presence of uncertainty over future preferences ought to impact on optimal environmental policy today (Heal and Kriström 2002; Ayong Le Kama and Schubert 2004; Krysiak and Krysiak 2006). While the present paper shares the concern for impacts on policy with these papers, its positive spirit sets it apart from this literature and is more in line with strategic focus spearheaded by Becker (1974). In doing so, it addresses a gap in environmental economics where intergenerational conflict and strategic distortions have so far received little attention.

The third literature to which the paper is directly related studies the interface between technology and the environment. A number of recent papers (Goeschl and Perino 2009, 2007; Hart 2009) study the implications of technological change generating technologies that are environmentally problematic. These papers adopt a normative position and therefore do not consider possible strategic effects of an intergenerational type. The present paper differs in that two decision-makers jointly determine whether the technology is deployed, and that the desirability of deployment depends on external circumstances and preferences rather than being exogenously given. The present paper also differs in the type of uncertainty considered: In contrast to Goeschl and Perino (2009) and Hart (2009) that focus on technological uncertainty, and in addition to the preference asymmetry considered in Krysiak and Krysiak (2006), the source of uncertainty in the present paper is of a scientific nature (Newell and Pizer 2003).

We proceed as follows: First we introduce a simple model of a stock pollution problem with abatement and intergenerational technology transfer, illustrating the general ideas in a climate change setting. The model consists of a simple and concrete four-period sequence of abatement choice, TEI choice, learning about climate sensitivity, and technology deployment. Section 3 determines a parameterization of the model in which strategic conflicts - like in Lindbeck and Weibull (1988) - take the shape of a time inconsistency problem. This is followed by proofs of existence for the strategic equilibria that arise instead. Section 4 completes the analysis with a full characterization of the outcomes of the intergenerational game. Section 5 extends the analysis to uncertainty over the future generation's preferences and provides numerical illustrations. Section 6 provides a concluding discussion and directions for future research.

2 Abatement and intergenerational technology transfer - a simple model

2.1 Model set-up

The following model provides a parsimonious setting that isolates the salient strategic implications of the intergenerational decision problem. It consists of a simple four-period setting. Figure 1 provides a graphical representation.

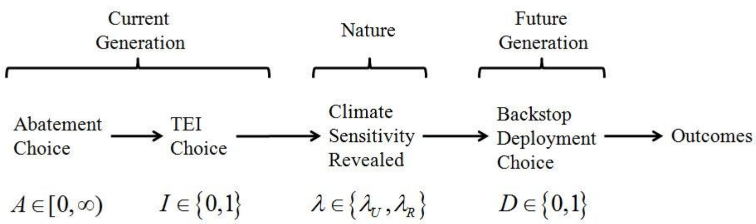


Figure 1: Four-period intergenerational transfer program

There are two non-overlapping generations, current and future. The current generation chooses pollution abatement level A in the first period and decides on whether to incur the costs of technology-enabling investments (TEI) in period 2 ($I = 1$) or not ($I = 0$).[1] The TEI involve technological as well as regulatory and institutional costs that allow the future generation to use the backstop technology. The future generation decides whether to deploy the backstop in period 4 to undo the damages T from the unabated pollution stock. If it deploys the backstop ($D = 1$) , the future generation avoids pollution damages, but incurs other environmental damages G associated with the backstop. If it does not ($D = 0$), it suffers damages T. Between these decisions, in period 3, nature resolves the scientific uncertainty regarding the parameter λ that determines the marginal damages of the pollution stock.

For simplicity and to make the model specific from the start, consider climate change as an example of the stock pollution problem and some form of 'geoengineering' (Barrett 2008) as the imperfect backstop. Like much of the stock pollution literature, stock-related damages T are quadratic in pollution. In period 4 damages caused by increased temperatures are of the form

$$T = \lambda^2 (R_0 - A)^2 , \tag{1}$$

[1]The sequentiality of the decisions on A and I is purely for ease of presention. A simultaneous choice of A and I leads to identical results.

capturing the convex impact of the amount of the stock pollutant net of abatement efforts $(R_0 - A)$ and of the carbon sensitivity of the climate to a doubling of CO_2, λ (Moreno-Cruz and Keith 2009). The scientific uncertainty over the severity of the stock pollution problem then takes the form of uncertainty about the carbon sensitivity. This is modeled as a random variable that is Bernoulli distributed and takes a responsive value of $\lambda_r > 0$ with probability p and an unresponsive value of $0 < \lambda_u < \lambda_r$ with probability $(1 - p)$. As expression (1) makes clear, abatement is productive as it reduces the expected value of pollution damages associated with climate change. Along with all moves $\{A, I\}$, the pollution damage function (1) is common knowledge.

Again, in line with the literature, abatement costs are convex in abatement efforts. As a simple approximation, the abatement cost function is of the quadratic type

$$X = \alpha A^2, \tag{2}$$

with increasing marginal abatement cost $2\alpha A$. For a fixed sum of TEI, K, investment I in period 2 activates the backstop technology that fully neutralizes pollution damages, such that $T = 0$ if the backstop is deployed. The cost of no TEI ($I = 0$) is zero.

The implementation of the backstop technology causes collateral damages by having unintended net negative impacts. In the case of a climate change backstop such as stratospheric aerosol dispersal, typical changes are the alterations of precipitation patterns and increases in air pollution. Assume, in line with Moreno-Cruz and Keith (2009), that the altruistic current generation values these collateral damages G according to

$$G = \rho (R_0 - A) \tag{3}$$

which details an essentially linear relationship between damages and the magnitude of additional stock of pollutants $(R_0 - A)$ that needs to be offset by the backstop, evaluated at constant marginal value ρ.

Preference asymmetry enters into the model in the form of the successor generation diverging from the current generation in terms of its valuation of collateral damages: From its vantage point, damages are $\tilde{G} = \theta\rho (R_0 - A)$, with the asymmetry parameter $\theta > 1$ indicating higher assessment of marginal colateral damages and $\theta < 1$ weaker preferences than the current generation. In reality, θ will be uncertain, but to make the results stark, we assume initially that the current generation believes it knows θ before studying uncertainty in section 5. The assumption is productive for characterizing the strategic conflicts, but it is also less radical if one accepts that the current generation has reasons to think that it broadly understands the processes that determine the formation of the relevant social preferences vis-a-vis the environment.[2] Clearly, the preference asymmetry parameter

[2]One could readily think of theories that endogenize θ as a function of the pollution stock $(R_0 - A)$, the state of the environment $\lambda^2 (R_0 - A)^2$, the state of technology I, or a combination thereof. The exogenous variant here is a

captures a significant number of possible hypotheses about the presence and direction of a preference shift from generation to generation such as those postulated by Krutilla (1967) and Krieger (1973). [3] Incidentally, a completely different, but equally valid explanation for preference asymmetry could arise out of the presence of hyperbolic discounting: Even generations with identical preferences will come to a different assessment of damages because the difference in the time horizon between long-term stock pollution damages and short-term collateral damages of using a backstop. While the long-term effects would be evaluated by the current generation using a relatively lower discount rate, the negative impacts of the backstop would be evaluated by the future generation using higher relative discounting. This provides an alternative mechanism supporting the modeling assumptions. We begin our analysis by assuming θ is known by the current generation. We discuss the effects of explicitly considering uncertainty over the preference parameter θ in section 5.

2.2 Objectives and equilibrium concept

Turning to the objective functions, payoffs, and strategies of both generations, we start with the future generation. Its objective is to minimize the sum of pollution damages and collateral damages from using the backstop, taking the choices of the current generation, A and I, and the realization of λ as given. The future generation's strategy set is reduced to a simple choice regarding the deployment of the backstop, i.e. $D \in \{0, 1\}$. Formally, the future generation's problem is

$$\min_{\{D\}} \quad \lambda^2 (R_0 - A)^2 (1 - D) + \theta\rho (R_0 - A) D, \tag{4}$$

capturing the essential choice by the future generation between climate change damages and geoengineering damages.

The current generation's problem is to optimally choose a strategy consisting of A in period 1 and I in period 2. In doing so, it aims to minimize expected total cost to itself (in the form of abatement A and TEI I) and the future generation (in the form of damages). The current generation takes decisions under uncertainty regarding the true value of λ and in anticipation of the expected deployment decision, $D \in \{0, 1\}$, by the future generation. Formally, the current generation's problem is

$$\min_{\{A,I\}} \quad \alpha A^2 + IK + \mathbb{E}\left[\lambda^2 (R_0 - A)^2 (1 - D(\lambda,\theta)) + \rho (R_0 - A) D(\lambda,\theta)\right] \ . \tag{5}$$

The objective function (5) captures - along the lines of Pollak (1988) - the two concurrent motives underpinning the current generation's decision. One is strong altruism: The damages suffered by the

simplifying case.

[3] The literature on technology, environment, and values (Barbour 1980) provides a rich set of narratives, some predicated on cognitive mechanisms, some on political economy (see the exchange of arguments between Mesthene (1968) and McDermott (1969))

future generation enter into the current generation's decision undiscounted. This strong altruism ensures that the current generation does consider intergenerational transfers in kind (abatement A) and in technologies (TEI effort I) to offset damages from pollution. The concurrent motive to altruism is a form of paternalism: The damages suffered by the future generation enter into the current generation's decision without adjusting for the preference asymmetry θ. The formulation in (5) shares with Cox (1987) an element of non-market service provision by the successor generation (here in the form of selective deployment D), but the spirit that the current generation's altruism is tempered by notions of merit behavior is closer to Pollak (1988): The current generation has most to gain from the successor generation behaving as the current generation would in the same circumstances given by A, I, and λ. Specifically, this means deploying the imperfect backstop only if the current generation would also do so. In the present example, this provides the current generation with a reason for thinking strategically about whether to pass on a geoengineering capability to the future generation or not.

The model set-up and objectives define a sequential game with incomplete information. Its basic structure, in particular the technology transfer decision, is a variant of the trust game by Kreps (1990). However, the intergenerational decision problem here features two important differences: One is the availability of the second instrument in the form of abatement, the other the presence of exogenous uncertainty in the form of the random variable λ. The proper solution concept for determining the equilibrium played by the current and future generations is that of sub-game perfection (SP). The current generation, looking forward, employs backward induction to solve problem 5: By determining the optimal play of future generation $(D^*|A, I, \lambda)$ in period 4 contingent on current generation's choices in periods 1 and 2 and nature's move in period 3, it identifies its own optimal play $\{A^*, I^*\}$ in periods 1 and 2. However, it has to do so not knowing λ and D. The SP equilibrium characterizes the expected welfare position of both generations.

2.3 Sum-up

The model is designed to capture - as parsimoniously as possible - a setting in which the intergenerational decision problem of whether to provide a future generation with different preferences with an imperfect backstop to solve a stock pollution problem can be productively discussed. The four-period model intentionally accentuates the strict sequentiality as well as the intertemporal structure of costs and benefits between current and future generation: The current generation sacrifices current consumption in favor of costly abatement activities and costly TEI, the benefits of which accrue to the future generation. The future generation, on the other hand, is forced to accept the carbon stock and the technologies handed down from the current generation, but benefits from the fact that the scientific uncertainty regarding the pollution problem has been resolved.

It is useful to re-emphasize two unusual features of the model: One is that in contrast to much of the intergenerational literature, the transfers here are not *inter vivos*. Once the the pollution and innovation choice have been made, the current generation has no further means of ensuring that the next generation "will be doing the right thing". The second is the presence of intergenerational technology transfer and that its use by the successor generation has a merit dimension: Depending on the circumstances, there is a course of action by the future generation that minimizes costs for the current generation, but may not be in the interest of its successors. These modeling features jointly determine the presence of a strategic conflict between the current and future generation.

3 Strategic equilibria

In the following, we first establish the existence and characterize the specific nature of the strategic conflict between the current and future generation. As in Lindbeck and Weibull (1988), the strategic conflict manifests itself as an incidence of time inconsistency. Specifically, we focus on the parameter space for which the strategy profile that maximizes the current generation's payoff is not subgame perfect. Starting from a benchmark in which the strategy maximizing the current generation's payoff is time inconsistent, we then study the subgame perfect equilibria of the game and characterize the strategic distortions that these equilibria imply.

3.1 Time inconsistency: Existence and characterization

Time inconsistency arises when the future generation deviates from the current generation's preferred play. Deviation requires choice. In the intergenerational decision problem, the later generation has a choice if and only if the current generation decides to incur the TEI and develops the backstop ($I = 1$). Otherwise, $D = 0$ by definition.[4] The search for instances of time inconsistency can therefore be restricted to sequences of play in which the current generation opts for TEI in period two, i.e. chooses $I = 1$. TEI is rational only if the current generation's expected payoff can increase on account of the backstop being available in the final period. This implies meaningful restrictions on the parameters of the model: In environmental terms, a necessary condition for the backstop to generate net benefits is that $\rho < \lambda_u^2 R_0$. Otherwise, collateral damages rule out deploying the backstop even in a world in which no abatement takes place ($A = 0$). In economic terms, a necessary condition is that $\rho < 2\alpha R_0$. Otherwise, the marginal cost of even the last unit of abatement is lower than the marginal damage of the backstop, implying that the backstop is never a competitive substitute for abatement. Both restrictions define necessary conditions for the existence of time inconcistency.

The necessary conditions can be refined further: The existence of time inconsistency requires not only that the backstop is developed by the current generation ($I = 1$), but also that there

[4] It could be possible that the future generation could have available a CE technology that could deploy even if TEI were absent. Although this question is of interest, we are not concerned here with that possibility.

is an associated abatement level A_C such that (5) is maximized. Given appropriate parameters (see appendix), it is easy to show that this current welfare-maximizing transfer of abatement A_C and of the geoengineering technology is predicated on a rule of discriminate use of the backstop. Specifically, it requires that $D = 0$ if $\lambda = \lambda_u$ and $D = 1$ if $\lambda = \lambda_r$: Under this rule, the deployment of the backstop is reserved for those circumstances in which the damage by the pollution stock is revealed to be high. This captures the basic idea that the backstop technology serves as a fallback option for the future generation for the case of a deleterious carbon sensitivity. Discriminate use, however, is only subgame perfect if the future generation finds it rational to follow the rule once the game arives at its decision node. Two cases can be distinguished and characterized.

Case 1 (Backstop Abuse) *Given (4), the future generation finds it beneficial to deviate from the stipulated rule, choosing $D = 1$ for any λ if*

$$\theta\rho < \lambda_u^2 (R_0 - A_C). \tag{6}$$

This is the case of indiscriminate use of the backstop by the future generation despite the discriminate spirit in which it was developed by its predecessor. Specifically, the future generation chooses to deploy the dirty backstop even if the realization of the pollution damage parameter is low. A necessary condition for this subgame imperfection to arise is that $\theta < 1$, i.e. the future generation cares less about environmental damages from the geoengineering backstop than the current one. Together with the above restrictions, (6) defines the feasible set of parameters for which backstop misuse arises.

Case 2 (Backstop Abandonment) *Given (4), the future generation finds it beneficial to deviate from the stipulated rule, choosing $D = 0$ for any λ if*

$$\theta\rho > \lambda_r^2 (R_0 - A_C). \tag{7}$$

This is the case of indiscriminate abandonment of the backstop by the future generation. Even if the damage parameter associated with the pollution stock is high, the future generation does not avail itself of the technological capability provided - at a cost - by its predecessor. A necessary condition for this case of subgame imperfection is that $\theta > 1$, i.e. the future generation cares more about the negative collateral effects of using the dirty backstop than the current generation. Together with the above restrictions, (7) defines the feasible set of parameters for which backstop abandonment arises.

In both cases, the current generation's preferred strategy is not implementable and the welfare optimum cannot be attained due to a deviation by the future generation. The deviation exists because the current generation delivers a technological capability that is under the exclusive control of its successor, deciding under a different set of preferences. This establishes the existence and nature of the intergenerational conflict.

3.2 Subgame-perfect equilibria

In order to study the time-consistent equilibria of the intergenerational game, it is useful to fix a benchmark to highlight the distortional effect of strategic behavior. As in Lindbeck and Weibull (1988), the benchmark maximizes the combined welfare of both generations. In addition, the benchmark achieves this maximum through a combination of TEI ($I = 1$) and abatement A_C that replicates the discriminate strategy. The heuristic benefit is that deviations from $I = 1$ and A_C have meaningful interpretations as measures of the distortionary effects induced by preference asymmetry.

The benchmark with the desired properties arises by setting the parameters K and ρ as follows:

$$0 < K < R_0^2 \alpha \frac{E[\lambda^2]}{\alpha + E[\lambda^2]} - \frac{4(1-p)R_0^2 \alpha \lambda_u^2 + 4pR_0 \alpha \rho_c - p^2 \rho^2}{4\left(\alpha + (1-p)\lambda_u^2\right)} \equiv \bar{K} \tag{8}$$

$$\rho \in \left(\underline{\rho}\,,\, \bar{\rho}\right)\ , \tag{9}$$

where $\underline{\rho} = 2\alpha R_0 \frac{\alpha + \lambda_u^2 - \sqrt{\alpha^2 + \alpha(1-p)\lambda_u^2}}{\alpha(1+p) + \lambda_u^2}$ and $\bar{\rho}$ is the minimum of the economical and environmental bound, $\bar{\rho} \;=\; \min\left\{2\alpha R_0\,,\, \lambda_u^2 R_0\right\}$.

When ρ and K are restricted to values satisfying equations (8) and (9), there is an initial abatement level $A_C = \frac{2(1-p)R_0\lambda_u^2 + p\rho}{2(\alpha + (1-p)\lambda_u^2)}$ that solves the problem in (5) and that generates the highest combined welfare of both generations (see appendix). This initial abatement level maximizes joint welfare if the play by the current and future generation is $(A_C, I = 1, \lambda = \lambda_r, D = 1)$ and $(A_C, I = 1, \lambda = \lambda_u, D = 0)$. With the benchmark established, we study the strategic equilibria that arise. We start with the *Backstop Abuse* case; that is, when the future generation is less concerned with the impacts of the backstop.

3.2.1 Strategic equilibria under Backstop Abuse ($\theta < 1$)

In this section we derive the three sub-game perfect equilibria between current and future generation that can arise in response to backstop abuse: In the first equilibrium, the current generation responds to the indiscriminate use of the backstop by reducing its level of abatement in order to minimize current costs. We refer to this equilibrium as the *underabatement* case. A second possibility arises if, considering the unconditional use of the backstop, the current generation finds it optimal to commit to not providing the technology to the future generation. We refer to this equilibrium as the *technology denial* case. In the third equilibrium, the current generation induces discriminate use by the future generation by increasing the optimal level of abatement, and in this way restricting the use of the dirty backstop to the case of an unfavorable state of the world. We refer to this equilibrium as the *overabatement* case. The equilibrium that arises depends on parameters of the model. We show in the next section the parameter constellation for which the technology denial dominates the underabatement case and the overabatement case dominates the technology denial.

The Underabatement Equilibrium From (6) it is clear that for backstop abuse to occur, the level of abatement by the current generation must be smaller than some critical level $A_{\text{crit}} = R_0 - \frac{\theta\rho}{\lambda_u^2}$. For $A < A_{\text{crit}}$, the future generation will deploy the backstop indiscriminately, while for $A > A_{\text{crit}}$, the future generation will use the backstop, at most, in the deleterious state of the world where $\lambda = \lambda_r$. Now consider the case that $I = 1$ and $A_C = \frac{2(1-p)R_0\lambda_u^2 + p\rho}{2(\alpha + (1-p)\lambda_u^2)} < A_{\text{crit}}$. This is equivalent to a constraint on θ in the form of an upper bound such that

$$\theta < \frac{\lambda_u^2(2\alpha R_0 - p\rho)}{2(\alpha + (1-p)\lambda_u^2)\rho} \equiv \bar{\theta} < 1. \tag{10}$$

If condition (10) is met, then a current generation that develops the backstop ($I = 1$) prefers reducing its abatement level to $A_U = \frac{\rho}{2\alpha}$ over shouldering the relatively high abatement level A_C, which is only optimal given that *conditional* use will take place. The subscript U refers to unconditional use of the backstop and reports the level of abatement that minimizes the problem in (5) when the choice of the future generation is $D = 1$ for all λ. The intuition behind this result is that in the face of a future generation offsetting stock pollution damages through the backstop in any state of the world, it is optimal for the current generation to reduce the level of abatement and save abatement costs. This strategy is subgame-perfect: Once abatement is reduced to A_U, the dirty backstop will be used independently of the climate sensitivity outcome.

The Technology Denial Equilibrium As the underabatement equilibrium demonstrates, given the right set of parameters, a rational decision-maker will decide to switch to a low abatement in stage 1 to counter the time inconsistency of technology use policies. However, one part of the current generation's strategy in the underabatement equilibrium is that it undertakes technology transfer. This transfer need not be optimal: A denial of technology may leave the current generation better off.

If current society commits to $I = 0$, then the level of abatement that minimizes the costs of the current generation is given by $A_B = \frac{R_0\mathbb{E}[\lambda^2]}{\alpha + \mathbb{E}[\lambda^2]}$. In this case, the current generation incurs higher abatement costs since its concern for future welfare require a compensation in abatement terms to a future generation deprived of the possibility of a backstop. The natural problem with this course of action is that in the case of an unfavorable state of the world (a carbon-sensitive climate), the future generation is left without the technological fallback option.

We show in the appendix that the constellation of parameters that supports the possibility of technology denial is given by a lower bound on the cost of enabling the backstop technology K such that:

$$K > R_0^2\alpha \cdot \frac{E[\lambda^2]}{\alpha + E[\lambda^2]} - R_0\rho + \frac{\rho^2}{4\alpha} \equiv \underline{K}. \tag{11}$$

That is, if the cost of TEI is large enough, the current generation decides to preclude strategically the use of the backstop and to impose instead pollution stock damages consistent with its level of abatement A_B on the future generation. This strategy by the current generation is subgame-perfect in a degenerate sense: By not developing the technology, the future generation is denied the opportunity of deviation from equilibrium play.

The Overabatement Equilibrium The third equilibrium is closely related to the underabatement equilibrium in that the current generation decides to provide the technology and adjusts its abatement level strategically. As before, then $I = 1$ and $A_C = \frac{2(1-p)R_0\lambda_u^2 + p\rho}{2(\alpha+(1-p)\lambda_u^2)} < A_{\text{crit}}$. In the third equilibrium, however, the current generation's play is to increase the level of abatement to a level above A_{crit} and, hence, above A_C. As the appendix shows, this equilibrium requires as a necessary condition a lower bound on θ such that

$$\theta \in [\underline{\theta}, \bar{\theta}] \tag{12}$$

where $\underline{\theta} = \bar{\theta} - \Delta$ and $\Delta = \lambda_u^2 \sqrt{\frac{1-p}{\alpha}} \frac{\sqrt{4R_0\alpha(\alpha+\lambda_u^2)\rho - 4R_0^2\alpha^2\lambda_u^2 - (\alpha(1+p)+\lambda_u^2)\rho^2}}{2(\alpha+(1-p)\lambda_u^2)\rho}$.

The intuition underpinning the overabatement equilibrium is that in order to induce the desired discriminate behavior in the future generation, the current generation needs to increase abatement beyond the cost-minimizing level such that the criterion (6) is not triggered. This increase in abatement transfers rents to the future generation in order to compensate it for the stock pollution damages if the state of the world turns out to be favorable and the backstop is not used. Condition (12) ensures that this subgame-perfect strategy leaves the current generation still better off relative to the indiscriminate use.

3.2.2 Strategic equilibria under Backstop Abandonment ($\theta > 1$)

Building on the intuition developed in the previous section, we now turn to the strategic equilibria that arise when $\theta > 1$. In this case the current generation's preferred course of action is to develop the technology in order to provide its successor with a fallback option; however, it also knows that the future generation will fail to use it given the stock pollution damages associated with the welfare-maximizing strategy. In the example of climate change and a geoengineering option, the future generation decides to suffer the climate change damages even in a highly carbon-sensitive world rather than using the backstop of stratospheric aerosol dispersion.

In this section we derive the two subgame perfect equilibria that can arise in response to backstop abandonment: One is the technology denial equilibrium, equivalent to that under the backstop abuse case. The second is an underabatement equilibrium that returns the future generation to a discriminate deployment regime.

Technology Denial Equilibrium Incurring the cost of TEI is wasteful for the current generation if the backstop will not be used under any circumstances, i.e. if $D(\lambda = \lambda_r) = D(\lambda = \lambda_u) = 0$. From (7) it is clear that for backstop abandonment to occur, the level of abatement by the current generation must be larger than some critical level $A^*_{\text{crit}} = R_0 - \frac{\theta\rho}{\lambda_r^2}$. A sufficient condition for this distortion to emerge at the initial abatement level A_C is hence given by

$$\theta > \frac{\lambda_r^2 \,(2\alpha R_0 - p\rho)}{2\,(\alpha + (1-p)\lambda_u^2)\,\rho} \equiv \underline{\theta}^* > 1 \tag{13}$$

If (13) holds, the current generation denies the future generation the technology. It is substantially costly to induce the future generation to behave consistently with the benchmark case of discriminate use. As under the $\theta < 1$ case, this solution is subgame perfect in a degenerate way. The difference is, however, that the reason here is not prevent the deployment of the dirty backstop, but is simple cost-saving.

Underabatement Equilibrium Technology denial resolves the subgame imperfection of the welfare-maximizing strategy $\{A_C, I = 1\}$, but is not necessarily optimal. Instead, the current generation may prefer to provide the technological option to the future and combine the technology transfer with a strategic inducement for deployment. The use of the backstop can be triggered if abatement levels are reduced. Technically this results in a condition

$$K < -\frac{R_0^2\alpha^2}{\alpha + \mathbb{E}[\lambda^2]} + \theta\rho\frac{(2\alpha R_0 - p\rho)}{\lambda_r^2} - \theta^2\rho^2\frac{\alpha + (1-p)\lambda_u^2}{\lambda_r^4} \equiv \hat{K}^*(\theta) \tag{14}$$

Given (14), the current generation chooses abatement of magnitude A_D *at most* equal to $A^*_{\text{crit}} = R_0 - \frac{\theta\rho}{\lambda_r^2} < A_C$ such that the future generation chooses $D = 1$ whenever $\lambda = \lambda_r$. In other words, the backstop technology is deployed when the state of the world is unfavorable. The subgame-perfection is accomplished by imposing more pollution stock on the successor generation such that deployment becomes preferred over accepting pollution damages.

4 The full characterization

Two obvious questions follow from demonstrating the existence of the benchmark, the overabatement, the underabatement, and the technology denial equilibrium. What parameter combinations give rise to the different equilibria? And are these equilibria unique given parameter settings? In this section, we provide a characterization of the intergenerational technology transfer game that answers these questions.

Figure 2 provides a graphical rendering of this characterization. Outcomes are determined in K-θ space. K are the fixed costs of TEI for the current generation and θ is the degree of asymmetry of preferences between the two generations. Conditions (8) and (9) hold such that the benchmark case of discriminate use is welfare-maximizing and such that K and θ fully determine the decision on the level of abatement.

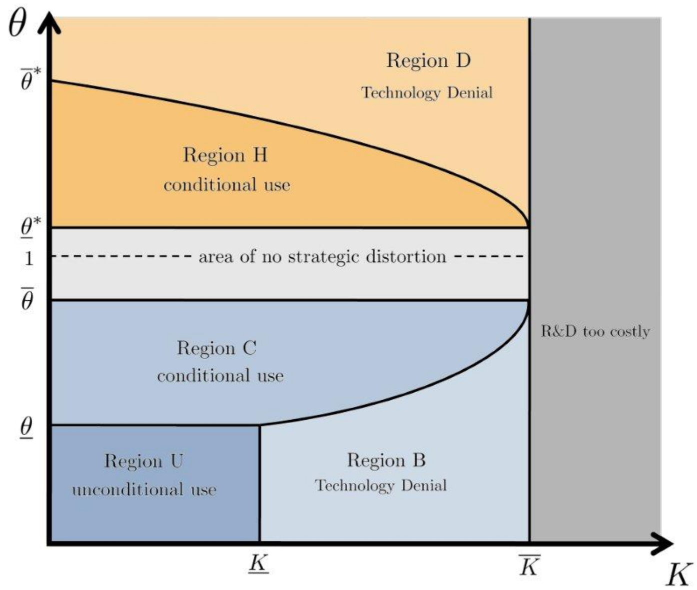


Figure 2: Subgame Perfect Equilibria in $K - \theta$ space

The first insight that Figure 2 captures is that for a number of parameter constellations, the benchmark strategy is implementable since it is subgame-perfect. On the one hand, the implementation of the current generation's optimal policy is ensured if the costs of TEI are high, $K > \bar{K}$: The current generation will not incur the fixed cost of developing the backstop. Trivially, then, $I = 0$ and the current generation does not deliver the technology to the future generation. The current decision-maker therefore faces the simpler problem of determining optimal abatement only. The outcome here is to choose an abatement level A_B, that coincides with that under TEI denial. On the other hand, the implementation of the current generation's optimal policy is ensured if the preference asymmetry falls within $\underline{\theta}^* > \theta > \bar{\theta}$. In such a world, current and future generation differ in their valuation of backstop-induced damages, but not sufficiently to make the future generation deviate from the first-best policy $(A_C, I = 1, \lambda = \lambda_r, D = 1)$ and $(A_C, I = 1, \lambda = \lambda_u, D = 0)$.

The second insight from Figure 2 is that for any parameter combination (K, θ), the subgame-perfect equilibrium is unique. In other words, the benchmark, the overabatement, the under-abatement, and the technology denial equilibria are mutually exclusive. The parameter space $[0, \bar{K}] \times [0, \bar{\theta}]$ in the lower portion of Figure 2 contains the strategic equilibria under the backstop abuse. The space between $\left[0, \bar{K}\right] \times [\underline{\theta}^*, \infty)$ in the upper portion contains the strategic equilibria under the backstop abandonment.

Starting with the lower portion, region U contains the combinations leading to the under-abatement equilibrium in response to time inconsistency. Region B contains those leading to the technology denial equilibrium. Region C contains combinations that give rise to the overabatement equilibrium. The borders separating the regions have intuitive explanations. The boundary between regions B and U is given by $\underline{K}$ and does not depend on θ. The reason is that the abatement levels in B and U, A_B and A_U, respectively, do not depend on θ. As a result, only variations in K determine the choice between the underabatement and technology denial equilibria. The boundary separating regions U and C is given by $\underline{\theta}$ and does not depend on K. The reason is that in both the overabatement and the underabatement equilibrium the technology is provided at the same cost. The absolute level of K is therefore irrelevant for the relative welfare position associated with the two equilibria. The boundary between regions C and B, on the other hand, is more involved: Whether the current generation prefers overabatement to technology denial depends on

$$K < -\frac{R_0^2 \alpha^2}{\alpha + \mathbb{E}[\lambda^2]} + \theta\rho\frac{(2\alpha R_0 - p\rho)}{\lambda_u^2} - \theta^2\rho^2\frac{\alpha + (1-p)\lambda_u^2}{\lambda_u^4} \equiv \hat{K}(\theta). \tag{15}$$

Observe that $\hat{K}'(\theta) > 0$ if $\theta < \bar{\theta}$ and $\hat{K}''(\theta) < 0$. Moreover, $\hat{K}(\bar{\theta}) = \bar{K}$ and $\hat{K}(\underline{\theta}) = \underline{K}$. This traces out a boundary $\hat{\theta}(K)$ in K-θ space. For values of K between $\underline{K}$ and $\bar{K}$, the current generation's decision depends nontrivially on the degree of preference asymmetry θ. The abatement level in region C is determined by A_{crit}, which decreases linearly in θ, leading to the costs of overabatement decreasing more than linearly in θ. As a result, a decision-maker previously indifferent between the overabatement and the technology denial equilibrium will accept more than proportionately higher costs of the technology to remain indifferent between the equilibria as θ approaches $\bar{\theta}$ rather than switching to overabatement.

The upper portion of Figure 2 contains the underabatement equilibrium in region H and the technology denial equilibrium in region D. The lower bound on the upper portion given by $\underline{\theta}^*$ is intuitive as it defines the region where the benchmark strategy fails subgame-perfection. The boundary between regions H and D is given by the condition

$$K < \hat{K}^*(\theta) \tag{16}$$

where $\hat{K}^*(\theta)$ is defined in (14). This traces out a function $\hat{\theta}(K)$ in K-θ space, with a threshold point at $\overline{\theta}^*$ and a minimum at $\theta = \underline{\theta}^*$. Between $\overline{\theta}^*$ and $\underline{\theta}^*$, the boundary is decreasing in θ in an intuitive way: Abatement levels in region H are determined by A^*_{crit}, which decreases linearly in θ, leading to the costs of underabatement decreasing more than linearly. As a result, a decision-maker previously indifferent between the underabatement and technology denial equilibrium will accept more than proportionately higher costs of the backstop to remain indifferent between the equilibria as θ approaches $\underline{\theta}^*$ rather than switching to underabatement.

5 Uncertainty over future preferences

Assuming the current generation knows the future generation's preference parameter θ with certainty provides an instructive starting point. A more general approach is to assume that the current generation regards future preferences as random and assigns a probability density function $f(\theta)$ to different possible realizations of θ. The objective function and the decision problem of the current generation (5) then remain formally unchanged, but the expectation operator now includes expectations over both climate sensitivity λ and future preferences θ. As a result, when deciding on abatement A and TEI I, the current generation needs to take into account the consequences of its choice $\{A, I\}$ on its welfare under all possible realizations of θ and λ. We first undertake an analytical characterization at the general level, followed by a specific numerical illustration for a set of parameters and probability functions.

5.1 Analytical characterization

The current generation will provide the backstop (choose $I = 1$) only if the costs are lower compared to the case in which the backstop is not provided (choose $I = 0$). The outcome under technology denial is simple: With $I = 0$, the objective function reads

$$\min_{\{A\}} \quad \alpha A^2 + \int_0^\infty \mathbb{E}[\lambda^2](R_0 - A)^2 f(\theta) d\theta \ . \tag{17}$$

The integrand does not depend on θ; thus, the abatement level A_B from the case under certainty is the minimizer. The fixed benefit level under $I = 0$ provides the benchmark for the maximum benefit level attainable for an optimal choice of A given that $I = 1$.

With $I = 1$, the objective function is

$$\begin{aligned} \min_{\{A\}} \quad & \alpha A^2 + K + \int_0^{\theta_{\text{crit}}(A)} \rho(R_0 - A) f(\theta) d\theta \, + \\ & \int_{\theta_{\text{crit}}(A)}^{\theta^*_{\text{crit}}(A)} \left[p\rho(R_0 - A) + (1-p)\lambda_u^2 (R_0 - A)^2 \right] f(\theta) d\theta + \int_{\theta^*_{\text{crit}}(A)}^{\infty} \mathbb{E}[\lambda^2](R_0 - A)^2 f(\theta) d\theta \ . \end{aligned} \tag{18}$$

To understand this expression, first note that there are three distinct intervals over which the expression is evaluated. These intervals represent three distinct use regimes of the backstop, depending on the specific realization of θ: indiscriminate use, discriminate use, and desisting from using the backstop. The critical levels $\theta_{\text{crit}}(A)$ and $\theta^*_{\text{crit}}(A)$ determine, for a given level A, the threshold at which an increase in θ leads to a change from indiscriminate to discriminate use ($\theta_{\text{crit}}(A)$) and a change from discriminate use to desistance ($\theta^*_{\text{crit}}(A)$). The thresholds are given by (see appendix):

$$\theta_{\text{crit}}(A) = \frac{\lambda_u^2}{\rho}(R_0 - A) \quad , \quad \theta^*_{\text{crit}}(A) = \frac{\lambda_r^2}{\rho}(R_0 - A) \tag{19}$$

It is clear from equation (19) that the thresholds depend negatively on A. That is, increasing abatement levels will render the indiscriminate use case less likely and the desistance case more likely. Figure 3 provides a graphical representation of the use regimes and thresholds in $A-\theta$ space.

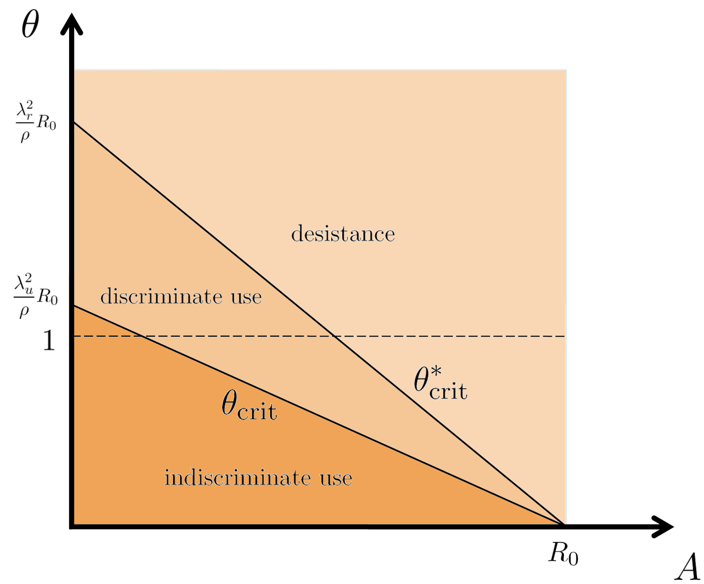


Figure 3: Use Regimes in A-θ space

Determining the optimal abatement level A for $I = 1$ and comparing the resulting benefits with that under technology denial $\{A_B, I = 0\}$ is a numerical exercise to which we turn - with specific examples of probability distributions - in the next section. We complete the analytical characterization by exploiting the nature of the intervals and the specific form of expression (18) to make some general statements on how increases in the variance of the probability distribution, σ^2, impact on the TEI decision and the optimal abatement level. To aid intuition, we assume all distributions of θ to have mean 1, implying an unbiased ex-ante belief about future generation, while the variance σ^2 is a measure of how uncertain the current generation is about future generation's attitude towards the backstop.

The first observation is that mean-preserving spreads of the distribution unambiguously decrease the benefits generated by technology provision. With the welfare maximizing benchmark located in the interval $[\theta_{\text{crit}}(A), \theta^*_{\text{crit}}(A)]$, increases in variance imply a redistribution of mass in the probability distribution from that interval to the outlying intervals $[0, \theta_{\text{crit}}(A)]$ and $[\theta^*_{\text{crit}}(A), \infty)$. Note that each of the three definite integrals is minimized by a different abatement level; hence, this redistribution invariably leads to a choice of A that in expected terms provides less welfare than a choice under a lower σ.

The second observation is that since the benefits of a technology denial policy $\{A_B, I = 0\}$ are fixed and independent of σ, an increase in σ renders technology denial ($I = 0$) relatively more beneficial than technology provision. The difference in benefits captures the quasi-option value that would be associated with the opportunity to learn the realization of θ before committing to a TEI and abatement policy. Given the fixed benchmark of technology denial, numerical implementations will find that a policy $\{A_B, I = 0\}$ is preferred to technology provision as soon as σ becomes large enough.

The third observation, that derives from the first and second observations together, is that optimal abatement levels under $I = 1$ can only deviate within bounds delimited by the outside option of $I = 0$. This third observation leads to the prediction that numerical implementations will find narrow deviations in A from the optimum in response to increases in σ.

The fourth and final observation is that the direction of deviations from the optimal level of A can be explained by the nature of the strategic equilibria characterized in the previous section and Figure 2. Since the thresholds in equation (19) are not equidistant from 1 unless $A = R_0 - 2\frac{\rho}{(\lambda_r^2+\lambda_u^2)}$, symmetric changes in the shape of the distribution have asymmetric impacts on the optimal level of abatement, A: For example, for $|1 - \theta_{\text{crit}}(A)| < |1 - \theta^*_{\text{crit}}(A)|$ and for σ increasing from 0 to a positive value, the first region of strategic distortion to receive positive mass in the distribution is region C. The positive weight given to region C involves an abatement level A_{crit} that decreases with θ, leading to increases in the optimal A the more probability is assigned to lower realizations of θ in this region. Further increases in σ assign more mass to regions such as U and H, associated with lower levels of A. Hence, abatement levels can vary non-monotonically with σ. However, it might be worth noting that it is clearly not the case that the optimal abatement level can be determined by simply avering the abatement levels derived in previous sections. The optimization problem current generation faces is rather complicated and in particular the qualitative behavior of the optimal abatement level for high values of σ is sensitive to the choice of probability distribution.

Determining the optimal abatement level A for $I = 1$ and comparing the resulting benefits with that under technology denial $\{A_B, I = 0\}$ is a numerical exercise to which we turn now.

5.2 Numerical illustration

The specific examples we discuss here are the case of a Bernoulli distribution with $\theta = 1 \pm \sigma$ with equal probability and a Gamma distribution that is not symmetric around the mean. We choose parameters such that the benchmark equilibrium exists:

$$p = 0.5 \quad , \quad R_0 = 4 \quad , \quad \lambda_u = 2 \quad , \quad \lambda_r = 3 \quad , \quad \alpha = 1 \quad , \quad \rho = 5 \; . \tag{20}$$

Applying the results of section 3 and 4, the critical levels for the strategic equilibria are then

$$\underline{K} = 0.12 \quad , \quad \bar{K} = 0.39 \quad , \quad \underline{\theta} = 0.49 \quad , \quad \bar{\theta} = 0.73 \quad , \quad \underline{\theta}^* = 1.65 \quad , \quad \bar{\theta}^* = 2.30 \tag{21}$$

and the resulting abatement levels for the various strategic equilibria are

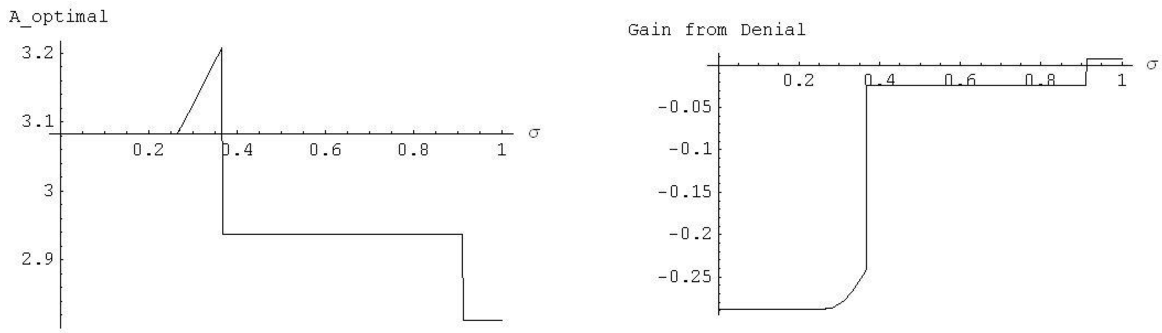
$$A_C = 3.08 \quad , \quad A_U = 2.50 \quad , \quad A_B = 3.47 \quad , \quad A_{\text{crit}} = \frac{16 - 5 \cdot \theta}{4} \quad , \quad A^*_{\text{crit}} = \frac{36 - 5 \cdot \theta}{4} \; . \tag{22}$$

The numerical results are contained in Figure 4. The top row shows the results for the case of a Bernoulli distribution with $\theta = 1 \pm \sigma$ with equal probability and the bottom row shows the results for the case of a Gamma distribution with expected value 1 and variance σ^2. The left column shows the optimal levels of abatement and the right column shows the benefits from denying the backstop technology, for the different values of σ .

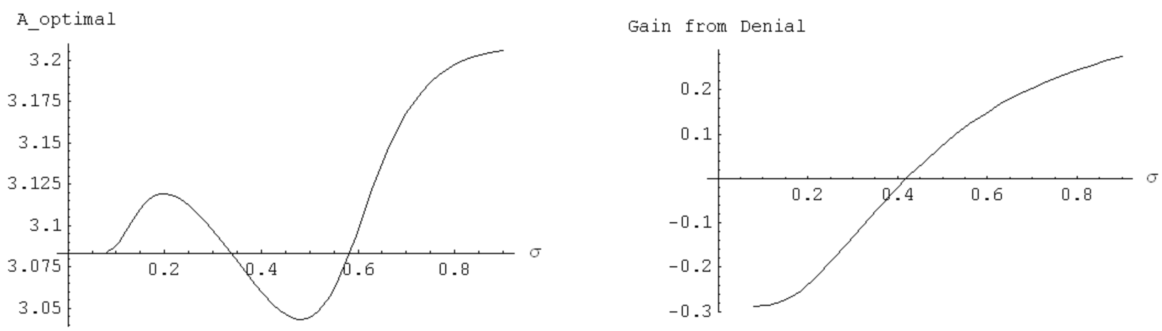
The top-left panel in Figure 4 confirms the intuition of our analytical discussion of expression (18): Abatement initially remains unchanged at $A_C = 3.08$ until region C has enough weight for the required increases in A to change the optimal level of abatement. Then abatement increases in line with A_{crit} before dropping off as more probability is shifted to regions characterized by underabatement equilibria. The top-right panel depicts the difference between current generation benefits under technology denial or technology provision under the Bernoulli distribution. The gains from technology denial rise quickly as the variance increases. This reflects the decrease in benefits associated with choosing an abatement level that commits the current generation to a given policy, independent of the realization of θ, rather than accruing the fixed benefits of the denial strategy.

The bottom-left panel shows that the basic insights regarding the narrow range of variation in abatement levels and their non-monotonicity as a function of variance holds for a different density function. The bottom-right panel shows, just as in the case of the Bernoulli distribution, that the net benefits of technology denial are increasing with the variance. That is, for large enough values of σ^2, technology denial becomes optimal.

The analytical and numerical discussion together emphasize three points: The first is that uncertainty over the future generation's preferences raises the attractiveness of technology denial since technology provision entails conditioning abatement levels to possible realizations of θ and therefore deviating more and more from the benefit level attainable under certainty. By introducing preference uncertainty, the argument in favor of technology denial gets stronger the more risk is admitted. This adds an additional dimension to the small literature on optimal environmental policy under preference uncertainty. The second point is that the optimal abatement level varies



(Bernoulli distribution case)



(Gamma distribution case)

Figure 4: Numerical Results.

only over a relatively small range for those levels of uncertainty under which the technology is provided. The third point is that variations in abatement levels have an intuitive explanation based on the strategic equilibria derived in section 4.

6 Concluding discussion

The starting point of this paper is that the impacts of long-lived stock pollutants and the abatement and technology measures supposed to address them link current and future generations together. Altruism towards successor generations is a prerequisite for resolving the resulting intergenerational equity issues. Preference asymmetry and typical imperfections of altruism, however, introduce the possibility of important strategic conflicts between generations. The presence and nature of these conflicts and the strategic distortions thus introduced are the subject of this inquiry.

The paper's result, derived in a stark setting, demonstrate that preference asymmetry and imperfect altruism raise the possibility of a rich set of outcomes in this type of intergenerational transfer game. The future generation may be deliberately denied an imperfect backstop, either because of a perception by the current generation that the technology will be used indiscriminately or not at all. The current generation may also decide that it is rational to accompany the development of the backstop with a reduction in abatement efforts, either because it believes that, with the backstop being used regardless of the level of damages, abatement is not productive or because it needs to 'turn up the heat' on a more cautious successor generation for it to choose to 'enjoy' the benefits of technology. Finally, comparing abatement levels under certainty and uncertainty, the current generation will prefer to avoid a costly smoothing of abatement levels across different realizations of the future generation's preferences once the variance is significant by simply reverting to the safe option of technology denial.

The paper's assumptions are stark, but we believe that they are productive for capturing something salient about the problem of intergenerational technology transfer. The rich set of outcomes not only provides a full characterization of this parsimonious setting, it also highlight a few counterintuitive and troubling findings. For example, a future generation known to assess collateral damages by the technology higher than current generation may be left with a higher stock of pollution for strategic reasons in order to induce use of the backstop. What the results highlight then is that the natural coexistence of intergenerational altruism with a notion of proper use of the technological capabilities generates rather striking results. Also, while the idea of a technology denial and of an underabatement equilibrium have been raised outside economics when discussing problematic backstops to climate change such as geoengineering solutions (Shepherd et al. (2009) for the Royal Society, Solomon et al. (2009) for the U.S. National Academy of Sciences), the equilibria established in this paper add a novel possibility: This is the presence of an overabatement equilibrium in a world in which the future generation is believed to be less concerned about the environment. Finally, uncertainty provides additional powerful arguments for leaving a future generation to suffer stock pollution damages by denying the available backstop technology. In sum, the paper illustrates that while altruism is a sine-qua-non for resolving the long-term environmental problems that are at the basis of intergenerational conflicts, differences in preferences and the paternalistic dimension inherent in much of altruistic behavior raise the possibility of strategic dimensions so far unconsidered.

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A Objectives and Abatement levels

We differentiate between three combinations of I and D and look at the associate objective function (equation (4)) of current generation:

$$\begin{aligned} C_{\text{always}} &= \alpha A^2 + p\rho(R_0 - A) + (1-p)\lambda_u^2(R_0 - A)^2 + K && \text{(A.1)} \\ &\quad \text{if } I = 1 \text{ and } D(\lambda = \lambda_u) = 1,\ D(\lambda = \lambda_r) = 1 \\ C_{\text{cond}} &= \alpha A^2 + \rho(R_0 - A) + K && \text{(A.2)} \\ &\quad \text{if } I = 1 \text{ and } D(\lambda = \lambda_u) = 0,\ D(\lambda = \lambda_r) = 1 \\ C_{\text{noCE}} &= \alpha A^2 + (p\lambda_r^2 + (1-p)\lambda_u^2) \cdot (R_0 - A)^2 && \text{(A.3)} \\ &\quad \text{if } I = 0\ . \end{aligned}$$

The optimal level of abatement can be derived by means of FOC

$$\frac{dC_{\text{always}}}{dA}(A_U) = 0 \ \Rightarrow\ A_U = \frac{\rho}{2\alpha} \quad , \tag{A.4}$$

$$\frac{dC_{\text{cond}}}{dA}(A_C) = 0 \ \Rightarrow\ A_C = \frac{2(1-p)R_0\lambda_u^2\ +\ p\rho}{2\left(\alpha + (1-p)\lambda_u^2\right)} \quad , \tag{A.5}$$

$$\frac{dC_{\text{noCE}}}{dA}(A_B) = 0 \ \Rightarrow\ A_B = \frac{R_0\mathbb{E}[\lambda^2]}{\alpha + \mathbb{E}[\lambda^2]} \quad . \tag{A.6}$$

These are minimizers because

$$\frac{d^2C_{\text{always}}}{dA^2} = 2\alpha > 0 \quad , \quad \frac{d^2C_{\text{cond}}}{dA^2} = 2\alpha + 2(1-p)\lambda_u^2 > 0 \quad , \quad \frac{d^2C_{\text{noCE}}}{dA^2} = 2\alpha + 2\mathbb{E}\left[\lambda^2\right] > 0\ . \tag{A.7}$$

It is useful to have the values of the objective functions at the minimizers at hand:

$$C_{\text{always}}(A_U) = R_0\rho - \frac{\rho^2}{4\alpha} + K \tag{A.8}$$

$$C_{\text{cond}}(A_C) = \frac{4(1-p)R_0^2\,\alpha\lambda_u^2\ +\ 4pR_0\alpha\rho\ -\ p^2\rho^2}{4\big(\alpha + (1-p)\lambda_u^2\big)} + K \tag{A.9}$$

$$C_{\text{noCE}}(A_B) = R_0^2\,\alpha\frac{\mathbb{E}[\lambda^2]}{\alpha + \mathbb{E}[\lambda^2]} \tag{A.10}$$

There is a level of abatement which separates de facto always use from de facto conditional use. To calculate this critical level A_{crit} we note that future generation will prefer indiscriminate use to conditional use iff damages by CE in the unresponsive case are smaller than temperature damages, i.e.

$$\theta\rho(R_0 - A) < \lambda_u^2(R_0 - A)^2 \ \Leftrightarrow\ A < R_0 - \frac{\theta\rho}{\lambda_u^2} \quad \equiv\ A_{\text{crit}}\ . \tag{A.11}$$

Similar, we find an abatement level that separates conditional use and renunciation of the CE-option. The latter will be the case for all

$$A > R_0 - \frac{\theta\rho}{\lambda_r^2} \quad \equiv \; A^*_{\text{crit}} \; . \tag{A.12}$$

It is obvious that $A^*_{\text{crit}} > A_{\text{crit}}$.
In addition to (A.8) - (A.10) we calculate

$$C_{\text{cond}}(A_{\text{crit}}) \;=\; R_0^2\alpha \,-\, \theta\rho\frac{2R_0\alpha - p\rho}{\lambda_u^2} \,+\, \theta^2\rho^2\frac{\alpha + (1-p)\lambda_u^2}{\lambda_u^4} \tag{A.13}$$

and

$$C_{\text{cond}}(A^*_{\text{crit}}) \;=\; R_0^2\alpha \,-\, \theta\rho\frac{2R_0\alpha - p\rho}{\lambda_r^2} \,+\, \theta^2\rho^2\frac{\alpha + (1-p)\lambda_u^2}{\lambda_r^4} \tag{A.14}$$

B Benchmark Equilibrium

The abatement level A_C is by definition optimal given that conditional use of CE will be reality. For this combination (A_C,conditional use) to be preferred over unconditional use and banning R&D two conditions must hold

$$C_{\text{cond}}(A_C) < C_{\text{always}}(A) \quad \forall A \tag{Start1}$$

$$C_{\text{cond}}(A_C) < C_{\text{noCE}}(A) \quad \forall A \tag{Start2}$$

We analyze both conditions separately

(Start1) is, by definition of A_U (cf. A.4), equivalent to $C_{\text{cond}}(A_C) < C_{\text{always}}(A_U)$. Using (A.8) and (A.9) lead to $\rho^2 - \frac{4R_0\alpha(\alpha+\lambda_u^2)}{\alpha(1+p)+\lambda_u^2}\rho + \frac{4R_0^2\alpha^2\lambda_u^2}{\alpha(1+p)+\lambda_u^2} < 0$. The roots are $\rho_1 = 2\alpha R_0\frac{\alpha+\lambda_u^2 - \sqrt{\alpha^2+\alpha(1-p)\lambda_u^2}}{\alpha(1+p)+\lambda_u^2}$, $\rho_2 = 2\alpha R_0\frac{\alpha+\lambda_u^2 + \sqrt{\alpha^2+\alpha(1-p)\lambda_u^2}}{\alpha(1+p)+\lambda_u^2}$ and since the quadratic function is larger 0 at $\rho = 0$ we see that (Start1) is equivalent to $\rho \in (\rho_1, \rho_2)$. But it is straightforward to show that ρ_2 is larger than the economic bound $2\alpha R_0$, hence $\rho > 2\alpha R_0 \frac{\alpha+\lambda_u^2 - \sqrt{\alpha^2+\alpha(1-p)\lambda_u^2}}{\alpha(1+p)+\lambda_u^2}$ is the relevant c! ondition.

(Start2) is, by definition of A_B (cf. A.6), equivalent to $C_{\text{cond}}(A_C) < C_{\text{noCE}}(A_B)$. By making use of (A.9) and (A.10) we get $K < R_0^2\alpha\frac{\mathbb{E}[\lambda^2]}{\alpha+\mathbb{E}[\lambda^2]} - \frac{4(1-p)R_0^2\alpha\lambda_u^2 + 4pR_0\alpha\rho - p^2\rho^2}{4\left(\alpha+(1-p)\lambda_u^2\right)} \equiv \bar{K}$.

As we do not allow for negative costs of enabling the technology, $K \geq 0$, we get an extra condition on the parameter to ensure that $\bar{K} > 0$. This can be translated into a condition on ρ,

$$\rho^2 - \frac{4R_0\alpha}{p}\rho + \frac{4R_0^2\alpha^2\lambda_r^2}{p(\alpha + \mathbb{E}[\lambda^2]} > 0 \; . \tag{B.1}$$

The roots are

$$\frac{2R_0\alpha}{p\sqrt{\alpha + \mathbb{E}[\lambda^2]}}\left(\sqrt{\alpha + \mathbb{E}[\lambda^2]} \pm \sqrt{\alpha + (1-p)\lambda_u^2}\right) \; .$$

Making use of $\rho \geq 0$, the fact that (B.1) holds for $\rho = 0$ and that the smaller root is positive shows that the condition $\bar{K} > 0$ is equivalent to

$$\rho \; < \; \frac{2R_0\alpha}{p\sqrt{\alpha + \mathbb{E}[\lambda^2]}}\left(\sqrt{\alpha + \mathbb{E}[\lambda^2]} - \sqrt{\alpha + (1-p)\lambda_u^2}\right) \equiv \rho_{\text{technical}} \; . \tag{B.2}$$

In summary, the initial equilibrium is characterized by

$$\rho \; \in \; \left(\underline{\rho}, \bar{\rho}\right) \quad \text{and} \quad K < \bar{K} \tag{B.3}$$

with

$$\underline{\rho} \; = \; 2\alpha R_0 \, \frac{\alpha + \lambda_u^2 \; - \; \sqrt{\alpha^2 + \alpha(1-p)\lambda_u^2}}{\alpha(1+p) + \lambda_u^2} \quad , \quad \bar{\rho} \; = \; \min\left\{2\alpha R_0 \, , \, \lambda_u^2 R_0 \, , \, \rho_{\text{technical}}\right\}$$

and

$$\bar{K} \; = \; R_0^2\alpha\frac{\mathbb{E}[\lambda^2]}{\alpha + \mathbb{E}[\lambda^2]} - \frac{4(1-p)R_0^2\alpha\lambda_u^2 \; + \; 4pR_0\alpha\rho - p^2\rho^2}{4\big(\alpha + (1-p)\lambda_u^2\big)} \; .$$

C Backstop Abuse

Underabatement

Condition (8)[use the right label/ref here] is easy to replicate: Use (A.5) and (A.11) to rearrange $A_C < A_{\text{crit}}$.

Remark 3 *It is not always true that switching to* A_U *is a* reduction *of abatement*

$$A_U < A_C \quad \textit{if and only if} \quad \rho < 2\alpha R_0 \frac{\lambda_u^2}{\alpha + \lambda_u^2} \; . \tag{C.1}$$

Note that whether the last condition is always fulfilled (since $\rho_{\text{technical}} < 2\alpha R_0 \frac{\lambda_u^2}{\alpha + \lambda_u^2}$*) or this anomaly might take place depends on the parameters. As it is not essential for our results we do not determine the relevant parameter constellations here.*

Proof of the Lemma. (Start1) is in particular true for $A = A_C$. Thus,

$$\begin{aligned} \alpha A_C^2 + p\rho(R_0 - A_C) + (1-p)\lambda_u^2(R_0 - A_C)^2 &< \alpha A_C^2 + p\rho(R_0 - A_C) + (1-p)\rho(R_0 - A_C) \\ \Rightarrow \lambda_u^2(R_0 - A_C) &< \rho \\ \Rightarrow \bar{\theta} &< 1 \,. \end{aligned}$$

■

Technology Denial

The condition for the technology denial to be preferred over the underabatement case is $C_{\text{noCE}}(A_B) < C_{\text{always}}(A_U)$. It is easy to rearrange it by means of (A.10) and (A.9) to $K > R_0^2\,\alpha \cdot \frac{\mathbb{E}[\lambda^2]}{\alpha + \mathbb{E}[\lambda^2]} - R_0\rho + \frac{\rho^2}{4\alpha} \equiv \underline{K}$.
The abatement level A_B is larger than A_C since simple algebra shows that $A_B > A_C$ iff $1 < \frac{\lambda_r^2(2\alpha R_0 - p\rho)}{(\alpha + (1-p)\lambda_u^2)\rho} = 2\underline{\theta}^*$. As $\underline{\theta}^* > 1$ (see Appendix D) the statement is proven.

Overabatement

There are two conditions that must hold for the Overabatement equilibrium to arise. Overabatement has to be preferred over, firstly, Underabatement and, secondly, technology denial.

(i) We compare $C_{\text{always}}(A_U)$ with C_{cond} at $A = A_{\text{crit}}$. If the latter is lower, such that conditional use at A_{crit} is preferred to unconditional use at A_U, by continuity an abatement level larger than A_{crit} exists at which this is still true - and conditional use will be strictly preferred over unconditional use.
The condition for this is, by (A.8) and (A.13),

$$\theta^2 - \frac{\lambda_u^2(2R_0\alpha - p\varrho)}{(\alpha + (1-p)\lambda_u^2)\rho}\theta + \frac{\lambda_u^4(2R_0\alpha - \varrho)^2}{4\alpha(\alpha + (1-p)\lambda_u^2)\rho^2} < 0 \,. \tag{C.2}$$

Calculating the roots yields $\theta_1 = \bar{\theta} - \Delta$, $\theta_2 = \bar{\theta} + \Delta$, where $\bar{\theta} = \frac{\lambda_u^2(2\alpha R_0 - p\rho)}{2(\alpha + (1-p)\lambda_u^2)\rho}$ and $\Delta = \lambda_u^2\sqrt{\frac{1-p}{\alpha}}\frac{\sqrt{4R_0\alpha(\alpha+\lambda_u^2)\rho - 4R_0^2\alpha^2\lambda_u^2 - (\alpha(1+p)+\lambda_u^2)\rho^2}}{2(\alpha+(1-p)\lambda_u^2)\rho}$.
It is straightforward to show that $\theta_1, \theta_2 > 0$. Because the quadratic equation (C.2) is not fulfilled at $\theta = 0$ condition (C.2) is equivalent to $\theta \in \left(\bar{\theta} - \Delta, \bar{\theta} + \Delta\right)$. Finally, because $\theta < \bar{\theta}$ by (8) [again, find right ref here], it reads $\theta \in (\underline{\theta}, \bar{\theta})$, where $\underline{\theta} = \bar{\theta} - \Delta$.

Remark 4 *Again, it is not a priori clear that $A_{\text{crit}} > A_U$, i.e.* over*abatement deserving of that appellation. We have $A_{\text{crit}} > A_U$ iff $R_0 - \frac{\theta\rho}{\lambda_u^2} - \frac{\rho}{2\alpha} > 0$. To get a* sufficient *condition we* *plug in $\bar{\theta}$ instead of θ. Some algebra yields*

$$A_{\text{crit}} > A_U \quad \Leftarrow \quad \rho < 2\alpha R_0 \frac{\lambda_u^2}{\alpha + \lambda_u^2} \ . \tag{C.3}$$

Note that this bound is also the one that decides whether $A_U < A_C$ or not (cf. C.1).

(ii) The Overabatement is preferred over the technology denial iff $C_{\text{cond}}(A_{\text{crit}}) < C_{\text{noCE}}(A_B)$. It is easy to verify, by (A.13) and (A.10), that this is in fact condition (11) [put right ref], $K < \frac{2R_0\alpha\rho}{\lambda_u^2}\theta - \frac{\theta(\alpha\theta+(p+(1-p)\theta)\lambda_u^2)\rho^2}{\lambda_u^4} - \frac{R_0^2\alpha^2}{\alpha+\mathbb{E}[\lambda^2]} \equiv \hat{K}(\theta)$. The first derivative reads $\hat{K}'(\theta) = \frac{\rho(2\alpha R_0 - p\rho)}{\lambda_u^2} - \frac{2(\alpha+(1-p)\lambda_u^2)\rho^2}{\lambda_u^4}\theta$, hence $\hat{K}''(\theta) = -\frac{2(\alpha+(1-p)\lambda_u^2)\rho^2}{\lambda_u^4} < 0$. Thus, a maximum of the function $\theta \mapsto \hat{K}(\theta)$ is at $\theta = \frac{\lambda_u^2(2\alpha R_0 - p\rho)}{2(\alpha+(1-p)\lambda_u^2)\rho} = \bar{\theta}$.
Finally, it is simply a lengthy calculation to show that $\hat{K}(\bar{\theta}) = \bar{K}$ and $\hat{K}(\underline{\theta}) = \underline{K}$.
Since $\hat{K}$ is monotonically increasing in $\left[\underline{\theta}, \bar{\theta}\right]$ with values in $\left[\underline{K}, \bar{K}\right]$, the inverse function $\hat{\theta} = \hat{K}^{-1}$ exists on $\left[\underline{K}, \bar{K}\right]$.

D Backstop Abandonment

Technology Denial

The possible outcome in this case is that future generation will prefer to *never* use the technology while current generation wants it to be used conditionally. The initial equilibrium is subject to this distortion if and only if (cf definition A.12) $A_C > A^*_{\text{crit}}$. This reads

$$\theta \ > \ \frac{\lambda_r^2(2\alpha R_0 - p\rho)}{2(\alpha + (1-p)\lambda_u^2)\rho} \equiv \underline{\theta}^* \ . \tag{D.1}$$

Proof of the Lemma. (Start2) is in particular true for $A = A_C$. Thus,

$$\begin{aligned} \alpha A_C^2 + p\rho(R_0 - A_C) + (1-p)\lambda_u^2(R_0 - A_C)^2 \ &< \ \alpha A_C^2 + p\lambda_r^2(R_0 - A_C)^2 + (1-p)\lambda_u^2(R_0 - A_C)^2 \\ \Rightarrow \ \rho \ &< \ \lambda_r^2(R_0 - A_C) \\ \Rightarrow \ 1 \ &< \ \underline{\theta}^* \ . \end{aligned}$$

■

Underabatement

The Underabatement is preferred over the technology denial if and only if $C_{\text{cond}}(A^*_{\text{crit}}) < C_{\text{noCE}}(A_B)$. It is easy to verify, by (A.14) and (A.10), that this in fact condition [???] $K < \frac{2R_0\alpha\rho}{\lambda_r^2}\theta - \frac{\theta(\alpha\theta+(p+(1-p)\theta)\lambda_u^2)\rho^2}{\lambda_r^4} - \frac{R_0^2\alpha^2}{\alpha+\mathbb{E}[\lambda^2]} \equiv \hat{K}^*(\theta)$. The first derivative reads $\frac{d}{d\theta}\hat{K}^*(\theta) = \frac{\rho(2\alpha R_0 - p\rho)}{\lambda_r^2} -$

$\frac{2(\alpha+(1-p)\lambda_u^2)\rho^2}{\lambda_r^4}\theta$, hence $\frac{d^2}{d\theta^2}\hat{K}^*(\theta) = -\frac{2(\alpha+(1-p)\lambda_u^2)\rho^2}{\lambda_r^4} < 0$. Thus, a maximum of the function $\theta \mapsto \hat{K}^*(\theta)$ is at $\theta = \frac{\lambda_r^2(2\alpha R_0 - p\rho)}{2(\alpha+(1-p)\lambda_u^2)\rho} = \underline{\theta}^*$.

Finally, it is simply a lengthy calculation to show that $\hat{K}^*(\underline{\theta}^*) = \bar{K}$.

Since $\hat{K}^*$ is monotonically increasing in $\left[\overline{\theta}^*, \underline{\theta}^*\right]$ with values in $\left[0, \bar{K}\right]$, the inverse function $\hat{\theta}^* = \left(\hat{K}^*\right)^{-1}$ exists on $\left[0, \bar{K}\right]$.

E Uncertainty over future preferences

To verify the expressions for θ_{crit} and θ^*_{crit} please cf. the Appendix A, where A_{crit} and A^*_{crit} are derived for given θ.

The Gamma Distribution is defined as

$$\gamma(x; k, \xi) \ := \ x^{k-1} \frac{\exp(-x/\xi)}{\Gamma(k)\xi^k} \quad , \ x \geq 0 \, , \ k, \xi > 0$$

where Γ is the gamma function. This distribution has the properties $\mathbb{E}[x] = k\xi$, $\mathrm{Var}[x] = k\xi^2$. As we want to have $\mathbb{E}[x] = 1$ we set $k = 1/\xi$ and use the distribution in the form

$$\gamma(\theta; \sigma) \ = \ \theta^{\frac{1-\sigma^2}{\sigma^2}} \frac{\exp(-\theta/\sigma^2)}{\Gamma(\frac{1}{\sigma^2})\sigma^{\frac{2}{\sigma^2}}} \ , \tag{E.1}$$

where $\sigma = \sqrt{\mathrm{Var}[\theta]}$.