Multi-Objective and Multi-Stage Reliability Growth Planning in Early Product-Development Stage

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Zhaojun Li, Mohammadsadegh Mobin, and Thomas Keyser

Abstract—This paper proposes a multi-objective multi-stage reliability growth planning method in the early product-development stage. Multi-stage reliability growth planning is common in practice, and it aligns well with multiple developmental stages of a new product such as concept design, detail design, prototype design, and final production version design. The multi-objective formulation reflects the needs of product development's multiple objectives, such as program cost, schedule, and reliability. Pareto optimal solutions of the multi-objective multi-stage formulation for reliability growth planning are searched using a modified nondominated sorting genetic algorithm (NSGA-II). To reduce the large size of Pareto optimal solutions to a workable size of efficient solutions for plan implementation, both constant return-to-scale and variable return-to-scale data envelopment analysis (DEA) methods are used for determining the efficient solutions. Based on tradeoff and sensitivity analysis, insights and guidelines are presented for choosing appropriate reliability growth plans in terms of optimal allocation of testing time and testing units, and the timing for new technology introduction. The growth rate in each product-development stage and its impact on the development cost, schedule, and reliability are also discussed. An illustrative example is given to demonstrate the approach for planning the reliability growth for a next-generation engine development.

Index Terms—Data envelopment analysis, genetic algorithm, multi-objective, multi-stage, reliability growth planning.

I. INTRODUCTION

In the process of developing modern complex engineering systems, reliability growth planning (RGP) becomes critical to management for decision support in terms of the timing

\begin{eqnarray}
\alpha_i & \text{ Growth rate in stage } i. \\
\tau_i & \text{ Number of test units for subsystem } j \text{ in stage } i. \\
N_i & \text{ Total number of test units in stage } i. \\
N_{\ell(i);} & \text{ Lower bound for the number of available test units in stage } i. \\
N_u(i) & \text{ Upper bound for the number of available test units in stage } i. \\
N & \text{ Total number of test units.} \\
\ell_j & \text{ Planned test time for each subsystem } j \text{ in stage } i. \\
T_i & \text{ Total planned test time for stage } i. \\
T & \text{ Total planned test time for all stages.} \\
\tau_u & \text{ Upper bound of total test time for reliability growth test.} \\
\tau_i & \text{ Development time for stage } i, \text{ which is the maximum planned test time in stage } i, \text{ i.e., } \tau_i = \max\{\ell_j\}, \text{ for all } j. \\
\tau & \text{ Total development time, } \tau = \sum_{i=1}^{n} \tau_i. \\
c_{ij} & \text{ Fixed cost of subsystem } j \text{ in stage } i. \\
C_{u(i)} & \text{ Variable cost which is a function of total test time } T_i \text{ for stage } i. \\
C_i & \text{ Total test cost of stage } i, \text{ including fixed cost and variable cost.} \\
C & \text{ Total test cost.} \\
\lambda_{ij} & \text{ Failure rate of subsystem } j \text{ in stage } i. \\
\theta_i & \text{ Percentage of introduced new contents in stage } i \text{ for a subsystem, } (\sum_{i=1}^{n} \theta_i = 100\%). \\
\lambda_{n(i)} & \text{ Failure rate of all new contents in stage } i. \\
\lambda_{I(i)} & \text{ Initial failure rate in stage } i, \text{ which is the summation of new contents } \lambda_{n(i)} \text{ and residual failure rate from previous stage } \lambda_{i-1}. \\
\lambda & \text{ The failure rate at the end of stage } i. \\
M_{I(i)} & \text{ Initial MTBF in stage } i. \\
M\{T_i\} & \text{ MTBF at the end of stage } i. \\
\end{eqnarray}
of a new product release, reliability performance monitoring and prediction, and product development cost estimation. An accurate and realistic reliability growth plan can provide tradeoff information between product development schedule, program budget, and achievable reliability level of a new product. Thus, RGP becomes an integral part of a rigorous reliability program in the early stage of new product introduction. RGP is even more important for developing complex engineering systems such as aerospace systems, modern vehicle systems, and aviation systems, since such complex systems usually involves multiple development stages across multiple disciplines.

In the literature, reliability growth modeling has been extensively investigated, and early research works focus on reliability growth modeling and prediction. Duane [1] introduced the first reliability growth model based on the development testing data of aircraft mechanical and electromechanical systems. Duane's empirical model is based on the learning curve characteristic. Crow [2] developed a statistical reliability growth model, i.e., Crow/AMSAA (Army Materiel Systems Analysis Activity) model, in which the improvement in reliability was modeled by a nonhomogeneous Poisson process (NHPP). Crow extended the original model by introducing confidence intervals on the failure intensity and reliability functions when failure data are generated by multiple repairable systems [3], [4]. Key parameters to reliability growth such as reliability goal setting, growth potential design margin, two types of failure modes, design correction effectiveness, and reliability management strategy have also been discussed [5]. Quigley and Walls [6] investigated the confidence intervals of reliability growth models when the sample sizes are small. Lloyd [7] used the binomial model and introduced a model for estimating and forecasting reliability from attribute data. Robinson and Dietrich [8] introduced a nonparametric reliability growth model to analyze the failure rate of a system under the assumptions of one unknown parameter and a unimodal likelihood function. The method was extended to analyze system-level reliability growth and development progress [9]. Smith and Oren [10] proposed a sufficient statistic based estimator for Duane model parameters by tabulating the number of failures between fixed points in time. The goodness-of-fit of this estimation method is compared with the Crow estimation method under varying and limited available failure sample data. Xie and Zhao [11] proposed a “first-model-validation-then-parameter-estimation” approach to simplify model validation and parameter estimation in software reliability analysis. Xie and Zhao [12] introduced alternative reliability growth models to accommodate the cases when Duane model is not well fitted. Ebrahimi [13] pointed out that one specific reliability growth model is usually valid for certain finite amount of development time and proposed to apply a specific reliability growth model to each design modification when the timing of such design improvements is known. However, most reliability growth models focus on reliability growth modeling and monitoring purpose rather than planning purpose.

In early product development stage, reliability growth planning becomes critical to support decision making for the overall product development program. Researchers start to pay more attention to budget and test time allocation for achieving desired reliability goal under a single-phase framework. Kracht et al. [14] compared the modified power law model with the other reliability growth models in the product design phase. They pointed out that reliability growth efforts should be shifted from the test stage to the design stage to achieve a cost and time effective reliability improvement program. Walls and Quigley [15], [16] studied product development stage reliability growth by integrating experts engineering judgments into a mathematical reliability growth model. Coit [17] investigated how to optimally allocate limited reliability growth testing time into different subsystems to maximize the overall system reliability under testing budget constraint. The method also considers minimizing the failure rate uncertainty for each subsystem and demonstrates the significance of intelligently allocating limited testing time to subsystems with various growth potentials. Johnston et al. [18] investigated how to select reliability improvement tasks under a new system development cost and time constraints in concept design stage. A formulated integer programming approach is used to sequence and schedule reliability improvement tasks. More recently, reliability growth planning has extended to multiobjective and multiple phase formulations, e.g., in addition to maximizing projected reliability, other objectives such as minimizing reliability estimation uncertainty has also been considered. Jin and Wang [19] considered three types of effectiveness functions for corrective actions and used it in a failure intensity model to predict mean time between failures (MTBF) of in-service systems. The formulated bi-objective reliability growth model maximizes system reliability and minimizes the reliability uncertainty with the constraint of limited corrective actions budget. Jin et al. [20] studied a stochastic model for predicting the reliability growth for field or in-service electronic systems considering latent failure modes. Jin et al. [21] proposed a multi-phase reliability growth model that sequentially determines and implements corrective actions (CAs). Failure modes are categorized into two modes, surfaced failures that occur during the in-house RGT process and latent failure modes which occur post the system installation or if the in-house testing time could be extended. They developed a CA effectiveness function to assess the tradeoff between the failure removal rate and the required resources and then applied Rosen's gradient projection algorithm to optimally allocate resources for each phase. Therefore, in each phase two iterative steps are considered: reliability prediction and CA resource allocation. Their proposed approach can be applied for product reliability estimation when extended in-house testing is infeasible in early design and prototyping phase.

The existing reliability growth models emphasize the reliability growth modeling by utilizing the actual testing data, which is under the assumption that initial reliability growth testing data are available. However, in the early product development stages such as in the concept design stage and product portfolio selection stage, actual testing data are rarely available for reliability growth planning. In addition, most existing RGP models consider a single product development stage by maximizing the single reliability objective, which may not be able to well model the different reliability growth profiles over a product’s multiple development stages. In this paper, we investigate multi-stage RGP for new product development in early product develop-
ment stages. The proposed multi-stage RGP is able to model each stage's unique reliability growth profile due to different growth rates, new contents/technology, time and budget allocation for each specific stage. In addition to maximizing the reliability objective, the projected total reliability growth testing time and the projected reliability testing cost are considered as optimization objectives. A multi-objective optimization tool called fast Non-dominated sorting genetic algorithm (NSGA-II) is modified and utilized to solve the proposed multi-stage multi-objective reliability growth planning problem. For the generated Pareto optimal solutions from NSGA-II, Data Envelopment Analysis (DEA) which is based on relative efficiency assessment is applied to eliminate the inefficient solutions. The new multi-objective and multi-stage RGP enables decision makers to intelligently allocate limited and expensive testing units/sub-systems, and testing time to each individual development stage such that Pareto optimality can be achieved in terms of reliability, program schedule, and product development cost.

The remainder of the paper is organized as follows. Section II provides a brief review of reliability growth models and the motivation for the proposed multi-objective and multi-stage reliability growth planning method. In Section III, the multi-objective and multi-stage RGP is formulated and a modified genetic algorithm is used to solve the RGP problem. The large size of Pareto optimal solutions are reduced to workable size efficient Pareto solutions by applying two types of data envelopment analysis (DEA) models. In Section IV, the reliability growth planning of a next generation diesel engine development is illustrated using the proposed multi-objective and multi-stage reliability growth planning approach. Guidelines and insights about the RGP model and trade-off analyses for Pareto solutions are presented in this section. Section V concludes the paper and discusses future research areas.

II. RELIABILITY GROWTH MODELING AND PLANNING

The basic assumption for reliability growth is that the product development team makes design changes to correct any discovered failure modes during product development and test. Reliability growth is first modeled by Duane based on the learning curve properties. Built on Duane’s basic model, more advanced reliability growth models have been developed for the purposes of quantifying reliability estimation uncertainties, modeling specific system applications, such as software reliability growth [22]–[25] and single mission system reliability growth [7], [26]–[28]. In this section, we review basic RGP models, and then present the motivation for the proposed multi-objective and multi-stage RGP.

A. Duane Reliability Growth Model

Duane [1] plotted failure data on a log-log scale from several systems under reliability growth testing and observed that the cumulative failure rate is approximately linearly decreasing over the accumulated testing time. Denote $N(t)$ as the cumulative number of failures up to time $t$ during the reliability growth testing, the cumulative failure rate is $C(t) = N(t)/t$ [1]. Based on the learning curve characteristic, Duane’s empirical observations can be mathematically expressed as: $\ln[C(t)] = \delta - \alpha \ln(t)$, where $\delta, \alpha > 0$. Duane interpreted the parameter $\alpha$ as the “Growth Rate”. The log-linear relationship can be rewritten as $C(t) = \lambda t^{-\alpha}$, where $\lambda = e^\delta$. Since $C(t) = N(t)/t$, the cumulative number of failures by time $t$ is $N(t) = \lambda t^{(1-\alpha)}$. Thus, the instantaneous failure rate $r(t)$ at time $t$ is: $r(t) = \frac{dN(t)}{dt} = \lambda (1-\alpha) t^{-\alpha}$. The MTBF is: $M(t) = 1/r(t) = |\lambda (1-\alpha)| t^{\alpha-1}$. One of the important properties of the learning curve growth rate is that reliability growth takes place as the instantaneous failure rate becomes smaller than the average failure intensity, i.e., when $\alpha > 0$. The cumulative failure rate $C(t)$ and instantaneous failure rate $r(t)$ differs by a factor of $(1-\alpha)$ for $\alpha \in (0, 1)$. This implies that the instantaneous failure rate $r(t)$ is less than the cumulative failure rate $C(t)$.

B. Crow (AMSAA) Reliability Growth Model

L. H. Crow [2] provided a non-homogeneous Poisson process (NHPP) interpretation of Duane model. Crow modeled the Duane postulate stochastically as an NHPP with corresponding maximum likelihood estimators (MLE) for model parameters and goodness-of-fit tests. Crow pointed out that during reliability growth testing (RGT), failures occur as NHPP; but after that, failures occur as a homogeneous Poisson process (HPP). The Crow/AMSAA reliability growth model can be summarized as follows [17]: $f(N(t)) = \lambda^\beta t^{\beta-1}$ and $r(t) = \lambda t^{\beta-1}$, where $N(t)$ represents the expected number of observed failures in $(0, t)$, $r(t)$ is the failure intensity or the instantaneous failures rate, $\lambda$ is the scale parameter, $\beta$ is the shape parameter for Crow AMSAA model ($\lambda, \beta > 0$), and $t$ is total reliability growth test time. When $0 < \beta < 1$, failures during development testing occur as a NHPP with a decreasing failure rate $r(t)$. For $\beta = 1$, there is no reliability growth. During development testing, $r(t)$ is decreasing because of design fixes which contribute to eliminating certain failure modes during reliability growth testing. After completion of the reliability growth test, the failure inter-arrival times follow the exponential distribution, i.e., failures occur following a HPP with constant failure rate $r(T)$, and it is not cost-effective to continue reliability growth testing. The MTBF at time $T$ and system reliability $R(t)$ after total reliability growth time $T$ are $MTBF = 1/\lambda^\beta T^{\beta-1}$, and $R(t) = \exp(-r(T) t) = \exp\left(\frac{-\lambda^\beta T^{\beta-1}}{t}\right)$, respectively. It can be seen that the Duane model and the Crow model share a similar power-law functional form for modeling reliability growth.

In the proposed multi-objective and multi-stage reliability growth planning model, we assume reliability growth follows the basic Duane and Crow models assumptions, i.e., the failure rate is linearly decreasing over cumulative testing time or the mean time to failure is linearly increasing over cumulative testing time on a log-log scale.

C. Motivation for Multi-Stage Reliability Growth Planning

For modern complex engineering systems, a new product usually goes through multiple developmental stages such as concept design, detailed design, and production design stages. Due to the needs of developing next-generation products or meeting regulatory requirements, new technologies are usually introduced to the product design concurrently or sequentially.
Reliability growth rates are usually very different due to the unique design contents and characteristics in each stage. To achieve more accurate and realistic reliability growth planning, reliability growth needs to be planned in multiple stages in order to align with the multiple product developmental stages. Fig. 1 shows an example of a three stages of product development process. In each stage, certain subsystems and/or a part of a subsystem is designed, built, and tested until the system-level product verification and validation.

**D. Key Elements in Planning Multi-Stage Reliability Growth During Early Product Development**

For a single-stage reliability growth planning, typical concerns and key parameters include how to set the reliability growth starting time and how to determine an appropriate growth rate for a specific product design [10], [29]. For multiple-stage reliability growth planning, other challenges exist: how the remaining failure modes from previous stage are carried over to the next stage and appropriately modeled; how to allocate test units and time to individual stage; when and what percentage of new contents/technologies should be introduced at each stage. These questions need to be addressed during new product development and are integrated into the proposed multi-stage reliability growth planning model. The schematic in Fig. 2 shows the outcomes of a three-stage reliability growth planning with key planning factors such as the growth rate, initial mean time to failure, test unit, and test time for each stage.

More specifically, the following major elements need to be determined for a multi-stage reliability growth plan.

1. Reliability growth rate ($\alpha_i$), which is usually empirically determined and can vary from stage to stage during a new product's development process. The characteristics of the system and the effectiveness of reliability program are major factors in determining the reliability growth rates.
Historical reliability growth data from previous product development experiences can also be used as references.

2) New contents analysis is the other major factor for accurate reliability growth planning, which is usually based on the amount of introduced new technologies. In this research, the initial failure rate for each subsystems are estimated based on the identified new contents from the bill of materials (BOM), other factors such as the design complexity, process maturity, and manufacturability have also been taken into account. However, software related reliability issues are not considered in the initial failure estimation and it could be one of our future research direction.

In the proposed model, the failure rate at the end of stage $i$ ($\lambda_i$) is a function of initial failure rate of the stage ($\lambda_{i-1}$), growth rate ($\alpha_i$) and the planned testing time ($T_i$). The initial failure rate of each stage ($\lambda_{i-1}$) is the summation of new contents failure rates ($\lambda_{n(i)}$) and the residual failure rate from the previous stage ($\lambda_{i-1}$).

3) New technology introduction and its timing also have significant impacts on reliability projection. In the process of developing a new product, a new technology can be sequentially introduced in different stages. The parameter $\theta_i$ in the model represents the proportion of introduced new contents for a specific subsystem $j$ in each stage, and it is also used to determine the development cost of subsystem $j$ in stage $i$.

III. MULTI-OBJECTIVE AND MULTI-STAGE RELIABILITY GROWTH PLANNING

A. Mathematical Formulation for Multi-Stage Reliability Growth Planning

With the assumptions and notations from Section II, the multi-objective and multi-stage RGP problem can be formulated. Three objectives are considered in this optimization problem. The first objective is to minimize the failure rate at the last stage for $n$ ($\lambda_i$), which is a function of the residual failure rate from previous stage ($\lambda_{i-1}$), new contents or introduced technologies in current stage ($\lambda_{n(i)}$), reliability growth rate ($\alpha_i$), and planned testing time ($T_i$) ($\lambda_i = \lambda_{i-1} + \alpha_i \ln T_i$). The second objective ($\tau$) is to minimize the total development time ($\tau$) which is the summation of the development time for all stages ($\tau = \sum_{i=1}^{n} \tau_i$). The third objective is to minimize the total test cost ($\mathcal{C}$), which is the summation of all fixed and variable costs from all stages ($\mathcal{C} = \sum_{i=1}^{n} \mathcal{C}_{i}$):
continues for a pre-determined number of generations or until no additional improvement is observed [31]. To solve a multi-objective optimization problem, extensive research has been carried out to develop multi-objective heuristic algorithms. Specially, nondominated sorting GA or NSGA, developed by Srinivas and Deb [32], is a popular nondomination based GA which uses a non-dominated sorting procedure and applies a ranking method that retains good solutions in the population. Through a sharing method, this algorithm maintains the diversity in the population. The algorithm explores different regions in the Pareto front and is very efficient in obtaining Pareto optimal sets. Although NSGA is a very effective algorithm, it has been generally criticized for its computational complexity, lack of elitism, and difficulty of choosing the optimal sharing parameter values. NSGA-II is a modified version of NSGA developed by Deb et al. and it utilizes a fast nondominated sorting genetic algorithm [33]. This method is more computationally efficient, non-elitism preventing, and less dependent on sharing parameter for diversity preservation.

The major difference of single-objective GA and multi-objective GA is in obtaining the survival probabilities of solutions. For multi-objective GA, the fitness function is insufficient for solutions comparison. For example, in NSGA-II, both fitness value or the rank and crowding distance which consider all objective values are used to evaluate and compare the solutions. In this paper, the multi-objective multi-stage reliability growth planning is solved using NSGA-II with appropriate modifications to represent the solutions and search the Pareto front. The specific aspects of NSGA-II are described as follows. A random parent population \( (P_0) \) is initialized and sorted by using the fast non-dominated sorting algorithm into each front \( (F_i) \) in order to identify the non-dominated fronts. The first front \( (F_1) \) is completely non-dominant solution set in the current population and the second front \( (F_2) \) is dominated by the solutions in the first front only and the front iterates so on. The solutions in one front are assigned a specific rank (fitness value). Besides the fitness value, the crowding distance is calculated for each solution in each front \( (F_i) \) which is the measure of population density around a solution. The crowding distance parameter helps to obtain a uniform spread of solutions along the best-known Pareto front without using a sharing parameter as in NSGA [30].

The crowding distance is evaluated based on the Euclidian distance between each pair of solutions in each front. The solutions in the boundary are always selected since they have infinite distance assignment. Large average crowding distance will result in better diversity in the population.

Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. The solutions with the lower rank are first selected. If the solutions are in the same non-dominated front, the solution with higher crowding distance value is selected. In this paper, the simulated binary crossover (SBX) [34] and polynomial mutation operators [35] are used for generating offspring from the selected population. Based on the selection process, the individuals of next generation are selected from the current population and current offspring. As long as all the previous and current best solutions (or elitist solutions) are maintained in the next population, elitism is ensured. This population, which includes both current population and current offspring, is sorted again based on rank and crowding distance and only the best \( N \) solutions are selected, where \( N \) is the population size [30]. The eight-step procedure of NSGA-II is summarized as follows [30].

### B. Pareto Optimal Solution Reduction Using DEA

Even though the results of NSGA-II are informative and can provide tradeoff information for the multiple objective decision making, the large number of solutions can be prohibitive for a decision maker to make choices. At this point, selecting representative solutions from all solutions obtained from NSGA-II itself can be considered as a multi-objective optimization problem, also called multiple objective selection optimizations (MOSOs) problem [36], [37]. In fact, the appropriate application of an MOSO method can significantly reduce the size of the original Pareto optimal solutions. A special MOSO method is the DEA method. From the perspective of relative efficiency, DEA is able to eliminate those inefficient Pareto optimal solutions. In the context of the design of multi-objective reliability growth plans, those plans with high projected reliability and low testing cost and time are preferred and need to be selected for plan implementation.

DEA, originally introduced by Charnes et al. [38], is a technique for measuring the relative performance of decision units (DMUs). This method is based on linear programming methods and it addresses the difficulties of comparing DMUs which use multiple inputs (i.e., cost type criteria) to produce multiple outputs (i.e., benefit type criteria) [39], [40]. For MOSO, each alternative solution is considered as a DMU in the DEA method, and the DMUs are usually assumed to be homogeneously comparable such that the resulting relative efficiencies are meaningful. In comparing their efficiencies, the relative efficiency incorporating multiple inputs and outputs can be defined [38].

In DEA, a ratio of a weighted sum of outputs to a weighted sum of inputs is calculated as a measure of efficiency of each DMU. Consider a set of \( n \) DMUs, with each DMU \( j \), \( j = 1, \ldots, n \) using \( m \) inputs \( x_{ij} \) \( i = 1, \ldots, m \) and generating \( s \) outputs \( y_{rj} \) \( r = 1, \ldots, s \). If the weights (price or multipliers) \( \bar{w}_r \) and \( \bar{v}_i \) associated with output \( r \) and input \( i \), respectively, are known, the efficiency \( \bar{e}_j \) of DMU \( j \) as the ratio of weighted outputs to weighted inputs is equal to \( \bar{e}_j = \sum_r w_r y_{rj} / \sum_i v_i x_{ij} \). Charnes et al. [38] proposed a constant return to scale DEA model or CCR model in the absence of known multipliers. Their model measures the efficiency of DMU \( j \) by solving the following fractional programming problem, known as the original CCR input-oriented model:

\[
\begin{align*}
0_0 &= \max \frac{\sum_r w_r y_{r0}}{\sum_i v_i x_{i0}} \\
\text{s.t. } & \sum_r w_r y_{rj} - \sum_i v_i x_{ij} \leq 0, \ \forall j \\
& u_r, v_i \geq \varepsilon, \ \forall r, i
\end{align*}
\] (20)

where \( \varepsilon \) is a non-Archimedean element smaller than any positive real number. Since this model involves the ratio of outputs to inputs, it is referred to as the input-oriented model. The output
oriented model is the inverted form of this ratio with minimization objective.

Using change of variables \( t = \frac{1}{\sum v_i x_{i0}} \), and \( v_i = t v_i \), where \( t = \left( \sum v_i x_{i0} \right)^{-1} \), the previous fractional programming problem can be converted to the linear programming (LP) model, known as the envelopment or primal problem, shown as follows:

\[
e_0 = \max \sum r \mu_r y_{r0}
\]

s.t. \( \sum_i v_i x_{i0} = 1 \)

\[
\sum r \mu_r y_{rj} - \sum i v_i x_{ij} \leq 0, \quad \forall j
\]

\( \mu_r, v_i \geq \varepsilon, \quad \forall r, i. \) \hspace{1cm} (21)

The duality of the previous formulation is a linear programming problem known as the multiplier or dual problem which provides detailed information for relative efficiency measure and is given by

\[
\min \theta_0 - \varepsilon \left( \sum r S_r^+ + \sum r S_i^- \right)
\]

s.t. \( \sum_j \lambda_j x_{ij} + S_i^- = \theta_0 x_{i0}, \quad i = 1, \ldots, m \)

\( \sum_j \lambda_j y_{rj} + S_r^+ = y_{r0}, \quad r = 1, \ldots, s \)

\( \lambda_j, S_i^-, S_r^+ \geq 0, \quad \forall i, j, r \)

\( \theta_0 \) unconstrained \hspace{1cm} (22)

where \( S_i^- \) and \( S_r^+ \) are slack variables [41], [42].

The other DEA model is introduced by Banker et al. [43], i.e., the BCC model, which is the extension of CCR model and is fundamentally the variable returns to scale model which allows to provide more flexibility for efficiency evaluation. The BCC model differs from CCR model by adding a variable \( \theta \). The linear programming of the BCC model is

\[
e_\theta = \max \sum r \mu_r y_{r0} - \mu g
\]

s.t. \( \sum_i v_i x_{i0} = 1 \)

\[
\sum r \mu_r y_{rj} - \mu g - \sum i v_i x_{ij} \leq 0, \quad j = 1, \ldots, n
\]

\( \mu_r, v_i \geq \varepsilon, \quad \forall r, i. \) \hspace{1cm} (23)

The dual of this BCC model is

\[
\min \theta_0 - \varepsilon \left( \sum r S_r^- + \sum i S_i^+ \right)
\]

s.t. \( \sum_j \lambda_j x_{ij} + S_i^- = \theta_0 x_{i0}, \quad i = 1, \ldots, m \)

\( \sum_j \lambda_j y_{r0} - S_r^+ + y_{r0}, \quad r = 1, \ldots, s \)

\( \sum_j \lambda_j \leq 1 \)

\( \lambda_j, S_i^-, S_r^+ \geq 0 \quad \forall i, j, r \)

\( \theta_0 \) unrestricted in sign. \hspace{1cm} (24)

The dual of the BCC model differs from the dual of CCR model in a way that it has additional convexity constraints on the \( m \) and \( s \). In both CCR and BCC model, the performance of DMU \( k \) is fully (100%) efficient if and only if both \( \theta_0^k = 1 \) and \( \lambda_j^k = 0 \) and weakly efficient if and only if both \( \theta_0^k = 1 \) and \( \lambda_j^k \neq 0 \) and/or \( S_r^k \neq 0 \) for some \( r \) in some alternative optima. Clearly, any CCR-efficient DMU is also BCC-efficient, but BCC-efficient solutions may not be CCR-efficient. Thus we would expect more efficient solutions from BCC model and fewer efficient solutions from CCR model. The CCR model is referred to as giving a radial projection. Particularly, each input is reduced by the same proportionality factor \( \theta \). The BCC model provides more flexible projection by providing decreasing, increasing and constant return to scale frontier [41].

IV. APPLICATION OF MULTI-OBJECTIVE MULTI-STAGE RGP FOR NEXT GENERATION DIESEL-GAS DUEL FUEL ENGINE DEVELOPMENT

In the early product development stage, strategic decisions such as the amount of new technology introduction and product portfolio selection usually need to be made based on projected program budget, schedule, and reliability of the new product. Accurate and realistic reliability growth planning under limited available product information is a challenge but very beneficial for product development program-level decision making.

In this section, we demonstrate how the proposed multi-objective and multi-stage reliability growth planning method can be applied to the case of developing a next generation diesel-gas dual fuel turbine engine. Diesel engine was invented over one hundred years ago; however the advancement and progress to develop new generation diesel engines have never been slowed down. These new engine developments can be attributed to factors such as the continuously increasing standard of emission requirements from the U.S. Environmental Protection Agency (EPA), expected higher mission reliability from customers, and technology advancements of electrical engine control systems. For example, the Tier-4 emissions standard from EPA will be enforced for all newly manufactured heavy duty locomotive engines in 2017. Recently, due to the prediction of abundant natural gas reserve in the US as well as predicted cheaper price of liquid natural gas (LNG) than that of diesel fuel, a few companies including Caterpillar and General Electrics are developing the new diesel-gas duel fuel engines. These new developments and requirements bring many challenges to maintain high reliability performance under more stringent emission requirements and the introduction of advanced engine control systems.

A. Multi-Stage RGP for Next Generation Engine Development

Based on the overall product development schedule, there are three major development stages for the next generation engine: concept engine, the prototype engine, and the pilot engine development. From previous product development experiences and historical reliability data, growth rates for the three stages are 0.4, 0.3, and 0.2, respectively. It is common to apply a relatively higher growth rates at the earlier product development stages than those at the latter product development stages.

For example, the prototype development is conducted in-house
and discovered failure modes can be more effectively resolved, while as the pilot product can be tested and verified at customers’ facilities, failure modes may not be resolved as effectively as in-house developments. In practice, new contents/technologies are introduced in different stages during new product development. In this case, represents the percentage of introduced new contents in each stage, which also affects the amount of fixed cost of new contents introduction. In each stage, different number of subsystems ($m_i$) with different failure rate ($\lambda_{ij}$) for each subsystem are tested for reliability growth purpose. To avoid cost overrun in product development, we impose an upper bound for the number of testing units for each subsystem. Table I shows the parameters for multi-objective multi-stage RGP problem formulation. The failure rate value for each subsystem in each stage ($\lambda_{ij}$) represents the total estimated failure rate of the introduced contents, and a proportion of this total failure rate of the new technology, defined as $\theta_i$ of the entire subsystem, is introduced to the existing subsystem in each specific stage $i$. For example, 80% of fuel system new contents are added to the system in stage 2, and 20% of the remaining fuel system new contents are introduced to the system in stage 3 (Table I).

The upper bound for total product development time $T_{ul}$, the effective work hours in each year and the variable cost per hour are assumed to be 3.5 years, 2000 h, and $2000$, respectively. Based on these assumptions, the RGP optimization is formulated as follows:

$$
\min : \lambda_{i=3} = f (\lambda_{i-1}, \lambda_{n(i)}, \alpha_i, T_i)
$$

$$
\min : \sum_{i=1}^{n} C_i = C_1 + C_2 + C_3
$$

$$
- (60 \times n_{11} + 45 \times n_{12}) + 2 \times (T_1 \times 2000)
$$

$$
+ (50 \times n_{21} + 30 \times n_{22})
$$

$$
+ 40 \times n_{23} + 20 \times n_{24})
$$

$$
+ 2 \times (T_2 \times 2000)
$$

$$
+ (12.5 \times n_{31} + 10 \times n_{32} + 5 \times n_{33})
$$

$$
+ 2 \times (T_3 \times 2000)
$$

$$
\min : \tau = \sum_{i=1}^{n} \tau_i = \tau_1 + \tau_2 + \tau_3
$$

s.t. $0 \leq \tau \leq \tau_n \Rightarrow \tau_1 + \tau_2 + \tau_3 \leq 3.5$

$4 \leq n_{11} + n_{12} \leq 8$

$8 \leq n_{21} + n_{22} + n_{23} + n_{24} \leq 16$

$6 \leq n_{31} + n_{32} + n_{33} \leq 20$. (25)

There are 18 decision variables in this multi-objective multi-stage RGP formulation including the number of test units for subsystem $j$ in stage $i(n_{ij})$ and the planned testing time for each
subsystem $j$ in stage $i(t_{ij})$. The NSGA-II algorithm explained in Section III.B is used to solve the problem. The algorithm is modified to deal with both continuous ($t_{ij}$) and discrete ($n_{ij}$) decision variables. The initial population with 18 decision variables is randomly generated using appropriate probability distributions and given constraints. In each generation, the solutions are sorted using a binary tournament selection method based on rank and crowding distance (diversity of solutions) of the solution. The rank and crowding distance of a solution are evaluated using the values of three objective functions. The simulated binary crossover (SBX) and polynomial mutation operators are then used to generate new offspring for next generation solutions. The detailed parameter setting and outcomes of the RGP problem are presented in next section.

B. RGP Pareto Solutions and Tradeoff Analysis

The parameters used in the NSGA-II algorithm for solving the RGP problem are as follows. The size of population and the number of generations are set to be 100 and 40 respectively. The distribution index for the simulated binary crossover is set to be 0.2, which determines how well spread the offspring solutions will be from their parent solutions [34]; the distribution index for polynomial mutation is set to be 0.2, which determines the deviation of mutated child to its parent [35]; Multiple runs of the NSGA-II algorithm to the RGP formulation generate a very stable Pareto optimal frontier as shown in Fig. 4, which represents 100 Pareto optimal solutions with given growth rates and new contents for each stage.

Pareto optimal solutions in the knee area of the Pareto front are presented in Table II. These 20 solutions are obtained from the original 100 solutions by restricting the failure rate of the final stage to be within $[0.9, 1.3]$. The parameters used in the NSGA-II algorithm for solving the RGP problem are as follows. The size of population and the number of generations are set to be 100 and 40 respectively. The distribution index for the simulated binary crossover is set to be 0.2, which determines how well spread the offspring solutions will be from their parent solutions [34]; the distribution index for polynomial mutation is set to be 0.2, which determines the deviation of mutated child to its parent [35]; Multiple runs of the NSGA-II algorithm to the RGP formulation generate a very stable Pareto optimal frontier as shown in Fig. 4, which represents 100 Pareto optimal solutions with given growth rates and new contents for each stage.

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C. Sensitivity Analysis

The sensitivity of the objectives with respect to major parameters in reliability growth planning is investigated by changing the growth rates $\alpha_i$ for each stage and the failure rates $\lambda_{n(i)}$ of new contents/technologies in each stage while keeping the other parameters constant. Figs. 5–7 illustrate the sensitivity of objectives to the growth rate changes in each stage ($\alpha_i \pm 0.1$). The sensitivity analysis results show that growth rate changes in stage

![Fig. 4. Pareto optimal frontier for RGP.](image-url)

<table>
<thead>
<tr>
<th>DMUs ($n_{ij}$)</th>
<th>Decision variables ($t_{ij}$)</th>
<th>objectives ($\lambda_0, T, C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>4 2 4 5 8 8 8 7</td>
<td>14 0.092 0.00 0.232</td>
</tr>
<tr>
<td>03</td>
<td>4 2 4 5 8 8 8 7</td>
<td>15 0.114 0.186 0.153 0.195 0.095 0.089 0.320 0.389 0.482</td>
</tr>
<tr>
<td>05</td>
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<td>14 0.115 0.199 0.320 0.339 0.184 0.208 0.325 0.462 0.556</td>
</tr>
<tr>
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<td>4 2 4 5 8 8 6 6</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>14 0.115 0.175 0.255 0.235 0.117 0.151 0.324 0.404 0.541</td>
</tr>
<tr>
<td>24</td>
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</tr>
<tr>
<td>32</td>
<td>4 2 4 5 8 8 9 7</td>
<td>14 0.105 0.171 0.321 0.320 0.167 0.206 0.333 0.466 0.562</td>
</tr>
<tr>
<td>48</td>
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<td>53</td>
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</tr>
<tr>
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</tr>
<tr>
<td>68</td>
<td>4 2 4 5 8 8 8 7</td>
<td>15 0.110 0.193 0.101 0.233 0.169 0.141 0.322 0.363 0.479</td>
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<tr>
<td>77</td>
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</tr>
<tr>
<td>83</td>
<td>4 2 4 5 8 8 8 9 7</td>
<td>14 0.112 0.197 0.320 0.319 0.190 0.196 0.327 0.466 0.556</td>
</tr>
</tbody>
</table>
three ($\alpha_3 \pm 0.1$) has bigger effects on the RGP objectives than growth rate changes in stage two ($\alpha_2 \pm 0.1$) and stage one ($\alpha_1 \pm 0.1$). Similar results are observed for the sensitivity of failure rates with respect to the three RGP objectives. As shown in Figs. 8–10, the failure rate changes of new contents in stage three ($\lambda_{n(3)} \pm 0.3$) have a bigger impact on RGP objective values than that by changing failure rates of previous two stages. These sensitivity analysis results indicate that: (1) decision makers
need be cautious to introduce new technologies in the pilot development stage since it can delay the overall product development plan, (2) Earlier efforts should be emphasized to improve the effectiveness in failure modes discovery and correction such that reliability growth will not be solely dependent on the less controllable final stage growth rate during new product development.

D. Pareto Optimal Solutions Reduction Using DEA

In this section, two types of data envelopment analysis (DEA) models are performed to compare the relative efficiency of the 100 Pareto optimal solutions obtained from NSGA-II method such that a few workable Pareto optimal solutions can be presented for decision making for implementing reliability test plan. When applying the DEA method, the Pareto optimal solutions are considered as DMUs. Time and cost objectives are considered as input variables and the reliability objective is considered as output variable. To obtain the efficiencies of the 100 DMUs, a linear program needs to be solved for each DMU. Obviously, as the objective function changes from one linear program to the other, the weights for each DMU may be different. Furthermore, in DEA method, there may be more than one efficient DMU with relative efficiency equal to one, as each individual DMU is trying to select a preferable weight set when evaluating the efficiency of this DMU. The higher relative efficiency value represent that a higher output value can be obtained under a relatively lower amount of weighted inputs.

Both CCR and BCC data envelopment analysis models are applied to evaluate the relative efficiency of the 100 solutions from NSGA-II. Four DMUs were identified as fully efficient \((\theta^*_i = 1\) and \(S_i^- = S_i^+ = 0\)) in all four methods including the input-oriented (I-O) and output oriented (O-O) models under both CCR and BCC models. Table II shows the inputs (cost and time objectives), output (reliability objective) and efficiency results from the four different DEA models. The fully efficient, weakly efficient and non-efficient DMUs are denoted by \((1^*)\), \((1)\), and \((0)\), respectively. The weakly efficient DMUs \((\theta^*_i = 1\) and at least one \(S_i^- * \) or \(S_i^+ * \neq 0\)) appear in BCC input and output oriented models only. The number of fully efficient DMUs in both BCC input and output oriented models are larger than that from the CCR models, which can be justified by the variable scale to return assumption of BCC model with a more flexible frontier selection [42]. The solutions from CCR input and output oriented models are the same and are plotted in Fig. 11. The efficient units obtained by BCC input and output oriented models are slightly different and are plotted in Figs. 12 and 13, respectively.

In summary, both DEA models can significantly reduce the large Pareto optimal solutions to a few implementable efficient solutions from an economic perspective of considering product development time and cost objectives as inputs and reliability objective, which is measured using mean time between failures (MTBF), as an output. The information from the original Pareto frontier and the efficient solutions from DEA can be used to select final solutions for RGP implementation. For example, RGP solutions represented by DMUs 15, 86, 96, and 98 (Table II) may be used for the final RGP implantation.
V. Conclusion

In this paper, we propose a multi-objective and multi-stage reliability growth planning approach with the goal of providing accurate and realistic reliability prediction in early product development stage. Trade-off analysis among multiple product development objectives including development cost, testing time, and projected reliability provides decision makers with the insights in terms of the amount and timing of new technologies introduction, optimal testing time and units allocations, and program management efforts for growth rate improvements. The proposed reliability growth planning method does not need new product testing data. The initial failure rates are estimated by using previous product development experiences as well as failure rates estimation of new technologies based on the Bill of Materials. The proposed reliability growth planning method has the benefits of providing the management and product development team with critical information such as projected reliability performance, program cost, and product release time in the earlier stage of new product development. Through sensitivity analysis, the impact of each stage’s reliability growth rate on the Pareto optimal reliability growth plan is investigated. The growth rates in later stages are more influential to RGP objectives than the growth rates in earlier development stages. Similar results are also observed for the impact of introduced new contents/technologies on projected reliability, cost, and development time objectives. To reduce the large number of Pareto optimal reliability growth plan alternatives, the DEA method is performed to reduce the number of Pareto solutions to further facilitate decision making.

The traditional reliability growth testing method is usually very time and cost consuming, and reliability growth modeling needs to focus more on planning purpose, especially in early product development stage. In our future research, we will integrate other proactive product verification and validation activities, such as increasingly used engineering simulation techniques and engineering design analytics, into reliability growth modeling and prediction.

References


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