2007

Brokers and Competitive Advantage

Michael D Ryall, *Melbourne Business School*
Olav Sorenson, *University of Toronto*

MANAGEMENT SCIENCE
Vol. 53, No. 4, April 2007, pp. 566–583
ISSN 0025-1909 | EISSN 1526-5501 | 07 | 5304 | 0566

DOI 10.1287/mnsc.1060.0675

Brokers and Competitive Advantage

Michael D. Ryall
Melbourne Business School, University of Melbourne, 200 Leicester Street, Carlton, VIC 5053, Australia,
m.ryall@mbs.edu

Olav Sorenson
Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5S 3E6
and London Business School, Regent's Park, London NW1 4SA, United Kingdom,
olav.sorenson@rotman.utoronto.edu

The broker profits by intermediating between two (or more) parties. Using a biform game, we examine whether such a position can confer a competitive advantage, as well as whether any such advantage could persist if actors formed relations strategically. Our analysis reveals that, if one considers exogenous the relations between actors, brokers can enjoy an advantage but only if (1) they do not face substitutes either for the connections they offer or the value they can create, (2) they intermediate more than two parties, and (3) interdependence does not lock them into a particular pattern of exchange. If, on the other hand, one allows actors to form relations on the basis of their expectations of the future value of those relations, then profitable positions of intermediation only arise under strict assumptions of unilateral action. We discuss the implications of our analysis for firm strategy and empirical research.

Key words: games–group decisions; bargaining; organizational studies; networks–graphs; social networks; biform games; strategy
History: Accepted by Bruno Cassiman and Pankaj Ghemawat, special issue editors; received February 15, 2006. This paper was with the authors 3 months for 1 revision.

1. Introduction

One of the most active domains of research in management today concerns the ways in which ongoing relations influence the patterns of exchange and the distributions of resources in and among organizations.[1] Although any attempt to summarize this literature would require its own paper-length treatment, recent studies have, for example, found that prior relations influence the recruitment, retention, and utilization of employees (Fernandez et al. 2000), the terms of exchange between buyers and suppliers (Kollock 1994), and the patterns of trade between nations (Ingram et al. 2005). Indeed, this large and growing literature has usefully documented a plethora of cases in which the patterns of existing relations appear to favor some firms at the expense of others.

Despite the substantial progress that has been made, many have criticized this literature as being overly static (e.g., Salancik 1995): Although certain positions appear valuable, research has remained relatively silent on how actors might come to occupy them. As a consequence, many questions remain open: Do patterns of relations themselves confer benefits, or do they merely reflect underlying actor-level heterogeneity in endowments? Perhaps only those with valuable resources come to occupy positions of intermediation; the apparent benefit of being a broker might then stem from those resources rather than the position. To the extent that actors do profit from their positions, should these returns prove long-lasting or fleeting? To answer these and other questions, research must adopt a more dynamic perspective on how relational networks affect exchange.[2]

Although any progress on this front will require both empirical investigations and theoretical elaboration, here, we focus on the latter by building an analytical model of how firms form relations and then use those relations to create and capture value. Our analysis is premised on the notion that firms attempt to maximize their expected profits both when forming connections to other parties and when cooperating

[1] Although we doubt that many would question this assertion, for evidence one needs look no further than the dozens of papers appearing in the journals each year and the theme of the 2002 Academy of Management Meetings: "Building Effective Networks."

[2] Some economists have developed analytic models of exchange networks, but most of these models address questions of overall social efficiency rather than distributional issues (e.g., which positions afford a competitive advantage). For an excellent review, see Jackson (2007).

and competing with them.[3] Although we recognize that this purely instrumental view of how firms form and manage their relationships almost certainly overstates the actual extent of profit-motivated behavior—at least some interfirm relations undoubtedly emerge from, or come to embody, more social interactions—it nonetheless provides a useful baseline for understanding the assumptions required for a theory of competitive advantage based on a particular position in a relational network.

Our analytical approach could address almost any type of "positional" advantage, but space constraints prevent us from considering them all here. We therefore focus on understanding one of the positional advantages most commonly considered in the literature: the "broker"—one that profits by intermediating between two or more parties (Simmel 1950, Burt 1992; see Burt 2000, for a review of the empirical literature). Individuals, for example, in positions of intermediation have been found to have greater influence (Padgett and Ansell 1993, Fernandez and Gould 1993, Burt 2004) and to receive larger bonuses and faster promotions (Burt 1992, Podolny and Baron 1997) than the average employee. Similarly, firms in such positions earn larger margins (Burt 1983, Talmud 1994) and appear able to protect those margins (Fernandez-Mateo 2006). In addition to being one of the most widely studied types of positions, it is also one particularly well suited to analysis by game theory because competitive scarcity—an issue that coalitional game theory addresses quite precisely—sits at the heart of both the control and information benefits posited as the sources of the broker's advantage.

We use the "biform game" (a two-stage model) as methodological scaffolding.[4] Our paper begins by describing the model. We then turn to the analysis of the second stage of the model, considering the conditions necessary for a broker to enjoy a competitive advantage. Our model allows us to distinguish the effects of position from those of actor-level endowments. Finally, we build in the first stage, allowing actors to form relationships endogenously (on the basis of the incentives implied by the second stage), to determine whether the broker's competitive advantage remains stable in a world of rational relational investments.

[3] This assumption places scope conditions on the applicability of our results. Few, for example, engage in explicit calculation when choosing their friends. Those that do, moreover, may find it difficult to act on their analysis because a relationship requires the acquiescence of the other party and most find repulsive the very thought of forming relations on wholly instrumental grounds; as Burt (1992, pp. 24–25) colorfully asserts: "Judging friends on the basis of their efficiency is an interpersonal flatulence from which friends will flee."

[4] Although this approach has not been applied to positional advantages, applications to other strategic issues appear in Brandenburger and Stuart (2007) and Chatain and Zemsky (2007).

Although our formal model examines stylized versions of the exchange relations one typically observes, we nonetheless believe that the insights it generates point to the value of such an approach. In particular, we find that several conditions must hold for brokers to profit from their positions: (1) The other actors involved must not have equally attractive alternatives for creating value that do not include the broker. (2) The broker's position must allow it to mediate between at least three parties. (3) The broker must enjoy sufficiently attractive alternatives so that its threat not to engage in any particular coalition remains credible. If these conditions hold, brokers can exploit their ability to intermediate in agreements among rational actors. If, however, actors build their networks with full understanding of how these relations could affect the creation and distribution of value, brokers cannot expect to profit from their positions under most conditions. Some actor always has an incentive to "close" any "structural hole." After we introduce our modeling framework, we discuss the intuitions behind and the implications of our propositions in §4 through §6.

2. Analytical Framework

Our model builds on the intellectual infrastructure of the biform game, introduced by Brandenburger and Stuart (2007). Biform games synthesize the two branches of game theory—noncooperative and coalitional—in a manner that retains the strengths of each.[5]

Although a recent development, this approach has enormous potential as a formal methodology for understanding the foundational issues of business policy and strategy because firm performance typically depends on both noncooperative (strategic) and coalitional (competitive) interactions. On the one hand, performance depends on the firm's ability to capture value. Firms, suppliers, and customers compete with one another in markets to both produce and appropriate value. Coalitional game theory maps well onto these situations and provides tools for determining which transactions should occur, how much value the system should create, and who should receive what share as a result. On the other hand, a firm anticipating its ability to compete can initiate a variety of moves—such as entering a new market, introducing a new product, or changing pricing

[5] We use the label "coalitional game theory" instead of the more common "cooperative game theory" because the math neither requires nor implies "cooperation" (in the popular sense of the word) on the part of the actors involved. The cooperative moniker therefore seems a misnomer that can only lead intuitions astray. In a similar vein, we refer to "relational networks" rather than the more commonly used "social networks" because we do not want to convey the misleading impression that these relations have any basis, in our model, beyond mutual self-interest.

policy—designed to alter the competitive landscape to its advantage. Strategic behavior of this sort fits better with the assumptions of noncooperative game theory. By splitting the analysis into two stages, one corresponding to the strategic phase and the other to the competitive phase, the biform game incorporates both types of interactions. In the first stage, actors engage in strategic actions (modeled using noncooperative game theory) with the express purpose of establishing themselves as effective competitors in the second, competitive stage (modeled using coalitional game theory).

In our case, the initial stage involves the creation of relations as a result of strategic machinations on the part of the actors, while the later stage allows those actors to compete over participation in value-creating projects—via the relationships formed in the first stage—in return for shares of the value created. Because our actors' first-stage, relationship-building activities depend on the anticipated value of those relationships, we begin with an analysis of the second stage. In other words, treating the network as a given, we first identify the conditions necessary for brokers to capture value in the market. In §5, we then introduce the first stage and analyze the stability of these positions in the presence of strategic relationship formation.

3. Competitive Stage

In the second stage, coalitional game, actors liaise and bargain with one another to create and capture value. As with any coalitional game, the "inputs" consist of the set of actors and an enumeration of the various ways in which they might interact to produce value. In our case, the latter stem from the combination of the network—formed in the first stage but considered fixed in the second stage—with a set of value-generating "projects" that require the participation of sets of connected actors. For those less familiar with coalitional games, we begin by reviewing some of their general features in §3.1 and then, in §3.2, explain how these features emerge from our setup.

3.1. Coalitional Game Preliminaries

A set of actors, N, combined with, for each group $G \subseteq N$, a number, v_G, define a coalitional game.[6] Each v_G, or *value available* to the group, represents the total economic value the actors in G can produce by transacting only among themselves—in other words, by ignoring any and all opportunities outside the group. In most applications, analysis assumes these values. In some cases, such as ours, however, where the details of value creation play a central role, the v_G arise from other, more basic, assumptions. Game theorists refer to the list of all the values available, $v \equiv (v_G)_{G \subseteq N}$, as a game's *characteristic function*; together with the set of agents, N, this characteristic function completely describes a coalitional game.

In these games, one can quantify any actor's overall contribution to the production of value. Specifically, an actor's *added value* is the difference between the value available to the group that includes all actors and the value produced when that particular actor does not participate (i.e., $av_i \equiv v_N - v_{N_{-i}}$, where N_{-i} denotes the set of all agents except for i).

The *output* of a coalitional game includes the aggregate value produced (v_N) and the various ways that the actors involved might split that value. A *distribution of value*, $\pi = (\pi_1, \ldots, \pi_n)$, is a vector of real numbers in which π_i indicates the amount of value received by actor i in exchange for its participation in value production. Which distributions one considers possible outcomes depends on the solution concept applied—the core, the nucleolus, Shapley value, kernal, stable set, Myerson value, etc.[7]

We adopt the preferred approach in strategy applications and focus on the core. We consider a distribution of value a *competitive outcome* if it satisfies: (i) $\sum_{i \in N} \pi_i \le v_N$, and (ii) for all $G \subseteq N$, $\sum_{i \in G} \pi_i \ge v_G$. The first condition imposes a budget constraint; actors cannot split among themselves more value than they produce. The second condition, meanwhile, ensures that every actor, and every potential group of actors, receives sufficient rewards to prevent them from defecting from the generation of v_N to produce value on their own. The *core* ($\mathscr{C}$), then, is the set of all competitive outcomes. Note that, in some cases, the aggregate value, v_N, cannot support any competitive distributions (in which case, $\mathscr{C} = \varnothing$).[8]

When a nonempty core exists, each actor faces a *range* of competitive outcomes $[\pi_i^{\min}, \pi_i^{\max}]$, where $\pi_i^{\min} = \pi_i^{\max}$ and even $\pi_i^{\min} = \pi_i^{\max} = 0$ are possibilities. At least one competitive distribution results in actor i receiving $\pi_i^{\min}$, at least one results in actor i receiving $\pi_i^{\max}$, and other distributions deliver all values between these bounds. The core offers an attractive solution concept for strategy applications because it allows one to distinguish the effects of competition from those due to extra-competitive forces (e.g., norms of fairness and reciprocity, institutional arrangements, and bargaining acumen); competition determines the bounds of the range, while

[6] When one assumes, as we do, that a group can freely divide the value it produces, (N, v) is a transferable utility (TU) game.

[7] These concepts vary in terms of their assumptions. Gans and de Fontenay (2006), for example, demonstrate that the Meyerson value has a noncooperative foundation. For a discussion of the differences between solution concepts in relation to strategy, see Lippman and Rumelt (2003).

[8] Bondareva (1962) and Shapley (1967) characterize the conditions governing the existence of a nonempty core. We provide an existence result for our specific setting in Appendix A.

Table 1 Graph Theory Terms

Terminology/notation	Definition
(N, R) is *complete*	$\forall\, i, j \in N, (i, j) \in R$ (i.e., all pairs of actors in N have relationships)
A *path* between i and t	$i, t \in N$, $\{(i, j), (j, k), \ldots, (r, s), (s, t)\} \subseteq R$ (i.e., a chain of relationships in R links i and t)
(N, R) is a *component*	(i) a path connects all pairs of actors in R_G, and (ii) no relationships connect the group to nongroup members
(G, R_G)—*subgraph induced by G*	$G \subseteq N$ and $R_G \equiv \{(i, j) \in R \mid i, j \in G\}$ (i.e., G and those relationships in R linking its members)
R_{-G}	$R_{-G} \equiv R_{N \setminus G}$ (i.e., all relationships in R minus those involving a member of G)

extra-competitive factors establish the specific payoff received within this range.

3.2. Our Setup

We build our second stage, coalitional game, from two elements: the set of relationships formed in the first stage and a set of value-producing projects (an exogenous feature of the second stage). The value that a group of actors can create depends on the projects available to it, and whether or not the relationships linking them permit the completion of those projects. Our description begins by elaborating the details of value creation. We then turn to an explication of our assumptions regarding value appropriation. Our description uses standard graph-theoretic terminology; for those unfamiliar with this vocabulary, please see Table 1.

We define the network, inherited from the first stage, as an undirected graph (N, R) composed of a set of actors, indexed by $N \equiv \{1, \ldots, n\}$, and a set of dyadic relationships, R. Typical element (i, j) signals that actors i and j have a relationship.[9] The set of actors can include a wide range of economic agents whose interaction generates economic value: producers, customers, employees, etc. On the relationship side, meanwhile, our imagery is more of an acquaintance than a friend—an actor with whom one might interact regularly and therefore have private information on or access to, but not one to whom one would attach emotional feelings.

Let P represent a set of available *projects*, with typical project p. For each project, we assume that its completion requires the involvement of a set of participant contributors, $X_p \subseteq N$, and that upon completion, the project delivers net economic value in the amount of u_p, a scalar (so, $p = (X_p, u_p)$). To allow for the possibility that actors do nothing, we also include a single "null" project, $p_\varnothing \equiv (\varnothing, 0)$.

[9] Unless otherwise indicated, we assume all sets to be finite.

Although a simple formulation, our notion of a "project" accommodates a wide variety of value-creating activities. For example, it might represent a project in the common sense of the word, such as bringing a film to market. It that case, X_p would reflect the specific combinations of inputs required to do so, not only capabilities and resources across the value chain—production, distribution, and exhibition—but also potentially a variety of competencies at each level—production, for instance, requires producers, writers, directors, actors, cameramen, grips, editors, etc. Alternatively, projects might also represent the economic opportunities available to partnerships, buyer-supplier relationships, or other forms of bilateral or multilateral exchange.[10]

Note that the net values, u_p, implicitly account for all costs, including those required to facilitate the transaction (e.g., communication costs, coordination costs, contract writing costs, monitoring costs, transportation costs, etc.).[11] Moreover, the number u_p need not reflect a single payment; it can just as easily represent an expected value, thereby allowing for uncertainty, or a present value, permitting returns over time. We nonetheless must assume that all actors agree on the amount of value each project creates. Our results therefore do not necessarily extend to situations in which actors differ markedly in their costs of capital, risk preferences, or subjective probabilities of the likelihoods of various outcomes.

Not all groups can complete all projects. To connect competition in our second stage to the network generated in the first stage, we only allow groups to engage in projects if: (1) the group includes all the skills and resources necessary for the project, and (2) the requisite members have the ability to coordinate with one another—either directly or indirectly—through their existing relationships. One might justify this second condition in multiple ways. For example, it could capture the fact that firms search for partners through their networks rather than via some open forum (Granovetter 1973). Alternatively, relationships might mitigate problems associated with agreeing to exchange. To the extent that they beget trust, for example, relationships can overcome the problems associated with incomplete contracts (i.e., not being able to anticipate all future possibilities when drawing up the terms of an agreement; see Granovetter

[10] "Assignment" games, in which the matching of buyers, sellers, and suppliers creates value, represent a special case of our model (e.g., Shapley 1962).

[11] In earlier versions of this paper, projects included relationship-dependent costs. In the analysis of broker performance, however, these more complicated cost structures failed to yield additional insight. We therefore omit this added intricacy in the interest of transparency.

1985). Similarly, they can facilitate the post-agreement monitoring of exchange partners (Coleman 1988).

In addition, we find it useful in terms of model transparency to restrict each actor to participation in only one project. Because of the flexibility of our definition of a project, however, this restriction should not impair the generality of our results. One can, for example, easily construct collections of projects containing small transactions for a set of agents (e.g., p_1 indicates the project in which IBM and Microsoft produce one personal computer for a single buyer) as well as composite transactions built up from smaller ones (e.g., p_q indicates the cost and value associated with IBM and Microsoft producing computers for q buyers).

Definition 1 (Realizability). Given a network, (N, R), a collection of projects, $F \subseteq P$, is realizable by $G \subseteq N$ under R if, for all $p \in F$, the following conditions hold:

(1) *Viability*. $X_p \subseteq G$.

(2) *Connectedness*. For all $i, j \in X_p$, a path in R_G connects i and j.

(3) *Mutual compatibility*. For all $p' \neq p$, $X_p \cap X_{p'} = \varnothing$.

Viability guarantees that a group has the resources needed to complete the projects. *Connectedness* meanwhile ensures that those with the required resources can coordinate via some path of relationships. Let us note here that these paths can include actors not in X_p; some group members could contribute to the projects only as critical components in the chains connecting the necessary participant contributors. (Our definition of a broker implies just such an actor.) *Mutual compatibility* imposes the one-project-per-agent restriction. Note that the "do nothing" project ($p_\varnothing$) is trivially realizable by all groups under all sets of relationships. Let $\mathcal{F}_G$ denote the set of all collections of projects realizable by G under R.

To simplify notation and interpretation, we introduce two additional assumptions. First, we exclude dominated projects by assuming uniqueness for every set of required participants (i.e., if $p \neq p'$, then $X_p \neq X_{p'}$). In reality, a particular group of actors might have a plethora of projects available to them. Without loss of generality, we restrict p to representing the project that the actors in X_p would most prefer. Second, we assume that every project except $p_\varnothing$ requires the participation of at least two actors. Single-actor groups therefore generate no value (via $p_\varnothing$), thereby normalizing the characteristic function such that one can interpret the values as economic surplus.

Now that we have detailed the conditions governing project completion, let us turn to a description of how groups produce and distribute value. Intuitively, a group of actors can always produce, on its own, value equivalent to that generated by its best collection of feasible projects. That is, given a group, G, a set of relationships, R, and a set of projects, P, the group can produce

$$v_G \equiv \max_{F \in \mathcal{F}_G} \sum_{p \in F} u_p. \tag{1}$$

Because all groups can complete $p_\varnothing$, $v_G \geq 0$. Using (1) to compute the value available to each group results in the coalitional game, (N, v).[12] Let us refer to a collection of projects $F \subseteq P$ as *attractive* if it maximizes (1) for v_N and does not contain $p_\varnothing$. We also call an individual project p attractive if it belongs to some attractive collection.

To avoid trivial cases, we assume, at least for the complete network, that the core exists. We do not require that every network has a nonempty core; only the one in which every actor can coordinate directly with every other (i.e., when all potential dyadic relationships exist).

Assumption 1 (Existence). *If* (N, R) *is complete, then* $v_N > 0$ *and the associated coalitional game has at least one competitive outcome.*

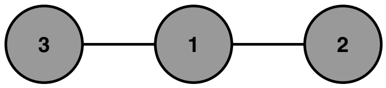
Recall, from the coalitional game preliminaries, that the core describes a range of potential outcomes. Loosely speaking, the more valuable an actor to various groups—that is, the better the alternatives available to it—the higher its minimum appropriation. Following MacDonald and Ryall (2004), we therefore say that an actor enjoys a "competitive advantage" if competition to include that actor in value-producing groups guarantees it an economic profit.

Definition 2 (Competitive Advantage). Given a coalitional game (N, v), an actor i has a *competitive advantage* if and only if $\pi_i^{\min} > 0$.

Our definition of competitive advantage differs somewhat from that commonly used in the management literature, where researchers frequently treat competitive advantage as synonymous with being profitable (or perhaps, "unusually" profitable). Two issues lead us to focus instead on the minimum surplus that an actor can expect. First, our approach provides greater precision in isolating the sources of advantage. Whereas a firm might garner returns from a variety of extra-competitive sources, $\pi_i^{\min}$ represents the amount it can expect purely as a function of its opportunities to exchange with other actors. Second, given that past research has pointed toward competitive scarcity as the underlying source of the broker's

[12] On a technical note, the object $\{N, R, P\}$ is, in essence, a generalized coalitional game. Under the complete network, we have a "standard" coalitional game that imposes no restrictions on group interaction. The removal of edges of the graph produces versions of this game with correspondent restrictions on value production. By varying the set of relations, we can then examine the effects of network position on value appropriation. (We thank an anonymous referee for bringing this point to our attention.)

Figure 1 **A Simple Network**



edge, focusing on this minimum value seems doubly important here.

Example 1 (Simple Network). To understand better how the model operates, let us walk through an example. Suppose that (N, R) includes three actors and two relationships as depicted in Figure 1. The relationships might represent prior exchanges, strategic alliances, transportation routes, or any other nonownership form of relation. Actor 1 is a monopolist with one unit of capacity. Actors 2 and 3, buyers, each value that unit of output at \$2, thereby implying two projects: $p_{12} = (\{1, 2\}, 2)$ and $p_{13} = (\{1, 3\}, 2)$.

From these two projects (and $p_{\varnothing}$), we can derive the characteristic function using (1). Table 2, where each column corresponds to a group (1 in Actor i's column indicates that i belongs to the group), details the characteristic function. Because N includes three actors, a complete enumeration involves seven groups $(=2^n - 1)$. In this example, the three actors can produce a maximum of \$2 of value (i.e., $v_N = 2$) by completing either of the two projects. Both projects are attractive. The only competitive outcome (i.e., stable and feasible distribution of value) is the one in which the monopolist (Actor 1) appropriates all the value (\$2).[13]

Before continuing, let us briefly summarize. In the second stage, the network formed in the first stage, together with an exogenously specified set of projects (P), allows us to compute the maximum value (v_G) that each group (G) can produce. Each project (p) requires a set of actors (X_p), and upon completion delivers some net value (u_p). Given R, G, and p, therefore, we can determine: (1) whether G can execute p, and, if so, (2) how much net value it would generate. A group can complete a collection of projects if its members include all of the required actors, those actors can reach one another through the network, and no actor joins more than one project.

4. Brokers and Competition

Having outlined the model, let us now turn to analyzing the broker. The notion of the broker appears in a variety of literatures. Simmel's (1950) discussion of the role of the tertius gaudens, Granovetter's (1973) identification of the importance of "weak ties," Burt's (1992) imagery of "structural holes," and the organizations and technology management literature on "boundary spanning" (Allen 1977, Tushman 1977) all point to settings in which an actor brings together two parties that normally would not interact because they have no common connections (except through the broker). Related empirical research has notably found positive associations between being in a position of intermediation and a number of indicators of individual and organizational performance, including higher compensation (Burt 1992), faster promotion (Burt 1992, Podolny and Baron 1997), and larger margins (Burt 1983, Talmud 1994).

In our analysis, we define a broker as an individual whose only contribution to the production of value stems from the connections it provides to other actors. By excluding the actor from the required participant sets, we decouple the value of the broker's position from whatever capabilities and resources it may hold.

Definition 3 (Broker). Given a network (N, R), we regard i as a *broker* in that network if: (1) for all projects $p \in P$, $i \notin X_p$, and (2) for at least one attractive p, there exists a group G including i such that p is feasible by G under R but not under $R_{-\{i\}}$.

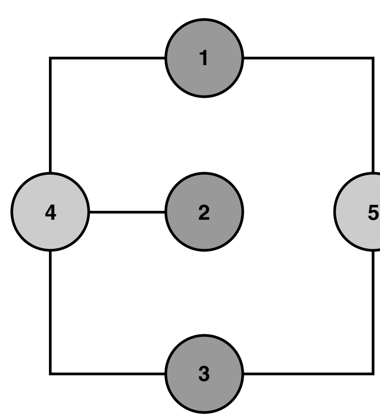
The first condition ensures that no project requires the broker's participation. The second item nonetheless implies that some groups cannot complete one or more attractive projects without the broker's connections.

Example 2 (Brokers). Let us extend Example 1 by incorporating two independent sales agents, Actors 4 and 5, both of whom represent the monopolist. Suppose that one buyer has a prior relation with Actor 4, while the other has connections to both Actors 4 and 5. Figure 2 depicts the resulting five-actor network. By our definition, then, Actors 4 and 5 qualify

Table 2 **Characteristic Function for Example 1**

	N	G_1	G_2	G_3	G_4	G_5	G_6
Actor 1 (Firm)	1	1	1	0	1	0	0
Actor 2 (Buyer A)	1	1	0	1	0	1	0
Actor 3 (Buyer B)	1	0	1	1	0	0	1
v_G	2	2	2	0	0	0	0

Figure 2 **A Network with Brokers (4, 5, and 6)**



[13] The competitive outcome is $\pi = (\pi_1, \pi_2, \pi_3) = (2, 0, 0)$.

as brokers. To see this fact, note that although neither project requires either of these actors, Actors 1 and 2 cannot complete p_{12} without Actor 4 (i.e., $p_{12} = (\{1, 2\}, 2)$ is feasible by $\{1, 2, 4\}$, but not $\{1, 2\}$), and Actors 1 and 3 cannot complete p_{13} without either Actor 4 or Actor 5.

The value appropriated by an actor in the second stage depends on three factors: (1) its contribution to the value produced, (2) the value-producing alternatives available to it and others, and (3) extra-competitive factors, such as its ability to persuade others to part with value for reasons unrelated to the first two items. If the first two guarantee positive surplus, we consider the actor to have a competitive advantage.

4.1. Uniqueness Proposition

Looking at the first of these factors, one notices that we have proposed a somewhat weak definition of a broker in the following sense: Even if the broker intermediates one attractive project for one group, the other actors may have the ability to generate the overall value (v_N) without the broker. Two types of situations might arise. On the one hand, a substitute may exist for the broker itself. In other words, some other individual or organization, also not required by the project, may have the connections necessary to enable completion of the project. Although some might contend that such an individual does not meet the intuitive meaning of being a broker, we still see value in pointing out that the potential advantages of intermediation only accrue to those in unique positions. On the other hand, a substitute may exist for the project that the broker can facilitate (i.e., the other actors may have available an equally attractive project that does not require the broker's connections). In either case, the broker does not add value to the interaction ($av_i = 0$), and therefore cannot profit from its position (i.e., $\pi_i^{\max} = 0$).

Proposition 1. *Given broker i in a network* (N, R), $av_i > 0$ *if and only if every attractive collection of projects* $F \subseteq P$ *contains a project p not feasible by N under* $R_{-\{i\}}$.

Example 3 (Substitutable Projects). To gain a better intuition for why a broker might not add value, let us return to Example 2. This example has two attractive "collections": the singleton sets $\{p_{12}\}$ and $\{p_{13}\}$. Even if we remove all of Actor 4's connections, the latter is still feasible (by $\{1, 3, 5\}$); hence, $av_4 = 0$. Although the other actors cannot complete p_{12} without Actor 4, they have access to an equally valuable project that does not require its participation. Competition therefore prevents the broker from appropriating any surplus.

Example 4 (Substitutable Brokers). Suppose that Actor 2, one of the buyers, no longer values Actor 1's product (i.e., only project p_{13} remains). Once again, $v_N = 2$, and Actor 4 remains a broker as does Actor 5. Actors 1 and 3, therefore, can complete the project two ways—either through Actor 4 or through Actor 5. Hence, the group still does not require Actor 4 (nor Actor 5) to produce v_N. Neither broker therefore can obtain any surplus from its position.

Although on the surface these conditions may seem obvious, they raise important issues for empirical research. Consider first the possibility of alternative pathways. In some cases, researchers have not explicitly considered the possibility of substitution. For example, research on boundary spanning and weak ties generally defines these roles with respect only to the dyad. In other cases, such as the literature on structural holes, the conceptual definition of the broker conforms to Proposition 1, but the measures typically used to assess these positions do not account for substitutes. Burt's (1992) constraint measure, for example, only incorporates the relations between an ego, its alters, and those alters' direct connections to each other. Potential substitutes fall outside its purview. For instance, in Figure 2, Actor 4 would have a low constraint measure despite the fact that Actor 5 can easily substitute for it and therefore reduces its added value to zero. In both cases, the lack of concordance between measurement and the necessary conditions for a broker to hold a competitive advantage risks underestimation of the benefits to being a broker. Apparent differences in the value of intermediation from one situation to the next, moreover, may reflect the distribution of substitutes rather than real differences across settings in the strength of positional advantage.

Consequently, this proposition points to the need for alternate measures of intermediation, particularly ones that account for relationships beyond the one-step radius of the actor. Calculating an appropriate measure, however, is not trivial because the substitute for the broker could easily reside multiple links away from the broker. Conceptually, one would want a measure that captures the extent to which an actor sits between two (or more) otherwise unconnected components. At a dyadic level, the betweenness measures of centrality, based on (weighted) counts of the number of geodesics that include an actor (see Freeman 1977), capture this notion well. These centrality measures nevertheless do not account for potentially important interactions outside the dyad. Hence, Moody and White's (2003, pp. 123–124) "cohesive blocking" algorithm appears to be a more attractive alternative.[14]

[14] This algorithm based on cut-points in the graph evaluates the minimum number of actors that one must remove to disconnect a group. Although computing it requires more time than betweenness centrality, this added cost appears well worthwhile given the better integration it offers between theory and measure.

The possibility of equally attractive projects—ways of creating value—poses a more vexing challenge. The problem arises because these alternate means of creating value can affect the profit that the broker receives but from the point of view of the researcher they typically represent unobserved counterfactuals. In other words, when one sees an actor in an apparent position of intermediation, one usually cannot determine whether the intermediated actors could have produced equivalent value without the broker, by producing in two independent groups. This issue seems most problematic in horizontal settings, such as strategic alliances among potentially competing firms, because outside opportunities commonly exist in theses cases. One might find, therefore, that brokers frequently cannot add (and consequently cannot appropriate) value in these contexts.

4.2. Competitive Scarcity Proposition

Although situations almost certainly occur in which brokers add no value (in terms of economic surplus), it is probably more common that brokers face a range of potential payoffs. Let us therefore consider how competition affects brokers' minimum appropriation (i.e., the amount that competition "guarantees" that they earn).

According to a general proposition in MacDonald and Ryall (2004), two conditions must hold for competition to ensure an actor positive surplus (i.e., $\pi_i^{\min} > 0$). First, at least two groups other than N must exist that could capitalize on the inclusion of the actor. Second, these groups must not share any members other than that actor. Applying this result to our setting yields the following proposition:

Proposition 2. *Given a network (N, R) and an actor $i \in N$, $\pi_i^{\min} > 0$ only if two groups $G \subset N$ and $G' \subset N$ exist such that*

(1) *$G \cap G' = \{i\}$ (only actor i belongs to both groups), and*

(2) *For all project collections F (F') that deliver v_G ($v_{G'}$), there exists a $p \in F$ ($p' \in F'$) such that p (p') is not feasible by G (G') under $R_{-\{i\}}$.*

Perhaps more clearly than the first proposition, this one illustrates the intuition behind the second stage: a broker's ability to profit from its position depends jointly on the alternatives available to it as well as on the value it adds. Here, it becomes clear that if the broker does not have multiple alternative projects competing for its attention, then competition does not guarantee it any surplus. We should also note that, while necessary, these conditions are not sufficient for a competitive advantage based on position.

Recall that neither broker in the last two examples had a competitive advantage. Consistent with this fact, condition (1) of Proposition 2 fails for both brokers in both examples: Actor 1, the monopolist, belongs to every value-creating group. For competition to ensure either broker a share, it would need to intermediate different deals involving distinct sets of buyers and sellers.

An interesting implication follows almost immediately from this proposition. Let n_i denote the number of actors in i's "neighborhood" in (N, R)—in other words, the number of other actors to whom i can trace a path.

Corollary 1. *Broker i has a competitive advantage only if $n_i \geq 3$.*

Juxtaposed against the intellectual history of the theory of the broker's advantage, this corollary strikes us as quite interesting. Simmel (1950), by reasoning through how three individuals might interact, argued that the individual connecting two others could benefit by bargaining them off against each other. That conjecture, however, would hold only if the broker also had some valuable capability or resource that it could offer to these other parties. In the absence of such endowments, the broker could not have a competitive advantage. Why? Because, without a third actor, the pure broker—who adds nothing to the project—does not have an alternate means of creating value and, hence, no credible threat for leaving to produce value in some other project.

4.3. Value Tension Proposition

Although alternatives usually increase the value an actor can expect to appropriate, not all outside options provide credible threats. The final proposition from the second stage concerns whether these alternatives introduce sufficient tension for the broker to have a competitive advantage.

Consider the following thought experiment. Imagine that actor i faces the possibility of receiving no more than its outside option (i.e., $\pi_i = 0$). In our setting, Actor i can and would then attempt to convince other actors to defect from participation in the projects that deliver v_N and to join it in some alternative project. This attempt will succeed if the alternative generates more value than the defectors can expect to appropriate in the creation of v_N because the alternative could then leave all defectors—including i—better off. The question thus becomes: Is v_N sufficiently large to compensate all actors well enough that i cannot tempt any group to join it? Posed in this way, we see MacDonald and Ryall's (2004) insight that whether or not i has a competitive advantage depends only on the values available to groups including i.

Because our game arises from more basic elements, we can refine this insight. Given a network (N, R) and an actor $i \in N$, we define a group G as *i-minimal* under R if it contains i and some project p is realizable by G under R but not, for all j in G, under $R_{-\{j\}}$.

In other words, given a network, an i-minimal group contains i as part of the smallest set of actors required to complete some value-producing project p. Intuitively, we need only focus on these groups because if i intends to use p as a lever to dissuade others from participating in the production of v_N, the associated i-minimal group identifies the fewest individuals in need of convincing. Let $\mathscr{G}_i$ denote the set of all i-minimal groups under R.

We define the *network value of i* as

$$nv_i \equiv \min_{\pi_{-i}\in\mathbb{R}_+^{n-1}} \left\{ \sum_{j\in N_{-i}} \pi_j \,\middle|\, \text{for all } G \in \mathscr{G}_i,\ \sum_{j\in G_{-i}} \pi_j \geq v_G \right\}, \quad (2)$$

where $\mathbb{R}_+^{n-1}$ is the set of all $(n-1)$-dimensional, nonnegative value distributions. The network value of i represents the minimum amount required to close all alternatives available to i while holding its appropriation to zero.[15]

Proposition 3. *Given a network (N, R), actor $i \in N$ has a competitive advantage ($\pi_i^{\min} > 0$) if and only if at least one competitive outcome exists ($\mathscr{C} \neq \varnothing$) and $nv_i > v_N$.*

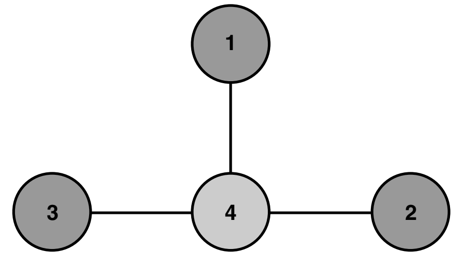
In other words, i (a broker or otherwise) must appropriate positive surplus, independent of any extra-competitive factors, if and only if the aggregate value required to neutralize competition for i (nv_i) exceeds the total value actually available (v_N).[16]

Example 5 (Value Tension). To understand better the intuition behind this proposition, imagine that three movie stars (Actors 1, 2, and 3) have exclusive contracts with an agent (Actor 4), as depicted in Figure 3. Suppose that the agent has been contacted by the producers of two movies. The first has roles for all three stars, with a budget for this talent of \$100. The second, meanwhile, has parts for two stars; the studio has budgeted \$90 to their hiring and has expressed indifference as to which two get the parts. The following projects describe the situation:

$$p_{123} = (\{1,2,3\}, 100), \qquad p_{12} = (\{1,2\}, 90),$$
$$p_{13} = (\{1,3\}, 90), \qquad p_{23} = (\{2,3\}, 90).$$

In this case, $v_N = 100$. Actor 4 occupies a position of brokerage; the necessary contributor participants cannot complete the only attractive project, p_{123}, without Actor 4 even though it does not participate directly in any project. It also enjoys a competitive advantage ($\pi_4^{\min} = 70$). One can easily verify that Actor 4's position satisfies Propositions 1 and 2. It also meets Proposition 3: Three i-minimal groups exist for Actor 4

Figure 3 **A Broker with Competitive Advantage**



($\{1,2,4\}$, $\{1,3,4\}$, and $\{2,3,4\}$). Calculating (2) yields $nv_4 = 135$; to ensure that no group would defect with Actor 4 (when $\pi_4 = 0$), each star would need to receive at least \$45, thereby requiring \$135 altogether. Because $v_N = 100$, no such arrangement could transpire. Actor 4 therefore has a competitive advantage: the actors in the game can only produce $v_N = 100$, and not some lesser amount, if Actor 4 receives positive surplus. Competition alone guarantees the broker a profit—even though it provides nothing beyond its connections.

To see a counterintuitive implication of Proposition 3, let us introduce a fourth star, Actor 5, and a third movie with a budget of \$135 for all four actors. Specifically, add the project $p_5 = (\{1,2,3,5\}, 135)$ to those in the previous example. Actor 4 remains a broker and now sits at the locus of a more valuable network (as depicted in Figure 4). Indeed, its added value increases from \$100 to \$135. Ironically, this increase leads to the loss of its competitive advantage. The network value of Actor 4 has not changed, but the increase in v_N means that the agent can no longer credibly threaten to drop the big project in favor of a smaller one.[17]

5. Stability of the Broker's Advantage

Having demonstrated that positions of intermediation can bestow competitive advantage, one might naturally wonder whether forward-looking actors would ever allow such a situation to arise. After all, by definition, our brokers contribute nothing beyond their ability to connect productive actors. Would these productive actors not try to configure their relations to avoid brokers? Although we will demonstrate that some actors actually benefit from the addition of a broker, the probability that such situations would prove stable seems low.

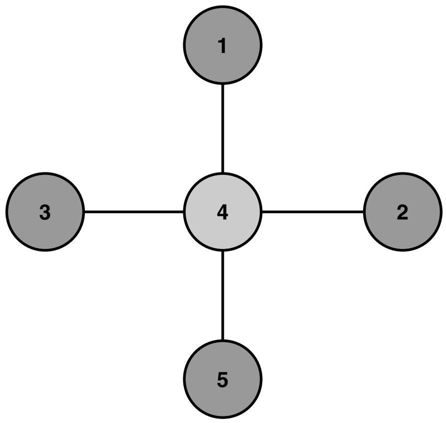
Let us therefore turn to a description of the relationship formation stage. Intuitively, relationships should

[15] The right-hand side of (2) is a linear program.

[16] This result relates closely to the existence of a nonempty core in our setting. See Appendix A.

[17] Note that increasing v_N generally expands actors' competitive ranges; minimum appropriation levels drop while maximum levels rise. Actors confident that extra-competitive factors will work in their favor therefore may willingly accept a lower minimum for a higher maximum.

Figure 4 Competitive Advantage Lost



only form when both parties agree to them. We implement this idea as follows: In the first stage, each actor proposes a vector of transfers, $s^i = (s^i_1, \ldots, s^i_n)$, in which s^i_j denotes the value offered by actor i to actor j to form a relationship. Ultimately, their proposals depend on actors' expectations of how these relationships affect their likelihoods of capturing value in the second stage. Hence, we refer to s^i as actor i's *relationship strategy*. A relationship forms if the actors involved mutually agree to it.

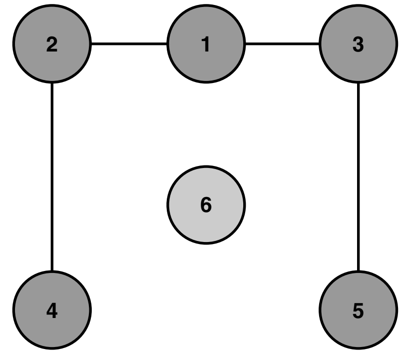
Assumption 2 (Mutual Agreeability). *Given* s^i_j *and* s^j_i, *we assume that* i *and* j *form a relationship* (*i.e.*, $(i, j) \in R$) *if and only if*

$$s^i_j + s^j_i \geq 0. \tag{3}$$

Note that, in equilibrium (discussed in detail below), $s^i_j + s^j_i = 0$. If both actors want the relationship, then obviously this condition holds. But it also accommodates less restrictive cases. For example, one of the actors may wish to invest in the relationship in the expectation of reaping future rewards, while the other requires inducement (e.g., $s^i_j > 0$ and $s^j_i < 0$). Consider, for instance, a supplier that volunteers to provide a good or service for free to gain access to a buyer, or a low-status actor that pays a premium to associate with a higher-status actor (such as entrepreneurs accepting less equity to affiliate with high-status venture capitalists; see Hsu 2004). As long as the actor that desires the relationship places greater value on it than the inducement required by the other actor, they can presumably find a mutually satisfactory transfer payment to align their interests. Our condition nonetheless allows any actor to avoid a relationship with any other simply by setting sufficiently low its value of the relationship.

Let us refer to a list of relationship strategies across all actors, $s \equiv (s^1, \ldots, s^n)$, as a *strategy profile*. By the mutual agreeability assumption, each such profile implies a set of dyadic relationships. Let us denote

Figure 5 Six-Agent Example



this implied set by $R^s \equiv \{(i, j) \in N^2 \mid s^i_j + s^j_i \geq 0\}$, and call the associated undirected graph (N, R^s), the *network implied by s*.[18] Please note that the network implied by s need not form a single component; in other words, pairs of agents within N may not have a means of reaching one another through a path in R^s. As described in §3.2, (N, R^s) and the set of projects P imply a coalitional game (N, v^s); let π^s represent the distribution of value expected under s. When $\mathscr{C}^s \neq \varnothing$, $\pi^s \in \mathscr{C}^s$, otherwise, we assume that $\pi^s = (0, \ldots, 0)$.[19]

Because brokers do not contribute directly to project completion, it may appear obvious that the intermediated actors would want to cut these middlemen out. But that intuition is not always correct. Proposition 3 implies that reducing the value available for appropriation by those that might defect with an actor increases its own competitive minimum. An actor may therefore find it advantageous to bring in a broker; by reducing the amount available to the actors required for value production, a broker can intensify competition in a way that favors some actors.[20] Consider the following example.

Example 6 (Broker Benefits Performance). Imagine that the network in Figure 5 depicts collaborations among six investment banks. Further, suppose that IPOs come in two sizes: small deals, that require two banks and generate surplus of \$50, and large deals, that require five banks and generate \$100. Although each bank can participate in only one deal, they differ

[18] We identify the objects associated with strategy profile s by an "s" superscript.

[19] In other words, actors expect to appropriate zero surplus when a competitive outcome does not exist. This assumption stems from the notion that bargaining instability in such situations results in the complete dissipation of any surplus. In cases with more than one competitive outcome, payoffs can vary by strategy profile. In other words, if s and $\hat{s}$ both generate the same network, then $\mathscr{C}^s = \mathscr{C}^{\hat{s}}$, but we allow $\pi^s \neq \pi^{\hat{s}}$.

[20] As noted above, reducing v tends to narrow each actor's competitive range. Thus, those that rely more on competitive forces for their appropriation may welcome the introduction of a broker. (We thank Glenn MacDonald for suggesting this interesting effect.)

in their capabilities and client access, so clients cannot always substitute between them. We abstract away these issues in the interests of simplicity and simply assume that the following six projects summarize the available opportunities:

$$p^* = (\{1,2,3,4,5\}, 100),$$

$$p_1 = (\{1,2\}, 50), \quad p_2 = (\{1,3\}, 50), \quad p_3 = (\{2,5\}, 50),$$

$$p_4 = (\{3,4\}, 50), \qquad p_5 = (\{4,5\}, 50).$$

Note that Bank 6 does not belong to any network, nor does any project require its participation. In this example, Bank 1 can expect to appropriate something between $\pi_1^{\min} = 0$ and $\pi_1^{\max} = 100$. But imagine that Bank 1 does not negotiate well and therefore always receives something near the bottom of this range; maybe the actual competitive outcome is $\pi^s = (\pi_1^s, \ldots, \pi_6^s) = (0, 50, 50, 0, 0, 0)$.

Observe the effect of introducing a broker. Consider the same set of projects and banks, but with the network depicted in Figure 6. Bank 6 has become a broker; the other banks cannot complete the attractive projects, p^*, without the broker's links. With this modification, the game now has a unique competitive outcome $\pi^{\hat{s}} = (50, 0, 0, 0, 0, 50)$. Both the broker—and Bank 1!—have competitive advantages. Although Bank 1 faces the same set of projects and has the same dyadic relationships, the introduction of a broker reduces the value available for softening competition for Bank 1's services, producing the tension required by Proposition 3.

Because not all actors gain from the elimination of brokers, it appears that the obvious intuition fails. Or does it? In our model, relationships form as a result of the strategic behavior of all actors. Although some might wish to introduce a broker, others may attempt to thwart them. Here, for example, it would appear that Banks 1 through 5 could split an additional \$50 if they could find a way of removing the broker. In our model, actors can arrange such side payments by altering their relationship strategies (i.e., vectors of transfers). An actor's overall payoff under s includes both what it appropriates in the second stage and all transfers resulting from the formation of relationships. Therefore, let us define i's net allocation:

$$\xi_i^s \equiv \pi_i^s - \sum_{(ij)\in R^s} s_j^i.$$

Example 7 (Broker Stability). Let us return to our example using the network in Figure 6, with actors appropriating $\pi^{\hat{s}} = (50, 0, 0, 0, 0, 50)$. Can the intermediated banks find a means of eliminating the broker—for example, through the relationships in Figure 5—that would leave them all more prosperous? Recall that we assumed that this alternate set of relations would lead to $\pi^s = (0, 50, 50, 0, 0, 0)$. Anticipating this outcome, Banks 1 through 5 might arrange up-front transfers that would precipitate the removal of the broker—for example, imagine a set of transfers that would result in net allocations of $\xi^s = (52, 20, 20, 4, 4, 0)$. The issue, however, becomes one of whether actors can coordinate these changes.

To illustrate, let us examine two extremes. Consider first the case in which actors can only act unilaterally. Assume that the strategy profile, $\hat{s}$, behind the Figure 6 network has the following structure: (1) all relationships formed arise from offers of $\hat{s}_j^i = \hat{s}_i^j = 0$, and (2) the "missing" relationships had offers of $\hat{s}_j^i = \hat{s}_i^j = -200$. Note that no bank can improve its situation through a unilateral change in its strategy. Although a bank could form a relationship unilaterally by meeting the \$200 demand of the potential partner, the second stage only produces \$100, so no actor would ever wish to do so. A bank i could also unilaterally eliminate one of its existing relations by changing $\hat{s}_j^i = 0$ to $\check{s}_j^i < 0$, but no bank would benefit from such an action. Table 3 reports the effects on each agent's competitive range of dropping one of the relationships in Figure 6 (labeled [a], [b], and [c]). For example, either Bank 1 or Bank 2 could remove relationship [a]. Although this action would alter the competitive ranges of several banks, neither Bank 1 nor Bank 2 benefits. In fact,

Figure 6 **Adding a Broker**

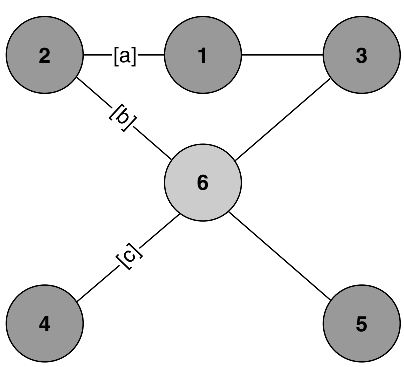
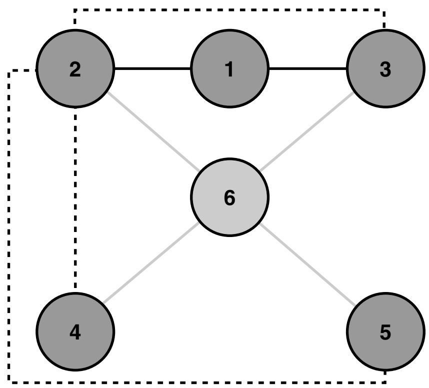


Table 3 **The Effect of Relationship Changes on Competitive Allocations in Example 7**

	Status quo		Remove [a]		Remove [b]		Remove [c]	
	$\pi_i^{\min}$	$\pi_i^{\max}$	$\pi_i^{\min}$	$\pi_i^{\max}$	$\pi_i^{\min}$	$\pi_i^{\max}$	$\pi_i^{\min}$	$\pi_i^{\max}$
Bank 1	50	50	**0**	**50**	50	50	0	0
Bank 2	0	0	**0**	**0**	**0**	**0**	0	0
Bank 3	0	0	0	50	0	0	0	0
Bank 4	0	0	0	0	0	0	**0**	**0**
Bank 5	0	0	0	50	0	0	0	0
Bank 6	50	50	0	50	**0**	**0**	**0**	**0**

Figure 7 **Disintermediation**



as the table demonstrates, no bank can improve its prospects by dropping a relationship.[21]

At the other extreme, if the intermediated banks can synchronize their strategies, then they can always find a means of eliminating the broker while improving their own expected payoffs—for example, by implementing a profile of transfer payments s such that $\xi^s = (52, 20, 20, 4, 4, 0)$ as suggested above.[22] Where actors can coordinate, brokers thus find it more difficult to sustain a competitive advantage.

Between unilateral moves and joint action, how much coordination do the banks require to remove the broker? Not much. Suppose that Bank 2 wishes to form relationships with Banks 3, 4, and 5, resulting in Figure 7 (new relationships are depicted as dashed lines). By Proposition 1, Bank 6, the broker, no longer adds value; hence, $\pi_6^{\check{s}\,\min} = \pi_6^{\check{s}\,\max} = 0$. In fact, this network educes a unique competitive distribution: Banks 1, 2, 4, and 5 receive \$25, while Banks 3 and 6 get none. Achieving this outcome simply requires that: (1) Bank 2 unilaterally changes its offers to Banks 3, 4, and 5 to $\check{s}_3^2 = \check{s}_4^2 = \check{s}_5^2 = 1$, and (2) those banks adjust their relationship strategies to accept (e.g., $\check{s}_2^3 = \check{s}_2^4 = \check{s}_2^5 = -1$). Everyone required for the change ends up ahead, and it requires no coordination beyond allowing actors to accept the updated offers of a single deviator.

To demonstrate this result generally, let us formally define this equilibrium concept. Given s, define a *neighborhood deviation by* i as a set of actors G, including i, and a strategy profile $\hat{s} = (\hat{s}^i, \hat{s}^{-i})$, in which the actors in G accept a revised set of offers from i (for all $j \in G_{-i}$, $\hat{s}_i^j = -\hat{s}_j^i$). Other than these acceptances, $\hat{s}^{-i}$ remains identical to s^{-i}. Intuitively, an actor targets a group of other actors for relationship formation, all of whom accept, but all other proposals remain unchanged. We handle these deviations by identifying the group with which actor i would wish to establish relationships and then simply assuming that they form (the rationality of such behavior enters in our concept of equilibrium). Hence, a single actor sits at the locus of every deviation; other members merely accept that deviator's offers.

We consider a strategy profile an equilibrium if no actor can identify a neighborhood and revised strategy such that everyone who would need to change their strategy would prefer the alternative (also accounting for the effects of those strategies on appropriation in the second stage).[23]

Definition 4 (Neighborhood Equilibrium). A strategy profile s is a *neighborhood equilibrium* if, for all $i \in N$, no neighborhood deviation by i, $(G, \hat{s})$ exists such that for all $k \in G$, $\xi_k^{\hat{s}} > \xi_k^s$.

Our definition implies that mutual agreeability (Assumption 2) produces an equality. It also means that no broker can ever have a competitive advantage in an endogenously-formed network. Although few empirical studies have considered the stability of positions of intermediation, consistent with this result, one of the notable exceptions finds that they decay rapidly (Burt 2002).

Proposition 4. *Given P, let (N, R^s) be a network in which i is a broker and $\pi_i^s > 0$. Then, s is not a neighborhood equilibrium.*

Corollary 2. *Given P, let s be a neighborhood equilibrium such that (N, R^s) is a network in which i is a broker. Then, i does not have a competitive advantage.*

A careful consideration of our result reveals some subtleties. On the one hand, our proof points to something stronger than the absence of a competitive advantage (i.e., $\pi^{\min} = 0$). The threat of disintermediation ensures that the broker must receive zero surplus in the second stage (i.e., $\pi^{\max} = 0$). On the other hand, this fact does not preclude a broker from profiting in the first stage of the game. A broker can accept transfer offers to form relationships, and those transfer payments could sum to more than zero. To the extent that a broker benefits, it gains not from the strength of its position under competition, but rather from its ability to convince others to provide up-front compensation for its relationship-providing role.

[21] Thus, $\hat{s}$ constitutes a Nash equilibrium.

[22] In game-theoretic terminology, s is not a strong Nash equilibrium.

[23] Note that every neighborhood equilibrium is also a Nash equilibrium. Our concept is similar to Bloch and Jackson's (2006) "pairwise" equilibrium. Pairwise equilibrium rules out strategy profiles in which some pair of actors wishes to form a relationship not implied by the profile. We introduce neighborhood equilibrium because pairwise equilibrium seems too restrictive: If an actor can revise an offer to one neighbor (in the graph-theoretic sense), why should we not allow it to do so for multiple neighbors? Both our definition and pairwise equilibrium nevertheless preclude simultaneous deviations by groups of actors.

Although quite subtle, this logic may inform our understanding of the contracts one typically sees with brokers in the real world. In all cases that we could find, these contracts involve prenegotiated payments with one of the parties involved. Consider, for instance, the case of a talent agent (a broker at least informally, and perhaps even consistent with our definition). A scriptwriter might retain an agent to help her find films for which to write (usually for a fixed percentage of her income). When bringing the writer to a project, the agent typically cannot negotiate with the producer for any additional payment as a "finder's fee"—although he may negotiate on his client's behalf. The agent also cannot renegotiate with his client after arranging a particularly good deal. Yet, if agents actually had a strong bargaining position because of their connections, one might expect both to occur; instead, agents only extract their predetermined fees. One sees similar arrangements among others that we might think of as brokers, such as real-estate agents, investment bankers, headhunters, and matchmakers.

Despite these subtleties, our final proposition reveals the fragility of positional advantage, and illustrates the fact that researchers should probably only find it in a limited range of situations. On the one hand, we do find that brokers can enjoy a competitive advantage purely through their positions. In this sense, our results fit well with the frequent finding that, at the level of an individual, rewards accrue to those in positions of intermediation. If relationships arise exogenously to the rewards associated with particular positions, then we see no reason why actors could not exploit these fortuitous circumstances.

On the other hand, our results raise doubts as to whether brokers should ever emerge when performance-motivated actors choose their relations strategically, and, if they do, how long such positions can persist. Hence, when considering whether firms—that presumably at least try to maximize their profitability—can benefit from being brokers, we find ourselves skeptical. Proposition 4 suggests that they probably cannot. Not surprisingly then, empirical research at the firm level has been more equivocal in its findings. Ahuja (2000) and Bae and Gargiulo (2004), for example, both fail to find positive relationships between intermediation and performance.

In response to our claims, some might contend that a broker should have the ability to maintain the integrity of its position by keeping secret the identity of the other parties. Although such a strategy might have merit in some contexts, the broker's advantage then would not stem from competitive scarcity. In coalitional games, it is precisely the other participants' awareness and understanding of an actor's outside options that allow an actor to appropriate positive surplus. In the absence of such alternatives, the broker could easily receive zero, and any positive surplus would depend entirely on extra-competitve factors (such as negotiating ability).

A more compelling source of sustained advantage might come from settings that do not permit the coordination required for neighborhood equilibrium. As Example 7 demonstrates, if actors can only deviate in pairs, brokers can enjoy a competitive advantage in equilibrium. Such a condition might arise in the real world if actors' opportunities for forming relations arrive sequentially. A venture capital firm building its coinvestment network, for example, might only have need for one syndicate partner on any particular investment. Because each dyad would then need to decide whether or not to create a relationship given the set of existing relations, even permitting a single deviator to propose simultaneously more than one new relationship might allow too much coordination. In such cases (consistent with Nash and pairwise equilibrium), intermediation can yield a persistent profit. Contexts with sequential relationship formation, however, seem more the exception than the rule; firms can typically build and maintain most types of ties in parallel.

6. Conclusion

Two primary issues in strategy concern: (1) What accounts for variation in firm performance? and (2) Can managers control these factors? Answers to these questions abound, and we would not attempt to address such broad issues in a single paper. We nonetheless do consider these questions carefully in the context of one potential source of competitive advantage: a firm's (or other economic actor's) position as an intermediary between other parties in a relational network—whether that network represents sourcing relations, alliances, syndicated investments, joint ventures, or some other form of collaboration or exchange.

Although this subject has garnered ample empirical attention, these studies have raised as many questions as they answer. Critics question whether positions of intermediation themselves can convey a competitive advantage or whether they merely reflect the valuable capabilities and/or resources that these actors control. Similarly, even if brokers do profit from their positions, these (frequently cross-sectional) studies leave open the question of the durability of these positional advantages. Although some studies have investigated intermediation with longitudinal data, they have typically used the panel structure of the data more to address the issue of unobserved heterogeneity described above than to investigate the temporal stability of positional advantages.

To address these issues, we have developed a biform game that allows the analysis of the dynamics of value appropriation when the topology of a relational network restricts the options available to actors. By analyzing the second, competitive, stage of our model, we have identified three conditions that must hold for competitive scarcity to guarantee that a broker profits: (1) The actors being intermediated must not have other means of coordinating among themselves (i.e., substitute brokers), or equally attractive alternatives that do not require the broker's assistance. (2) The broker must have connections that allow it to intermediate between at least three other actors. If the broker links only two actors, then it cannot produce value (through brokering) outside the pair and therefore cannot credibly threaten to exit the coalition. (3) The outside options available to the broker must offer sufficient value that the broker can wield them as credible threats in its negotiations with other parties. If these conditions hold, brokers can profit from their positions.

If, however, these positions emerge endogenously—in other words, if actors form relationships in response to expectations of how they will influence value appropriation—then network topologies that bestow competitive advantages on brokers should usually not materialize. To the extent that "structural holes" do open, one should see others quickly move to "close" them. Although we recognize that this condition does not hold in a variety of situations—for example, few form friends on the basis of such rational calculation—the recognition that it does form a scope condition to positional advantage has important consequences. Most notably, strategic efforts to benefit through intermediation alone seem destined to fail.

Beyond the results presented here, we believe that this model has promise as an infrastructure for considering generally how features of the environment might interact with a relational network to determine the distribution of rewards. Several potential extensions come to mind. For example, although we consider a world in which actors know that they will face a particular set of projects, most empirical settings in which researchers have focused on social networks experience considerable uncertainty. Adding some level of randomness to the available projects in the second stage therefore might offer an interesting route for extension. More generally, intermediation is not the only aspect of relational structure that sociologists have examined. To the extent that any position has been proposed as a source of advantage in performance, substantial untapped value lies in formal examination of the conditions necessary for it to exist and persist.

Acknowledgments

Melbourne Business School and London Business School have generously supported this research. The first author gratefully acknowledges financial support from an Australian Research Council discovery grant. The authors thank Kira Fabrizio, Isabel Fernandez-Mateo, Glenn MacDonald, Bill McEvily, Anita McGahan, Richard Makadok, Scott Rockart, Pai-Ling Yin, and participants in Boston University, Fuqua, Goizueta, and Rotman seminars as well as the Atlanta Competitive Advantage Conference and Academy of Management conferences for useful comments.

Appendix A. Core Existence

Given that we use the core as a solution concept, we must demonstrate that the core (i.e., a set of feasible competitive distributions of value) exists. As noted above, this issue relates quite closely to Proposition 4. The same competitive forces that give an actor a competitive advantage result in an empty core if they become too strong.

Given the coalitional game implied by s, (N, v^s), define the *minimum total value* as

$$mv^s \equiv \min_{\pi \in \mathbb{R}^n_+} \left\{ \sum_{j \in N} \pi_j \,\middle|\, \text{for all } G \subset N, \sum_{j \in G} \pi_j \geq v^s_G \right\}, \tag{A1}$$

where $\mathbb{R}^n_+$ is the set of all nonnegative value distributions. Note that a solution to this linear program, π^*, always exists. In essence, mv^s represents the minimum aggregate value required to compensate each actor sufficiently so that no group of actors could do better by producing value on their own. Meanwhile, note that the definition of the core of the coalitional game implied by s is

$$\mathscr{C}^s \equiv \left\{ \pi \in \mathbb{R}^n_+ \,\middle|\, \sum_{j \in N} \pi_j = v_N \text{ and for all } G \subset N, \sum_{j \in G} \pi_j \geq v^s_G \right\}. \tag{A2}$$

One can easily see that any solution to (A1) satisfies everything but the feasibility constraint in (A2). Therefore, we move immediately to the following result.

Proposition 5. *Given a network* (N, R^s), $C^s \neq \varnothing$ *if and only if* $mv^s_i \leq v^s_N$.

Not only is this proposition simple, but also it has a clear interpretation: An empty core arises when actors have such strong competitive alternatives, relative to the total value available to all actors producing together, that no feasible distribution of value can prevent at least one group from acting on its own.

Although important as a component of our theory, we also believe that this interpretation sheds light broadly on the meaning of core existence in coalitional games. Compare, for example, our characterization and interpretation to one commonly found (the version below comes from Moulin 1995, pp. 412–414).

Definition 5. Given N, a *balanced family of coalitions* is a subset $\mathscr{B} \in 2^N \backslash \{N\}$ such that there exists for each $G \in \mathscr{B}$ a "weight" δ_G, $0 \leq \delta_G \leq 1$, such that for all $i \in N$,

$$\sum_{G \in \mathscr{B}_i} \delta_G = 1,$$

where $\mathscr{B}_i \equiv \{G \in \mathscr{B} \mid i \in G\}$. A vector of balanced weights is a $\delta \equiv (\delta_G)_{G \subsetneq N}$ satisfying these equations.

Theorem 1 (Bondareva 1962, Scarf 1967). *Given* (N, v^s), $C^s \neq \varnothing$ *if and only if for every vector of balanced weights* δ,

$$\sum_{G \subsetneq N} \delta_G v^s_G \leq v^s_N. \tag{A3}$$

"A general vector of balanced weights represents a pattern of coalition formation more involved than (but similar to) partition of the set N. Agent i joins coalition G for a fraction δ_G of its time; all members of G devote the same fraction of their time to this particular coalition; coalition G produces $\delta_G \cdot v_G$ units of surplus. Inequality (A3) says that in any such coalition pattern, the overall surplus does not exceed v_N." (Moulin 1995, p. 414)

Although correct and equivalent (as a dual of our primal), the standard characterization strikes us as somewhat less intuitive and more detached from the model (e.g., actors do not allocate their time among various projects, all at the same percentage, etc.).

Appendix B. Proofs

B.1. Proof of Proposition 1

We prove the result by showing that removing the relationships to an actor within a network is equivalent to removing that actor from the game.

Lemma 1. *Given a network* (N, R^s), *consider* $i \in N$ *and let* $\hat{s}$ *be such that* $R^{\hat{s}} = R^s_{-i}$. *Then,* $v^s_{G_{-i}} = v^{\hat{s}}_G$.

Assume that $p \in F^s_{G_{-i}}$. By the definition of feasibility by G_{-i} under R^s, this assumption implies that $X_p \subseteq G_{-i}$ and, for all $j, k \in X_p$, that there exists a path

$$\{(x_1, x_2), (x_2, x_3), \ldots, (x_{w-1}, x_w)\} \subseteq R^s \tag{B1}$$

such that $x_1 = j$ and $x_w = k$. Clearly then, $X_p \subseteq G$. By the definition of R^s_{-i},

$$R^{\hat{s}} = \{(j, k) \in R^s \mid j \neq i \text{ or } k \neq i\}. \tag{B2}$$

Together $X_p \subseteq G_{-i}$, $i \notin X_p$, and (B2) imply that every path of the form (B1) is also a subset of $R^{\hat{s}}$, and therefore that $p \in F^{\hat{s}}_G$. Hence, $F^s_{G_{-i}} \subseteq F^{\hat{s}}_G$.

Now, assume that $p \in F^{\hat{s}}_G$. By the definition of feasibility by G under $R^{\hat{s}}$, this assumption implies that $X_p \subseteq G$ and, for all $j, k \in X_p$, that there exists a path

$$\{(x_1, x_2), (x_2, x_3), \ldots, (x_{w-1}, x_w)\} \subseteq R^{\hat{s}}. \tag{B3}$$

Suppose that $i \in X_p$; then there exists a path of the form (B3) with $x_1 = i$. But this implication contradicts the assumption that $R^{\hat{s}} = R^s_{-i}$. Therefore, $i \notin X_p$ and, hence, $X_p \subseteq G_{-i}$. Because $R^{\hat{s}} \subseteq R^s$, $p \in F^s_{G_{-i}}$, thereby implying that $F^{\hat{s}}_G \subseteq F^s_{G_{-i}}$. Therefore, for all $G \subset N$, $F^s_{G_{-i}} = F^{\hat{s}}_G$.

For all $G \subset N$, by (1),

$$v^s_{G_{-i}} = \max_{F \subseteq F^s_{G_{-i}}} \sum_{p \in F} u_p$$

and

$$v^{\hat{s}}_G = \max_{F \in \mathcal{F}^{\hat{s}}_G} \sum_{p \in F} u_p$$

because $F^s_{G_{-i}} = F^{\hat{s}}_G$, $v^s_{G_{-i}} = v^{\hat{s}}_G$.

Assume that $i \in N$ is a broker in (N, R^s) and define $R^{\hat{s}}$ as in the preceding lemma. By definition, $av^s_i \equiv v^s_N - v^s_{N_{-i}}$. Also, by (1),

$$v^s_N = \max_{F \in \mathcal{F}^s_N} \sum_{p \in F} u_p. \tag{B4}$$

($\Rightarrow$) Suppose that every attractive collection F contains an attractive project p that is not feasible by N under R^s_{-i}. Then, every maximizer of (B4) contains a p such that $p \in \mathcal{F}^s_N$ but $p \notin F^s_{N_{-i}}$. Because $u_p > 0$ for all attractive projects, this implies $v^s_N > v^s_{N_{-i}}$ and hence, $av^s_i > 0$.

($\Leftarrow$) Assume that $av^s_i > 0$. Now, suppose that every attractive project is feasible by N under R^s_{-i}. This supposition implies that, if p is attractive, $p \in F^{\hat{s}}_N$. Therefore, every F maximizing (B4) is contained in $F^{\hat{s}}_N$. By (1),

$$v^{\hat{s}}_N = \max_{F \subseteq F^{\hat{s}}_N} \sum_{p \in F} u_p.$$

Therefore, $v^{\hat{s}}_N \geq v^s_N$. By definition, $F^{\hat{s}}_N \subseteq \mathcal{F}^s_N$, so $v^s_N \geq v^{\hat{s}}_N$. Therefore, $v^s_N = v^{\hat{s}}_N$. By the preceding lemma, $v^s_{N_{-i}} = v^{\hat{s}}_N$, and by the previous equality, $v^s_N = v^s_{N_{-i}}$, yielding $av^s_i = 0$, a contradiction.

B.2. Proof of Proposition 2

MacDonald and Ryall (2004) demonstrate that, in a general coalitional game (N, v), agent $i \in N$ has $\pi^{\min}_i > 0$ only if there exist two groups G, G', strict subsets of N, with the properties:

(1) $G \cap G' = \{i\}$;

(2) $v_G > v_{G_{-i}}$ and $v_{G'} > v_{G'_{-i}}$.

Item 1 of Proposition 2 repeats item 1 above. Item 2 follows using identical logic to the ($\Rightarrow$) part of the proof to Proposition 1.

B.3. Proof of Corollary 1

Take (N, R^s) as given. If $\mathcal{C}^s = \varnothing$, then $\pi^s = 0$ regardless of any other considerations. Therefore, assume that $\mathcal{C}^s \neq \varnothing$ and consider three cases. Finally, let $n^s_i = 2$. For some distinct j, $k \in N_{-i}$, $\{i, j, k\}$ is a component in (N, R^s)). By assumption, no single-agent projects exist (i.e., other than $p_\varnothing$), so at most, one project exists with $X_p = \{j, k\}$ (as i is a broker, no $X_p = \{i, j, k\}$ *exists*). Thus, $v_{\{i,j,k\}} = v_{\{j,k\}} \geq 0$. Let $C_1 = \{i, j, k\}$.

Because components, by definition, have no connections across them, they cannot complete any projects together that each of the components could not produce independently. Therefore, for any s,

$$v^s_N = \sum_{j=1,\ldots,k} v^s_{C_j}, \tag{B5}$$

where $(C_j, R^s_{C_j})$ represents one of the $k \geq 1$ components of the network implied by s.

By (B5), then

$$v^s_N = v_{\{i,j,k\}+} \sum_{l=2,\ldots,m} v^s_{C_l},$$

where m is the number of components in the network. Removing i entirely,

$$v^s_{N_{-i}} = v_{\{j,k\}+} \sum_{l=2,\ldots,m} v^s_{C_l}.$$

Because $v_{\{i,j,k\}} = v_{\{j,k\}}$, we have $v_N^s = v_{N_{-i}}^s$, and therefore, $av_i = 0$, implying $\pi_i^{s\max} = \pi_i^{s\min} = 0$. The argument for $n_i^s = 1$ is identical, except the component is of the form $C_1 = \{i, j\}$ and $v_{\{i,j\}} = v_{\{j\}} = 0$. Suppose that $n_i^s = 0$. Then, by Lemma 1, for all $G_{-i} \subset N$, $v_G = v_{G_{-i}}$, because this holds for N_{-i}, $av_i = 0$, etc.

B.4. Proof of Proposition 3

Given a general coalitional game, (N, v), define the *i-minimum total value*

$$mv_i \equiv \min_{\pi_{-i}\in\mathbb{R}_+^{n-1}} \left\{ \sum_{j\in N_{-i}} \pi_j \,\middle|\, \text{for all } G \subset N_{-i}, \sum_{j\in G} \pi_j \geq v_{G\cup\{i\}} \right\}. \tag{B6}$$

The following result comes from MacDonald and Ryall (2004).

Proposition 6. *Assume that* (N, v) *is a coalitional game such that*: (i) $\mathscr{C} \neq \varnothing$; (ii) *for all* $j \in N$, $v_{\{j\}} = 0$; *and* (iii) *for all* $G \subset N_{-i}$, $v_{G\cup\{i\}} \geq v_G$. *Then*, $(\pi_i^{\min} > 0) \Leftrightarrow (mv_i > v_N)$.

First, note that any second-stage game in our model satisfies premises (ii) and (iii). Item (ii) holds by direct assumption, while item (iii) holds because, under (1), adding actors never reduces the number of projects that a group can complete.

Item (i) does not necessarily hold. By assumption, $\mathscr{C}^s = \varnothing$ implies $\pi_i^{\min} = 0$. Conversely, $\pi_i^{\min} > 0$ implies $\mathscr{C}^s \neq \varnothing$. Moreover, given a network (N, R^s), $mv_i^s > v_N^s$ alone does not guarantee $\mathscr{C}^s \neq \varnothing$.

Therefore, in our model, one can restate Proposition 6 as: *Given* (N, R^s) *and an actor* $i \in N$,

$$(\pi_i^{\min} > 0) \Leftrightarrow (\mathscr{C}^s \neq \varnothing \wedge mv_i^s > v_N^s).$$

The key, then, is to demonstrate that $nv_i^s = mv_i^s$. First, note that LPs (2) and (B6) are identical except for the constraints. Because the i-minimal groups under R^s contain i by definition, $\mathscr{G}_i^s \subseteq \mathscr{G}_{+i}$, so consider $G \in \mathscr{G}_{+i} \backslash \mathscr{G}_i^s$. If $v_G^s = 0$, then the constraint associated with G becomes redundant due to the requirement that π_{-i} be nonnegative (i.e., an element of $\mathbb{R}_+^{n-1}$). Suppose that $v_G^s > 0$ and let $F \subseteq P$ deliver v_G^s. Define

$$P_i^s \equiv \{p \in P \mid \exists G \in \mathscr{G}_i^s,\, p \in \mathscr{F}_G^s\}.$$

Suppose that $p \in F$ but $p \notin P_i^s$. Because $p \in F$, $p \in \mathscr{F}_G^s$. Because $p \in \mathscr{F}_G^s$ and $p \notin P_i^s$, p is also feasible for some $G' \subset G$, under $R_{-G'}^s$. Let G' represent the largest group that satisfies this condition; then, $G \backslash G' \in \mathscr{G}_i^s$. But, this implies $p \in P_i^s$, a contradiction. Therefore, $p \in F$ implies $p \in P_i^s$. It also implies, because $F \in \mathscr{F}_G^s$, that we can partition G such that each project in F is feasible by some i-minimal subgroup in the partition. Let $\Xi_F \subset \mathscr{G}_i^s \cap 2^G$ be a disjoint set of subsets in G such that for all $p \in F$, there exists a $G' \in \Xi_F$ such that p is feasible by G'. Then,

$$v_G^s = \sum_{G'\in\Xi_F} v_{G'}^s = \sum_{p\in F} u_p. \tag{B7}$$

The constraint in LP (B6) associated with G is

$$\sum_{j\in G_{-i}} \pi_j \geq v_G^s. \tag{B8}$$

However, LP (2) contains constraints,

$$\text{for all } G' \in \Xi_F, \quad \sum_{j\in G'_{-i}} \pi_j \geq v_{G'}^s. \tag{B9}$$

Because the groups in Ξ_F are disjoint subsets of G, and given (B7), (B9) implies (B8). Therefore, all the constraints in LP (B6) are either contained in or implied by LP (2). Hence, $nv_i^s = mv_i^s$.

B.5. Proof of Proposition 4

Suppose that (N, R^s) is a supported network that includes a broker $b \in N$ with $\pi_b^s > 0$. Note the implication that $v_N^s > 0$. By Corollary 1, $n \geq 4$.

1. If any $(j, k) \in R^s$ exist such that $s_k^j + s_j^k > 0$, then (N, R^s) is not supported. Therefore, for all $(j, k) \in R^s$,

$$s_k^j + s_j^k = 0. \tag{B10}$$

2. By Proposition 3, $\pi_b^{s\min} > 0$ implies $\mathscr{C}^s \neq \varnothing$. Together with (B10), we have

$$\sum_{j\in N} \xi_j^s = \sum_{j\in N} \pi_j^s = v_N^s. \tag{B11}$$

3. Let $i \in N\backslash\{b\}$ deviate to the neighborhood $N_{-\{i,b\}}$ via $\hat{s}$, which we construct momentarily. Note that $(N, R^{\hat{s}})$ is such that i establishes (or re-establishes) relations with all agents except b. Because b is, by definition, not required by any project and because i now connects all required agents previously connected by b, $av_b = 0$. Therefore,

$$\pi_b^{\hat{s}} = 0. \tag{B12}$$

4. Because $(N, R^{\hat{s}})$ is connected, $v_N^{\hat{s}}$ is maximal. Hence,

$$v_N^{\hat{s}} \geq v_N^s. \tag{B13}$$

5. To ensure that $\pi^{\hat{s}} \neq (0, \ldots, 0)$, we must show that $\mathscr{C}^s \neq \varnothing$. The existence condition requires the complete network (let us denote it (N, R^c)) to have $v_N^c > 0$ and $\mathscr{C}^c \neq \varnothing$. Because $(N, R^{\hat{s}})$ is connected, $\mathscr{F}_N^{\hat{s}} = \mathscr{F}_N^c$ and, as a result, $v_N^{\hat{s}} = v_N^c$. However, because $(N, R^{\hat{s}})$ is connected but not necessarily complete, some project collections realizable for some G under R^c may not be so under $R^{\hat{s}}$. Therefore, for all $G \subsetneq N$, $v_G^c \geq v_G^{\hat{s}}$. Therefore, by Proposition 5,

$$\mathscr{C}^{\hat{s}} \neq \varnothing. \tag{B14}$$

6. Assume that actor i does not change its offer to b (i.e., $\hat{s}_b^i = s_b^i$). Then, by the construction of a neighborhood equilibrium (i.e., $\hat{s}_i^j = -\hat{s}_j^i$), (B10) and (B14) imply that $\pi^{\hat{s}}$ is such that

$$\sum_{j\in N} \xi_j^{\hat{s}} = \sum_{j\in N} \pi_j^{\hat{s}} = v_N^{\hat{s}}. \tag{B15}$$

7. Combining (B11) and (B15),

$$\begin{gathered} \sum_{j\in N} \xi_j^{\hat{s}} - \sum_{j\in N} \xi_j^s = \sum_{j\in N} \pi_j^{\hat{s}} - \sum_{j\in N} \pi_j^s \quad \text{implies} \\ \sum_{j\in N} \xi_j^{\hat{s}} - \sum_{j\in N} \xi_j^s = v_N^{\hat{s}} - v_N^s. \end{gathered} \tag{B16}$$

8. Because $b \notin N_{-\{i,b\}}$, by the construction of $\hat{s}$,

$$\sum_{(b,k)\in R^{\hat{s}}} \hat{s}_k^b = \sum_{(b,k)\in R^s} s_k^b. \tag{B17}$$

By the premise, $\pi_b^s > 0$, and by (B12), $\pi_b^{\hat{s}} = 0$; combined with (B17), they imply

$$\xi_b^s - \xi_b^{\hat{s}} = \pi_b^s > 0. \tag{B18}$$

9. Using (B16) and rearranging some terms,

$$\begin{aligned}\sum_{j\in N_{-b}} \xi_j^{\hat{s}} - \sum_{j\in N_{-b}} \xi_j^s &= (v_N^{\hat{s}} - \xi_b^{\hat{s}}) - (v_N^s - \xi_b^s)\\ &= (v_N^{\hat{s}} - v_N^s) + (\xi_b^s - \xi_b^{\hat{s}}).\end{aligned}$$

Now, using (B18),

$$\sum_{j\in N_{-b}} \xi_j^{\hat{s}} - \sum_{j\in N_{-b}} \xi_j^s = v_N^{\hat{s}} - v_N^s + \pi_b^s. \tag{B19}$$

10. Define $\Delta \equiv v_N^{\hat{s}} - v_N^s + \pi_b^s - \varepsilon$, where $\varepsilon > 0$. Note that $\pi_b^s > 0$ and (B13) imply that ε exists such that $\Delta > 0$. Now construct the deviation offers to $N_{-\{i,b\}}$. For each $j \in N_{-\{i,b\}}$ such that $(i,j) \in R^s$, set

$$\hat{s}_j^i = \pi_j^s - \pi_j^{\hat{s}} - s_i^j + \frac{\Delta}{n-2}. \tag{B20}$$

If $(i,j) \notin R^s$, set

$$\hat{s}_j^i = \pi_j^s - \pi_j^{\hat{s}} + \frac{\Delta}{n-2}. \tag{B21}$$

11. Consider, for $j \in N_{-\{i,b\}}$ such that $(i,j) \in R^s$,

$$\xi_j^{\hat{s}} - \xi_j^s = \left(\pi_j^{\hat{s}} - \sum_{(jk)\in R^{\hat{s}}} \hat{s}_k^j\right) - \left(\pi_j^s - \sum_{(jk)\in R^s} s_k^j\right).$$

For all $k \neq i$, $\hat{s}_k^j = s_k^j$. Taking (B20) and setting $\hat{s}_i^j = -\hat{s}_j^i$, we have

$$\begin{aligned}\xi_j^{\hat{s}} - \xi_j^s &= \pi_j^{\hat{s}} - \hat{s}_i^j - \pi_j^s + s_i^j\\ &= \pi_j^{\hat{s}} - \pi_j^s + s_i^j + \left(\pi_j^s - \pi_j^{\hat{s}} - s_i^j + \frac{\Delta}{n-2}\right)\\ &= \frac{\Delta}{n-2}.\end{aligned}$$

By similar reasoning, this also holds for $j \in N_{-\{i,b\}}$ such that $(i,j) \notin R^s$. Therefore, for all $j \in N_{-\{i,b\}}$,

$$\xi_j^{\hat{s}} - \xi_j^s = \frac{\Delta}{n-2}.$$

12. For actor i,

$$\begin{aligned}\xi_i^{\hat{s}} - \xi_i^s &= \left(\pi_i^{\hat{s}} - \sum_{(i,k)\in R^{\hat{s}}} \hat{s}_k^i\right) - \left(\pi_i^s - \sum_{(i,k)\in R^s} s_k^i\right)\\ &= \pi_i^{\hat{s}} - \pi_i^s + \sum_{(i,k)\in R^s} s_k^i - \sum_{(i,k)\in R^{\hat{s}}} \hat{s}_k^i.\end{aligned}$$

If $j \in N_{-\{i,b\}}$ is such that $(i,j) \in R^s$, then $s_j^i = -s_i^j$ and

$$\begin{aligned}s_j^i - \hat{s}_j^i &= s_j^i - \left(\pi_j^s - \pi_j^{\hat{s}} - s_i^j + \frac{\Delta}{n-2}\right)\\ &= \pi_j^{\hat{s}} - \pi_j^s - \frac{\Delta}{n-2}.\end{aligned}$$

For $j \in N_{-\{i,b\}}$ such that $(i,j) \notin R^s$, it is nevertheless true that $(i,j) \in R^{\hat{s}}$, so we simply subtract (B21). Because $\hat{s}_b^i = s_b^i$, we have

$$\begin{aligned}\xi_i^{\hat{s}} - \xi_i^s &= \pi_i^{\hat{s}} - \pi_i^s + \sum_{j\in N_{-\{i,b\}}} \left(\pi_j^{\hat{s}} - \pi_j^s - \frac{\Delta}{n-2}\right)\\ &= \sum_{j\in N_{-b}} \pi_j^{\hat{s}} - \sum_{j\in N_{-b}} \pi_j^s - \Delta\\ &= \sum_{j\in N_{-b}} \pi_j^{\hat{s}} - \sum_{j\in N_{-b}} \pi_j^s - \left(\sum_{j\in N} \pi_j^{\hat{s}} - \sum_{j\in N} \pi_j^s + \pi_b^s - \varepsilon\right)\\ &= \sum_{j\in N_{-b}} \pi_j^{\hat{s}} - \sum_{j\in N} \pi_j^{\hat{s}} + \sum_{j\in N} \pi_j^s - \left(\pi_b^s + \sum_{j\in N_{-b}} \pi_j^s\right) + \varepsilon\\ &= -\pi_b^{\hat{s}} + \varepsilon\\ &= \varepsilon.\end{aligned}$$

13. Therefore, this deviation results in everyone in N_{-b} being strictly better off. Therefore, (N, R^s) is not supported, a contradiction.

References

Ahuja, G. 2000. Collaboration networks, structural holes and innovation: A longitudinal study. *Admin. Sci. Quart.* **45**(3) 425–455.

Allen, T. J. 1977. *Managing the Flow of Technology*. MIT Press, Cambridge, MA.

Bae, J. H., M. Gargiulo. 2004. Partner substitutability, alliance network structure, and firm profitability in the telecommunications industry. *Acad. Management J.* **47**(6) 843–859.

Bloch, F., M. O. Jackson. 2006. The formation of networks with transfers among players. *J. Econom. Theory*. Forthcoming.

Bondareva, O. 1962. The theory of the core in an *n*-person game. *Vestnik Leningrad University* **13** 141–142. (In Russian.)

Brandenburger, A., H. Stuart. 2007. Biform games. *Management Sci.* **53**(4) 537–549.

Burt, R. S. 1983. *Corporate Profits and Cooptation*. Academic Press, New York.

Burt, R. S. 1992. *Structural Holes*. Harvard University Press, Cambridge, MA.

Burt, R. S. 2000. The network structure of social capital. *Res. Organ. Behav.* **22** 345–423.

Burt, R. S. 2002. Bridge decay. *Soc. Networks* **24**(4) 333–363.

Burt, R. S. 2004. Structural holes and good ideas. *Amer. J. Sociol.* **110**(2) 349–399.

Chatain, O., P. Zemsky. 2007. The horizontal scope of the firm: Organizational tradeoffs vs. buyer-supplier relationships. *Management Sci.* **53**(4) 550–565.

Coleman, J. S. 1988. Social capital in the creation of human capital. *Amer. J. Sociol.* **94** S95–S120.

Fernandez, R. M., R. V. Gould. 1993. A dilemma of state power: Brokerage and influence in the national health policy domain. *Amer. J. Sociol.* **99**(6) 1455–1491.

Fernandez, R. M., E. Castillo, P. Moore. 2000. Social capital at work: Networks and employment at a phone center. *Amer. J. Sociol.* **105**(5) 1288–1356.

Fernandez-Mateo, I. 2006. Who pays the price of brokerage? Transferring constraint through price-setting in the staffing sector. *Amer. Sociol. Rev.* Forthcoming.

Freeman, L. C. 1977. A set of measures of centrality based on betweenness. *Sociometry* **40**(1) 35–41.

Gans, J. S., C. C. de Fontenay. 2006. Bilateral bargaining with externalities. Working paper, Melbourne Business School, University of Melbourne, Victoria, Australia.

Granovetter, M. S. 1973. The strength of weak ties. *Amer. J. Sociol.* **78**(6) 1360–1380.

Granovetter, M. S. 1985. Economic action and social structure: The problem of embeddedness. *Amer. J. Sociol.* **91**(3) 481–510.

Hsu, D. H. 2004. What do entrepreneurs pay for venture capital affiliation? *J. Finance* **59**(4) 1805–1844.

Ingram, P., J. Robinson, M. L. Busch. 2005. The intergovernmental network of world trade: IGO connectedness, governance and embeddedness. *Amer. J. Sociol.* **111**(3) 824–858.

Jackson, M. O. 2007. The economics of social networks. R. Blundell, W. K. Newey, T. Persson, eds. *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, Vol. 1. Cambridge University Press, Cambridge, UK, 1–56.

Kollock, P. 1994. The emergence of exchange structures: An experimental study of uncertainty, commitment and trust. *Amer. J. Sociol.* **100**(2) 313–345.

Lippman, S. A., R. P. Rumelt. 2003. A bargaining perspective on resource advantage. *Strategic Management J.* **24**(11) 1069–1086.

MacDonald, G., M. D. Ryall. 2004. How do value creation and competition determine whether a firm appropriates value? *Management Sci.* **50**(10) 1319–1333.

Moody, J., D. R. White. 2003. Structural cohesion and embeddedness: A hierarchical concept of social groups. *Amer. Sociol. Rev.* **68**(1) 103–127.

Moulin, H. 1995. *Cooperative Microeconomics: A Game-Theoretic Introduction*. Princeton University Press, Princeton, NJ.

Padgett, J. F., C. K. Ansell. 1993. Robust action and the rise of the Medici, 1400–1434. *Amer. J. Sociol.* **98**(6) 1259–1319.

Podolny, J. M., J. N. Baron. 1997. Resources and relationships: Social networks and mobility in the workplace. *Amer. Sociol. Rev.* **62**(5) 673–693.

Salancik, G. R. 1995. Wanted: A good network theory of organization. *Admin. Sci. Quart.* **40**(2) 345–349.

Scarf, H. 1967. Core of an n person game. *Econometrica* **35**(1) 50–59.

Shapley, L. S. 1962. Complements and substitutes in the optimal assignment problem. *Naval Res. Logist. Quart.* **9**(1) 45–48.

Shapley, L. S. 1967. On balanced sets and cores. *Naval Res. Logist. Quart.* **14**(4) 453–460.

Simmel, G. 1950. *The Sociology of Georg Simmel*. Free Press, New York.

Talmud, I. 1994. Relations and profits: Imperfect competition and its outcome. *Soc. Sci. Res.* **23**(2) 109–135.

Tushman, M. L. 1977. Special boundary roles in the innovation process. *Admin. Sci. Quart.* **22**(4) 587–605.