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# Competitive intensity and its two-sided effect on the boundaries of firm performance

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**Abstract**

The new perspective emerging from strategy's value-capture stream is that the effects of competition are two-fold: competition for an agent bounds its performance from below, while that for its transaction partners bounds from above. Thus, assessing the intensity of competition on either side is essential to understanding firm performance. Yet, the literature provides no formal notion of "competitive intensity" with which to make such assessments. Rather, some authors use *added value* as their central analytic concept, others the *core*. Added value is simple, but misses the crucial, for-an-agent side of competition. The core is theoretically complete, but difficult to interpret and empirically intractable. This paper formalizes three, increasingly general notions of competitive intensity, all of which improve upon added value while avoiding the complexity of the core. We analyze markets characterized by disjoint networks of agents (e.g., supply chains), providing several insights into competition and new tools for empirical work.

# 1 Introduction

Today, strategy boasts a rich and growing vein of theoretical findings based upon the application of coalitional game theory.[1] This growth is due to a number of important features of the coalitional game formalism, including its: explicit distinction between the value creation and capture aspects of market transactions; ability to represent the sort of unstructured dealings that are typical in real-world markets; built-in scalability with respect to market size; and operation at a level of abstraction that frees theorists from the need to commit to a myriad of special assumptions on the underlying details of value creation. The findings in this stream have resulted in numerous insights into *the* central question in strategy: what drives the economic performance of agents in competitive markets?[2] Of course, new insights invariably lead to new questions. In this paper, we tackle several such questions with the aim of providing further contributions. While our focus is on the stream within strategy proper, we believe several of our findings will be of interest to researchers outside the field. Let us begin by identifying our motivating research questions.

Strategy is a field dominated by empirical work. Thus, one of the most important questions arising from this new stream is how its theoretical claims translate into empirical hypotheses. Presently, the papers in this stream are mostly categorized into work that uses added-value as its essential analytical tool (e.g., beginning with Brandenburger and Stuart, 1996), versus that based upon the core (e.g., beginning with MacDonald and Ryall, 2004). Both approaches provide insights into the effect of competition on the distribution of value within a market. The appeal of added-value is its great simplicity. However, this simplicity comes at a cost: by ignoring the lion's share of competitive relations embedded in a coalitional game, analytical claims based upon added-value tend to be coarse at best and misleading at worst. Use of the core solves the problem of theoretical incompleteness, but does so at the cost of information requirements that increase exponentially with the number of agents in the market. Thus, empiricists (or practitioners) who wish to apply the theory face a dilemma: opt for the manageable but loose-to-misleading approach of added-value or, instead, for the fully refined but potentially intractable path of the core. This dilemma raises the question of whether it is possible to construct an alternative theoretical tool that, on the one hand, provides greater analytical traction than added-value while, on the other,

[1] Gans and Ryall (2015) provide a recent review of this line of work.

[2] We say "the" because a suitable answer to this question is needed to answer others, such as: Why are there persistent differences in firm performance?; Is firm performance the result of management choice or other factors?; What capabilities must a firm cultivate in a given situation to maximize performance?; and so on.

mitigating the untenable information requirements of the core.

There is another issue that is closely related to the preceding one. The abstract nature of the core can impede one's grasp of the economics driving theoretical results. For example, Aumann (1973) sets up a simple buyer/seller market and shows that, according to the core, the buyers are worse off forming a bargaining syndicate. So egregious is this violation of our economic sensibily, that Aumann concludes that the core is a flawed solution concept. Later, Postlewaite and Rosenthal (1974) show that Aumann's conclusion was premature. With a bit of work, we see that reasonable economics underpin this "paradoxical" result after all – at least in the simple case of discrete buyer/seller markets. Yet, if the core obscures the economics in relatively straightforward settings (to intellectual giants the likes of Aumann, no less), strategy might be better off avoiding it altogether. In an applied field such as ours, theoretical clarity is important. The question raised is whether some other, more easily interpretable analytical tools can be developed to provide greater transparency to the competitive forces at work in cooperative games.

A third question arises from informal discussions in the literature (e.g., Gans et al., 2008; Gans and Ryall, 2015) surrounding the indeterminacy of the core. Typically, even considering all the competitive relations implied by a coalitional game (i.e., by using the core), the amount of value captured by each agent is – at best – narrowed down to within a nontrivial interval. Although intervals are less analytically convenient than the point-estimates arising in more familiar settings like Cournot and Bertrand, such indeterminacy may nevertheless be a feature of real-world markets:

> Taking the premises of the model seriously ... competition determines an *interval* within which an agent's value capture lies; where, within that interval, actual capture lands *is due to factors other than competition* ... the theory points toward a new conception of "competitive intensity" as well as the existence and possible importance of "persuasive" resources. [Gans and Ryall (2015, p. 5)]

That is, if competition's role in an agent's performance is to determine a *range* of performance possibilities, then the question arises whether that range is wide or narrow: that is, to what extent is competition the determining factor for a given agent's performance?[3] This leads to the suggestion that the right way to think about "competitive intensity" is not in terms of downward pressure on

[3] If wide, then the importance of supra-competitive sources of value capture is substantial. If narrow, such resources have little influence on agent performance. Note that scholarly interest in the existence and effects of supra-competitive resources (e.g., Gans et al., 2008; Chatain, 2013; Gans and Ryall, 2015), is recent – and entirely in response to the aforementioned theoretical findings.

profitability (the traditional understanding) but, rather, in terms of the extent to which profitability is determined by competition per se.[4] Moving beyond informal speculation, the question is what form a mathematical implementation of this suggestion might take.

The contribution of this paper is the answers it provides to these questions. After a more detailed discussion of how the added-value and core approaches have been used in strategy (next section), we proceed to develop several variations on a new, formal notion of competitive intensity (Sec. 4). We begin with a formalization that is based on unilateral deviations only (akin in spirit to the notion of Nash equilibrium for non-cooperative games) that we simply refer to as *competitive intensity*.[5] It is based upon the insight that added-value has different implications depending upon whether the group considered is one within which the focal agent *will* transact versus one with which the agent *could* transact. This answers all three challenges: 1) it extends the added-value approach in a significant way without approaching anywhere near the information demands of the core; 2) it helps illuminate the competitive economics in more general settings than, e.g., buyer/seller markets; 3) it provides a formal response to the informal speculation in strategy connecting competitive intensity to bounds on value appropriation. Due to its inherent simplicity, we imagine this notion of competitive will be the one most conducive to empirical work in strategy.

What follows are increasingly complex variations on the initial idea. In the first of these, we show (Sec. 4.2) that the direct effects of competition imply certain indirect effects. This leads to a more refined definition of *indirect* competitive intensity. We show that indirect competitive intensity illuminates and extends our understanding to more general settings, of the economics at work in the simple example of Postlewaite and Rosenthal (1974). Though, more information is demanded by this definition, the implications in certain settings may be sufficiently promising to warrant its use in some empirical studies. Our third and final variation, which we label *generalized* competitive intensity, pushes insight captured by the first two definitions to its natural limit (Sec. 4.3). While this last idea may be too demanding for empirical work, we provide it as a matter of theoretical completeness, noting that it does provide an alternative solution concept to the core for cooperative games. Throughout the analysis, we use the core interval as a benchmark and, thus, are careful to provide formal results that tie our concepts back to it.

Finally, we believe that the ideas explored in this paper help provide insights not only into

[4] Under the latter conception, extreme competitive intensity describes situations in which the interval determined by competition collapses to a single point: competition fully determines profit – high, low or otherwise.

[5] The label "*direct* competitive intensity" is more descriptive, but overly cumbersome.

cooperative game analyses per se but, more importantly, into how competition actually works in the real world. To assist in the development of these insights, the second part of the paper (Sec. 5) sets up a differentiated products model with capacity constraints. We use it to explore some familiar industry settings and show the important role played by competitive intensity in shaping performance.

## 2 Coalitional games in strategy: added-value versus the core

By way of selective literature review and an introduction to the coalitional game formalism, we use this section to discuss added-value and the core – the two points on the dimension of theoretical complexity that bound the middle ground we wish to explore later in the paper. Essentially, all applications of coalitional game theory in strategy adopt one or the other of these two concepts as the foundation of their analyses. Both approaches have been used to provide important economic insights for strategy research. We highlight several of these as the discussion unfolds.

We begin by defining the objects in a coalitional game, then quickly move on to examine the first concept, added-value. The simplicity of added-value is why it appeals as an analytical tool. Yet, it is also why added-value analysis is incomplete and can lead to problems. By illustrating these, we motivate our following examination of the core. As we show, while the core solves the problem of incompleteness, it does so by introducing substantial complexity and other barriers to practical use. Throughout the paper, we adopt the following notational conventions. Sets are indicated with capital letters, while elements of sets, scalars, vectors and functions are all represented by small letters. Technical terms are italicized at the point of definition.

**Value creation** A *coalitional game* is a pair $(N, v)$ in which $N \equiv \{1, \dots n\}$, $n < \infty$, is the set of *agents* and $v : 2^n \to \mathbb{R}_+$ is the *characteristic function*. An arbitrary agent is denoted $i \in N$ and group of agents $G \subseteq N$. For any group $G$, $v(G)$ is the economic value that the group $G$ can create via some collection of mutually agreeable transactions.[6] Let $v(\emptyset) \equiv 0$ and assume $v$ is superadditive.[7] There is no reason for the members of $G$ to produce less than $v(G)$ since, were that

[6] Keep in mind that the transactions required to produce $v(G)$ are meant to be "arms-length" in the sense that they are undertaken only if each member of $G$ receives a share of $v(G)$ sufficient to induce it to engage with $G$ and not some other group. In particular, central coordination or collusion is not implied (and is typically inappropriate for the market settings of interest to strategy researchers).

[7] Superadditivity requires, for all $G, G' \subseteq N$ such that $G \cap G' = \emptyset$, $v(G \cup G') \geq v(G) + v(G')$: adding more agents to a group does not destroy value. This restricts attention to markets in which negative externalities cannot be avoided via an appropriate organization of activities.

the case, everyone in $G$ could be made strictly better off by producing the full amount. Therefore, under superadditivity, $v(N)$ is interpreted as *the economic value that is actually created*. This interpretation is highlighted via the special notation $V \equiv v(N)$.[8] In what follows, we simplify notation for singleton sets by dropping the brackets; e.g., $v(i)$ rather than $v(\{i\})$. Also, if $i \in G$, then $G_{-i}$ indicates $G$ with $i$ removed; if $i \notin G$, then $G_{+i}$ denotes the set $G$ with $i$ added. Finally, without loss of generality, we assume that, for all $i \in N$, $v(i) = 0$ (a simple normalization).

**Value capture** A *distribution of value* is defined as an $n$-tuple, $\pi \equiv (\pi_1, \ldots, \pi_n) \in \mathbb{R}^n$. Here, $\pi_i$ is a number indicating the amount of value captured (or "appropriated") by agent $i$ in return for its participation in the value-creating activities that contribute to the production of $V$. In the case of a firm, $\pi_i$ represents its economic profit. The objective in CGT applications is to understand how the productive alternatives elaborated by $v$ shape $\pi$. A simple set of assumptions that relates $v$ to $\pi$ leads to the concept of added-value.

**Added-value** Added-value analysis relies on three assumptions: i) the aggregate value captured by the agents in $N$ is $V$ (essentially, a feasibility condition); ii) the agents in $N_{-i}$ must capture at least $v(N_{-i})$, else they could abandon $i$ and, in the process, make themselves all strictly better off; and iii) agents cannot be forced to take less than their outside value, lest they not participate in the market to begin with. Formally,

$$\sum_{i \in N} \pi_i = V, \tag{1}$$

$$\sum_{j \in N_{-i}} \pi_j \geq v(N_{-i}) \text{ and } \pi_i \geq 0 \text{ for all } i \in N, \tag{2}$$

where $N_{-i}$ denotes the set $N$ with $i$ removed (everyone but $i$) and $\pi_i \geq 0$ assumes outside values are normalized to zero (e.g., $v(i) = 0$). Agent $i$'s *added-value* is defined as $av_i \equiv V - v(N_{-i})$. If $\pi$ is such that (1) and (2) hold, it immediately follows that $\pi_i \leq av_i$ for all $i \in N$. Note well the analytical simplicity: for a given agent $i$, only two quantities – $V$ and $v(N_{-i})$ – are required to compute added-value. Moreover, the economic insight is immediate: when participation cannot be forced, as is the case in a free market, an agent cannot expect to capture more than the incremental

[8] Coalitional game theory dates back to von Neumann and Morgenstern (1944). Far earlier, Edgeworth (1881) developed the idea that the arm's-length alternatives available to agents in a market influences not only the amount of value produced but also how that value is distributed.

value associated with its participation. This insight motivates the idea that firm strategies should aim to enhance added-value.[9]

Unfortunately, the simplicity of added-value is also its major drawback. To see why, consider a market with two firms, $a$ & $b$, and two symmetric buyers, 1 & 2. The firms each have one unit of capacity and produce at zero cost. Buyers each demand one unit of product: they value firm $a$'s product at 1 and $b$'s at 0.3. The value created is $V = 1.3$, say with buyer 1 dealing with $a$ and buyer 2 with $b$. If $a$ is removed, $b$ is left to transact with one buyer to produce 0.3. Thus, $av_a = 1$ and, immediately, $\pi_a \leq 1$. Yet, reasoning beyond added value, we also predict $\pi_a \geq 0.7$. To see why, say $a$'s buyer demands $\pi_1 = 0.3 + x$, with $0.7 > x > 0$. Then, even if $b$'s buyer stands to appropriate *all* the value in its deal with $b$ ($\pi_2 = 0.3$), $a$ could offer it $\pi_2 = 0.3 + \frac{1}{2}x$, which would make both ($a$ and buyer 1) strictly better off. Thus, $\pi_a \in [0.7, 1.0]$, with the final outcome dependent upon non-competitive factors, such as haggling ability.

Suppose $a$ adds a social networking feature for its product that creates a complementarity with $b$'s. Specifically: each buyer values $b$'s product at 1, provided that the other buyer has a unit of $a$'s product. Otherwise, everything is exactly as before. Now (using primes to indicate the new game), $V' = 2.0$ and $av'_a = 1.7$. This is a successful added-value strategy in the sense that $a$ increased its added value. Does greater added value imply greater value capture? Consider the distribution in which the buyers capture all the value ($\pi_1 = \pi_2 = 1$ and $\pi_a = \pi_b = 0$). Is competition in this new situation enough to rule this out? The answer is: no. Even though $\pi_a = 0$, $a$ has no mutually-improving offer to make to $b$'s buyer (together, they can create 1, but $\pi_2 = 1$ already). Here, $\pi_a \in [0, 1.7]$: relative to its original position, the added value strategy increased its maximum, but decreased its minimum. If $a$ is a bad haggler, then it may care more about competition's slackening effect on its minimum. This effect is simply not picked up by added-value.

**The core** In the preceding example, conditions (1) and (2) limits attention to five groups. This leaves out ten other groups – some of which have important implications, as we saw. To ensure all

[9]Brandenburger and Stuart (1996) are the first to apply CGT to strategy and argue in favor of added-value as a strategic goal ("added-value strategies"). Economists use the term "agent marginal product" for the same concept (interested readers are referred to the excellent survey by Makowski and Ostroy, 2001). Important contributions that focus upon added-value in strategy include Adner and Zemsky (2006); Chatain and Zemsky (2007); Adegbesan (2008); Chatain and Zemsky (2011).

such implications are taken into consideration, (2) can be generalized to:

$$\sum_{j \in G} \pi_j \geq v(G) \text{ for all } G \subset N. \tag{3}$$

A distribution $\pi$ is said to be *consistent with competition* when it satisfies (1) and (3). The set of all such $\pi$ is commonly referred to as the *core*. For every $i \in N$, the core implies a *competitive interval*, denoted $[\pi_i^{\min}, \pi_i^{\max}]$, such that $\pi$ is consistent with competition only if $\pi_i \in [\pi_i^{\min}, \pi_i^{\max}]$ and, for every $\pi_i \in [\pi_i^{\min}, \pi_i^{\max}]$, there is at least one $\pi$ containing it that is consistent with competition.[10]

The core has several positive features with respect to strategy theory. First, it eliminates the problem of omission associated with added-value. Second, taking the possibility of nontrivial competitive intervals seriously leads to important insights, not the least of which are: i) competition bounds appropriation from *below* as well as from above; and ii) beyond competition, *persuasive* resources – those that result in appropriation above one's competitive minimum – may also play an important role in firm performance. Finally, the elegant mathematics of the core (convex analysis) is well understood and extensively developed.[11]

In practice, however, the core suffers from some major drawbacks. The most significant of these is that its complexity increases exponentially in the size of $N$. To be clear, this is not a theoretical issue. As mentioned above, whether $n$ equals two or a quadrillion, the math is elegant and tractable.[12] Rather, the combinatorial complexity is a problem in the *application* of the theory to practice and in the testing of its claims via empirical analysis. For example, a market with just forty participants implies over a *trillion* possible groups.[13] Thus, even when $N$ is relatively small, the sheer amount of data required to test the theory seems prohibitive. This problem is compounded by the fact that the lion's share of the groups, because they represent alternatives to the activities that generate $V$, are not observed. Thus, the $v(\cdot)$'s for these groups (counterfactuals) must be estimated in some way (perhaps, from other primitives). Then, even if these data issues are surmounted, a computational hurdle remains: identifying the competitive interval for a single, focal firm, requires the solution of two linear programming problems (for $\pi_i^{\min}$ and $\pi_i^{\max}$), each

[10] This assumes a nonempty core – see the following discussion.

[11] MacDonald and Ryall (2004) are the first to introduce core analysis to strategy. Since then, a number of important papers have appeared that also rely on the full analytical weight of the core. These include Stuart (2005); MacDonald and Ryall (2006); Brandenburger and Stuart (2007); Ryall and Sorenson (2007); Stuart (2011)

[12] An actual theoretical concern is that the core may be empty; i.e., given $(N, v)$, there may be no $\pi$ that satisfy (1) and (3). The conditions for core existence are well-known (see Bondareva, 1962) and always amount to $V$ being too meagre to satisfy (3). Stuart (1997) shows that this is not an issue in a wide range of market settings.

[13] We thank R. Casadesus-Masanell for helpfully doing the math and sharing this fact.

of which requires the inclusion of all the group-related inequalities. Even three-letter government agencies might have trouble solving LPs with over a trillion constraints.

This raises a further concern: apparently, to satisfy (3) in the real-world, the agents in $N$ are not only required to grasp an enormous number of different, potential ways of organizing their economic activities, but also to know both the values that these would produce and, simultaneously, the shares of $V$ captured by all the other market participants for the activities actually undertaken. In most settings, this seems far beyond the ken of any real human being. Here, however, caution must be exercised since the binding constraints in (3) may be small in number and salient to those they affect. Additionally, consistent with theories of learning, agents may "grope their way" into the core over time.[14] Even so, the rationality demands of the core do seem excessive.

**Actual empirical work** One of the exciting, recent developments in strategy is the entry of a number of authors whose work focuses upon empirical analysis of the claims issuing from the theory stream. Not surprisingly given the preceding discussion, most of these use added-value as their focal theoretical construct.[15] A notable exception is Chatain (2013), who extends the analysis substantially beyond added-value.[16] As Chatain points out, the problem facing the empiricist is that the number of inequalities required is, "very large and unmanageable for the researcher and for the agents themselves." His solution is to include inequalities only for groups that are "one perturbation away" from the observed transactions. While this selection criterion seems reasonable, its theoretical implications are unknown. Thus, it is difficult to square the findings with existing theoretical results. If substantial progress is to be made, new theoretical tools must be developed for empirical scholars. These should provide guidance on the identification of a subset of groups that, on the one hand, is empirically tractable and, on the other, is theoretically meaningful. It is to the development of such tools that we now turn our attention.

## 3 Value partitions

We begin our analysis with a practical observation. When the agents engaged in one set of economic activities contemplates reorganizing to engage in some other, they must consider all the costs

[14] See, e.g., Fudenberg and Levine (1998).

[15] These include Adegbesan and Higgins (2011), Chatain (2011), Bennett (2013), Jia et al. (2012), Obloj and Capron (2011) and Grennan (2014).

[16] He builds on the empirical approach to inequality constraints found in Fox (2006).

associated with that reorganization. Unfortunately, this distinction is not always explicated in a cooperative game setting. In a model, $V$ and $v(G)$ are typically said to be "the" economic value that can be created by $V$ and $v(G)$, respectively. In an a real-world empirical setting, $V$ is *data* – value actually created. If the agents in $G$ transacted among themselves to contribute to $V$ (say, as a supply chain), then $v(G)$ is also data. However, if the agents in $G$ did not transact with one another, then $v(G)$ is a *counterfactual*, the outcome of transaction possibilities that could have occurred but did not. In the latter case, $v(G)$ must account for all of the economic costs associated with its production (i.e., switching costs). Thus, the real-world analyst must identify those groups actively engaged in the production of $V$ versus those that do not form but could have done (and ensure that the values of $v$ reflect these distinctions).

We respond to this need by taking the distinction between realized and unrealized groups as our starting point. A collection of groups in $N$, denoted $\mathcal{P}$, is called a *value partition* if: (i) $\mathcal{P}$ is a partition of $N$, i.e., division of $N$ into disjoint subsets; and (ii) $V = \sum_{G \in \mathcal{P}} v(G)$. We refer to each element in a value partition as a *value network* and, henceforth, use $X$ to denote such a group (as distinguished from an arbitrary group $G$). Agents in value networks are linked to one another via the transactions that give rise to some portion of the overall value created in the market, $V$. The *trivial value partition* is the one composed of a single value network; i.e., $\mathcal{P} = \{N\}$.

For a given coalitional game $(N, v)$, there may be multiple value partitions; i.e., different configurations of disjoint groups may have values that add up to $V$. As discussed above, however, in the real world only one of these is realized as the collection of groups that actually forms to produce $V$. Therefore, we associate one partition with this role, denote it $\mathcal{P}^*$, and refer to it as the *focal value partition*. The elements of $\mathcal{P}^*$ are similarly highlighted with asterisks ($X^*$) and referred to as the *focal value networks*. The *focal value network containing* agent $i$ is denoted $X_i^*$.

When $(N, v)$ is understood from the context, the set of all value partitions is labeled $\mathbf{P}$. At least one element of $\mathbf{P}$ corresponds to a partition that is "maximal" in the sense of set cardinality. The idea of a maximal partition is that the agents in each value network are all explicitly linked to one another by chains of transactions so that any further decomposition of a value network into smaller groups results in the production of less value. Formally, a value partition $\mathcal{P} \in \mathbf{P}$ is said to be *maximal* if, for each $X \in \mathcal{P}$, there do not exist disjoint $S, T \subset X$ such that $v(X) = v(S) + v(T)$. The results that follow apply to any value partition, including those that are neither focal nor maximal. However, when focal partitions correspond to the most refined (maximal) parsing of observed transactions, they will yield the most precise conclusions.

# 4 Quantifying three shades of "competitive intensity"

We now present several analytical tools that embrace the logic of (3) to a degree that substantially tightens the notion of added-value while, simultaneously, retaining a workable level of complexity for actual applications. Our approach builds upon the idea of a focal value partition, which is amenable to real-world interpretation and useful in accounting for the relative frictions faced by agents considering alternatives to some status-quo market structure. We begin with our notion of competitive intensity, which is the simplest and, we conjecture, most useful idea from an application standpoint. We expand this idea to indirect intensity which, while somewhat more complex, highlights an important, "indirect" effect of direct competition. Extending these ideas to the maximum number of groups leads to our notion generalized competitive intensity, which we include for theoretical completeness. This sequence of definitions is increasing in both computational difficulty and refinement with respect to the bounds on a firm's set of possible payoffs.

## 4.1 Competitive intensity

Assume the focal value partition $\mathcal{P}^*$ is given.[17] Using this as our guide and retaining the added-value constraints (1) and (2), we wish to identify a further subset of core constraints (3) that provides meaningful analytical traction while yet retaining much of the simplicity of the added-value construct. Obvious first candidates are those associated with the focal value networks themselves:

$$\sum_{i \in X^*} \pi_i \geq v(X^*) \text{ for all } X^* \in \mathcal{P}^*. \tag{4}$$

Invoking (4) means assuming that, in aggregate, the members of a focal network cannot be forced to capture in aggregate less than the value they produce. Feasibility (1) combined with (4) implies:

$$\sum_{i \in X^*} \pi_i = v(X^*) \text{ for all } X^* \in \mathcal{P}^*. \tag{5}$$

[17]In the spirit of providing tools for real-world situations, we assume the analyst has a specific value partition in mind. From a purely theoretical point-of-view, our results apply to *any* value partition.

That is, the agents of a focal value network capture among themselves precisely the value produced via their joint economic activities – an uncontroversial assumption. Note also, since $\pi_i \leq av_i(N)$,

$$v(X^*) = \sum_{i \in X^*} \pi_i \leq \sum_{i \in X^*} av_i(N) \text{ for all } X^* \in \mathcal{P}^*. \tag{6}$$

Typically, "the" added-value for an agent refers to the added-value as defined above (i.e., with respect to the market as a whole, $N$). However, the notion of added-value can be extended to any group: for any $G \subseteq N$, define $av_i(G) \equiv v(G) - v(G_{-i})$ if $i \in G$; and, $av_i(G) \equiv v(G_{+i}) - v(G)$ otherwise. With our sights still on $\mathcal{P}^*$, an obvious quantity of interest is $av_i(X_i^*)$. The relevant core constraints are:

$$\sum_{j \in X_{i-i}^*} \pi_j \geq v(X_{i-i}^*) \text{ for all } i \in N, \tag{7}$$

where $X_{i-i}^*$ is $i$'s focal value network with $i$ removed. Expressions (5) and (7) imply:

$$\pi_i \leq av_i(X_i^*) \text{ for all } i \in N. \tag{8}$$

In other words, just as an agent's added-value to the market as a whole caps its ability to appropriate, so too does its added-value to its *own* value network. This is consistent with our intuition about added-value, emphasized as it is with the value capture stream.[18]

Contrary to this intuition is the effect of $i$'s added-value to networks *outside* its own. To the extent $i$ has positive added-value to a value network besides its own, that added-value places a *floor* beneath its ability to capture value. Reaching this conclusion requires the consideration of the following constraint:

$$\pi_i + \sum_{j \in X^*} \pi_j \geq v(X_{+i}^*) \text{ for all } i \in N \text{ and } X^* \in \mathcal{P}^* \backslash X_i^*. \tag{9}$$

We refer to the value networks outside of $i$'s own ($\mathcal{P}^* \backslash X_i^*$) as $i$'s *competitive periphery.* The elements of an agent's competitive periphery are value networks outside its own that compete with the agent to create value with its transaction partners and, simultaneously, with its transaction partners to create value with it. External value networks compete implicitly with the agents who transact with

[18]Since $v(N_{-i}) \geq \sum_{j \neq i} v(X_j^*) + v(X_{i-i}^*)$ by superadditivity, it is not difficult to show that $av_i(N) \leq av_i(X_i^*)$.

one another within a given network. Thus, (9) includes the constraints associated with the groups constructed by adding each $i$ to each value network in its competitive periphery. When these hold as well,

$$\pi_i \geq av_i(X^*_{+i}) \text{ for all } i \in N \text{ and } X^* \in \mathcal{P}^* \backslash X^*_i, \tag{10}$$

which follows from (5) and (9). When agent $i$ has added-value to a network in its periphery, then those in $X^*_i$ must impart to $i$ a share of $v(X^*_i)$ sufficient to prevent it from leaving for a better deal in the alternate network – at least an amount equal to $i$'s added-value to it.

As mentioned, others have emphasized the importance of the two aspects of competition highlighted by the core (i.e., its two-sided effect and its ambiguous nature).[19] Findings (8) and (10) are consistent with these observations. They do, however, impart an additional insight: the effect of a agent's ability to create value with a given group critically depends upon whether the group is a focal value network containing the agent (i.e., the active network in which the agent participates), or is an alternative to that network. The added value of an agent to the focal network containing it represents an *upper bound* on its ability to capture. For any group in which its membership is only potential, its added-value represents a *lower bound*. Thus, the agents with whom the firm *is* transacting are qualitatively different than those with whom it *could* transact.

This refinement is missed in coarser treatments such as Porter (1979) which, e.g., lumps all of a firm's buyers together as "competitors." What the preceding analysis shows is, first, that it matters whether the buyers are actual or potential. When the firm's actual buyers have the potential to add value to a peripheral value network, that network competes *for the buyers* (and against the firm). A firm's buyers do not compete with it – rather, external value networks do. Second, potential buyers (e.g., those who would like to transact with the firm but cannot, say, due to capacity constraints) create competition *for the firm* and, thereby, may assure it positive appropriation.

With a bit of further reflection, we come to our central idea. As we have seen, a firm's added-value to a network in its periphery implies competition for it – the greater the added-value, the greater the implicit intensity of that competition. Thus, scanning the firm's periphery and taking its maximum added-value is one way of measuring competitive intensity for the firm.

[19] For example, Gans et al. (2008); Gans and Ryall (2015)

**Definition 1.** *The* competitive intensity for $i \in N$ (CI) *in a focal value partition* $\mathcal{P}^*$ *is:*

$$w_i^* \equiv av_i(C_i^*), \text{ where } C_i^* \equiv \underset{X \in \mathcal{P}^* \setminus X_i^*}{\arg\max}\, av_i(X). \tag{11}$$

$C_i^* \in \mathcal{P}^*$ is any value network in $i$'s competitive periphery to which $i$ adds maximum value (there may be more than one). We point out that computing $w_i^*$ requires assessing $i$'s added-values to no more than $\frac{n}{2}$ groups.[20] In actual industries, with large chains of raw material suppliers, parts manufacturers, OEMs, and distributors, the number of focal value networks will be much smaller. Note as well that $i$ need know nothing about what other agents are appropriating to compute $w_i^*$. Since the logic of Definition 1 applies symmetrically for all the agents in the firm's focal value network, and since the effects across agents are cumulative (whatever value is guaranteed to the firm's transaction partners is value prohibited to it), the aggregate of all the other intensities is a measure of the intensity of competition against the firm. This leads to the following.

**Definition 2.** *Given* $\mathcal{P}^*$, *the* competitive intensity for $i$'s partners *is:*

$$\sum_{j \in X_i^* \setminus i} w_j^*.$$

*Then, define* $i$'s competitive residual (CR) *as:*

$$w_{-i}^* \equiv \min \left\{ v(X_i^*) - \sum_{j \in X_i^* \setminus i} w_j^*, av_i(X_i^*) \right\}. \tag{12}$$

In other words, $i$'s competitive residual is what is left over from $v(X_i^*)$ after paying $i$'s partners exactly their CI, but no more than $i$'s added-value to $X_i^*$. One may think of a firm's competitive residual as the most "optimistic" payoff it can expect should it succeed in limiting its transaction partners to appropriating no more than warranted by their own competitive intensities.[21] This leads to the following proposition linking CI to the added-value and core approaches common in the strategy literature.

[20] $\mathcal{P}^*$ is a disjoint collection of subsets of $N$ which require at least two members to produce value. Agents who do not transact should be collected into a zero-value, "no-transaction" group. This latter possibility, which requires rounding $\frac{n}{2}$ up, is offset by the fact that $X_i^*$ is not included in the computation of $w_i^*$.

[21] Notice that these definitions embody the spirit of immunity to unilateral deviations associated with the noncooperative game concept of Nash equilibrium.

**Proposition 1.** *Given a focal value partition $\mathcal{P}^*$, for all $i \in N$,*

$$\pi_i^{\min} \geq w_i^* \geq v(i), \text{ and} \tag{13}$$

$$\pi_i^{\max} \leq w_{-i}^* \leq av_i(N). \tag{14}$$

Definitions 1 and 2 represent an operationalization of "competitive intensity" that has the desirable features of: being conceptually consistent with core-based notions of competition emphasized in related strategy research, while using a much smaller set of constraints than required to compute the core, in a way that sheds new light on the underlying economics.[22] From Proposition 1, we see that the competitive intensity for a firm provides a conservative lower bound, and an optimistic upper bound, on competition's effect on its ability to capture value. If the maximal partition is the trivial one, i.e. is $\{N\}$, then this approach coincides with conventional added-value analysis.

How complex are the competitive intensity bounds versus calculations of added-value or the core interval? Suppose there are $m \leq \frac{n}{2}$ value networks in the market. To compute a firm's added-value requires 2 quantities ($V$ and $v(N_{-i})$). Computing its core interval requires $(2^n - 1)$ values ($V$ and one for every nonempty subset of $N$). Calculation of the firm's CI requires $2m$ values – two for each of the $(m-1)$ value networks in $i$'s periphery plus two for its added-value to its own network. Computing the firm's CR requires CI to be calculated for the other $(n-1)$ agents – which adds $m(n-1)$ values. Thus, altogether, $m(n+1)$ values are required to compute the bounds implied by competitive intensity. For example, if $n = 10$, a firm's added-value requires 2 pieces of data, its core interval $1,023$, and its CI no more than $55$.[23]

Some readers may find it surprising that neither the lower nor the upper bound on an agent's appropriation need directly depend upon its added-value to the market as a whole. However, it has long been known that the correspondence between added-value and appropriation does not always behave according to intuition. Indeed, in some reasonable cases increasing a firm's added-value actually *decreases both endpoints* of its core interval – a result so counterintuitive that some have taken it as a prima facie argument against the use of the core as a solution concept. One of the contributions of CI is that it provides insight into these objectionable "paradoxes," showing that they are not paradoxes at all but, instead, the outcome of perfectly reasonable economics. Indeed,

[22]Proposition 1 holds for all value partitions. Therefore, our notion of competitive intensity can be strengthened by considering not only $\mathcal{P}^*$, but all maximal value partitions; i.e., all maximal $\mathcal{P} \in \mathbf{P}$.

[23]Remember, $m$ is at most $\frac{n}{2}$ and, we conjecture, much smaller in most markets.

grasping the economics of these special cases is a major step toward understanding competition more generally. A concrete example will be instructive.

**Example 1.** *Consider a market for a homogeneous good sold by two firms, labeled a and b. Each of them has capacity of two units, which can be produced at zero marginal cost. There are three identical buyers, labeled 1, 2 and 3: each has unit demand and values the firms' products identically at* $u > 0$*. In this situation,* $V = 3u$*. The added value of each player, buyer or seller, is* $u$*. It can be shown that there is a single distribution of value in the core:* $\pi_a = \pi_b = 0$ *and* $\pi_i = u$ *for each buyer* $i \in \{1, 2, 3\}$*. Intuitively, the slack capacity in the market causes the firms to compete for each other's buyers – to the point of driving prices to zero. This is precisely the example of Postlewaite and Rosenthal (1974).*

*Let us see how close the bounds implied by CI come to capture this outcome. Among the six non-trivial value partitions in* $\mathbf{P}$*, suppose the focal one is* $\mathcal{P}^* = \{\{a, 1, 2\}, \{b, 3\}\}$*, firm a sells to buyers 1 and 2, firm b sells to buyer 3 (all possible value partitions are the same up to agent relabelling). To see that this is a value partition simply note that* $v(\{a, 1, 2\}) = 2u$ *and* $v(\{b, 3\}) = u$*, so* $V = v(\{a, 1, 2\}) + v(\{b, 3\})$*. Starting with buyers 1 and 2,* $w_1^* = w_2^* = u$ *since both have added-value of u with respect to b's value network. Given the market configuration* $\mathcal{P}^*$*, firm b has excess capacity with which to attract one of a's buyers were that buyer to capture an amount of value less than u. For example, if firm a demanded a price of p from buyer 1 such that* $u > p > 0$*, then firm b could offer 1 a price of* $p/2$*, thereby making both it and the buyer strictly better off. Under this configuration,* $w_3^* = 0$ *because firm a is selling at full capacity. Finally,* $w_a^* = w_b^* = 0$ *because firms add no value outside their own networks: the products are homogeneous and all the demand in each network is satisfied.*

*Proposition 1 says* $u \leq \pi_1^{min}, \pi_2^{min}$*. Given* $v(\{a, 1, 2\}) = 2u$ *and that the sum of the amount of value captured within a network must exactly equal the value it produces, we deduce* $\pi_1 = \pi_2 = u$ *and* $\pi_a = 0$*. Moving on to firm b's value network, all we get from Proposition 1 is* $0 \leq \pi_b, \pi_3 \leq u$ *(i.e., firm b and buyer 3 split the value they produce in some way). This is where the analysis stops based only upon competitive intensity. Still, with a small number of calculations, the value captured by firm a and its buyers is pinned down exactly. Moreover, this improves on the standard added-value approach, which indicates only that the payoff of each agent is somewhere between* 0 *and* 1*.*

Summing up this section, we use the structure implied by a market value partition to show that having added-value to a network outside one's own implies a floor to the quantity of value one must capture. This identifies network-level competition for an agent. We define the "intensity" of this sort of competition as the maximum of such added-values. Summing the intensities of competition for the others in one's own network provides a measure of the network-level competition arrayed against oneself. Proposition 1 shows that these measures provide an upper and a lower bound on an agent's appropriation. These bounds require far less information to compute than the core. Indeed, if one is willing to use the firm's added-value to its own network as the upper bound, then the number of constraints required collapses to the number of value networks in the market plus one. Just how much traction competitive intensity actually provides will depend upon the particulars of the situation at hand. Still, given the extreme simplicity of this approach, clear economic interpretation, and relatively low information demands, we conjecture that the preceding material is that which empiricists and practitioners will find most useful in this paper.

### 4.2 Indirect competitive intensity

Once the effect of competitive intensity is grasped, a further insight arises. Suppose a firm and a rival operate in separate value networks. The firm adds no value to the rival's network and, hence, enjoys no benefit of competitive intensity. Even so, what if the competitive intensity for the rival is so strong that the rival captures the lion's share of the value produced in its network? If the firm can create value with some of the rival's transaction partners above-and-beyond the meagre amount they must receive *once the CI for the rival is taken into account*, then competition for the firm does, in fact, exist. If the firm is offered less value in its own network than the difference between the value it can create with its rival's transaction and the amount they look to capture in their own network, then the firm can cut a deal with them that makes them all strictly better off. Competition for the rival indirectly creates competition for the firm.

Formally, suppose $i$ considers joining $X \in \mathcal{P}^* \setminus X_i^*$ as a replacement to some subset $Y \subset X$. Let $av_Y(X)$ denote the value added by $Y$ to $X$. By Proposition 1 agent $j \in Y$ cannot receive less than $w_j^*$. Moreover the the agents in $Y$ cannot, in aggregate, receive less than what they can produce on their own, $v(Y)$. Therefore, in aggregate, the agents in $X \setminus Y$ cannot capture more than

$$v(X) - \max\left\{\sum_{j \in Y} w_j^*, v(Y)\right\} \tag{15}$$

Expression (15) presents an upper bound on the opportunity cost that the agents in $X\backslash Y$ bear by foregoing transactions with those in $Y$. Thus, the agents in $X\backslash Y$ should be willing to replace $Y$ with $i$, provided $i$ asks for no more than

$$v(i \cup X\backslash Y) - \left[ v(X) - \max\left\{ \sum_{j\in Y} w_j^*, v(Y) \right\} \right] =$$

$$av_i(i \cup X\backslash Y) - \left( av_Y(X) - \max\left\{ \sum_{j\in Y} w_j^*, v(Y) \right\} \right). \tag{16}$$

Equation (16) is the value added by $i$ to the agents in $X\backslash Y$ above-and-beyond the upper bound on their opportunity cost of abandoning the agents in $Y$. When this value is positive and larger than $w_i^*$, then the group $X\backslash Y$ generates further competition for $i$ beyond $w_i^*$. Note that CI corresponds to the particular case in which $Y = \emptyset$ ($i$ joins $X$ but replaces none of its agents). This leads to a definition that includes both indirect competitive intensity and indirect competitive residual.

**Definition 3.** *Given* $\mathcal{P}^*$*, the* indirect competitive intensity for $i \in N$ *is*:

$$s_i^* \equiv \max_{Y\subset X, X\in\mathcal{P}^*\backslash X_i^*} \left[ av_i(i \cup X\backslash Y) - \left( av_Y(X) - \max\left\{ \sum_{j\in Y} w_j^*, v(Y) \right\} \right) \right]. \tag{17}$$

*The* indirect competitive intensity for $i$'s partners *is*:

$$\sum_{j\in X_i^*\backslash i} s_j^*,$$

*and the* indirect competitive residual of $i$ *is:*

$$s_{-i}^* \equiv \min\left\{ v(X_i^*) - \sum_{j\in X_i^*\backslash i} s_j^*, av_i(X_i^*) \right\}. \tag{18}$$

Once again, an agent's external alternatives create a lower bound on the value it must capture. Symmetrically, the intensity of direct competition for its partners places an upper bound on its ability to capture value. As always, an agent can never capture more than its added-value to its focal value network. This leads to bounds that are (weakly) tighter than those in Proposition 1.

**Proposition 2.** *Given a focal value partition $\mathcal{P}^*$, for all $i \in N$,*

$$\pi_i^{\min} \geq s_i^* \geq w_i^*, \text{ and} \tag{19}$$

$$\pi_i^{\max} \leq s_{-i}^* \leq w_{-i}^*. \tag{20}$$

By increasing the scope of competition for an agent, we tighten our estimate of its appropration – at the cost of increased information requirements. At this point, we stop tracking the number of groups required to compute the bounds of interest. They are fewer than those required to estimate the core interval but more (possibly by a significant number) than those required to compute competitive intensity. Thus, indirect intensity may or may not be of practical use in real applications. Whether or not the data requirements are prohibitive will vary from situation to situation. Even so, we believe the theoretical insight into how competition for one agent indirectly generates competition for another is quite valuable. To illustrate, we return to Example 1 above.

**Example 2.** *Under the focal value partition $\mathcal{P}^* = \{\{a, 1, 2\}, \{b, 3\}\}$, the notion of competitive intensity identified the exact competitive intervals for some but not all players. In particular, $w_3^* = 0$ left ambiguous the split of value between firm b and buyer 3. Given $\mathcal{P}^*$, buyer 3 adds no value to the network $\{a, 1, 2\}$ because, under the deals implied by that network, firm a has no capacity remaining to make a sale to buyer 3.*

*However, firm a does compete for buyer 3 in a way that is not captured by $w_3^*$. The high competitive intensities for buyers 1 and 2 imply zero value capture by firm a. This leaves firm a highly motivated (in the sense of being willing to take even the smallest quantity of value) to replace either of its buyers with buyer 3. Indeed, firm a becomes indifferent to such a deal only when $\pi_3 = u$. Thus, competition from firm a drives up buyer 3's minimum level of appropriation in its transaction with firm b.*

*The competition for buyer 3 that is so generated by the high competitive intensities for buyers 1 and 2 is captured by our notion of indirect competitive intensity: we have that $s_3^* = u-(u-w_i^*) = w_i^*$ with $i \in \{1, 2\}$. Recall that $w_i^* = u$, and thus by Proposition 2 we have $\pi_3^{\min} \geq u$. However as $av_3(\{b, 3\}) = u$ we have $\pi_i^{\max} \leq u$. Thus, $\pi_3 = u$ and, since the value captured within a value network must be shared among its members ($\pi_b + \pi_3 = u$), we also have $\pi_b = 0$*

Examples such as this have garnered considerable interest in economics when contrasted with the case in which all the buyers are merged into a single, monopsonist. In our example, it can be shown that a merged buyer faces a core interval of $[u, 3u]$ while, in the disaggregated case, the core indicates that each buyer is guaranteed to capture exactly $u$ (or, $3u$ in aggregate). According to the core, then, the merger of buyers cannot be advantageous. As Aumann (1973, p. 1) says, "It seems intuitively obvious that in a monopolistic market, the monopolist has an advantage because he can avoid competition." Indeed, so obvious does Aumann consider this intuition that he regards counterexamples like this to be strong evidence against the intelligibility of the core as a solution concept. In response, Postlewaite and Rosenthal (1974, by means of an example similar to ours) show that the core *accurately* captures key differences in the underlying economics – competition may, indeed, work to the disadvantage of some syndicates.

Our notions of competitive intensity provide a transparent conceptual framework by which to understand the economics of these and more (i.e., viz the opacity of the core) as well as a nice means by which to qualify them. Without resorting to the core, competitive intensity alone indicates the disaggregated buyers must appropriate between $2u$ and $3u$ due to competition from firm $b$ for buyers 1 and 2. Adding the indirect competitive intensity that arises as a result (firm $a$ indirectly competes for buyer 3), pins down everyone's payoffs exactly. The economics are clear. When the buyers merge, both direct and indirect intensities vanish – there is only one value chain (both firms selling to the merged buyer), hence zero competitive intensity of any kind. Again, this is perfectly reasonable.

Before leaving this example, note that it also illuminates what competitive intensity leaves out (versus the core). According to the core, the merged buyer faces appropriation in the range $[u, 3u]$. The minimum is due to the surplus unit of capacity between the firms which creates a form of within-network competition (the firms jostle to fill their respective capacities) that assures the merged buyers capture of at least $u$. Since our competitive intensities are induced by external competition (which requires nontrivial value partitions), they do not pick this up. In the next section, we will allow for this.

Finally, we note that iterating this logic may lead to sharper estimates of the effect of competition on appropriation. That is, one can pursue the following process for agent $i$: at Step 0, compute $s_i^*(0)$ according to (17); at Step 1, set $w_i^*(1) = s_i^*(0)$, compute $s_i^*(1)$ according to (17) using $w_i^*(1)$; continue with this procedure. Since this sequence is monotonically increasing and bounded above by $\pi_i^{\min}$ of the core, it has a limit that can be used to define an iterated measure of competitive intensity for $i$ and for $i$'s partners. The preceding algorithm provides a tighter bound of an agent's competitive interval, yet convergence to the core interval is not guaranteed. Surprisingly, in some fairly general settings (such as the differentiated product markets model we explore in Section 5), one round of iteration is all that is required to completely pin down the agents' core intervals.

### 4.3 Generalized competitive intensity

Above, we considered competition for an agent generated by external networks (competitive intensity) and by subsets of such networks affected by competitive intensities (indirect competitive intensity). We believe this way of conceptualizing the effects of competition is natural, straightforward and enlightening. Yet, as we saw in the immediately preceding example, these definitions fail to account for some forms of competition (e.g., within-network competition). Therefore, we now drop the requirement for a value partition and focus, instead, on the entire set of subsets of $N$. This extension approaches the core in its information demands and, as a result, is unlikely to be especially useful in real-world applications. That said, we present it now for theoretical completeness. Readers uninterested in these technical theoretical details may wish to skip this section.

Our definition of generalized competitive intensity builds on the premise that no group $G \subseteq N$ can receive more than the value it adds to the market as a whole, $av_G(N)$ and, moreover, no more than the aggregate of its agents' added-values, $\sum_{j \in G} av_j(N)$. We define the general competitive intensity (GCI) for $i$ by the threat to join a subgroup by paying the agents in that subgroup their full added-value to the market (i.e., the most each could hope to appropriate). This provides an upper bound on the opportunity cost that the agents in the group bear should they forego transactions with $i$. We wish to identify the set of groups for which this value is maximal. Formally,

$$C_i \in \underset{G \subseteq N \setminus i}{\arg\max} \left[ v(G \cup i) - \min \left\{ av_G(N), \sum_{j \in G} av_j(N) \right\} \right].$$

How does the preceding expression generalize our earlier ideas? By the definition of a value partition $\mathcal{P}$, $V = \sum_{X \in \mathcal{P}} v(X)$. Therefore,

$$v(X) = av_X(N) \text{ for any } X \in \mathcal{P}. \tag{21}$$

Recalling expression (6), for all $X \in \mathcal{P}^*$,

$$av_X(N) \leq \sum_{i \in X} av_i(N).$$

The identification of $C_i^*$ in Definition 1 is similar to that of $C_i$, with the difference being that the maximization in the definition of the former is restricted to $X \in \mathcal{P}^*$. As before, no group can capture a share of value greater than its added-value to the market. This brings us to the following.

**Definition 4.** *The* generalized competitive intensity for $i \in N$ *is*:

$$\underline{w}_i \equiv v(C_i \cup i) - \min \left\{ av_{C_i}(N), \sum_{j \in C_i} av_j(N) \right\}. \quad (22)$$

*From (21), the natural extension of* $w^*_{-i}$ *is the* generalized residual of $i \in N$:

$$\overline{w}_i \equiv \min_{G \subseteq N} \left[ av_{G \cup i}(N) - \sum_{j \in G \setminus i} \underline{w}_j \right].$$

Denote the set of payoffs that are consistent with generalized competitive intensity by

$$GCI \equiv \left\{ \pi \in R^n | \sum_{i \in N} \pi_i = V, \underline{w} \leq \pi \leq \overline{w} \right\}, \quad (23)$$

where $\underline{w}$ and $\overline{w}$ are the vectors composed by the $\underline{w}_i$s and $\overline{w}_i$s, respectively. Stated this way, $CGI$ can be seen to represent an alternative solution concept for coalitional games. Hence, we now establish its relationship to the core.

**Proposition 3.** *The core is a subset of those payoffs that are compatible with generalized competitive intensity, i.e.,* $C \subseteq GCI$*, where* $C$ *is the set of value distributions satisfying (1) and (3). Moreover, for any (focal) value partition* $\mathcal{P}^*$*, for all* $i \in N$, $s^*_{-i} \geq \overline{w}_i \geq \pi_i^{\max} \geq \pi_i^{\min} \geq \underline{w}_i \geq s_i^*$.

Importantly, if the core for a market is nonempty, then the set of payoffs that are compatible with generalized competitive intensity for that market is also non-empty.

In our setting, the extreme points of $GCI$ can be described by vectors that, themselves, can be used to determine whether the bounds implied by generalized competitive intensity fully characterize an agent's core interval. Quant et al. (2005) describe how to do this with a set of vectors they call the "larginals." Here, we take a related approach to characterizing the set of payoffs that are consistent with the notion of generalized competitive intensity.

Let $\phi$ be an ordered profile of the agents in $N$, and $\phi(k) \in N$ be the agent in position $k$ according to $\phi$. Let $\Phi$ denote the set of all possible ordered profiles. A *greedy residual vector* associated with $\phi \in \Phi$ is a distribution of the total value $V$ constructed as follows: pay as many of the first agents indicated by $\phi$ their generalized residuals, provided that all agents receive at least their generalized competitive intensity. The set of agents in $\phi$ that receive exactly their generalized residuals is called the *front of* $\phi$; the set of agents that receive only their generalized competitive intensity is called the *end of* $\phi$. For any $\phi$, there is at most one player that does not belong to the front or the end.[24]

**Definition 5.** *The payoff vector $\pi^{\phi}$ is a* greedy residual vector associated with the ordering $\phi \in \Phi$ *if*

$$\pi^{\phi}_{\phi(k)} = \begin{cases} \overline{w}_{\phi(k)} \ \textit{if } f_{\phi(k)} + \overline{w}_{\phi(k)} \leq V \\ \underline{w}_{\phi(k)} \ \textit{if } f_{\phi(k)} + \underline{w}_{\phi(k)} \geq V \\ V - f_{\phi(k)} \ \textit{otherwise,} \end{cases}$$

*where*

$$f_{\phi(k)} = \sum_{j=1}^{k-1} \overline{w}_{\phi(j)} + \sum_{j=k+1}^{n} \underline{w}_{\phi(j)}.$$

Notice from (23) that the convex hull of the greedy residual vectors fully characterizes the set of payoffs that are compatible with competitive intensity. That is,

$$GCI = conv \left\{ \pi^{\phi} \middle| \, \phi \in \Phi \right\}. \tag{24}$$

Typically, there are fewer greedy residual vectors than orderings because any two vectors with the same front and end are equivalent – independent of the agent ordering. As we have emphasized, identifying the agents' core intervals in most real-world markets is computationally prohibitive. However, it is a much simpler task to check if a *particular distribution* is, itself, in the core. If, in addition to checking whether the $\pi^{\phi}$s are in the core, we have $\overline{w}_i$ and $\underline{w}_i$ for each $i \in N$, the set of greedy residual vectors can be used to check whether competitive intensities identify the core

[24] Our notion of greedy residual vector is inspired by the well-known concept of a "greedy vector." The *greedy vector associated with an ordering* $\phi \in \Phi$ is constructed by paying each agent her added-value to the group consisting of the players that precede her in $\phi$. The convex hull of the greedy vectors in $\Phi$ is called the *Weber set*. Greedy vectors have been used to characterize several solution concepts for coalitional games. For example, the Weber set coincides with the core when a coalitional game is convex. The Shapley value can be obtained by averaging over the set of greedy vectors.

intervals.

**Proposition 4.** *If some $\pi^{\phi}$ is in the core, i.e. if $\pi^{\phi} \in C$, then $\overline{w}_i = \pi_i^{\max}$ if $i$ belongs to the front of $\phi$ and $\underline{w}_i = \pi_i^{\min}$ if $i$ belongs to the end of $\phi$. If, for all $\phi \in \Phi$, $\pi^{\phi} \in C$ then $CI = C$: generalized competitive intensity captures the agents' competitive intervals exactly.*

We may now state necessary and sufficient conditions for the core interval of every agent to be identified by generalized competitive intensity. Our result follows by observing that a similar characterization provided by Quant et al. (2005) applies to any upper and a lower bounds of the core payoffs (which we have shown to be the case for the $\underline{w}_i$s and $\overline{w}_i$s).

**Proposition 5.** *The payoffs compatible with generalized competitive intensity coincide with those that are consistent with competition (i.e., $GCI = C$) if and only if for each $G \subseteq N$,*

$$v(G) \leq \max\left\{\sum_{i\in G} \underline{w}_i, V - \sum_{i\in N\setminus G} \overline{w}_i\right\}. \tag{25}$$

*Then, $\overline{w}_i = \pi_i^{\max}$ and $\underline{w}_i = \pi_i^{\min}$ for every $i \in N$.*

From Proposition 5 we make an important observation. If competitive intensity is sufficiently strong, then both elements on the right hand side of (25) are high. This means that the inequalities are satisfied for each $G \subseteq N$. Therefore generalized competitive intensity, when it is sufficiently strong, fully characterizes the core.[25]

# 5 Competition in a duopolistic market

We develop a version of the standard Hotelling model of duopolistic competition, augmented to permit vertical differentiation and capacity constraints. We explore the effect of competitive intensity in this setting, the relationship between added-value and firm performance, how competitive intensity explains the effects of capacity constraints on firm performance; and how vertical mergers alter the balance of competition in favor of or against the firms involved.

[25]Our concept of generalized competitive intensity is also related to the notion *core covers* ($CC$), Tijs and Lipperts (1982). While we omit the proposition due to space limitations, it can be shown that $GCI \subseteq CC$.

## 5.1 Setup

On the supply side, there are two single-product firms, labeled $a$ and $b$, that have the same constant marginal cost of production, normalized to zero. Product types for each firm, denoted $y_a$ and $y_b$, are represented as points in the interval $[0,1]$. Assume the firms have fixed locations: $y_a = 0$ and $y_b = 1$. Firm $r$'s capacity is in full units (integers), denoted $q_r$. We adopt the following conventions to simplify notation: $\Pi_r \equiv [\pi_r^{\min}, \pi_r^{\max}]$ denotes firm $r$'s competitive interval; $W_r^* \equiv [w_r^*, w_{-r}^*]$ is firm $r$'s interval of value capture implied by competitive intensity; and $S_r^* \equiv [s_r^*, s_{-r}^*]$ is the interval implied by indirect competitive intensity.

On the demand side, there are three buyers, labeled 1, 2 and 3. Each buyer demands one unit. Buyer $i$'s ideal product is denoted $x_i \in [0,1]$. Assume $x_1 = 0$, $x_2 = \frac{1}{2}$, and $x_3 = 1$.[26] The utility buyer $i$ obtains from consuming the product of firm $r$ is $u_i(y_r, l_r) \equiv l_r - t|y_r - x_i|$, where $|y_r - x_i|$ is the distance of buyer $i$'s most preferred product from the one firm $r$ offers. The scalar $l_r > 0$ is a vertical differentiation parameter – larger values of $l_r$ are preferred by all buyers. The degree of horizontal differentiation is captured by the parameter $t > 0$. Larger values of $t$ imply greater disutility from a product that is not ideally located.[27]

We assume $l_a > l_b \geq t$, so that firm $a$'s product is vertically superior to that of firm $b$; additionally, both firms' products generate nonnegative utility for all buyers. It will also be helpful to assume that $t \geq l_a - l_b$. This implies that buyer 3 prefers product $y_b$ over $y_a$ when confronted with the choice between them. Thus, firm $a$'s product may be vertically superior, but not "too much" so. Because $l_a - l_b > 0$, it is worth noting that $q_a + q_b \geq 3$ implies there is always a unique value partition that is not trivial. We note that the case with no vertical differentiation obtains as $l_a \downarrow l_b$.

## 5.2 Preliminary Results

With three buyers, it suffices to consider capacity levels of 1, 2 or 3 for each firm. Limiting attention to the cases in which both firms operate and all buyers purchase, results in six capacity scenarios: $(q_a, q_b) \in \{(1,2),(2,1),(2,2),(3,2),(2,3),(3,3)\}$.[28] The aggregate value produced in this market depends on the firm capacities and is denoted $V_{1,2}$. In capacity scenario $(1,2)$, firm $a$ sells to buyer 1 and firm $b$ to the other two. Adding up the utilities of the buyers yields $V = l_a + l_b - t|1 - .5| + l_b$, or

[26] This setup exhibits symmetric horizontal differentiation. Each firm faces: a buyer who is unit distant from its product, another who is half a unit away, and a third for whom its product is ideal.

[27] It is worth noting that our results below do not depend on the assumption that disutility is linear in the distance between a buyer and a firm. All that is needed is increasing disutility in the distance between them.

[28] Keep in mind that the firms are asymmetric, so the implications of, e.g., $(1,2)$ and $(2,1)$ are, likewise, asymmetric.

$V = l_a + 2l_b - .5t$. The value network that delivers this is unique – firm $a$ and buyer 1 form one value network ($v(X_a^*) = l_a$); firm $b$ and the other buyers form the other ($v(X_b^*) = 2l_b - .5t$). Once firm $a$ has two or more units of capacity (i.e., all the other cases), it always sells to buyers 1 and 2, with firm $b$ picking up buyer 3. In these cases, $V_{1,2} = l_a + l_a - t\,|0 - .5| + l_b$, or $V_{q_a,q_b} = 2l_a - t\,|0 - .5| + l_b$ ($q_a \neq 1$; $q_b \neq 2$). The value network that delivers this outcome is also unique – firm $a$ is in a value network with buyers 1 and 2 ($v(X_a^*) = 2l_a - .5t$); firm $b$ and buyer 3 is in the other ($v(X_b^*) = l_b$).

Given the relatively small number of agents in this market, it is possible to compute the exact competitive bounds for each firm. These are shown in Table 1. How well do our measures of competitive intensity capture the actual extent of competition implied by the core requirements? The following propositions provide the answers, along with some economic insights about how competition is operating in each case.

| **Capacities** | | $\mathbf{\Pi_a}$ | | $\mathbf{\Pi_b}$ | |
|---|---|---|---|---|---|
| $\mathbf{q_a}$ | $\mathbf{q_b}$ | **min** | **max** | **min** | **max** |
| 1 | 2 | $l_a - l_b$ | $l_a$ | 0 | $2l_b - .5t$ |
| 2 | 1 | 0 | $2l_a - .5t$ | 0 | $l_b$ |
| 2 | 2 | 0 | $t + 2(l_a - l_b)$ | 0 | $t$ |
| 2 | 3 | 0 | $t + 2(l_a - l_b)$ | 0 | $t$ |
| 3 | 2 | 0 | $t + 2(l_a - l_b)$ | 0 | $t - (l_a - l_b)$ |
| 3 | 3 | 0 | $t + 2(l_a - l_b)$ | 0 | $t - (l_a - l_b)$ |

Table 1: Competitive intervals for $a$ and $b$ in each case.

Competitive intensities are easy to compute. Doing so immediately reveals that they are sufficient to characterize the firms' competitive intervals in capacity cases $(1, 2)$, $(2, 1)$, $(3, 2)$ and $(3, 3)$. This is stated formally in the next lemma.

**Lemma 1.** *If* $(q_a, q_b) \in \{(1, 2), (2, 1), (3, 2), (3, 3)\}$*, then* $\Pi_r = W_r^*$ *for* $r \in \{a, b\}$*.*

For the other two cases, $(2, 2)$ and $(2, 3)$, competitive intensity pins down the exact values of firm $a$'s competitive interval, but it fails to do so for firm $b$. The following comments apply to both cases. As we know, the value partition is unique: firm $a$ sells to buyers 1 and 2, firm $b$ sells to buyer 3. Because firm $a$ is vertically superior, it sells its entire capacity to the two nearest buyers (1 and 2). However, because its capacity is exactly filled by those buyers, the competitive intensities for firm $b$ and buyer 1 are zero.[29] Putting firm $b$ in firm $a$'s value network adds no value. The

[29]That competitive intensity fails to provide an exact bound when firm $b$ has idle capacity but firm $a$ does not is no coincidence.

transactions between firm $a$ and buyers 1 and 2 create maximum value with those buyers. If firm $b$ were to join that value network, it would sit idle.[30] Thus, there is no direct competition for $b$. The same reasoning applies to buyer 3. We know these two will transact to create and divvy up $l_b$ in economic value. At this point, however, there is nothing we can say about competitive constraints on what that split will be.

This is where indirect competitive intensity becomes useful. We know buyer 2's minimum capture in her transaction with firm $a$ is at least $l_b - .5t$. This is the competitive intensity for buyer 2 created by the fact that firm $b$ has a unit of slack capacity with which to attempt to lure her away from firm $a$. Thus, from Proposition 1, we know $\pi_2 \geq l_b - .5t$. Now, suppose firm $b$ demands all the value from buyer 3, so $\pi_3 = 0$. Then, buyer 3 looks appealing to firm $a$ as a replacement for buyer 2. Under these circumstances, firm $a$ can offer buyer 3 $\pi_3 = \frac{1}{2}(l_b - t)$ to make them both strictly better off. This indirect competition for buyer 3 from firm $a$ implies buyer 3 captures $\pi_3^{\min} \geq l_b - t$ in her deal with firm $b$, since firm $a$ would lose $l_a - \dfrac{t}{2}$ and gain at least $l_b - \dfrac{t}{2} + l_a - t$ by replacing 2 with 3. Of course, this immediately implies $\pi_b^{\max} \leq t$, the indirect residual of firm $b$. It turns out that considering the indirect intensities for firm $b$ and buyer 3 are sufficient to identify these agents' competitive intervals.

**Lemma 2.** *If* $(q_a, q_b) \in \{(2,2),(2,3)\}$*, then* $\Pi_r = S_r^*$ *for* $r \in \{a, b\}$.

In this example, the two simple measures of competitive intensity were all that was needed to figure out the firms' competitive intervals. There was no need to solve the much more complicated linear optimization problem with $2^5 - 1 = 31$ constraints for each of the six capacity scenarios. Indeed, once the interval for $a$ and its buyers were characterized by competitive intensity, the indirect intensity measure had only to be computed for two agents – firm $b$ and buyer 3.

### 5.3 Managerial implications

**Competitive Advantage** Notice from Table 1 that the vertical superiority of firm $a$ over $b$ did not consistently result in greater profitability. In all cases, firm $a$'s competitive interval is nontrivial: here, there is always room for value capture above-and-beyond the minimum guaranteed by competition. This is true in spite of firm $a$'s superior product and, in the five cases in which it has two or more units of capacity, a market share that is twice the size of firm $b$'s. Indeed, in

[30] Keep in mind that firm $b$'s presence in the market is exactly what creates competition for $a$'s buyers. This is what helps nail down $a$'s range of value appropriation.

those cases, competition guarantees it no capture whatsoever. Whatever value firm $a$ does capture in those cases is entirely due to its persuasive resources – those capabilities that enhance its ability to persuade buyers 1 and 2 to part with value versus those that are used to produce value.

To see the problem this poses to empirical research on firm performance, consider the cases in which firm $a$ has at least two units of capacity and suppose that its persuasive capabilities are either nonexistent or ineffective with buyers 1 and 2. Then, if firm $b$ has even a slight persuasive advantage vis-à-vis buyer 3, the empiricist observes $\pi_a < \pi_b$, even though $l_a > l_b$.[31] Empirically, firm $a$ exhibits higher quality, greater market share, lower prices, and lower profit. An industrial organization economist might conclude that there are unobserved product quality or marginal cost parameters that, when taken into account, make $a$'s offering inferior or more costly. A behavioral economist might conclude that $a$'s managers are "leaving money on the table" due to some version of limited foresight. In this case, neither conjecture is correct – and, missing an important part of the story, would ultimately fail to achieve robust explanatory power or reliable normative implications across markets.

Strategy has had a long-running debate on the proper meaning of "competitive advantage" and the implications thereof. While a full treatment of this issue is beyond the scope of the present paper, our results do add an important refinement to our understanding of it. The fact that $l_a > l_b$ does have an important competitive implication (many scholars would *define* $a$ to have a competitive advantage on this basis, at least when $q_a = q_b$ and hence firms only differ on their vertical dimensions): when $a$ has two or more units of capacity, its competitive residual is greater than $b$'s, resulting in a greater maximum level of value capture consistent with competition. The competitive residual is a function of competition for the firm's customers from other firms. This fits the traditional conceptualization of competition. Yet, when the industry has slack capacity, this is far from the whole story. Industry capacity is such that buyers are free to migrate to their most preferred firms. This eliminates competition for firms from outside their value networks – which effectively kills that side of competition responsible for raising the minimum value they can capture. This side of competition is not so well appreciated. In this state of affairs, any "competitive" advantage – in the sense of being the source of sustained performance superiority – is actually a persuasive advantage. We conjecture that this distinction is crucial to any complete conception of competitive advantage (see the recent empirical work by Grennan, 2014, which

[31] When $(q_a, q_b) = (2, 2)$, there does exist a distribution of value consistent with competition in which $\pi_a$ is zero and $\pi_b$ is small yet positive, as shown in the Appendix.

suggests that this is, indeed, an important point).

**Strategic Commitments** The preceding analysis leads in a number of directions. One is toward the important topic of strategic commitments arising from sunk investments.[32] In the canonical analysis, an incumbent deters entry by making sunk capacity investments: for an incumbent with sufficient capacity, entry triggers it to switch from monopoly to marginal-cost pricing. The idea is that the incumbent creates an environment in which, post-entry, competition is so intense as to ensure such entry does not occur at all.

The preceding discussion suggests the story is more nuanced. The first question is how much capacity the monopolist adds when there are no potential entrants in sight. The competitive intervals for firm $a$ acting as a monopolist under 1, 2 or 3 units of capacity are detailed in Table 2. What is the optimal level of capacity? Clearly, it depends upon $a$'s ability to persuade its buyers to part with value. To keep it simple, assume $a$ gets to fix a capacity level once and for all at zero cost. If $a$ is able to capture value at the top end of its competitive range, it builds enough capacity to serve the entire market. At the other end of the spectrum, if the buyers' persuasive capabilities force $a$ to the bottom of its competitive range, the firm builds one or two units of capacity (two if $l_a \geq 1.5t$, one otherwise). The point is, competition alone is not sufficient to define "rational" monopolistic capacity choice.[33]

| | $\mathbf{\Pi_a}$ | |
|---|---|---|
| $\mathbf{q_a}$ | **min** | **max** |
| 1 | $l_a - .5t$ | $l_a$ |
| 2 | $2(l_a - t)$ | $2l_a - .5t$ |
| 3 | 0 | $3l_a - 1.5t$ |

Table 2: Competitive intervals for Monopolist $a$.

Things become more complicated as we add the possibility of entry. Suppose firm $b$ arrives on the scene as an entrant. Regardless of $a$'s capacity choice, there is no competition *for* $b$ – without persuasive capabilities, $b$ captures nothing. To put it differently, if entrants lack persuasive abilities, entry does not factor into $a$'s capacity decision. On the other hand, once $b$'s persuasive capabilities exert positive effect, it enters – regardless of $a$'s capacity choice. True, its maximum depends upon

[32]Here, we tread once again upon heavily-travelled ground – both in economics and strategy. Porter (1980) and Ghemawat (1991) are well-known examples in strategy.

[33]For a more elaborate example of capacity choice by a monopolist in the coalitional game context, see Brandenburger and Stuart (1996).

the exact configuration of capacity in the industry. However, to the extent it can convince $a$ to part with *some* value consistent with the limits imposed by competition, entry is profitable. Not to put too fine a point on it: sunk-cost/commitment reasoning fails in this scenario. Essentially, almost all the action is on the super-competitive dimension. This segues nicely to the next consideration.

**Added-value and Firm Performance** Managers are frequently exhorted to organize their activities so to maximize the net value their firms create.[34] Our results show that creating net value may, in some cases, *reduce* profitability. That is, if $a$'s product is sufficiently close to $b$'s both in terms of horizontal and vertical differentiation, then starting from the $(q_a, q_b) = (1, 2)$ scenario and costlessly adding one unit to $a$'s capacity results in the following: $a$'s overall added-value increases but, simultaneously, the endpoints of its competitive interval *both* decrease.

To see what is going on, we turn to our notions of competitive intensity. First, assume that $a$'s product is "sufficiently close" to $b$'s in the sense that $t \leq (2l_b - l_a)$. Then, start with the capacity scenario $(q_a, q_b) = (1, 2)$ and note the changes when $a$ adds one unit of capacity. First, $a$'s added-value increases from $l_a$ to $(2l_a - l_b)$. However, referring to Table 1, we see that $\pi_a^{\min}$ drops from $l_a - l_b$ to zero and $\pi_a^{\max}$ drops from $l_a$ to $t + 2(l_a - l_b))$.[35] The change in capacity has two effects: 1) it reduces competitive intensity for $a$; and 2) it increases competitive intensity for $a$'s buyers. Taken together, these changes result in a strict worsening of $a$'s competitive interval – even as $a$ creates strictly more value. Let us take each effect in turn.

First, consider the change in competitive intensity for $a$. The resultant drop in $\pi_a^{\min}$ is not too surprising. It is consistent with conventional models in which expanding capacity reduces the just-excluded buyer's willingness-to-pay, thereby changing the amount that included buyers can be charged. When $a$ has one unit of capacity, the just-excluded buyer is 2. Buyer 2 values $a$'s product $(l_a - l_b)$ more than $b$'s product. Thus, in capacity scenario $(1, 2)$, competitive intensity for $a$ is $w_a^* = (l_a - l_b)$.[36] Put simply, $a$ can credibly threaten to replace buyer 1 with buyer 2 under any deal in which $a$ gets less than $(l_a - l_b)$ from the transaction. When $a$'s capacity increases to 2, buyer 2 becomes a customer. The just-excluded buyer is buyer 3. Unfortunately for $a$, this changes competitive intensity for it to $w_a^* = 0$. Firm $a$ cannot create more value with buyer 3 than buyer 3 and firm $b$ create together. Firm $b$ does not clamor to replace either of $a$'s buyers. Again, this

[34] Where "net value created" is the firm's added-value less the cost required to create it.
[35] The change in $\pi_a^{\max}$ is a drop as a result of our product proximity assumption $t < (2l_b - l_a)$.
[36] Because adding $a$ to the value network $\{b, 2, 3\}$ creates this much additional value by switching buyer 2 to $a$.

effect conforms to the usual marginal-cost/marginal revenue trade-off that arises in, e.g., Cournot settings.

Next, consider the surprising part of the example – the simultaneous drop in $\pi_a^{\max}$. Intuitively, we associate an increase in added-value with an increase in an agent's competitive maximum. That is not the case here. The reason is that competitive intensity for $a$'s buyers increases, thereby reducing its competitive residual and, as a result, its maximum. Under capacity scenario $(1,2)$, adding buyer 1 to the value network $\{b,2,3\}$ creates no additional value: $b$'s capacity is already accounted for by buyers who value its product more. Therefore, $w_1^* = 0$ – there is no competition for buyer 1. When $a$ adds a unit of capacity, buyer 2 switches to $a$. This is socially efficient – more economic value is created within the market. However, the change *frees up a unit of firm b's capacity.* As a result, both buyers 1 and 2 add value to $b$'s new value network $\{b,3\}$. In particular, competitive intensity for buyer 1 increases to $w_1^* = (l_b - t)$. As a result, the most value firm $a$ can capture from buyer 1 is reduced.

Summing up, if one firm's added capacity attracts new customers and, thereby, frees the capacities of other firms, there is a twofold competitive effect. On the one hand, the highest willingness-to-pay buyer leave the expanding firm's competitive periphery and become actual buyers. This increases the firm's added-value as well as its market share but, most importantly, decreases competitive intensity for it. On the other hand, freeing up capacity of rival firms increases the competitive intensity for its customers, both old and new. Even a firm with maximal persuasive capabilities will experience a drop in profit. Competition can be subtle.

**Capacity Expansion Strategies** One of the nice features of this formalism is that strategic behavior on the part of firms can be analyzed explicitly by extending the model to include a front-end non-cooperative game that accounts for firms' investments in productive and persuasive resources (i.e., a biform game). Therefore, let us adopt this approach and add a first stage in which the firms choose capacity levels before competing in the market represented by the resulting coalitional game. Assume capacity choices are made simultaneously.

Instead of characterizing the equilibrium outcome of the biform game – which requires getting into the specifics of everyone's relative persuasive capabilities – it suffices for our purpose to limit our analysis to ruling out certain outcomes that cannot arise in equilibrium regardless of relative persuasive capabilities.[37] For this section, assume that capacity costs $\kappa$ per unit, where $\kappa > 0$ may

[37]The equilibrium concept is Nash in the non-cooperative capacity choice game where payoffs are defined by

be arbitrarily small. The idea is to consider cases in which a firm is willing to pay for an extra unit of capacity as long as competition does not rule out the possibility of capturing additional value.

Referring to the payoffs in Table 1, it is apparent that neither $(q_a, q_b) = (3, 2)$ nor $(q_a, q_b) = (3, 3)$ can arise as equilibria. The reason is that the interval for firm $a$'s payoff is the same regardless of whether $(q_a, q_b) = (2, 2), (3, 2), (2, 3)$, or $(3, 3)$. That is, firm $a$'s best reply to capacity choices of 2 or 3 on the part of firm $b$ never includes building 3 units of capacity. Similarly, neither can $(2, 3)$ arise in equilibrium, because the interval for firm $b$'s payoff is the same regardless of whether $(q_a, q_b) = (2, 2)$, or $(2, 3)$. We can also rule out $(2, 2)$ because, by reducing capacity by one unit, firm $b$ keeps its minimal payoff unchanged while increasing its maximal payoff. By like reasoning, $(3, 1)$ and $(1, 3)$ are not equilibria. If $t \leq 2l_b - l_a$ (this is the product proximity condition of the previous section), then $(1, 2)$ can also be discarded. In all of these cases, expenditures on capacity fail to shift competitive intensities in a sufficiently beneficial way.

We are then left with the following candidates for equilibrium outcomes: $(q_a, q_b) = (0, 0)$, (0,1), (1,0), (1,1), (3,0), (0,3), (2,1) and, if $t > 2l_b - l_a$, $(1, 2)$. Of these, which are sustainable as equilibria depends on the value of $\kappa$ and on behavioral assumptions about how firms view appropriation of subsequent payoffs. Even without getting into these details, there is something definitive that can be said: excess capacity is never an equilibrium. This is consistent with the finding of Kreps and Scheinkman (1983) for homogeneous goods with divisible capacity. We conjecture that, because competitive intensities drive it in a consistent way, this finding generalizes well beyond our simple three-buyer example.

### 5.4 Vertical mergers

Finally, we add a distribution layer to the market and consider the effects of vertical mergers.[38] Suppose firms $a$ and $b$ have one unit of capacity each and symmetric production costs, normalized to zero. There are two identical, unit-demand buyers, 1 and 2. Suppose all the action in buyer preferences is now in how the products are distributed. That is, the firms offer an identical product or service but differ in how they deliver it; e.g., store locations, online purchased options, installation assistance, ease of delivery scheduling, follow-up customer service, reputation, etc. The buyers'

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each agent's competitive interval in the resulting (second stage) coalitional game complemented by an agent-specific "appropriation factor." The appropriation factor is a parameter that can be interpreted as summarizing the firm's beliefs at the time of making its capacity choice regarding the relative strength of its persuasive capabilities.

[38]This highlights another benefit of the coalitional game approach – its ability to generalize beyond simple, bilateral markets.

willingness-to-pay to make a direct purchase from firm $r \in \{a, b\}$ is $l_r > 0$.

In addition, the market contains a distributor, labelled $d$. Firm $d$ has enhanced distribution capabilities. It can distribute at most one unit from either firm and has no production capabilities of its own. A buyer's willingness-to-pay for firm $r$'s product when sold through $d$ is $h_r > l_r$. To keep it simple, assume the distributor bears no cost for providing its service. Finally, it will be helpful to assume $h_a + l_b > h_b + l_a$ so that $V = h_a + l_b$. We place no restrictions on the relative size of $h_a$ to $h_b$, nor of $l_a$ to $l_b$. The competitive intervals are detailed in Table 3.

| | **Competitive Interval** | |
|---|---|---|
| **Firm** | **min** | **max** |
| $a$ | $\max\{l_a - l_b, 0\}$ | $h_a - (h_b - l_b)$ |
| $b$ | $\max\{h_b - h_a, 0\}$ | $l_b$ |
| $d$ | $h_b - l_b$ | $h_a - l_a$ |

Table 3: Competitive intervals for $a$, $b$ and $d$.

In this situation, competitive intensity fully identifies the firms' competitive intervals.

**Lemma 3.** $\Pi_r = W_r^*$, *for* $r \in \{a, b, d\}$.

Referring to Table 3, it should not be hard to see that $h_a > h_b$ implies $\pi_a^{\min} \geq \pi_b^{\min}$ and $\pi_a^{\max} > \pi_b^{\max}$. If, instead, $h_b \geq h_a$, then $\pi_a^{\min} < \pi_b^{\min}$ and $\pi_a^{\max} \leq \pi_b^{\max}$. Thus, $a$'s competitive interval dominates that of its direct rival $b$ if and only if $h_a > h_b$. The existence of this advantage does not depend on the size of the quality gap between $a$ and $b$ (indeed, $l_a - l_b$ may be positive or negative). Here, a firm's relative performance, other things equal, depends entirely on how the intermediary (firm $d$) enhances its value relative to that of its rival. Summing up,

**Proposition 6.** *If* $h_a > h_b$, *then* $\pi_a^{\min} \geq \pi_b^{\min}$ *and* $\pi_a^{\max} > \pi_b^{\max}$, *otherwise* $\pi_a^{\min} < \pi_b^{\min}$ *and* $\pi_a^{\max} \leq \pi_b^{\max}$.

Now, consider a merger between one of $r \in \{a, b\}$ and the distributor. Assume that the merged entity, called $r'$, scraps firm $r$'s pre-merger distribution capabilities. Specifically, if firm $b$ merges with $d$, assume that $h_b + l_a > h_a$, so that $V = h_b + l_a$; if $a$ and $d$ merge, $V = h_a + l_b$ as before. The competitive intervals in these two scenarios are elaborated in Table 4.[39]

Once again, Lemma 4 demonstrates that all the agents' competitive intervals are fully characterized by competitive intensity.

[39] The "$a$" refers to the merged entity in the first two columns and $a$ alone in the last two; "$b$" refers the un-merged firm in the first two columns and the merged entity in the last two.

| | Merger: $a+d$ | | Merger: $b+d$ | |
|---|---|---|---|---|
| **Firm** | **min** | **max** | **min** | **max** |
| $a$ | $\max\{h_a - l_b, h_b - l_b\}$ | $h_a$ | $\max\{l_a - l_b, 0\}$ | $h_a - (h_b - l_b)$ |
| $b$ | $\max\{h_b - h_a, 0\}$ | $l_b$ | $\max\{h_b - h_a, 0\}$ | $l_b$ |

Table 4: Competitive intervals under mergers.

**Lemma 4.** *Consider two cases: one in which $a$ represents a merger between $a$ and $d$ and competes with $b$, and one in which $b$ represents a merger between $b$ and $d$ and competes with $a$. In both cases, $\Pi_r = W_r^*$, for $r \in \{a, b\}$.*

The question of interest is whether either firm $a$ or $b$ has an actual incentive to merge with distributor $d$ (leaving possible efficiency gains aside). Consider first the case of firm $a$ merging with $d$. Prior to merging, firms $a$ and $d$ jointly captured the value produced by their value network net of that captured by their buyer. Prior to the merger, $w_i^* = 0$ and $w_{-i}^* = h_a + l_b - \max\{h_b, h_a\}$ for buyers $i \in \{1, 2\}$. Thus, the minimum combined payoff for $a$ and $d$ pre-merger is $\pi_{a'}^{\min} = h_a - (h_a + l_b - \max\{h_b, h_a\}) = \max\{h_b, h_a\} - l_b$; the maximum is $\pi_{a'}^{\max} = h_a$. The merger between $a$ and $d$ leaves both the minimum and maximum combined payoffs unchanged. Therefore, there is no mutual, competitive incentive for the merger. What about the case of $b$ merging with $d$? Before the merger, their minimum and maximum combined payoffs are $h_b - l_b + \max\{h_b - h_a, 0\}$ and $h_a - l_a + l_b$, respectively. Because $h_a + l_b > h_b + l_a$ implies that $h_a - l_a + l_b > h_b$ and $\max\{h_a, h_b\} - l_a > h_b - l_b + \max\{h_b - h_a, 0\}$, this merger decreases their maximum combined payoff, but increases their minimum. This is summed up below.

**Proposition 7.** *Suppose that $h_a + l_b > h_b + l_a > h_a$. A merger between firms $a$ and $d$ leaves their minimum and maximum combined capture levels unaffected. However, a merger between firms $b$ and $d$ increases their minimum combined payoff and decreases their maximum.*

The competitive implications of a vertical merger between firms $a$ and $d$ differs substantively from those of one between $b$ and $d$. In the former case, the balance of competitive intensities with respect to buyers remains unchanged. In the latter case, competitive intensities increase with respect to the two merging firms. When $b$ and $d$ are not merged, they create competition for each other. Merging intensifies competition for them by shifting superior distribution away from $a$ (since $l_a < h_a + l_b - h_b$). This makes the merged entity more attractive to $a$'s buyer, thereby increasing the minimum combined payoff of $b$ and $d$ (as the merged entity). At the top end, merging allows $b$ and $d$ to appropriate all the value created by their value network, $h_b$. However, this value is less than $h_a + l_b - l_a$, which is the overall value that $b$ and $d$ create when not merged, adjusted for the fact that $a$ and one of the buyers have the ability to create value with each other.

## 6 Conclusions

We examine markets in which economic value is created by disjoint subsets of agents (e.g., via independent supply or value chains). In this context, we define three notions of competitive intensity – all of which imply bounds on the amount of value agents may capture in a market setting. We demonstrate that, while these bounds are weaker than the ones provided by the core, they also require much less information. At the same time, they add substantial traction beyond added-value (the present workhorse for empirical work in this line of research). We apply these ideas in a more concrete market setting to show how competitive intensity performs in simpler, more familiar circumstances. Here, even our weakest notion of competitive intensity is sufficient to characterize the firm's competitive intervals. Throughout, competitive intensity provides strong intuition about how competition is actually operating in these cases.

There is much that remains to be done in this line of work. Our results make much use of the ability to divide a market into distinct value networks. While many markets may exhibit this structure, it is, nevertheless, special. Is there some notion of competitive intensity – one that simplifies the computation of competitive intervals and maintains the appealing logic of the core – that is applicable to markets that cannot be decomposed in this way? Presumably, one would have to consider *within-group* competition to begin to answer this question. This is a complementary question for future research.

We have argued that thinking in terms of competitive intervals is important for strategy scholars. Among other things, it raises the need to distinguish between resources deployed with productive intent versus those deployed with persuasive intent. This is a significant conceptual refinement to business strategy, one uniquely offered by coalitional game theory. What do "persuasive" resources look like in a real world context of unstructured bargaining? Presently, it is hard to say. The theory says this distinction is almost certainly central to explaining persistent performance differences among firms. Early empirical findings appear to support this claim (Grennan, 2014; Bennett, 2013). Thus, deeper theoretical inquiry into this issue appears promising for strategy work, ripe with the potential for substantial contribution.

# A Proof of Proposition 1

We first show that $\pi_i^{\min} \geq w_i^* = \max_{X \in \mathcal{P}^* \setminus X_i^*} av_i(X)$. If this were not the case, $i$ could always join any $Y \in \arg\max_{X \in \mathcal{P}^* \setminus X_i^*} av_i(X)$ (or refuse joining any value network if we do not have $w_i^* > v(i)$). Since we know that $v(Y) = \sum_{j \in Y} \pi_j$, a new value network formed by $i$ and the members of $Y$ would make all of them better off because of the positive value added by $i$, which would contradict the hypothesis that $\pi_i^{\min}$ is a possible core allocation.

Since it clearly holds that $\pi_i^{\max} \leq av_i(X_i^*)$ (otherwise, the other members of $X_i^*$ would improve by excluding $i$), we conclude the proof by demonstrating that $\pi_i^{\max} \leq v(X_i^*) - \sum_{j \in X_{i-i}^*} w_j^*$. To show this, note if $i \in X_i^*$ receives the maximal payoff it can get in a core allocation that we must still have that any other agent $j \neq i$ in $X_i^*$ gets a payoff $\pi_j$ at least as large as $\pi_j^{\min}$. Because $v(X_i^*) = \pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j$, we must then have that $v(X_i^*) \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j^{\min}$, so the result that $\pi_j^{\min} \geq w_j^*$ for all $j \in X_{i-i}^*$ yields that $\pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j^{\min} \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} w_j^*$. We therefore have $v(X_i^*) \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} w_j^*$, which ends the proof.

# B Proof of Proposition 2

Note from the definition of $s_i^*$ that choosing $Y = \emptyset$ would make $s_i^* = w_i^*$, so allowing $Y$ to differ from $\emptyset$ implies that $s_i^* \geq w_i^*$ and $w_{-i}^* \geq s_{-i}^*$, so we just need to show that $s_i^* \leq \pi_i^{\min}$ and $\pi_i^{\max} \leq s_{-i}^*$. To prove that $\pi_i^{\min} \geq s_i^*$, suppose to the contrary that $s_i^* - \pi_i^{\min} > 0$ and assume that $s_i^* > w_i^*$ to avoid triviality. Letting $X^* \in \mathcal{P}^* \setminus X_i^*$ and $Y^* \subset X^*$ be some maximizers of

$$av_i(i \cup X \setminus Y) - \left( av_Y(X) - \max \left\{ \sum_{j \in Y} w_j^*, v(Y) \right\} \right),$$

we must have

$$av_i(i \cup X^* \setminus Y^*) - \left( av_{Y^*}(X^*) - \max \left\{ \sum_{j \in Y^*} w_j^*, v(Y^*) \right\} \right) - \pi_i^{\min} > 0.$$

This means that the agents in $X^*$ could exclude a subset $Y^* \subset X^*$ of agents and replace them with $i$, thus making themselves better off together with agent $i$. Because such a mutually profitable deviation cannot happen if the value distribution is consistent with competition,[40] a contradiction is obtained and hence we have that $\pi_i^{\min} \geq s_i^*$.

We conclude the proof by demonstrating that $\pi_i^{\max} \leq s_{-i}^*$. Letting $s_{-i}^* < w_{-i}^*$ to avoid triviality, the proof parallels the one showing that $\pi_i^{\max} \leq w_{-i}^*$ in Proposition 1.

# C Proof of Proposition 3

From the generalized competitive intensity we have that for each $i \in N$, $S \subseteq N \backslash \{i\}$ and $\pi \in C$ that

$$\pi_i \geq v(S \cup i) - \sum_{j \in S} \pi_j,$$

and therefore in the particular case where $S = C_i$ we have that

$$\pi_i \geq v(C_i \cup i) - \sum_{j \in S} \pi_j \geq v(C_i \cup i) - \min \left\{ av_{C_i}(N), \sum_{C_i} av_j(N) \right\} = \underline{w}_i.$$

Moreover we also have, for any $S \subseteq N$,

$$\sum_{j \in S} \pi_j \leq av_S(N).$$

Thus, for every $i \in N$ and $S \subset N \backslash \{i\}$

$$\pi_i \leq V - \sum_{j \in N \backslash \{i\}} \pi_j \leq V - v(N \backslash S \cup i) - \sum_{j \in S} \pi_j \leq av_{S \cup i}(N) - \sum_{S \backslash \{i\}} \underline{w}_j.$$

Therefore we also have that $\pi_i \leq \overline{w}_i$. Hence for each $i \in N$ and $\pi \in C$ we have that $\overline{w}_i \geq \pi_i \geq \underline{w}_i$. Since in addition both the elements of $CI$ and $C$ are efficient, it follows that $C \subseteq CI$. Moreover, $\underline{w}_i \geq w_i^*$ since the set over which $C_i$ is obtained contains the set used to obtain $w_i^*$. Since residuals are obtained using the respective competitive intensities, it also follows that $w_{-i}^* \geq \overline{w}_i$.

[40]In terms of costs and benefits of such a deviation, note that the agents in $Y^*$ obtain no less than $\max\left\{\sum_{j \in Y^*} w_j^*, v(Y^*)\right\}$ out of the value $av_{Y^*}(X^*)$ they bring in to $X^*$, so excluding them from $X^*$ costs no more than

$$av_{Y^*}(X^*) - \max\left\{\sum_{j \in Y^*} w_j^*, v(Y^*)\right\},$$

whereas the opportunity cost of having $i$ join $X^* \backslash Y^*$ is $\pi_i^{\min}$. So the benefit of replacing $Y^*$ with $i$ for the subset of agents $X^* \backslash Y^*$, which equals $av_i(\{i\} \cup X^* \backslash Y^*)$, always exceeds the opportunity costs of such an action, which cannot happen if $\pi_i^{\min}$ is a payoff consistent with competition.

# D Proof of Proposition 4

We have seen above that $\overline{w}_i \geq \pi_i^{\max} \geq \pi_i^{\min} \geq \underline{w}_i$ for every $i \in N$. It follows naturally that if $\pi^\phi \in C$ then those agents in its front and end receive respectively the upper and lower bounds of their competitive intervals. Moreover, since $C \subseteq CI$, if the boundaries of the $CI$ lie in the $C$ it must be that $CI = C$.

# E Proof of Proposition 5

Suppose that $CI = C$, then for all $\phi \in N^\phi$ we have that $\pi^\phi \in C$ and therefore $v(S) \leq \sum_{i \in S} \pi_i^\phi$ for every $S \subseteq N$. We show that in this case (25) is satisfied. Denote the set of first elements of a given $\phi$ by $N \backslash S$ and the last by $S$. There are two possibilities: i) all the elements of $N \backslash S$ receive their generalized residuals or ii) not all elements of $N \backslash S$ receive their generalized residuals. In case i) we have that

$$v(S) \leq \sum_{i \in S} \pi_i^\phi = V - \sum_{i \in N \backslash S} \pi_i^\phi = V - \sum_{i \in N \backslash S} \overline{w}_i$$

while in case ii) we have that each player in $S$ receives only the value of their individual generalized competitive intensity, i.e.

$$v(S) \leq \sum_{i \in S} \pi_i^\phi = \sum_{i \in S} \underline{w}.$$

If we combine the two cases we obtain (25). Conversely, assume that (25) holds for each $S \subseteq N$, then since the Core is convex we only need to show that in each ordering $\phi$ the associated greedy residual vector $\pi^\phi \in C$. Thus

$$\begin{aligned} v(S) &\leq \max \left\{ \sum_{i \in S} \underline{w}_i, V - \sum_{i \in N \backslash S} \overline{w}_i \right\} \\ &\leq \max \left\{ \sum_{i \in S} \pi_i^\phi, V - \sum_{i \in N \backslash S} \pi_i^\phi \right\} = \sum_{i \in S} \pi_i^\phi \end{aligned}$$

So $\pi^\phi \in C$ and therefore the converse is also verified.

## F Proofs of Lemmas 1 and 2

**Cases in which** $(q_a, q_b) = (1, 2)$ Let us look first at the cases in which $(q_a, q_b) = (1, 2)$, so that $V_{1,2} = l_a + l_b + l_b - t/2$. Let $u_i$ ($i \in \{1, 2, 3\}$) denote the payoff of buyer $i$ in a core allocation. Similarly, let $\pi_j$ ($j \in \{a, b\}$) denote the payoff of firm $j$ in a core allocation. Recalling that $\pi_a + \pi_b + u_1 + u_2 + u_3 = V$, expression (3) used for all $G \subset N = \{1, 2, 3, a, b\}$ yields the following inequality constraints that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy in order to lie in the core (together with the condition that $\sum_{j\in G} \pi_j \leq V - v(N\backslash G)$ implied by both $V = \sum_{j\in N} \pi_j$ and (3)):

$$
\begin{aligned}
0 &\leq \pi_a \leq l_a \\
0 &\leq \pi_b \leq l_b + l_b - t/2 \\
0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_b - t/2 \\
l_a - t/2 &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_b - t/2 \\
l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a + l_b - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_b - t/2 \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_b - t/2 \\
l_a - t/2 + l_b &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + u_1 \leq l_a \\
l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_b - t/2 - (l_b - t) \\
l_a - t &\leq \pi_a + u_3 \leq l_a + l_b - (l_b - t)
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + u_1 + u_2 \leq l_a + l_b - t/2 \\
l_a &\leq \pi_a + u_1 + u_3 \leq l_a + l_b \\
l_a - t/2 &\leq \pi_a + u_2 + u_3 \leq l_a + l_b - t/2 + l_b - (l_b - t) \\
l_a &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_b - t &\leq \pi_b + u_1 \leq l_a + l_b + l_b - t/2 - (l_a - t/2) \\
l_b - t/2 &\leq \pi_b + u_2 \leq l_b + l_b - t/2 \\
l_b &\leq \pi_b + u_3 \leq l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_b - t + l_b - t/2 &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_b - t/2 - (l_a - t) \\
l_b + l_b - t &\leq \pi_b + u_1 + u_3 \leq l_a + l_b + l_b - t/2 - (l_a - t/2) \\
l_b + l_b - t/2 &\leq \pi_b + u_2 + u_3 \leq l_b + l_b - t/2 \\
l_b + l_b - t/2 &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 \leq l_a + l_b - t/2 - (l_a - t/2) \\
0 &\leq u_2 \leq l_b - t/2 \\
0 &\leq u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 + u_2 \leq l_a + l_b - t/2 \\
0 &\leq u_1 + u_3 \leq l_a + l_b + l_b - t/2 - (l_a - t/2) \\
0 &\leq u_2 + u_3 \leq l_b + l_b - t/2 \\
0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_b - t/2
\end{aligned}
$$

If we denote any nontrivial value partition given capacity vector $(q_a, q_b)$ by $\mathcal{P}^*_{q_a,q_b}$, we have that the unique nontrivial partition given $(q_a, q_b) = (1, 2)$ is $\mathcal{P}^*_{1,2} = \{\{a, 1\}, \{b, 2, 3\}\}$. Under $\mathcal{P}^*_{1,2}$, we now proceed to show that the competitive intensities/residuals, $w^*_a = l_a - t/2 - (l_b - t/2) = l_a - l_b$,

$w^*_{-a} = l_a$, $w^*_b = 0$, and $w^*_{-b} = 2l_b - t/2$, are sharp bounds. To do so, we assume that we indeed have either $\pi_a^{\min} = l_a - l_b$ or $\pi_a^{\max} = l_a$ or $\pi_b^{\min} = 0$ or $\pi_b^{\max} = 2l_b - t/2$ and find no violation of the conditions imposed by the set of inequality constraints above. This will show that the bounds are giving us admissible core allocations, and hence they must be exact, since they were shown to bound below/above an agent's competitive interval.

Thus, using the above inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = l_a - l_b$ can arise as a feasible core allocation for firm $a$ whenever it holds that

$$l_a - t - (l_a - l_b) > 0, \tag{26}$$

an inequality that is always satisfied given our assumption that $l_b - t > 0$. The core payoffs for the other players are $u_1 = l_b$, $u_2 = l_b - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq t$, $l_b - t \leq u_3 \leq t$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain.

Using the inequalities that define the core of this game, it can also be readily verified that $\pi_a^{\max} = l_a$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = 0$, and any tuple of values $(\pi_b, u_2, u_3)$ such that $l_b - t \leq \pi_b \leq 2l_b - t/2$, $0 \leq u_2 \leq t/2$, $0 \leq u_3 \leq t$, $2l_b - 3t/2 \leq \pi_b + u_2 \leq 2l_b - t/2$, $2l_b - t \leq \pi_b + u_3 \leq 2l_b - t/2$, and $\pi_b + u_2 + u_3 = 2l_b - t/2$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff is sharp and coincides with the maximal payoff it can obtain.

Using the inequalities that define the core of this game, it can further be shown that $\pi_b^{\min} = 0$ can arise as a feasible core allocation for firm $b$. The core payoffs for the other players are $u_2 = l_b - t/2$, $u_3 = l_b$, and any pair of values $(\pi_a, u_1)$ such that $l_a - l_b \leq \pi_a \leq l_a - l_b + t$, $l_b - t \leq u_1 \leq l_b$ and $\pi_a + u_1 = l_a$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $b$'s payoff is sharp and coincides with the minimal payoff it can obtain.

Using the inequalities that define the core of this game, it can finally be shown that $\pi_b^{\max} = l_b + l_b - t/2$ can arise as a feasible core allocation for firm $b$. The core payoffs for the other players are $u_2 = 0$, $u_3 = 0$, and any pair of values $(\pi_a, u_1)$ such that $l_a - t/2 \leq \pi_a \leq l_a$, $0 \leq u_1 \leq t/2$, and $\pi_a + u_1 = l_a$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $b$'s payoff is sharp and coincides with the maximal payoff it can obtain.

**Cases in which** $(q_a, q_b) = (2, 2)$ We consider next how the bounds work when firm $a$ adds one unit of capacity, so let now $q_a = 2$ and $q_b = 2$. Given that $V_{2,2} = l_a + l_b + l_a - t/2$, the conditions that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy are:

$$\begin{aligned}
0 &\leq \pi_a \leq l_a + l_a - t/2 - (l_b - t/2) \\
0 &\leq \pi_b \leq l_b \\
0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$\begin{aligned}
l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_a - t/2 \\
l_a - t/2 &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_a - t/2 \\
l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_a - t/2 \\
l_b + l_a - t/2 &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$\begin{aligned}
l_a &\leq \pi_a + u_1 \leq l_a + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_a - t/2 - (l_b - t) \\
l_a - t &\leq \pi_a + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) - (l_b - t)
\end{aligned}$$

$$\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 \leq l_a + l_a - t/2 \\
l_a + l_a - t &\leq \pi_a + u_1 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 + l_a - t &\leq \pi_a + u_2 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t) \\
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$l_b - t \leq \pi_b + u_1 \leq l_a + l_b - (l_a - t)$$

$$l_b - t/2 \leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t) \tag{27}$$

$$l_b \leq \pi_b + u_3 \leq l_b \tag{28}$$

$$\begin{aligned} l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 - (l_a - t) \\ l_b + l_b - t &\leq \pi_b + u_1 + u_3 \leq l_a + l_b \\ l_b + l_b - t/2 &\leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2 \\ l_b + l_b - t/2 &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned} \tag{29}$$

$$\begin{aligned} 0 &\leq u_1 \leq l_a \\ 0 &\leq u_2 \leq l_a - t/2 \\ 0 &\leq u_3 \leq l_b \end{aligned}$$

$$\begin{aligned} 0 &\leq u_1 + u_2 \leq l_a + l_a - t/2 \\ 0 &\leq u_1 + u_3 \leq l_a + l_b \\ 0 &\leq u_2 + u_3 \leq l_b + l_a - t/2 \\ 0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned}$$

The unique nontrivial partition given $(q_a, q_b) = (2, 2)$ is $\mathcal{P}^*_{2,2} = \{\{a, 1, 2\}, \{b, 3\}\}$. Under $\mathcal{P}^*_{2,2}$, do we get sharp bounds using $w^*_a = 0$, $w^*_b = 0$, $w^*_{-a} = l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) = 2(l_a - l_b) + t$, and $w^*_{-b} = l_b$? As we show next, the answer is affirmative, except for $\pi_b^{\max} = l_b$, in which case the upper bound on firm $b$'s maximal payoff is not sharp because $\pi_b = l_b$ cannot satisfy the set of inequality constraints above.

Thus, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = 0$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_a$, $u_2 = l_a - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq t + l_b - l_a$, $l_a - t \leq u_3 \leq l_b$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain.

Likewise, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\max} = 2(l_a - l_b) + t$ and $\pi_b^{\min} = 0$ can arise as feasible core allocations for firms $a$ and $b$. The core payoffs for the other players are $u_1 = l_b - t$, $u_2 = l_b - t/2$ and $u_3 = l_b$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff and the lower bound on firm $b$'s payoff are sharp.

It can finally be shown that the inequalities that should hold in order for $\pi_b^{\max} = l_b$ to arise as a core allocation for firm $b$ would be violated, so we must have $\pi_b^{\max} < l_b$.[41] We then have that the upper bound in not tight because buyer 3's competitive intensity fails to capture a credible threat that such a buyer has.

Such a credible threat is to offer herself to firm $a$ as a replacement for buyer 2 if she is getting nothing from firm $b$. This will work as long as $l_a - t/2 - u_2 < l_a - t$, that is, as long as $u_2 > t/2$. So we must have $u_2 \leq t/2$ in order for firm $a$ not to have an incentive to replace buyer 2 with 3. But if $u_2 \leq t/2$, the assumption that $l_b - t/2 > t/2$ implies that firm $b$ could offer buyer 2 some amount between $t/2$ and $l_b - t/2$ and be better off. Buyer 3 exploits this tension to offer $u_2 > t/2$ to buyer 2 in order to show firm $b$ that it could credibly replace buyer 2. In other words, buyer 3 can exploit to her benefit the fact that buyer 2's competitive intensity equals $l_b - t/2$. This free riding on buyer 2's positive competitive intensity is not internalized in the competitive intensity index. This happens because firm $a$ is at full capacity but firm $b$ is not. In these cases, we will need to use the indirect notion of competitive intensity for the firm with idle capacity.

In the current case, $s_3^* = l_a - t - (l_a - t/2 - w_2^*)$, where $w_2^* = l_b - t/2$, so $s_3^* = l_b - t$. This would lead to an upper bound on firm $b$'s maximal payoff equal to $s_{-b}^* = t$. It can indeed be shown now that $\pi_b^{\max} = t$ can arise as a feasible core allocation for firm $b$. The core payoffs for the other players are $u_2 = l_b - t/2$, $u_3 = l_b - t$, and any pair of values $(\pi_a, u_1)$ such that $2(l_a - l_b) \leq \pi_a \leq 2(l_a - l_b) + t$, $l_b - t \leq u_1 \leq l_b$ and $\pi_a + u_1 = 2l_a - l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $b$'s maximal payoff that arises from the

[41] This happens because we cannot possibly have $l_b \leq \pi_b + u_3 \leq l_b$ (see (28)), $l_b - t/2 \leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t)$ (see (27)) and $l_b + l_b - t/2 \leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2$ (see 29) be satisfied when $\pi_b = l_b$. Note that $l_b \leq \pi_b + u_3 \leq l_b$ and $\pi_b = l_b$ would imply that $u_3 = 0$ and hence the other constraints would become $l_b - t/2 \leq \pi_b + u_2 \leq l_b + t/2$ and $l_b + l_b - t/2 \leq \pi_b + u_2 \leq l_b + l_a - t/2$, which cannot possibly be fulfilled at the same time because $l_b > t$. What happens is that buyer 2 could always threaten to join the coalition formed by firm $b$ and buyer 3, compensate firm $b$ with $l_b$ and ensure herself a payoff of $l_b - t/2$. However, if firm $b$ and buyer 2 are respectively getting $l_b$ and no less than $l_b - t/2$, then the set of all agents could exclude them and be better off: what all agents lose by excluding them, $l_a - t/2 + l_b - (l_a - t) = l_b + t/2$, is less than their joint reward, which is no less than $l_b + l_b - t/2$.

indirect notion of competitive intensity is sharp.

**Cases in which** $(q_a, q_b) = (2, 1)$ We now focus on the situations in which $(q_a, q_b) = (2, 1)$. In such a case, we have that $V_{2,1} = l_a + l_b + l_a - t/2$. The conditions that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy are:

$$\begin{aligned} 0 &\leq \pi_a \leq l_a + l_a - t/2 \\ 0 &\leq \pi_b \leq l_b \\ 0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_a - t/2 \end{aligned}$$

$$\begin{aligned} l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_a - t/2 \\ l_a - t/2 &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_a - t/2 \\ l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned}$$

$$\begin{aligned} l_a + l_a - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 \\ l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_a - t/2 \\ l_a - t/2 + l_b &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned}$$

$$\begin{aligned} l_a &\leq \pi_a + u_1 \leq l_a + l_a - t/2 \\ l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_a - t/2 \\ l_a - t &\leq \pi_a + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \end{aligned}$$

$$\begin{aligned} l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 \leq l_a + l_a - t/2 \\ l_a + l_a - t &\leq \pi_a + u_1 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \\ l_a - t/2 + l_a - t &\leq \pi_a + u_2 + u_3 \leq l_a + l_a - t/2 + l_b - (l_b - t) \\ l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned}$$

$$
\begin{aligned}
l_b - t &\leq \pi_b + u_1 \leq l_a + l_b - (l_a - t) \\
l_b - t/2 &\leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t) \\
l_b &\leq \pi_b + u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
l_b - t/2 &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 - (l_a - t) \\
l_b &\leq \pi_b + u_1 + u_3 \leq l_a + l_b \\
l_b &\leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2 \\
l_b &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 \leq l_a \\
0 &\leq u_2 \leq l_a - t/2 \\
0 &\leq u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 + u_2 \leq l_a + l_a - t/2 \\
0 &\leq u_1 + u_3 \leq l_a + l_b \\
0 &\leq u_2 + u_3 \leq l_b + l_a - t/2 \\
0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

The unique nontrivial partition given $(q_a, q_b) = (2, 1)$ is $\mathcal{P}^*_{2,1} = \{\{a, 1, 2\}, \{b, 3\}\}$. Under $\mathcal{P}^*_{2,1}$, we now show that we get sharp bounds for $\pi_a^{\min}$, $\pi_b^{\min}$, $\pi_a^{\max}$ and $\pi_b^{\max}$ respectively using $w_a^* = 0$, $w_b^* = 0$, $w_{-a}^* = 2l_a - t/2$, and $w_{-b}^* = l_b$.

Thus, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = 0$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_a$, $u_2 = l_a - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain. Indeed, this also proves that $\pi_b^{\min} = 0$ is a sharp bound for firm $b$'s minimal payoff.

Using the inequalities that define the core of this game, it can also be shown that $\pi_a^{\max} = l_a + l_a - t/2$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = 0$, $u_2 = 0$, and any tuple of values $(\pi_b, u_3)$ such that $l_b - t/2 \leq \pi_b \leq l_b$, $0 \leq u_3 \leq l_b - (l_b - t/2)$, and $\pi_b + u_3 = l_b$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff is sharp and coincides with the maximal payoff it can obtain. In fact, this also shows that $\pi_b^{\max} = l_b$ is a sharp bound for firm $b$'s maximal payoff.

**Cases in which** $(q_a, q_b) = (3, 2)$ We consider next how the bounds work when $(q_a, q_b) = (3, 2)$, in which case $V_{3,2} = l_a + l_b + l_a - t/2$. The conditions that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy are:

$$
\begin{aligned}
0 &\leq \pi_a \leq l_a + l_a - t/2 - (l_b - t/2) \\
0 &\leq \pi_b \leq l_b - (l_a - t) \\
0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_a - t/2 \\
l_a - t/2 &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_a - t/2 \\
l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_a - t/2 \\
l_b + l_a - t/2 &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + u_1 \leq l_a + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_a - t/2 - (l_b - t) \\
l_a - t &\leq \pi_a + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) - (l_b - t)
\end{aligned}
$$

$$
\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 \leq l_a + l_a - t/2 \\
l_a + l_a - t &\leq \pi_a + u_1 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 + l_a - t &\leq \pi_a + u_2 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t) \\
l_a + l_a - t/2 + l_a - t &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_b - t &\leq \pi_b + u_1 \leq l_a + l_b - (l_a - t) \\
l_b - t/2 &\leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t) \\
l_b &\leq \pi_b + u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 - (l_a - t) \\
l_b + l_b - t &\leq \pi_b + u_1 + u_3 \leq l_a + l_b \\
l_b + l_b - t/2 &\leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2 \\
l_b + l_b - t/2 &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 \leq l_a \\
0 &\leq u_2 \leq l_a - t/2 \\
0 &\leq u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 + u_2 \leq l_a + l_a - t/2 \\
0 &\leq u_1 + u_3 \leq l_a + l_b \\
0 &\leq u_2 + u_3 \leq l_b + l_a - t/2 \\
0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

The unique nontrivial partition given $(q_a, q_b) = (3, 2)$ is $\mathcal{P}^*_{3,2} = \{\{a, 1, 2\}, \{b, 3\}\}$. Under $\mathcal{P}^*_{3,2}$, we now show that we get sharp bounds for $\pi_a^{\min}$, $\pi_b^{\min}$, $\pi_a^{\max}$ and $\pi_b^{\max}$ respectively using $w_a^* = 0$, $w_b^* = 0$, $w_{-a}^* = l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) = 2(l_a - l_b) + t$, and $w_{-b}^* = l_b - (l_a - t) = t - (l_a - l_b)$.

Thus, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = 0$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_a$, $u_2 = l_a - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain. Indeed, this also proves that $\pi_b^{\min} = 0$ is a sharp bound for firm $b$'s minimal payoff, whereas $\pi_b^{\max} = l_b - (l_a - t)$ is a sharp bound for firm $b$'s maximal payoff.

Using the inequalities that define the core of this game, it can also be shown that $\pi_a^{\max} = l_a + l_a - t/2 - (l_b - t/2 + l_b - t)$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_b - t$, $u_2 = l_b - t/2$, and any tuple of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$, and $\pi_b + u_3 = l_b$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff is sharp and coincides with the maximal payoff it can obtain.

**Cases in which** $(q_a, q_b) = (2, 3)$ We turn now to the situations in which $(q_a, q_b) = (2, 3)$. Given that $V = l_a + l_b + l_a - t/2$, the conditions that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy are:

$$\begin{aligned}
0 &\leq \pi_a \leq l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) \\
0 &\leq \pi_b \leq l_b \\
0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$\begin{aligned}
l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_a - t/2 \\
l_a - t/2 &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_a - t/2 \\
l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_a - t/2 \\
l_b + l_a - t/2 &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}$$

$$
\begin{aligned}
l_a &\leq \pi_a + u_1 \leq l_a + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_a - t/2 - (l_b - t) \\
l_a - t &\leq \pi_a + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) - (l_b - t)
\end{aligned}
$$

$$
\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 \leq l_a + l_a - t/2 \\
l_a + l_a - t &\leq \pi_a + u_1 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 + l_a - t &\leq \pi_a + u_2 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t) \\
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_b - t &\leq \pi_b + u_1 \leq l_a + l_b - (l_a - t) \\
l_b - t/2 &\leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t) \\
l_b &\leq \pi_b + u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 - (l_a - t) \\
l_b + l_b - t &\leq \pi_b + u_1 + u_3 \leq l_a + l_b \\
l_b + l_b - t/2 &\leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2 \\
l_b + l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 \leq l_a \\
0 &\leq u_2 \leq l_a - t/2 \\
0 &\leq u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 + u_2 \leq l_a + l_a - t/2 \\
0 &\leq u_1 + u_3 \leq l_a + l_b \\
0 &\leq u_2 + u_3 \leq l_b + l_a - t/2 \\
0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

The unique nontrivial partition given $(q_a, q_b) = (2, 3)$ is $\mathcal{P}^*_{2,3} = \{\{a, 1, 2\}, \{b, 3\}\}$. Under $\mathcal{P}^*_{2,3}$, we now show that we get sharp bounds for $\pi_a^{\min}$, $\pi_b^{\min}$, $\pi_a^{\max}$ and $\pi_b^{\max}$ respectively using $w_a^* = 0$, $w_b^* = 0$, $w_{-a}^* = l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) = 2(l_a + l_b) + t$, and $s_{-b}^* = l_b - (l_a - t) + l_a - t/2 - (l_b - t/2) = t$ (note that the bound given by firm $b$'s competitive intensity for buyer 3 does not work well, so it needs to be strengthened).

Thus, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = 0$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_a$, $u_2 = l_a - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain. Indeed, this also proves that $\pi_b^{\min} = 0$ is a sharp bound for firm $b$'s minimal payoff.

Using the inequalities that define the core of this game, it can also be shown that $\pi_a^{\max} = l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) = 2(l_a - l_b) + t$ can arise as a feasible core allocation for firm $a$.[42] The core payoffs for the other players are $u_1 = l_b - t$, $u_2 = l_b - t/2$, and any tuple of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t) + l_a - t/2 - (l_b - t/2) = t$, $l_a - t - (l_a - t/2) + (l_b - t/2) \leq u_3 \leq l_b$, and $\pi_b + u_3 = l_b$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff is sharp and coincides with the maximal payoff it can obtain. Indeed, this also proves that $\pi_b^{\max} = t$ is a sharp bound for firm $b$'s maximal payoff.

**Cases in which** $(q_a, q_b) = (3, 3)$ We consider finally cases in which $(q_a, q_b) = (3, 3)$. Taking into account that $V_{3,3} = l_a + l_b + l_a - t/2$, the conditions that $(u_1, u_2, u_3, \pi_a, \pi_b)$ must satisfy are:

$$\begin{aligned} 0 &\leq \pi_a \leq l_a + l_a - t/2 - (l_b - t/2) \\ 0 &\leq \pi_b \leq l_b - (l_a - t) \\ 0 &\leq \pi_a + \pi_b \leq l_a + l_b + l_a - t/2 \end{aligned}$$

$$\begin{aligned} l_a &\leq \pi_a + \pi_b + u_1 \leq l_a + l_b + l_a - t/2 \\ l_a - t &\leq \pi_a + \pi_b + u_2 \leq l_a + l_b + l_a - t/2 \\ l_b &\leq \pi_a + \pi_b + u_3 \leq l_a + l_b + l_a - t/2 \end{aligned}$$

[42] In verifying that $\pi_a^{\max} = 2(l_a - l_b) + t$ satisfies the constraints required by the core, we have used the fact that $l_a - t + l_b - t/2 - (l_a - t/2) > 0$, which always holds given that $l_b > t$.

$$
\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_3 \leq l_a + l_b + l_a - t/2 \\
l_b + l_a - t/2 &\leq \pi_a + \pi_b + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_a &\leq \pi_a + u_1 \leq l_a + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 &\leq \pi_a + u_2 \leq l_a + l_a - t/2 - (l_b - t) \\
l_a - t &\leq \pi_a + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) - (l_b - t)
\end{aligned}
$$

$$
\begin{aligned}
l_a + l_a - t/2 &\leq \pi_a + u_1 + u_2 \leq l_a + l_a - t/2 \\
l_a + l_a - t &\leq \pi_a + u_1 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t/2) \\
l_a - t/2 + l_a - t &\leq \pi_a + u_2 + u_3 \leq l_a + l_b + l_a - t/2 - (l_b - t) \\
l_a + l_a - t/2 + l_a - t &\leq \pi_a + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
l_b - t &\leq \pi_b + u_1 \leq l_a + l_b - (l_a - t) \\
l_b - t/2 &\leq \pi_b + u_2 \leq l_b + l_a - t/2 - (l_a - t) \\
l_b &\leq \pi_b + u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 \leq l_a + l_b + l_a - t/2 - (l_a - t) \\
l_b + l_b - t &\leq \pi_b + u_1 + u_3 \leq l_a + l_b \\
l_b + l_b - t/2 &\leq \pi_b + u_2 + u_3 \leq l_b + l_a - t/2 \\
l_b + l_b - t/2 + l_b - t &\leq \pi_b + u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 \leq l_a \\
0 &\leq u_2 \leq l_a - t/2 \\
0 &\leq u_3 \leq l_b
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq u_1 + u_2 \leq l_a + l_a - t/2 \\
0 &\leq u_1 + u_3 \leq l_a + l_b \\
0 &\leq u_2 + u_3 \leq l_b + l_a - t/2 \\
0 &\leq u_1 + u_2 + u_3 \leq l_a + l_b + l_a - t/2
\end{aligned}
$$

The unique nontrivial partition given $(q_a, q_b) = (3,3)$ is $\mathcal{P}^*_{3,3} = \{\{a,1,2\},\{b,3\}\}$. Under $\mathcal{P}^*_{3,3}$, we proceed to show that we get sharp bounds for $\pi_a^{\min}$, $\pi_b^{\min}$, $\pi_a^{\max}$ and $\pi_b^{\max}$ respectively using $w_a^* = 0$, $w_b^* = 0$, $w_{-a}^* = l_a + l_a - t/2 - (l_b - t/2) - (l_b - t) = 2(l_a - l_b) + t$, and $w_{-b}^* = l_b - (l_a - t) = t - (l_a - l_b)$.

Thus, using the inequalities that define the core of this game, it can be shown that $\pi_a^{\min} = 0$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_a$, $u_2 = l_a - t/2$, and any pair of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$ and $\pi_b + u_3 = l_b$ hold. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the lower bound on firm $a$'s payoff is sharp and coincides with the minimal payoff it can obtain. Indeed, this also proves that $\pi_b^{\min} = 0$ is a sharp bound for firm $b$'s minimal payoff, whereas $\pi_b^{\max} = t - (l_a - l_b)$ is a sharp bound for firm $b$'s maximal payoff.

Using the inequalities that define the core of this game, it can also be shown that $\pi_a^{\max} = 2(l_a - l_b) + t$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $u_1 = l_b - t$, $u_2 = l_b - t/2$, and any tuple of values $(\pi_b, u_3)$ such that $0 \leq \pi_b \leq l_b - (l_a - t)$, $l_a - t \leq u_3 \leq l_b$, and $\pi_b + u_3 = l_b$. Hence, our assumptions that $l_a \geq l_b \geq t$ and $-t \leq l_a - l_b \leq t$ imply that the upper bound on firm $a$'s payoff is sharp and coincides with the maximal payoff it can obtain.

## F.1 Proof of Lemmas 3 and 4

Suppose that neither firm $a$ nor firm $b$ are merged with $d$. Let $u_i$ ($i \in \{1,2,3\}$) denote the payoff of buyer $i$ in a core allocation. Similarly, let $\pi_j$ ($j \in \{a,b,d\}$) denote the payoff of firm $j$ in a core allocation. Recalling that $\pi_a + \pi_b + \pi_d + u_1 + u_2 = V$, expression (3) used for all $G \subset N = \{1,2,a,b,d\}$ yields the following inequality constraints that $(u_1, u_2, \pi_a, \pi_b, \pi_d)$ must satisfy in order to lie in the core (together with the condition that $\sum_{j \in G} \pi_j \leq V - v(N \backslash G)$ implied by both $V = \sum_{j \in N} \pi_j$ and (3)):

$$
\begin{aligned}
0 &\leq \pi_a \leq h_a + l_b - h_b \\
0 &\leq \pi_b \leq h_a + l_b - h_a \\
0 &\leq \pi_d \leq h_a - l_a \\
0 &\leq u_1 \leq h_a + l_b - \max\{h_a, h_b\} \\
0 &\leq u_2 \leq h_a + l_b - \max\{h_a, h_b\}
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq \pi_a + \pi_b \leq h_a + l_b \\
0 &\leq \pi_a + \pi_d \leq h_a \\
l_a &\leq \pi_a + u_1 \leq h_a + l_b - h_b \\
l_a &\leq \pi_a + u_2 \leq h_a + l_b - h_b \\
0 &\leq \pi_b + \pi_d \leq h_a + l_b - l_a \\
l_b &\leq \pi_b + u_1 \leq l_b \\
l_b &\leq \pi_b + u_2 \leq l_b \\
0 &\leq \pi_d + u_1 \leq h_a + l_b - \max\{l_a, l_b\} \\
0 &\leq \pi_d + u_2 \leq h_a + l_b - \max\{l_a, l_b\} \\
0 &\leq u_1 + u_2 \leq h_a + l_b
\end{aligned}
$$

$$\begin{aligned}
0 &\leq \pi_a + \pi_b + \pi_d \leq h_a + l_b \\
\max\{l_a, l_b\} &\leq \pi_a + \pi_b + u_1 \leq h_a + l_b \\
\max\{l_a, l_b\} &\leq \pi_a + \pi_b + u_2 \leq h_a + l_b \\
h_a &\leq \pi_a + \pi_d + u_1 \leq h_a \\
h_a &\leq \pi_a + \pi_d + u_2 \leq h_a \\
l_a &\leq \pi_a + u_1 + u_2 \leq h_a + l_b \\
h_b &\leq \pi_b + \pi_d + u_1 \leq h_a + l_b - l_a \\
h_b &\leq \pi_b + \pi_d + u_2 \leq h_a + l_b - l_a \\
l_b &\leq \pi_b + u_1 + u_2 \leq h_a + l_b \\
0 &\leq \pi_d + u_1 + u_2 \leq h_a + l_b
\end{aligned}$$

$$\begin{aligned}
\max\{h_a, h_b\} &\leq \pi_a + \pi_b + \pi_d + u_1 \leq h_a + l_b \\
\max\{h_a, h_b\} &\leq \pi_a + \pi_b + \pi_d + u_2 \leq h_a + l_b \\
l_a + l_b &\leq \pi_a + \pi_b + u_1 + u_2 \leq h_a + l_b \\
h_a &\leq \pi_a + \pi_d + u_1 + u_2 \leq h_a + l_b \\
h_b &\leq \pi_b + \pi_d + u_1 + u_2 \leq h_a + l_b
\end{aligned}$$

The unique nontrivial value partitions are $\mathcal{P}^*_{1,2} = \{\{a, d, 1\}, \{b, 2\}\}$ and $\mathcal{P}^*_{2,1} = \{\{a, d, 2\}, \{b, 1\}\}$. Take $\mathcal{P}^*_{1,2}$, so that $w^*_a = \max\{l_a - l_b, 0\}$, $w^*_b = \max\{h_b - h_a, 0\}$, $w^*_d = h_b - l_b$ and $w^*_1 = w^*_2 = 0$, whereas $w^*_{-a} = h_a - (h_b - l_b)$, $w^*_{-b} = l_b$, $w^*_{-d} = h_a - l_a$, $w^*_{-1} = h_a - \max\{l_a - l_b, 0\} - (h_b - l_b)$ and $w^*_{-2} = l_b - \max\{h_b - h_a, 0\}$.[43] The analysis for $\mathcal{P}^*_{2,1}$ is identical to that of $\mathcal{P}^*_{1,2}$ up to a relabeling of buyers, so recalling the assumption that $h_a + l_b > h_b + l_a$ yields

$$l_b - \max\{h_b - h_a, 0\} \leq h_a - \max\{l_a - l_b, 0\} - (h_b - l_b),$$

[43]Note that $w^*_{-d} = \min\{h_a - l_a, h_a - \max\{l_a - l_b, 0\}\} = h_a - l_a$.

and we therefore find that the competitive residual of buyer $i \in \{1,2\}$ is $w^*_{-i} = l_b - \max\{h_b - h_a, 0\} = h_a + l_b - \max\{h_b, h_a\}$.

We proceed as follows in order to show that these competitive intensities/residuals give exact bounds. We assume that we indeed have either $\pi_j^{\min} = w_j^*$ ($j \in \{a,b,d\}$) or $\pi_j^{\max} = w^*_{-j}$ ($j \in \{a,b,d\}$) or $u_i^{\min} = w_i^*$ ($i \in \{1,2\}$) or $u_i^{\max} = w^*_{-i}$ ($i \in \{1,2\}$), and find no violation of the conditions imposed by the set of inequality constraints above. This will show that the bounds are giving us admissible core allocations, and hence they must be exact, since they were shown to bound below/above an agent's competitive interval.

Using the inequalities that define the core of this game, it can also be readily verified that $\pi_a^{\min} = \max\{l_a - l_b, 0\}$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are any tuple of values $(\pi_b, \pi_d, u_1, u_2)$ such that $\max\{h_b - h_a, 0\} \leq \pi_b \leq l_b$, $h_b - l_b \leq \pi_d \leq h_a - l_a$, $0 \leq \pi_b + \pi_d \leq h_a + l_b - l_a$, $l_a - \max\{l_a - l_b, 0\} \leq u_1 \leq h_a + l_b - \max\{h_b, h_a\}$, $l_a - \max\{l_a - l_b, 0\} \leq u_1 + u_2 \leq h_a + l_b - \max\{l_a - l_b, 0\}$, $\pi_b + u_i = l_b$ for all $i \in \{1,2\}$ and $\pi_d + u_i = h_a - \max\{l_a - l_b, 0\}$ for all $i \in \{1,2\}$. Indeed, this also proves that $\pi_b^{\min} = \max\{h_b - h_a, 0\}$, $\pi_b^{\max} = l_b$, $\pi_d^{\min} = h_b - l_b$, $\pi_d^{\max} = h_a - l_a$ are sharp bounds, as are $u_i^{\min} = 0$ and $u_i^{\max} = h_a + l_b - \max\{h_b, h_a\}$.

Using the inequalities that define the core of this game, it can also be readily verified that $\pi_a^{\max} = h_a - (h_b - l_b)$ can arise as a feasible core allocation for firm $a$. The core payoffs for the other players are $\pi_b = l_b$, $\pi_d = h_b - l_b$ and $u_1 = u_2 = 0$.

Consider now the case in which firm $b$ merges with $d$, recalling that $V = \pi_a + \pi_{b'} + u_1 + u_2$ in this case, so $\pi_a + \pi_{b'} + u_1 + u_2 = h_b + l_a$. The following inequality constraints that $(u_1, u_2, \pi_a, \pi_{b'})$ must then satisfy:

$$
\begin{aligned}
0 &\leq \pi_a \leq l_a \\
0 &\leq \pi_{b'} \leq h_b \\
0 &\leq u_1 \leq h_b + l_a - \max(h_a, h_b) \\
0 &\leq u_2 \leq h_b + l_a - \max(h_a, h_b)
\end{aligned}
$$

$$
\begin{aligned}
0 &\le \pi_a + \pi_{b'} \le h_b + l_a \\
l_a &\le \pi_a + u_1 \le l_a \\
l_a &\le \pi_a + u_2 \le l_a \\
h_b &\le \pi_{b'} + u_1 \le h_b \\
h_b &\le \pi_{b'} + u_2 \le h_b \\
0 &\le u_1 + u_2 \le h_b + l_a
\end{aligned}
$$

$$
\begin{aligned}
\max(h_a, h_b) &\le \pi_a + \pi_{b'} + u_1 \le h_b + l_a \\
\max(h_a, h_b) &\le \pi_a + \pi_{b'} + u_2 \le h_b + l_a \\
l_a &\le \pi_a + u_1 + u_2 \le h_b + l_a \\
h_b &\le \pi_{b'} + u_1 + u_2 \le h_b + l_a
\end{aligned}
$$

It holds that $\mathcal{P}^* = \{\{a,1\},\{b',2\}\}$ is the unique nontrivial value partition (up to a relabeling of buyers). It is easy to show that $w_a^* = \max\{h_a - h_b, 0\}$, $w_{b'}^* = \max\{h_b - l_a, h_a - l_a\}$, and $w_1^* = w_2^* = 0$, whereas $w_{-a}^* = l_a$, $w_{-b'}^* = h_b$, $w_{-1}^* = l_a - \max\{h_a - h_b, 0\}$ and $w_{-2}^* = h_b - \max\{h_b - l_a, h_a - l_a\} = l_a - \max\{h_a - h_b, 0\}$. We proceed to show that these competitive intensities/residuals give exact bounds for competitive intervals of all agents.

Using the inequalities that define the core of this game, it can be readily verified that $\pi_{b'}^{\min} = \max\{h_a, h_b\} - l_a$ can arise as a feasible core allocation for firm $b'$. The core payoffs for the other players are $\pi_a = \max\{h_a - h_b, 0\}$ and $u_1 = u_2 = l_a - \max\{h_a - h_b, 0\}$, which also proves that $\pi_a^{\min} = \max\{h_a - h_b, 0\}$ and $u_i^{\max} = l_a - \max\{h_a - h_b, 0\}$ ($i \in \{1,2\}$) are sharp bounds. Similarly, it is easy to show that $\pi_{b'}^{\max} = h_b$ can arise as a feasible core allocation for firm $b'$, with the core payoffs for the other players being $\pi_a = l_a$ and $u_1 = u_2 = 0$, which also proves that $\pi_a^{\max} = l_a$ and $u_i^{\min} = 0$ ($i \in \{1,2\}$) are sharp bounds.

The case in which firm $a$ merges with $d$ is similar to the one in which firm $b$ is involved in the merger with $d$ after exchanging the subindices of firms $a$ and $b$, so the proof is complete.

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