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Abstract. The new perspective emerging from strategy's value-capture stream is that the effects of competition are twofold: competition for an agent bounds its performance from below, while that for its transaction partners bounds from above. Thus, assessing the intensity of competition on either side is essential to understanding firm performance. Yet, the literature provides no formal notion of "competitive intensity" with which to make such assessments. Rather, some authors use *added value* as their central analytic concept, others the *core*. Added value is simple but misses the crucial, for-an-agent side of competition. The core is theoretically complete but difficult to interpret and empirically intractable. This paper formalizes three, increasingly general notions of competitive intensity, all of which improve on added value while avoiding the complexity of the core. We analyze markets characterized by disjoint networks of agents (e.g., supply chains), providing several insights into competition and new tools for empirical work.

1. Introduction

Today, strategy boasts a rich and growing vein of theoretical findings based on the application of coalitional game theory.[1] This growth is due to a number of important features of the coalitional game formalism, including its explicit distinction between the value creation and capture aspects of market transactions, ability to represent the sort of unstructured dealings that are typical in real-world markets, built-in scalability with respect to market size, and operation at a level of abstraction that frees theorists from the need to commit to a myriad of special assumptions on the underlying details of value creation. The findings in this stream have resulted in numerous insights into *the* central question in strategy: What drives the economic performance of agents in competitive markets?[2] Of course, new insights invariably lead to new questions. In this paper, we tackle several such questions with the aim of providing further contributions. While our focus is on the stream within strategy proper, we believe several of our findings will be of interest to researchers outside of the field. Let us begin by identifying our motivating research questions.

Strategy is a field dominated by empirical work. Thus, one of the most important questions arising from this new stream is how its theoretical claims translate into empirical hypotheses. Presently, the theoretical work published in the strategy literature falls into two essential categories: work the primary analytical focus of which is on added value (e.g., beginning with the analysis of "added-value strategies" in Brandenburger and Stuart 1996) versus that with an explicit interest in the implications of the core (e.g., beginning with MacDonald and Ryall 2004). Both approaches provide insights into the effect of competition on the distribution of value within a market. The appeal of added value is its great simplicity. However, this simplicity comes at a cost: By ignoring the lion's share of competitive relations embedded in a coalitional game, analytical claims based on added value tend to be coarse at best and misleading at worst. Use of the core solves the problem of theoretical incompleteness but does so at the cost of information requirements that increase exponentially with the number of agents in the market. Thus, empiricists (and practitioners) who wish to apply the theory face a dilemma: opt for the easily-manageable-but-loose approach of added value or, alternatively, for the potentially-intractable-but-tight path of the core.[3] This dilemma raises the following question: Is it possible to construct an alternative theoretical tool that, on the one hand, provides greater analytical traction than added value while, on the other, mitigating the untenable information requirements of the core?

There is another issue that is closely related to the preceding one. The abstract nature of the core can

impede one's grasp of the economics driving theoretical results. For example, Aumann ([1973]) sets up a simple buyer/seller market and shows that, according to the core, the buyers are worse off forming a bargaining syndicate. So egregious is this violation of our economic sensibility that Aumann concludes that the core is a flawed solution concept. Later, Postlewaite and Rosenthal ([1974]) show that Aumann's conclusion was premature. As we will see, a reasonable explanation underpins this "paradoxical" result after all. Yet, if use of the core obscures the economics in the simple and straightforward setting of discrete buyer/seller markets—to a master of the theory the likes of Aumann, no less—one worries about our ability to grasp its implications in the kinds of more general settings of interest to strategy researchers. In an applied field such as ours, theoretical clarity is important. Thus, we ask: Can we develop new analytical tools designed to provide greater transparency to the economics at work under the general competitive logic of the core?

A third consideration arises from informal discussions in the literature regarding the indeterminacy of competition. Typically, even considering all of the competitive relations implied by a coalitional game under the core, the amount of value captured by each agent is—at best—narrowed down to a nontrivial interval. Although intervals are less convenient than the point estimates implied in more familiar settings (e.g., Cournot and Bertrand games), such indeterminacy may nevertheless be a feature of real-world markets:

> Taking the premises of the model seriously ... competition determines an *interval* within which an agent's value capture lies; where, within that interval, actual capture lands *is due to factors other than competition* ... the theory points toward a new conception of "competitive intensity" as well as the existence and possible importance of "persuasive" resources.
>
> (Gans and Ryall [2016], p. 5)

If competition's role in an agent's performance is to determine a *range* of performance possibilities, then the question arises whether that range is wide or narrow: That is, to what extent is competition the determining factor for a given agent's performance?[4] This has led to the suggestion (e.g., Gans et al. [2008]) that the right way to think about "competitive intensity" is not in terms of downward pressure on profitability (the traditional understanding) but, rather, in terms of the extent to which profitability is determined by competition per se.[5] Moving beyond informal speculation, the question is the following: What concrete form might a mathematical implementation of this suggestion take?

The contribution of this paper is that it provides answers to these questions. After a more detailed discussion of how the added value and core approaches have been used in strategy (next section), we proceed

to develop several variations on a new, formal notion of competitive intensity (Section [4]). We begin with a formalization that we refer to as *direct competitive intensity*. Our definition is based on simple unilateral deviations from a focal market structure.[6] It builds on the insight that added value to an economically productive group outside the firm's own implies competition *for* the firm—which, in turn, drives up the lower bound on the share of value the firm must capture from the economic activities of its group.[7] This answers all three of the preceding questions: (1) it extends the added-value approach in a significant way without approaching anywhere near the information demands of the core; (2) it helps illuminate the competitive economics at work in settings of interest to strategy scholars (e.g., more general than discrete buyer/seller markets); and (3) it provides a formal response to the informal speculation in strategy connecting competitive intensity to bounds on value appropriation. Due to its inherent simplicity, we imagine this notion will be the one most conducive to empirical work in strategy.

What then follows are two, increasingly complex variations on the initial notion of direct competitive intensity. In the first of these, we show (Section [4.2]) that the direct effects of competition imply certain indirect effects. This leads to a more refined definition, *indirect competitive intensity*. The underlying insight is that greater direct intensity for a rival makes it less attractive to its transaction partners (higher appropriation by the rival leaves less residual value for its partners). As a result, the relative attractiveness of the firm (to the rival's partners) tends to increase. This is also a form of competition for the firm, albeit an indirect one. We show that indirect competitive intensity illuminates and extends our understanding to more general settings, for example explaining the economics at work in the classic example of Postlewaite and Rosenthal ([1974]). Though more information is demanded by this definition, the implications in certain settings may be sufficiently promising to warrant its use in some empirical studies.

Our third and final variation, which we call *generalized* competitive intensity, pushes insight captured by the first two definitions to a natural limit (Section [4.3]). While this last concept may be too demanding for empirical work, it should be of interest to theorists as we use it to demonstrate conditions under which our approach leads to bounds that coincide with those of the core (though, as will be seen, the core is used as the primary benchmark against which our concepts are formally compared at all stages of our analysis).

Our three notions of competitive intensity provide alternative approaches to the assessment of value capture that, to varying degrees, stop short of the full computational requirements of the core—a feature that should be helpful for empirical applications. Like the

core, each identifies an interval of value capture for every agent. Moreover, each is consistent with the core in the sense that a core distribution also satisfies the conditions of each for all agents. Because the core accounts for *all* competitive relations in any situation described by a cooperative game, it is likely to remain popular in the development of general theory in strategy. Because they are closely related, our concepts provide useful insights into the economics at work in the core. The potential benefit of enhanced explanation is, itself, a theoretical contribution. Toward the recognition of this benefit, Section 5 uses a differentiated products model with capacity constraints to explore some familiar industry settings in which competitive intensity illuminates the essential competitive forces at work.

Finally, with respect to empirical analysis, we close this introduction with a word of caution. Any theoretical solution concept for cooperative games that is less information-intensive than the core runs a risk by leaving out some subset of competitive relations: in a given situation, the relations left out may have a substantial effect on the distribution of value. Thus, while our notions of competitive intensity are much simpler than the core and, typically, provide substantially stronger implications than added value alone, the intervals implied by our concepts are usually wider than those implied by the core. (As mentioned above, analyzing these differences explicitly is a central feature of our analysis.) Thus, care must be taken when applying our tools empirically. We say more about this in Section 4.4.

2. Coalitional Games in Strategy: Added Value vs. the Core

By way of selective literature review and an introduction to coalitional game theory (CGT), we use this section to discuss added value and the core—the two points on the dimension of theoretical complexity that bound the middle ground we wish to explore later in the paper. Essentially, all applications of coalitional game theory in strategy adopt one or the other of these two concepts as the foundation of their analyses. Both approaches have been used to provide important economic insights for strategy research. We highlight several of these as the discussion unfolds.

We begin by defining the objects in a coalitional game, then quickly move on to examining the first concept, added value. The simplicity of added value is why it appeals as an analytical tool. Yet, it is also why added-value analysis is incomplete and can lead to problems. By illustrating these, we set up our examination of the core which follows. As we show, while the core solves the problem of incompleteness, it does so by introducing substantial complexity and other barriers to practical use. Throughout the paper, we adopt

the following notational conventions. Sets are indicated with capital letters, while elements of sets, scalars, vectors, and functions are all represented by small letters. Technical terms are italicized at the point of definition.

Value creation. A *coalitional game* is a pair (N, v) in which $N \equiv \{1, \ldots n\}$, $n < \infty$, is the set of *agents* and $v: 2^n \to \mathbb{R}_+$ is the *characteristic function*. An arbitrary *agent* is denoted $i \in N$ and a *group* of agents by $G \subseteq N$. For any group G, $v(G)$ is the economic value that the group G can create via some collection of feasible, mutually agreeable transactions.[8] Let $v(\varnothing) \equiv 0$ and assume v is superadditive.[9] There is no reason for the members of G to produce less than $v(G)$ since, were that the case, everyone in G could be made strictly better off by producing the full amount. Therefore, under superadditivity, $v(N)$ is what the theory predicts is *the actual economic value created*. We highlight this feature via the special notation $V \equiv v(N)$.[10] We also adopt the convention of dropping set brackets when referring to specific group of agents as an argument of v; for example, given $\{i, j, k\} \subseteq N$, we abuse notation and write $v(i, j, k)$ rather than the more cumbersome $v(\{i, j, k\})$. If $i \in G$, then G_{-i} indicates G with i removed; if $i \notin G$, then G_{+i} denotes the set G with i added. Finally, without loss of generality, we assume that v is normalized: that is, for all $i \in N$, $v(i) = 0$.[11]

Value capture. A *distribution of value* is defined as an n-tuple, $\pi \equiv (\pi_1, \ldots, \pi_n) \in \mathbb{R}_+^n$. Here, π_i is a number indicating the amount of value captured (or "appropriated") by agent i in return for its participation in the value-creating activities that contribute to the production of V. In the case of a firm, π_i represents its economic profit. The objective in CGT applications is to understand how the productive alternatives elaborated by v shape π. A simple set of assumptions that relates v to π leads to the concept of added value.

Added value. Added-value analysis relies on three assumptions: (i) the aggregate value captured by the agents in N is V; (ii) the agents in N_{-i} must capture at least $v(N_{-i})$, else they could abandon i and, in the process, make themselves all strictly better off; and (iii) agents cannot be forced to take less than their outside value, lest they not participate in the market to begin with. Formally,

$$\sum_{i \in N} \pi_i = V, \tag{1}$$

$$\sum_{j \in N_{-i}} \pi_j \geq v(N_{-i}) \text{ and } \pi_i \geq 0 \quad \text{for all } i \in N. \tag{2}$$

Agent i's *added value* is defined as $av_i \equiv V - v(N_{-i})$. If π is such that (1) and (2) hold, it immediately follows that $\pi_i \leq av_i$ for all $i \in N$. Note well the analytical simplicity: for a given agent i, only two quantities, V and

$v(N_{-i})$, are required to compute added value. Moreover, the economic insight is immediate: when participation cannot be forced, as is the case in a free market, an agent cannot expect to capture more than the incremental value associated with its participation. This insight motivates the idea that firm strategies should aim to enhance added value.[12]

Unfortunately, the simplicity of added value is also its major drawback. To see why, consider a market with two firms, a and b, and two symmetric buyers, 1 and 2. The firms each have one unit of capacity and produce at zero cost. Buyers each demand one unit of product: they value firm a's product at 1 and b's at 0.3. The value created is $V = 1.3$, with either buyer dealing with either firm. If a is removed, b is left to transact with one buyer to produce 0.3. Thus, $av_a = 1$. Therefore, $\pi_a \leq 1$. Yet, reasoning beyond added value, we also predict $\pi_a \geq 0.7$. To see why, say a's buyer demands $\pi_1 = 0.3 + x$, with $0.7 > x > 0$. Then, even if b's buyer stands to appropriate *all* of the value in its deal with b ($\pi_2 = 0.3$), a could offer it $\pi_2 = 0.3 + \frac{1}{2}x$, which would make both (a and buyer 1) strictly better off. Thus, $\pi_a \in [0.7, 1.0]$, with the final outcome dependent on noncompetitive factors, such as haggling ability.

Suppose that a adds a social-networking feature for its product that creates a complementarity with b's. Specifically: each buyer values b's product at 1, provided that the other buyer has a unit of a's product. Otherwise, everything is exactly as before. Now (using primes to indicate the new game), $V' = 2.0$ and $av'_a = 1.7$. This is a successful "added-value strategy" in the sense that a increased its added value. Is greater value capture implied for a? Consider the distribution $\pi_1 = \pi_2 = 1$ and $\pi_a = \pi_b = 0$ (buyers capture all of the value). Does competition, in the sense of the alternatives elaborated by v, rule this out? The answer is no. Even though $\pi_a = 0$, a has no mutually improving offer to make to b's buyer (i.e., for buyer i, $v(a, i) = 1$, but $\pi_i = 1$ already). It can be shown that $\pi_a \in [0, 1.7]$: a wider range than before. Thus, if a is a bad haggler for example, it may be worse off as a result of this move—an issue missed by the added-value analysis.

The core. In the preceding example, conditions (1) and (2) limit attention to five groups. This leaves out 10 other groups—some of which have important implications, as we saw. To ensure that all such implications are taken into consideration, (2) can be generalized to

$$\sum_{j \in G} \pi_j \geq v(G) \quad \text{for all } G \subseteq N. \tag{3}$$

A distribution π is said to be *consistent with competition* when it satisfies (1) and (3). The set of all such π is commonly referred to as the *core*. For every $i \in N$, the core implies a *competitive interval*, denoted $[\pi_i^{\min}, \pi_i^{\max}]$, such that π is consistent with competition only if $\pi_i \in$

$[\pi_i^{\min}, \pi_i^{\max}]$ and, for every $\pi_i \in [\pi_i^{\min}, \pi_i^{\max}]$, there is at least one π containing it that is consistent with competition.[13]

The core has several positive features with respect to strategy theory. First, it eliminates the problem of omission associated with added value. Second, taking the possibility of nontrivial competitive intervals seriously leads to important insights, not the least of which are (i) competition bounds appropriation from *below* as well as from above; and (ii) beyond competition, *persuasive* resources—those that result in appropriation above one's competitive minimum—may also play an important role in firm performance. Finally, the elegant mathematics of the core (convex analysis) is well understood and extensively developed.[14]

In practice, however, the core suffers from its own drawbacks. The most significant of these is that its complexity increases exponentially with the size of N. To be clear, this is not a theoretical issue. As mentioned above, whether n equals two or a quadrillion, the math is elegant and tractable.[15] Rather, the combinatorial complexity is a problem in the *application* of the theory in practice and in the testing of its claims empirically. For example, a market with just 40 participants implies over a *trillion* possible groups.[16] Thus, even when N is relatively small, the sheer amount of data required to test the theory seems prohibitive. This problem is compounded by the fact that the lion's share of the groups, because they represent alternatives to the actual activities that generate V, are not observed. Thus, the $v(\cdot)$'s for these groups (counterfactuals) must be estimated in some way (perhaps, from other primitives). Then, even if these data issues are surmounted, a computational hurdle remains: identifying the competitive interval for a single, focal firm, requires the solution of two linear programming problems (for $\pi_i^{\min}$ and $\pi_i^{\max}$), each of which requires the inclusion of all of the group-related inequalities. Even three-letter government agencies might have trouble solving LPs with over a trillion constraints.

This raises a further concern: Apparently, to satisfy (3) in the real world, the agents in N are not only required to grasp an enormous number of different, potential ways of organizing their economic activities, but also to know both the values that these would produce and, simultaneously, the shares of V captured by all of the other market participants for the activities actually undertaken. In most settings, this seems far beyond the ken of any real human being.[17]

Actual empirical work. One of the exciting, recent developments in strategy is the entry of a number of authors whose work focuses on empirical analysis of the claims issuing from the theory stream. Not surprisingly given the preceding discussion, most of these use added value as their focal theoretical construct.[18] The empirical estimation problem associated with the

core is overwhelming, as Chatain and Mindruta (2017) point out. Therefore, if substantial progress is to be made, new theoretical tools must be developed for empirical scholars. These should provide guidance on the identification of a subset of groups that is, on the one hand, empirically tractable and, on the other, theoretically meaningful. It is to the development of such tools that we now turn our attention.

3. Value Partitions

We begin our analysis with a practical observation. When the agents engaged in one set of economic activities contemplate reorganizing to engage in some other, they must consider all of the costs associated with that reorganization. Unfortunately, this distinction is not always explicated in a cooperative game setting. In a model, V and $v(G)$ are typically said to be "the" economic value that can be created by N and G, respectively. In a real-world empirical setting, V is *data*—value actually created. If the agents in G transacted among themselves to contribute to V (say, as a supply chain), then $v(G)$ is also data. However, if the agents in G did not transact with one another, then $v(G)$ is a *counterfactual*, the outcome of transaction possibilities that could have occurred but did not. In the latter case, $v(G)$ must account for all of the economic costs associated with its production (e.g., switching costs). Thus, the real-world analyst must identify those groups actively engaged in the production of V versus those that do not form but could have done (and ensure that the values of v reflect these distinctions).

We respond to this need by taking the distinction between realized and unrealized groups as our starting point. A collection of groups in N, denoted $\mathscr{P}$, is called a *value partition* if: (i) $\mathscr{P}$ is a partition of N (i.e., division of N into disjoint subsets) and (ii) $V = \sum_{G \in \mathscr{P}} v(G)$. We refer to each element in a value partition as a *value network* and, henceforth, use X to denote such a group (as distinguished from an arbitrary group G). Agents in value networks are linked to one another via the transactions that give rise to some portion of the overall value created in the market, V.

For a given coalitional game (N, v), there may be multiple value partitions; that is, different configurations of disjoint groups may be capable of creating values that aggregate to V. As discussed at the beginning of this section, empirically, only one of these configurations instantiates to actualize V. Let us denote this configuration $\mathscr{P}^*$ and refer to it as the *focal value partition*. The elements of $\mathscr{P}^*$ are similarly highlighted with asterisks (X^*) and referred to as the *focal value networks*. The idea is that, in an empirical context, we have some way of identifying the X^*s, $v(X^*)$s, and, via aggregation, V. The *focal value network containing* agent i is denoted X_i^*.

With (N, v) understood from the context, the set of all value partitions is denoted $\mathbf{P}$. Note that the *trivial value partition*, $\{N\}$, is always an element of $\mathbf{P}$. At least one element of $\mathbf{P}$ corresponds to a partition that is "maximal" in terms of set cardinality—the aggregate value produced by any larger partition is less than V. Formally, a value partition $\mathscr{P} \in \mathbf{P}$ is said to be *maximal* if, for each $X \in \mathscr{P}$, there do not exist disjoint $S, T \subset X$ such that $v(X) = v(S) + v(T)$. Presuming that $\mathscr{P}^*$ is focal because it corresponds to empirical data does not necessarily imply that it is maximal (i.e., it may be possible to break an observed network of transacting agents into subgroups without changing the overall value produced). Therefore, it is worth mentioning that the results that follow apply to any value partition, including those that are not maximal. That said, a partition that corresponds to the most refined (maximal) parsing of observed transactions will yield the most refined conclusions.

4. Quantifying Three Shades of "Competitive Intensity"

We build on the idea of a focal value partition, which is amenable to real-world interpretation and useful in accounting for the relative frictions faced by agents considering alternatives to some status quo market structure. Our initial notion of direct competitive intensity is the simplest and, we conjecture, most useful idea from the standpoint of empirical applications. We expand this idea to indirect intensity which, while somewhat more complex, highlights an important implication of direct competitive intensity. Extending these implications to the theoretically maximum number of groups leads to our most general version of competitive intensity. This sequence of definitions is increasing in both computational difficulty and refinement of the bounds on a firm's range of competitive interval.

4.1. Direct Competitive Intensity

Assume $\mathscr{P}^*$ is given. Using this as our guide and retaining added-value constraints (1) and (2), we wish to identify a further subset of core constraints (3) that provides meaningful analytical traction while yet retaining much of the simplicity of the added-value construct. Obvious first candidates are those associated with the focal value networks themselves:

$$\sum_{i \in X^*} \pi_i \geq v(X^*) \quad \text{for all } X^* \in \mathscr{P}^*. \tag{4}$$

Invoking (4) means assuming that the members of a focal network cannot be forced to capture, in aggregate, less than the value produced by that network. The definition of a value partition combined with (4) implies

$$\sum_{i \in X^*} \pi_i = v(X^*) \quad \text{for all } X^* \in \mathscr{P}^*. \tag{5}$$

Thus, the agents of a focal value network capture among themselves precisely the value produced via their joint economic activities—an uncontroversial conclusion. Note also, since $\pi_i \leq av_i(N)$,

$$v(X^*) = \sum_{i \in X^*} \pi_i \leq \sum_{i \in X^*} av_i(N) \quad \text{for all } X^* \in \mathscr{P}^*. \quad (6)$$

When we say "the" added value for i, we refer to the added value as defined with respect to the market as a whole, $av_i(N)$. However, the notion of added value can be extended to any group: for any $G \subseteq N$, define $av_i(G) \equiv v(G) - v(G_{-i})$ if $i \in G$, and $av_i(G) \equiv v(G_{+i}) - v(G)$ otherwise. Given $\mathscr{P}^*$, an obvious quantity of interest is $av_i(X_i^*)$. The relevant core constraints are

$$\sum_{j \in X_{i-i}^*} \pi_j \geq v(X_{i-i}^*) \quad \text{for all } i \in N, \quad (7)$$

where X_{i-i}^* is i's focal value network with i removed. Expressions (5) and (7) imply

$$\pi_i \leq av_i(X_i^*) \quad \text{for all } i \in N. \quad (8)$$

In other words, just as an agent's added value to the market as a whole caps its ability to appropriate, so too does its added value to its *own* value network. This is consistent with our usual intuition about added value.

At this point, it is worth pausing to highlight the fact that $av_i(N) \leq av_i(X_i^*)$. The proof of this is simple. By the definition of added value, $av_i(N) = V - v(N_{-i})$. By the definition of the focal value partition and added value to a group, $av_i(X_i^*) = V - (v(X_{i-i}^*) + \sum_{X^* \in \mathscr{P}^* \setminus X_i^*} v(X^*))$. By superadditivity, $v(N_{-i}) \geq (v(X_{i-i}^*) + \sum_{X^* \in \mathscr{P}^* \setminus X_i^*} v(X^*))$. The conclusion follows immediately. For this reason, our formal definitions, which seek to identify the tightest bounds, use $av_i(N)$, rather than $av_i(X_i^*)$, as the relevant constraint on the upper limit of appropriation. That said, (8) should be kept in mind as it is often useful in working through the implications for a particular configuration of value networks.

Contrary to this intuition is the effect of i's added value to networks *outside* its own. To the extent that i has positive added value to a value network besides its own, that added value places a *lower bound* on its ability to capture value. This conclusion requires consideration of the following subset of core constraints:

$$\pi_i + \sum_{j \in X^*} \pi_j \geq v(X_{+i}^*) \quad \text{for all } i \in N \text{ and } X^* \in \mathscr{P}^* \setminus X_i^*. \quad (9)$$

We refer to the value networks outside of i's own ($\mathscr{P}^* \setminus X_i^*$) as i's *competitive periphery*. The value networks in an agent's competitive periphery compete against the agent to create value with its transaction partners but, simultaneously, against its transaction partners to create value with it. That is, external value networks compete implicitly with the agents who transact with one another within a given network. Thus, (9) includes

the constraints associated with the groups constructed by adding each i to each value network in its competitive periphery. When these hold as well,

$$\pi_i \geq av_i(X_{+i}^*) \quad \text{for all } i \in N \text{ and } X^* \in \mathscr{P}^* \setminus X_i^*, \quad (10)$$

which follows from (5) and (9). When agent i has added value to a network in its periphery, then those in X_i^* must impart to i a share of $v(X_i^*)$ sufficient to prevent i from leaving for a better deal in the alternate network—an amount no less than that added value.

As mentioned, others have emphasized the importance of the two aspects of competition highlighted by the core (i.e., its two-sided effect and its ambiguous nature).[19] Findings (8) and (10) are consistent with these observations. They do, however, impart an additional insight: the effect of an agent's ability to create value with a given group critically depends on whether the group is a focal value network containing the agent (i.e., the active network in which the agent participates) or an alternative to that network. The added value of an agent to the focal network containing it represents an *upper bound* on its ability to capture. For any group in which its membership is only potential, its added value represents a *lower bound*. Thus, the agents with whom the firm *is* transacting are qualitatively different from those with whom it *could* transact.

This refinement is missed in coarser treatments such as Porter (1979), which, for example, lumps all of a firm's buyers together as "competitors." What the preceding analysis shows is, first, it matters whether buyers are actual or potential. When the firm's actual buyers have the potential to add value to a peripheral value network, that network competes *for the buyers* (and against the firm). A firm's buyers do not compete with it—rather, external value networks do.[20] Second, potential buyers (e.g., those who would like to transact with the firm but cannot, say, due to capacity constraints) create competition *for the firm* and, thereby, may assure it positive appropriation.

With a bit of additional reflection, we come to our central idea. As we have seen, a firm's added value to a network in its periphery implies competition for it—the greater the added value, the greater the implicit intensity of that competition. Thus, scanning the firm's periphery and taking its maximum added value is one way of measuring competitive intensity for the firm.

Definition 1. The direct competitive intensity (DCI) for $i \in N$ in a focal value partition $\mathscr{P}^*$ is

$$w_i^* \equiv av_i(C_i^*), \quad \text{where } C_i^* \equiv \underset{X \in \mathscr{P}^* \setminus X_i^*}{\arg\max} \, av_i(X). \quad (11)$$

That is, $C_i^* \in \mathscr{P}^*$ is any value network in i's competitive periphery to which i adds maximum value (there may be more than one). We point out that computing w_i^* requires assessing i's added values to no more than

$n/2$ groups.[21] In actual industries, with large chains of raw material suppliers, parts manufacturers, original equipment manufacturers (OEMs), and distributors, the number of focal value networks will be much smaller. Note as well that i need know nothing about what other agents are appropriating to compute w_i^*. Since the logic of Definition 1 applies symmetrically for all of the agents in the firm's focal value network, and since the effects across agents are cumulative (whatever value is guaranteed to the firm's transaction partners is value prohibited to it), the aggregate of all of the other intensities is a measure of the intensity of competition against the firm. This leads to the following.

Definition 2. Given $\mathscr{P}^*$, the direct competitive intensity (DCI) for i's partners is

$$\sum_{j \in X_i^* \setminus i} w_j^*.$$

Then, define i's direct competitive residual (DCR) as

$$w_{-i}^* \equiv \min\left\{ v(X_i^*) - \sum_{j \in X_i^* \setminus i} w_j^*, av_i(N) \right\}. \quad (12)$$

In other words, i's competitive residual is what is left over from $v(X_i^*)$ after paying i's partners exactly their DCI, but no more than i's added value to N. One may think of a firm's competitive residual as the most "optimistic" payoff it can expect should it succeed in limiting its transaction partners to appropriating no more than warranted by their own competitive intensities.[22] This leads to the following proposition linking DCI to the added-value and core approaches common in the strategy literature.

Proposition 1. *Given a focal value partition* $\mathscr{P}^*$, *for all* $i \in N$,

$$\pi_i^{\min} \geq w_i^* \geq v(i), \quad and \quad (13)$$
$$\pi_i^{\max} \leq w_{-i}^* \leq av_i(N). \quad (14)$$

Proof. See Appendix A. □

Definitions 1 and 2 represent an operationalization of "competitive intensity" that has the desirable features of being conceptually consistent with core-based notions of competition emphasized in related strategy research, while using a much smaller set of constraints than required to compute the core, in a way that sheds new light on the underlying economics.[23] From Proposition 1, we see that a firm's DCI provides a conservative lower bound, and its DCR an optimistic upper bound, on competition's effect on its ability to capture value. If the maximal partition is the trivial one—i.e., is $\{N\}$—then this approach coincides with conventional added-value analysis.

How complex are the direct competitive intensity bounds versus calculations of added value or the core

interval? Suppose there are $n \geq 2$ agents participating in a market with $m \geq 2$ value networks.[24] Further, assume firm i's value network contains $z \geq 1$ agents. To compute av_i requires two quantities: V and $v(N_{-i})$. Calculating i's core interval requires $(2^n - 1)$ values: V and one for every nonempty, proper subset of N. In comparison, assessing the firm's DCI requires $2(m-1)$ values because the added value to each of the $(m-1)$ networks $X^* \neq X_i^*$ requires two pieces of information: $v(X^*)$ and $v(X_{-i}^*)$. To get i's DCR requires DCI to be calculated for the other $(z-1)$ agents in X_i^*. With the $v(X^*)$s already in hand, this requires an additional $(m-1)$ values. Additionally, the DCR requires three more values: $v(X_i^*)$ as well as V and $v(N_{-i})$ for av_i. Altogether, then, the intensity interval requires $K \equiv (z+1)(m-1) + 3$ values.

The *maximum* K arises when the networks in i's competitive periphery are all made up of pairs plus a one group (a singleton if n is odd or an empty group if the number is even, so $m = (n-z)/2 + 1$), in which case $K = (z+1)((n-z)/2) + 3$. This expression is concave in z and maximized at $z^* = (n-1)/2$, resulting in $K^{\max} = \frac{1}{8}(n+1)^2 + 3$. This means that, compared to $K^{\max}$, the number of values required to obtain the core is, approximately, four times greater when $n = 5$, 50 times greater when $n = 10$, and 1,000 times greater when $n = 15$. Moreover, keep in mind, $K^{\max}$ is an extreme upper bound; typically, K will actually be much smaller.

Some readers may find it surprising that neither the lower nor the upper bound on an agent's appropriation need directly depend on its added value to the market as a whole. However, it has long been known that the correspondence between added value and appropriation does not always behave according to intuition. Indeed, in some reasonable cases, increasing a firm's added value actually *decreases both endpoints* of its core interval—a result so counterintuitive that some have taken it as a prima facie argument against the use of the core as a solution concept. One of the contributions of DCI is that it provides insight into these objectionable "paradoxes," showing that they are not paradoxes at all but, instead, the outcome of perfectly reasonable economics. Indeed, grasping the economics of these special cases is a major step toward understanding competition more generally. A concrete example will be instructive.

Example 1. Consider a market for a homogeneous good sold by two firms, labeled a and b. Each of them has capacity of two units, which can be produced at zero marginal cost. There are three identical buyers, labeled 1, 2, and 3: each has unit demand and values the firms' products identically at $u > 0$. In this situation, $V = 3u$. The added value of each player, buyer or seller, is u. It can be shown that there is a single distribution of value in the core: $\pi_a = \pi_b = 0$ and $\pi_i = u$ for each buyer

$i \in \{1,2,3\}$. Intuitively, the slack capacity in the market causes the firms to compete for each other's buyers—to the point of driving prices to zero. This is precisely the example of Postlewaite and Rosenthal (1974).[25]

Let us see how close the bounds implied by DCI come to capture this outcome. Among the six nontrivial value partitions in **P**, suppose the focal one is $\mathscr{P}^* = \{\{a,1,2\},\{b,3\}\}$: firm a sells to buyers 1 and 2, firm b sells to buyer 3 (all possible value partitions are the same up to agent relabeling). To see that this is a value partition simply note that $v(\{a,1,2\}) = 2u$ and $v(\{b,3\}) = u$, so $V = v(\{a,1,2\}) + v(\{b,3\})$. Starting with buyers 1 and 2, $w_1^* = w_2^* = u$ since both have added value of u with respect to b's value network. Given the market configuration $\mathscr{P}^*$, firm b has excess capacity with which to attract one of a's buyers were that buyer to capture an amount of value less than u. For example, if firm a demanded a price of p from buyer 1 such that $u > p > 0$, then firm b could offer 1 a price of $p/2$, thereby making both it and the buyer strictly better off. Under this configuration, $w_3^* = 0$ because firm a is selling at full capacity. Finally, $w_a^* = w_b^* = 0$ because firms add no value outside their own networks: the products are homogeneous and all of the demand in each network is satisfied.

Proposition 1 states that $u \le \pi_1^{\min}, \pi_2^{\min}$. Given $v(\{a,1,2\}) = 2u$ and that the sum of the amount of value captured within a network must exactly equal the value it produces, we deduce $\pi_1 = \pi_2 = u$ and $\pi_a = 0$. Moving on to firm b's value network, all we get from Proposition 1 is $0 \le \pi_b, \pi_3 \le u$ (i.e., firm b and buyer 3 split the value they produce in some way). This is where the analysis stops based only on competitive intensity. Still, with a small number of calculations, the value captured by firm a and its buyers is pinned down exactly. Moreover, this improves on the standard added-value approach, which indicates only that the payoff of each agent is somewhere between 0 and u.

Summing up this section, we use the structure implied by a market value partition to show that having added value to a network outside one's own implies a floor to the quantity of value one must capture. This identifies network-level competition for an agent. We define the "intensity" of this sort of competition as the maximum of such added values. Proposition 1 shows that competitive intensity improves on added value in two ways. First, it provides a lower bound. This important, positive feature of competition is entirely missed by added-value analysis. Second, the upper bound is tighter than added value whenever the sum of the value added by individual members of a network to alternative networks is sufficiently high. Moreover, while the core interval is typically tighter, the information required to compute intensity bounds is bounded above by a quadratic function of n (rather than one that

is exponential in n). Just how much traction competitive intensity actually provides will depend on the particulars of the situation at hand. Still, given its simplicity and clear economic meaning, we conjecture that the preceding approach is one that empiricists and practitioners will find useful.

4.2. Indirect Competitive Intensity

Once the effect of competitive intensity is grasped, a further insight arises. Suppose a firm and a rival operate in separate value networks. The firm adds no value to the rival's network and, hence, enjoys no benefit of competitive intensity. Even so, what if the competitive intensity for the rival is so strong that the rival captures the lion's share of the value produced in its network? If the firm can create value with some of the rival's transaction partners above and beyond the meager amount that they must receive *once the DCI for the rival is taken into account*, then competition for the firm does, in fact, exist. If the firm is offered less value in its own network than the difference between the value it can create with its rival's transaction and the amount they look to capture in their own network, then the firm can cut a deal with them that makes them all strictly better off. Competition for the rival indirectly creates competition for the firm.

Formally, suppose agent i considers joining $X^* \in \mathscr{P}^* \setminus X_i^*$ as a replacement to some subset $Y \subset X^*$. Let $av_Y(X^*) \equiv v(X^*) - v(X^* \setminus Y)$, the value added by Y to X^*. By Proposition 1 agent $j \in Y$ cannot receive less than w_j^*. Moreover the agents in Y cannot, in aggregate, receive less than what they can produce on their own, $v(Y)$. Therefore, the agents in $X^* \setminus Y$ cannot capture more than

$$v(X^*) - \max\left\{\sum_{j \in Y} w_j^*, v(Y)\right\}. \quad (15)$$

Expression (15) presents an upper bound on the opportunity cost that the agents in $X^* \setminus Y$ bear by foregoing transactions with those in Y. Thus, the agents in $X^* \setminus Y$ should be willing to replace Y with i, provided i asks for no more than

$$v(i \cup X^* \setminus Y) - \left[v(X^*) - \max\left\{\sum_{j \in Y} w_j^*, v(Y)\right\}\right]$$

$$= av_i(i \cup X^* \setminus Y) - \left(av_Y(X^*) - \max\left\{\sum_{j \in Y} w_j^*, v(Y)\right\}\right). \quad (16)$$

Equation (16) is the value added by i to the agents in $X^* \setminus Y$ above and beyond the upper bound on their opportunity cost of abandoning the agents in Y. When this value is positive and larger than w_i^*, then the group $X^* \setminus Y$ generates further competition for i beyond w_i^*. Note that DCI corresponds to the particular case in which $Y = \varnothing$ (i joins X^* but replaces none of its agents). This leads to a definition that includes both indirect competitive intensity and indirect competitive residual.

Definition 3. Given $\mathscr{P}^*$, the indirect competitive intensity (ICI) for $i \in N$ is

$$s_i^* \equiv \max_{Y \subset X^*, X^* \in \mathscr{P}^* \setminus X_i^*} \left[av_i(i \cup X^* \setminus Y) \right.$$
$$\left. - \left(av_Y(X^*) - \max\left\{ \sum_{j \in Y} w_j^*, v(Y) \right\} \right) \right]. \quad (17)$$

The indirect competitive intensity for i's partners is

$$\sum_{j \in X_{-i}^*} s_j^*,$$

and the indirect competitive residual (ICR) of i is

$$s_{-i}^* \equiv \min\left\{ v(X_i^*) - \sum_{j \in X_{-i}^*} s_j^*, \, av_i(N) \right\}. \quad (18)$$

Once again, an agent's external alternatives create a lower bound on the value it must capture. Symmetrically, the competitive intensity for its partners places an upper bound on its ability to capture value. As always, an agent can never capture more than its added value. This leads to bounds that are (weakly) tighter than those in Proposition 1.

Proposition 2. *Given a focal value partition $\mathscr{P}^*$, for all $i \in N$,*

$$\pi_i^{\min} \geq s_i^* \geq w_i^*, \quad and \quad (19)$$
$$\pi_i^{\max} \leq s_{-i}^* \leq w_{-i}^*. \quad (20)$$

Proof. See Appendix B. □

By increasing the scope of competition for an agent, we can tighten our estimate of its appropriation—at the cost of increased information requirements. This is the case whenever there is sufficient substitutability between some agents in one network with some of the agents in another (which is likely to be the case, for example, with supply chains). At this point, we stop tracking the number of groups required to compute the bounds of interest. They are fewer than those required to estimate the core interval but more (possibly by a significant number) than those required to compute competitive intensity. Thus, indirect intensity may or may not be of practical use in real applications. Whether or not the data requirements are prohibitive will vary from situation to situation. Even so, we believe the theoretical insight into how competition for one agent indirectly generates competition for another is quite valuable. To illustrate, we return to Example 1.

Example 2. Under the focal value partition $\mathscr{P}^* = \{\{a, 1, 2\}, \{b, 3\}\}$, the notion of direct competitive intensity identified the exact competitive intervals for some but not all players. In particular, $w_3^* = 0$ left ambiguous

the split of value between firm b and buyer 3. Given $\mathscr{P}^*$, buyer 3 adds no value to the network $\{a, 1, 2\}$ because, under the deals implied by that network, firm a has no capacity remaining to make a sale to buyer 3.

However, firm a does compete for buyer 3 in a way that is not captured by w_3^*. The high competitive intensities for buyers 1 and 2 imply zero value capture by firm a. This leaves firm a highly motivated (in the sense of being willing to take even the smallest quantity of value) to replace either of its buyers with buyer 3. Indeed, firm a becomes indifferent to such a deal only when $\pi_3 = u$. Thus, competition from firm a drives up buyer 3's minimum level of appropriation in its transaction with firm b.

The competition for buyer 3 that is so generated by the high competitive intensities for buyers 1 and 2 is captured by our notion of indirect competitive intensity: we have that $s_3^* = u - (u - w_i^*) = w_i^*$ with $i \in \{1, 2\}$. Recall that $w_i^* = u$, and thus by Proposition 2, we have $\pi_3^{\min} \geq u$. However, as $av_3(\{b, 3\}) = u$, we have $\pi_i^{\max} \leq u$. Thus, $\pi_3 = u$ and, since the value captured within a value network must be shared among its members ($\pi_b + \pi_3 = u$), we also have $\pi_b = 0$.

Examples such as this have garnered considerable interest in economics when contrasted with the case in which all the buyers are merged into a single monopsonist. In our example, it can be shown that a merged buyer faces a core interval of $[u, 3u]$ while, in the disaggregated case, the core indicates that each buyer is guaranteed to capture exactly u (or $3u$ in aggregate). According to the core, then, the merger of buyers cannot be advantageous. As Aumann (1973, p. 1) states, "It seems intuitively obvious that in a monopolistic market, the monopolist has an advantage because he can avoid competition." Indeed, so obvious does Aumann consider this intuition that he regards counterexamples like this to be strong evidence against the intelligibility of the core as a solution concept. In response, Postlewaite and Rosenthal (1974, by means of an example similar to ours) show that the core *accurately* captures key differences in the underlying economics—competition may, indeed, work to the disadvantage of some syndicates.

Our notions of competitive intensity provide a transparent conceptual framework by which to understand the economics of these and more (cf. the opacity of the core), as well as a nice means by which to qualify them. Without resorting to the core, direct competitive intensity alone indicates that the disaggregated buyers must appropriate between $2u$ and $3u$ due to competition from firm b for buyers 1 and 2. Adding the indirect competitive intensity that arises as a result (firm a indirectly competes for buyer 3) pins down everyone's payoffs exactly. The economics are clear: merge the buyers and competitive intensities vanish. There is only one

value chain (both firms selling to the merged buyer) and hence zero competitive intensity of any kind. This is quite reasonable.

Before leaving this example, note that it also illuminates what competitive intensity leaves out (versus the core). According to the core, the merged buyer faces appropriation in the range $[u, 3u]$. The minimum is due to the surplus unit of capacity between the firms, which creates a form of within-network competition (the firms jostle to fill their respective capacities) that assures the merged buyer's capture of at least u. Since our competitive intensities are induced by external competition (which requires nontrivial value partitions), they do not pick this up. In the next section, we will allow for this.[26]

4.3. Generalized Competitive Intensity

Thus far, we have considered competition for an agent generated by external networks (direct competitive intensity) and by subsets of such networks affected by competitive intensities (indirect competitive intensity). We believe that this way of conceptualizing the effects of competition is natural, straightforward, and enlightening. Yet, as we saw in the immediately preceding example, these definitions fail to account for some forms of competition (e.g., within-network competition). Therefore, we now drop the requirement for a value partition and focus, instead, on the entire set of subsets of N. This extension approaches the core in its information demands and, as a result, is unlikely to be especially useful in real-world applications. That said, we present it now for theorists who may be interested in seeing a generalization of the basic ideas presented in Sections 4.1 and 4.2. Readers uninterested in these technical theoretical details may wish to skip this section.

Our definition of generalized competitive intensity builds on the premise that no group $G \subseteq N$ can receive more than the value it adds to the market as a whole, $av_G(N)$, and, moreover, no more than the aggregate of its agents' added values, $\sum_{j \in G} av_j(N)$. We define the general competitive intensity (GCI) for i by the threat to join a subgroup by paying the agents in that subgroup their full added value (i.e., to the market)—the most each could hope to appropriate. This provides an upper bound on the opportunity cost that the agents in the group bear should they forgo transactions with i. We wish to identify the set of groups for which this value is maximal. Formally,

$$C_i \in \arg\max_{G \subseteq N \setminus i}\left[v(G \cup i) - \min\left\{av_G(N), \sum_{j \in G} av_j(N)\right\}\right].$$

How does the preceding expression generalize our earlier ideas? By the definition of a value partition $\mathcal{P}$, $V = \sum_{X \in \mathcal{P}} v(X)$. Therefore,

$$v(X) = av_X(N) \quad \text{for any } X \in \mathcal{P}. \tag{21}$$

Recalling expression (6), for all $X \in \mathcal{P}^*$,

$$av_X(N) \le \sum_{i \in X} av_i(N).$$

The identification of C_i^* in Definition 1 is similar to that of C_i, with the difference being that the maximization in the definition of the former is restricted to $X \in \mathcal{P}^*$. As before, no group can capture a share of value greater than its added value to the market. This brings us to the following.

Definition 4. The generalized competitive intensity for $i \in N$ is

$$\underline{w}_i \equiv v(C_i \cup i) - \min\left\{av_{C_i}(N), \sum_{j \in C_i} av_j(N)\right\}. \tag{22}$$

From (21), the natural extension of w_{-i}^* is the generalized residual of $i \in N$:

$$\bar{w}_i \equiv \min_{G \subseteq N}\left[av_{G \cup i}(N) - \sum_{j \in G \setminus i} \underline{w}_j\right].$$

Denote the set of payoffs that are consistent with generalized competitive intensity by

$$GCI \equiv \left\{\pi \in R^n \;\middle|\; \sum_{i \in N} \pi_i = V, \underline{w} \le \pi \le \bar{w}\right\}, \tag{23}$$

where $\underline{w}$ and $\bar{w}$ are the vectors composed by the $\underline{w}_i$'s and $\bar{w}_i$'s, respectively. Stated this way, GCI can be seen to represent an alternative solution concept for coalitional games. Hence, we now establish its relationship to the core denoted by C.

Proposition 3. *The core is a subset of those payoffs that are compatible with generalized competitive intensity; i.e., $C \subseteq GCI$. Moreover, for any (focal) value partition $\mathcal{P}^*$, for all $i \in N$, $s_{-i}^* \ge \bar{w}_i \ge \pi_i^{\max} \ge \pi_i^{\min} \ge \underline{w}_i \ge s_i^*$.*

Proof. See Appendix C. □

Importantly, if the core for a market is nonempty, then the set of payoffs that are compatible with generalized competitive intensity for that market is also nonempty.

In our setting, the extreme points of GCI can be described by vectors that, themselves, can be used to determine whether the bounds implied by generalized competitive intensity fully characterize an agent's core interval. Quant et al. (2005) describe how to do this with a set of vectors they call the "larginals." Here, we take a related approach to characterizing the set of payoffs that are consistent with the notion of generalized competitive intensity.

Let ϕ be an ordered profile of the agents in N and $\phi(k) \in N$ be the agent in position k according to ϕ. Let Φ denote the set of all possible ordered profiles.

A *greedy residual vector* associated with $\phi \in \Phi$ is a distribution of the total value V constructed as follows: pay as many of the first agents indicated by ϕ their generalized residuals, provided that all agents receive at least their generalized competitive intensity. The set of agents in ϕ that receive exactly their generalized residuals is called the *front of* ϕ; the set of agents that receive only their generalized competitive intensity is called the *end of* ϕ. For any ϕ, there is at most one player that does not belong to the front or the end.[27]

Definition 5. The payoff vector π^ϕ is a greedy residual vector associated with the ordering $\phi \in \Phi$ if

$$\pi^\phi_{\phi(k)} = \begin{cases} \bar{w}_{\phi(k)} & \text{if } f_{\phi(k)} + \bar{w}_{\phi(k)} \leq V, \\ \underline{w}_{\phi(k)} & \text{if } f_{\phi(k)} + \underline{w}_{\phi(k)} \geq V, \\ V - f_{\phi(k)} & \text{otherwise,} \end{cases}$$

where

$$f_{\phi(k)} = \sum_{j=1}^{k-1} \bar{w}_{\phi(j)} + \sum_{j=k+1}^{n} \underline{w}_{\phi(j)}.$$

Notice from (23) that the convex hull of the greedy residual vectors fully characterizes the set of payoffs that are compatible with competitive intensity. That is,

$$GCI = \text{conv}\{\pi^\phi \mid \phi \in \Phi\}. \tag{24}$$

Typically, there are fewer greedy residual vectors than orderings because any two vectors with the same front and end are equivalent—independent of the agent ordering. As we have emphasized, identifying the agents' core intervals in most real-world markets is computationally prohibitive. However, it is a much simpler task to check if a *particular distribution* is, itself, in the core. If, in addition to checking whether the π^ϕs are in the core, we have $\bar{w}_i$ and $\underline{w}_i$ for each $i \in N$, the set of greedy residual vectors can be used to check whether competitive intensities identify the core intervals.

Proposition 4. *If some π^ϕ is in the core—i.e., if $\pi^\phi \in C$— then $\bar{w}_i = \pi_i^{\max}$ if i belongs to the front of ϕ and $\underline{w}_i = \pi_i^{\min}$ if i belongs to the end of ϕ. If, for all $\phi \in \Phi$, $\pi^\phi \in C$ then $DCI = C$: generalized competitive intensity captures the agents' competitive intervals exactly.*

Proof. Proof. See Appendix D. □

We may now state necessary and sufficient conditions for the core interval of every agent to be identified by generalized competitive intensity. Our result follows by observing that a similar characterization provided by Quant et al. (2005) applies to any upper and a lower bounds of the core payoffs (which we have shown to be the case for the $\underline{w}_i$'s and $\bar{w}_i$'s).

Proposition 5. *The payoffs compatible with generalized competitive intensity coincide with those that are consistent with competition (i.e., $GCI = C$) if and only if for each $G \subseteq N$,*

$$v(G) \leq \max\left\{\sum_{i \in G} \underline{w}_i, V - \sum_{i \in N \setminus G} \bar{w}_i\right\}. \tag{25}$$

Then, $\bar{w}_i = \pi_i^{\max}$ and $\underline{w}_i = \pi_i^{\min}$ for every $i \in N$.

Proof. See Appendix E. □

From Proposition 5, we make an important observation. If competitive intensity is sufficiently strong, then both elements on the right-hand side of (25) are high. This means that the inequalities are satisfied for each $G \subseteq N$. Therefore, generalized competitive intensity, when it is sufficiently strong, fully characterizes the core.[28]

4.4. Caveats

Earlier, we motivated the development of these notions of competitive intensity by asserting their usefulness on both empirical and theoretical grounds. We speculate that direct competitive intensity, the simplest of our objects, will be most useful for empirical work. For example, even when the added values to all of the value networks cannot be estimated, every added value that *can* be estimated has implications for value capture (i.e., our approach may provide useful empirical traction, even though implementation falls well short of its ideal). On the theoretical side, the core's implicit, abstract, and complex notion of competition often obscures the economics driving its conclusions. Our simpler concepts are more transparent and, as a result, may illuminate counterintuitive outcomes or bring newcomers to a speedier grasp of the essential economics associated with the core.

Still, the core is the present "gold standard" for strategy research in the area of value capture—precisely because it accounts for all of the relevant alternatives and, hence, for all of the competitive effects. Thus, our complaint about the incompleteness of added value and its potential to yield misleading results is also relevant with respect to our concepts. This danger is greatest with our simplest concept, direct competitive intensity, less so with indirect intensity, and least with generalized competitive intensity. How well our concepts approximate the true core bounds depends on the situation. Therefore, gaining a sense of the effects our concepts fail to pick up is important for those wishing to apply them. Since Proposition 5 fully characterizes the conditions for $GCI = C$, let us reflect on the limitations of our two simpler intensity concepts.

The first, and most obvious, concern is that none of DCI, DCR, ICI, or ICR account for *within-network* competition. To see the problem, return to our earlier example consisting of two firms, a and b, and two symmetric

buyers, 1 and 2. Recall that firms may sell one unit of product at zero cost. Buyers value firm a's product at 1 and b's at 0.3. Suppose the focal value partition has a transacting with 1 and b with 2, resulting in $V = 1.3$. The core interval for a is $[0.7, 1.0]$. Note that a has an added value to $\{b, 2\}$ of 0.7. Therefore, a's DCI is equal to 0.7. Its buyers have DCI equal to zero. Here, a's core interval corresponds to the one implied by DCI and DCR.

Suppose that firm i enters with a licensable technology that increases each buyer's utility by 0.5 for either product. To get the new aggregate value, V', i licenses to both a and b, who each sell a unit of product to a buyer. This generates $V' = 2.3$. What happens to a's competitive interval? Thus, $av'_a = 1.5$. Unfortunately, none of DCI, DCR, ICI, or ICR provide additional traction because the focal value partition is the trivial one—a single value network containing everyone. All we can say is $\pi'_a \in [0, 1.5]$. However, the actual core interval is $[0.2, 1.5]$. The true minimum, $\pi'^{\min}_a$, is driven up by the fact that a has valuable alternatives within its own network.

For example, suppose i initially demands 0.7 and each buyer insists on 0.8, leaving both a and b with nothing. Under this distribution of value, all agents meet their added value constraints. Even so, a can offer buyer 1 an alternative deal involving the two of them alone, resulting in $1 being created and shared between them. This is more than the $0 + 0.8 = 0.8$ they capture under the initial proposal. The extra $0.2 in value can be shared in a way that makes both better off. Moreover, since a can make this offer to either buyer, it must capture the difference, 0.2. Within-network competition for a drives up its minimum.

In cases like this, in which competitive alternatives exist for subsets of agents within a value network, an agent's core interval may be substantially narrower than the one implied by competitive intensity. The problem is worse the larger the values of these internal alternatives relative to the value created by the network. This also highlights the reason for choosing focal partitions that are as fine as possible (i.e., decomposing the market into the greatest number of value networks removes spurious, "within-network" alternatives). Note that our theory is easily extended to situations in which the alternatives available to subsets of within-network agents involve switching networks (buyers come to mind): such a subset must capture, in aggregate among its members, an amount equal to its maximum added value to the periphery.

Another, second class of problems arises when there are *multiple value partitions*. That is, suppose there are many ways of structuring deals in the market so as to result in the same quantity of aggregate value. Then, because our simpler concepts consider single-agent

deviations from the focal partition, implicit competition in the form of more sweeping reconfigurations are missed. For example, suppose there are five airlines in a market in which any three-airline alliance can produce $9 and any two-airline alliance can produce $6. Four- and five-airline alliances are not feasible. Suppose the alliances that actually form are $\{1, 2, 3\}$ and $\{4, 5\}$, producing $V = \$15$. Assume further that we are interested in the competitive interval for airline 4. Competitive intensity analysis tells us that airlines 1, 2, and 3 capture exactly $3. Each of them has competitive intensity of 3 (i.e., by joining $\{4, 5\}$) and what they capture must equal what they produce, $9. For them, DCI and DCR tell us everything we need to know. However, the same is not true for airline 4 (or airline 5): adding airline 4 to the three-airline alliance adds no value. Nor does indirect intensity refine our analysis. All we know is that airline 4's added value is 6. Of course, given all of the symmetry, airline 4's competitive range must be identical to that of the airlines in the larger group: $\pi^{\max}_4 = \pi^{\min}_4 = \$3$. However, to deduce this, we must consider a wholly different configuration than the one observed.

More generally, the problems that can arise are the situations in which the true binding alternatives in the core are not the ones contemplated by our concepts. Indeed, the alternatives restricting value capture (i.e., those constituting the binding core constraints) may involve a system of counterfactual networks that differs significantly from the ones observed. Therefore, applying these concepts in empirical settings does require thoughtful consideration, particularly on counterfactuals that are not included in the analysis but may, nevertheless, be shaping value capture. On the brighter side, keep in mind that the problem, should it arise at all, is always that the true bounds on value capture are tighter than the ones estimated using competitive intensity—to the extent our concepts indicate any bounds at all, they must be satisfied. Hence, competitive intensity should rarely fail to provide at least a certain measure of meaningful empirical content.

5. Application: Competition in a Duopolistic Market

We now employ a standard version of the Hotelling model of duopolistic competition, augmented to permit vertical differentiation and capacity constraints. We explore the implications of competitive intensity in this setting, the relationship between added value and firm performance, and how competitive intensity clarifies the effects of capacity constraints on firm performance.

5.1. Setup

On the supply side, there are two single-product firms, labeled a and b, that have the same constant marginal cost of production, normalized to zero. The range of possible product types is represented by the interval

[0, 1]. Denote the firms' product types as y_a and y_b, respectively. Assume $y_a = 0$ and $y_b = 1$. Also assume that capacities are denominated in whole units. We adopt the following notational conventions to simplify the discussion. Firm $r \in \{a, b\}$ has capacity q_r. In addition, $\Pi_r \equiv [\pi_r^{min}, \pi_r^{max}]$ denotes its competitive interval; $W_r^* \equiv [w_r^*, w_{-r}^*]$ its interval of value capture implied by DCI/DCR; and $S_r^* \equiv [s_r^*, s_{-r}^*]$ the interval implied by ICI/ICR.

On the demand side, there are three buyers, labeled 1, 2, and 3. Each buyer demands one unit. Buyer $i \in \{1, 2, 3\}$ has ideal product type $x_i \in [0, 1]$. Assume $x_1 = 0$, $x_2 = \frac{1}{2}$, and $x_3 = 1$. Thus, each firm faces a buyer who is one unit distant from its product, another who is half a unit away, and a third for whom its product is ideal. The utility buyer i obtains from consuming product type y_r is $u_i(y_r, l_r) \equiv l_r - t|y_r - x_i|$, where $|y_r - x_i|$ is the distance of buyer i's most preferred product from the one firm r offers. The scalar $l_r > 0$ is a vertical differentiation parameter—larger values of l_r are preferred by all buyers. The degree of horizontal differentiation is captured by the parameter $t > 0$. Larger values of t imply greater disutility from a product that is not ideally located.[29]

Assume the following with respect to utility. Product y_a is vertically superior to y_b: $l_a > l_b \geq t$. Both firms' products generate nonnegative utility for all buyers. Buyer 3 prefers product y_b over y_a given a choice between them: $t \geq l_a - l_b$. Thus, firm a's product may be vertically superior, but not "too much" so. Note: because $l_a - l_b > 0$, $q_a + q_b \geq 3$ implies that there is always a unique value partition that is not trivial. Also observe that vertical differentiation vanishes as l_a approaches l_b.

5.2. Preliminary Results

With three buyers, it suffices to consider capacity levels of 1, 2, or 3 for each firm. Limiting attention to the cases in which both firms operate and all buyers purchase results in six capacity scenarios: $(q_a, q_b) \in \{(1, 2), (2, 1), (2, 2), (3, 2), (2, 3), (3, 3)\}$.[30] The aggregate value produced in this market depends on the firm capacities; let V_{q_a, q_b} denote the value produced in scenario (q_a, q_b). Thus, in scenario $(1, 2)$, a sells to buyer 1 and b to the other two. Aggregate utility is $V_{1,2} = l_a + l_b - t|1 - 0.5| + l_b$, or $V_{1,2} = l_a + 2l_b - 0.5t$. The value network that delivers this is unique—a and buyer 1 form one network ($v(X_a^*) = l_a$), b and the other buyers form the other ($v(X_b^*) = 2l_b - 0.5t$). Once a has two or more units of capacity (i.e., all of the other cases), it always sells to buyers 1 and 2, with b picking up buyer 3. In these cases, $V_{q_a, q_b} = l_a + l_a - t|0 - 0.5| + l_b$, or $V_{q_a, q_b} = 2l_a - t|0 - 0.5| + l_b$ ($q_a \neq 1$; $q_b \neq 2$). The network values are $v(X_a^*) = 2l_a - 0.5t$ and $v(X_b^*) = l_b$.

Given the small number of agents in this market, it is relatively easy to compute the exact core intervals

Table 1. Competitive Intervals for a and b in Each Case

Capacities		Π_a		Π_b	
q_a	q_b	Min	Max	Min	Max
1	2	$l_a - l_b$	l_a	0	$2l_b - 0.5t$
2	1	0	$2l_a - 0.5t$	0	l_b
2	2	0	$t + 2(l_a - l_b)$	0	t
2	3	0	$t + 2(l_a - l_b)$	0	t
3	2	0	$t + 2(l_a - l_b)$	0	$t - (l_a - l_b)$
3	3	0	$t + 2(l_a - l_b)$	0	$t - (l_a - l_b)$

for each firm. These are shown in Table 1. How well do our measures of competitive intensity capture the full effects of competition as implied by the core? The following propositions provide the answer, along with several insights about how competition is operating in each case.

Computing direct competitive intensities immediately reveals that they are sufficient to characterize the firms' competitive intervals in capacity cases $(1, 2)$, $(2, 1)$, $(3, 2)$, and $(3, 3)$.

Lemma 1. *If* $(q_a, q_b) \in \{(1, 2), (2, 1), (3, 2), (3, 3)\}$, *then* $\Pi_r = W_r^*$ *for* $r \in \{a, b\}$.[31]

For cases $(2, 2)$ and $(2, 3)$, DCI/DCR pin down the exact values of a's competitive interval, but it fails to do so for b.[32] The following comments apply to both cases. Because a is vertically superior, it sells its entire capacity to the two nearest buyers (1 and 2). However, because its capacity is exactly filled by those buyers, the competitive intensities for firm b and buyer 1 are zero. Putting b in a's value network adds no value. The transactions between firm a and buyers 1 and 2 create maximum value with those buyers. If firm b were to join that value network, it would sit idle. Thus, there is no direct competition for b. The same reasoning applies to buyer 3. All we can say at this point is that b and buyer 3 divvy up l_b in some fashion.

This is where indirect competitive intensity becomes useful. Buyer 2's minimum capture in her transaction with a is at least $l_b - 0.5t$ as a result of b having a unit of slack capacity with which to attempt to lure her away from firm a. Thus, from Proposition 1, we know $\pi_2 \geq l_b - 0.5t$. Now, suppose that b demands all the value from buyer 3, so $\pi_3 = 0$. Then, buyer 3 looks appealing to a as a replacement for buyer 2: a can, for instance, offer buyer 3 $\pi_3 = \frac{1}{2}(l_b - t)$, making them both strictly better off. This indirect competition for buyer 3 implies that buyer 3 captures $\pi_3^{min} \geq l_b - t$ in her deal with b, since a would lose $l_a - t/2$ and gain at least $l_b - t/2 + l_a - t$ by replacing 2 with 3. Of course, this immediately implies $\pi_b^{max} \leq t$, the ICR for b. Considering ICI/ICR for b and buyer 3 is sufficient to identify these agents' core intervals.

Lemma 2. *If* $(q_a, q_b) \in \{(2, 2), (2, 3)\}$, *then* $\Pi_r = S_r^*$ *for* $r \in \{a, b\}$.

In this example, the two simple measures of competitive intensity were all that was needed to figure out the firms' competitive intervals. There was no need to solve the much more complicated linear optimization problem with $2^5 - 1 = 31$ constraints for each of the six capacity scenarios. Indeed, once the interval for a and its buyers were characterized by competitive intensity, the indirect intensity measure had only to be computed for two agents—firm b and buyer 3.

5.3. Managerial Implications
5.3.1. Competitive Advantage. Notice from Table 1 that the vertical superiority of firm a over b does not consistently result in greater profitability. In all cases, a's competitive interval is nontrivial: there is always room for value capture above-and-beyond the minimum guaranteed by competition. This is true in spite of a's superior product and, in the five cases in which it has two or more units of capacity, a market share that is twice the size of firm b's. Indeed, in those cases, competition provides no guarantee of capture whatsoever. Whatever value a does capture in those cases is entirely due to its persuasive resources—those capabilities that enhance its ability to persuade buyers 1 and 2 to part with value beyond that required by competition.

To see the problem this poses to empirical research on firm performance, consider the cases in which a has at least two units of capacity and suppose that its persuasive capabilities are either nonexistent or ineffective with buyers 1 and 2. Then, if b has even a slight persuasive advantage vis-à-vis buyer 3, the empiricist observes $\pi_a < \pi_b$, even though $l_a > l_b$.[33] Empirically, a exhibits higher quality, greater market share, lower prices, and lower profit. An industrial organization economist might conclude that there are unobserved product quality or marginal cost parameters that, when taken into account, make a's offering inferior or more costly. A behavioral economist might conclude that a's managers are "leaving money on the table" due to some version of limited foresight. In this case, neither conjecture is correct—by missing this important dimension of the story, we fail to achieve robust explanations or reliable normative implications across markets.

Central to the strategy agenda is understanding the causes of persistent performance heterogeneity among firms. Analysis of competitive intensities adds some depth to this understanding. Consider the assumption that $l_a > l_b$. When a has two or more units of capacity, its DCR is greater than b's, resulting in the potential for greater value capture. CR shrinks when other firms compete for one's own transaction partners. This fits the traditional conceptualization of competition. When the industry has slack capacity, however, the standard intuition is far from the whole story. Slack capacity permits buyers to migrate to their most preferred firms.

This reduces the firms' DCIs—ultimately killing the side of competition that guarantees value capture. In the extreme case, any performance advantage arises as a consequence of superior persuasive resources. This distinction is a crucial component to understanding of performance differences.[34]

5.3.2. Added Value and Firm Performance. Managers are frequently exhorted to organize their activities so to maximize their added value. The potential danger of such admonitions—i.e., that moves designed to increase added value can cause a *strict reduction* in profitability—is well appreciated in the value-capture stream. Our results provide a clear illustration. If a's product is sufficiently close to b's, both in terms of horizontal and vertical differentiation, then starting from the $(q_a, q_b) = (1, 2)$ scenario with costless capacity expansions, adding one unit to a's capacity results in an increase in av_a but a decrease in *both* endpoints of its core interval.

To understand this, assume that a's product is "sufficiently close" to b's in the sense that $t < (2l_b - l_a)$. What happens when a adds one unit of capacity under scenario $(q_a, q_b) = (1, 2)$? First, av_a increases from l_a to $(2l_a - l_b)$. Yet, referring to Table 1, we see that $\pi_a^{\min}$ drops from $l_a - l_b$ to zero, and $\pi_a^{\max}$ drops from l_a to $t + 2(l_a - l_b)$.[35] This is explained by the two effects induced by the change in capacity: (1) DCI for a is reduced and (2) DCI for a's buyers is increased, thereby reducing DCR for a. Together, these effects cause a strict worsening of a's competitive interval—even though a creates strictly more value. Let us take each effect in turn.

First, consider the change in DCI for a. The resultant drop in $\pi_a^{\min}$ is not surprising. It is consistent with conventional models in which expanding capacity reduces the just-excluded buyer's willingness to pay, thereby changing the amount that included buyers can be charged. When a has one unit of capacity, the just-excluded buyer is 2. Buyer 2 values a's product $(l_a - l_b)$ more than b's product. Thus, in capacity scenario $(1, 2)$, competitive intensity for a is $w_a^* = (l_a - l_b)$.[36] Put simply, a can credibly threaten to replace buyer 1 with buyer 2 under any deal in which a gets less than $(l_a - l_b)$. When a's capacity increases to two, buyer 2 becomes a customer. Then, the just-excluded buyer is buyer 3. Unfortunately for a, this changes its DCI to $w_a^* = 0$. Firm a cannot create more value with buyer 3 than buyer 3 and b create together. Firm b does not clamor to replace either of a's buyers. Again, this effect conforms to the usual marginal-cost/marginal revenue trade-off that arises in familiar—e.g., Cournot settings.

Now, consider the surprising part of the example—the simultaneous drop in $\pi_a^{\max}$. Under capacity scenario $(1, 2)$, adding buyer 1 to the value network $\{b, 2, 3\}$ creates no additional value: b's capacity is already accounted for by buyers who value its product more. Therefore, $w_1^* = 0$—there is no competition

for buyer 1. When a adds a unit of capacity, buyer 2 switches to a. This is socially efficient—more economic value is created within the market. However, the change *frees up a unit of firm b's capacity*. As a result, both buyers 1 and 2 add value to b's new value network $\{b, 3\}$. In particular, DCI for buyer 1 increases to $w_1^* = (l_b - t)$: the most value firm a can capture from buyer 1 is reduced.

In general, when one firm's added capacity attracts new customers and, thereby, frees the capacities of other firms, there is a twofold competitive effect. On the one hand, the highest willingness-to-pay buyers leave the expanding firm's competitive periphery and become actual buyers. This increases the firm's added value as well as its market share but—most importantly—it decreases competitive intensity for it. On the other hand, freeing up capacity of rival firms increases the competitive intensity for its customers, both old and new. Persuasive capabilities held constant, the firm will experience a drop in profit.[37] Competition can be subtle!

6. Conclusions

We examine markets in which economic value is created by value networks, disjoint subsets of agents (e.g., independent supply or value chains). In this context, we define three notions of competitive intensity—all of which imply bounds on the amount of value agents may capture in a market setting. We demonstrate that, while these bounds are weaker than the ones provided by the core, they also require much less information. At the same time, they add substantial traction beyond added value (the present workhorse for empirical work in this line of research). We apply these ideas in a more concrete market setting to show how competitive intensity performs in more familiar circumstances. In this specific setting, even our weakest notion of competitive intensity may be sufficient to characterize the firm's competitive intervals. Throughout, competitive intensity provides strong intuition about how competition is actually operating in these cases.

There is much that remains to be done in this line of work. Our results make much use of the ability to divide a market into distinct value networks. While many markets may exhibit this structure, it is, nevertheless, special. Is there some notion of competitive intensity—one that simplifies the computation of competitive intervals and maintains the appealing logic of the core—that is applicable to markets that cannot be decomposed in this way? Presumably, one would have to consider *within-group* competition to begin to answer this question. This is a complementary question for future research.

We argue, along with others before us, that thinking in terms of competitive intervals is important for

strategy scholarship. Our competitive intensity concepts bring the focus on intervals front and center. By doing so, we emphasize the need to distinguish between resources deployed with productive intent versus those deployed with persuasive intent. This is a significant conceptual refinement to business strategy, one uniquely provided by the value-capture stream. What do "persuasive" resources look like in a real-world context of unstructured bargaining? Presently, it is hard to say. The theory states that this distinction is almost certainly central to explaining persistent performance differences among firms. Early empirical findings appear to support this claim (Grennan 2014, Bennett 2013). Thus, deeper theoretical inquiry into this issue appears promising for strategy work, ripe with the potential to refine our grasp of the causes of persistent performance heterogeneity between firms.

Appendix A. Proof of Proposition 1
We first show that $\pi_i^{\min} \geq w_i^* = \max_{X \in \mathcal{P}^* \setminus X_i^*} av_i(X)$. If this were not the case, i could always join any $Y \in \arg\max_{X \in \mathcal{P}^* \setminus X_i^*} av_i(X)$ (or refuse joining any value network if we do not have $w_i^* > v(i)$). Since we know that $v(Y) = \sum_{j \in Y} \pi_j$, a new value network formed by i and the members of Y would make all of them better off because of the positive value added by i, which would contradict the hypothesis that $\pi_i^{\min}$ is a possible core allocation.

Since it clearly holds that $\pi_i^{\max} \leq av_i(X_i^*)$ (otherwise, the other members of X_i^* would improve by excluding i), we conclude the proof by demonstrating that $\pi_i^{\max} \leq v(X_i^*) - \sum_{j \in X_{i-i}^*} w_j^*$. To show this, note that if $i \in X_i^*$ receives the maximal payoff it can get in a core allocation that we must still have that any other agent $j \neq i$ in X_i^* gets a payoff π_j at least as large as $\pi_j^{\min}$. Because $v(X_i^*) = \pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j$, we must then have that $v(X_i^*) \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j^{\min}$, so the result that $\pi_j^{\min} \geq w_j^*$ for all $j \in X_{i-i}^*$ yields that $\pi_i^{\max} + \sum_{j \in X_{i-i}^*} \pi_j^{\min} \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} w_j^*$. We therefore have $v(X_i^*) \geq \pi_i^{\max} + \sum_{j \in X_{i-i}^*} w_j^*$, which ends the proof.

Appendix B. Proof of Proposition 2
Note from the definition of s_i^* that choosing $Y = \varnothing$ would make $s_i^* = w_i^*$, so allowing Y to differ from $\varnothing$ implies that $s_i^* \geq w_i^*$ and $w_{-i}^* \geq s_{-i}^*$, so we just need to show that $s_i^* \leq \pi_i^{\min}$ and $\pi_i^{\max} \leq s_{-i}^*$. To prove that $\pi_i^{\min} \geq s_i^*$, suppose to the contrary

that $s_i^* - \pi_i^{\min} > 0$ and assume that $s_i^* > w_i^*$ to avoid triviality. Letting $X^* \in \mathscr{P}^* \backslash X_i^*$ and $Y^* \subset X^*$ be some maximizers of

$$av_i(i \cup X \backslash Y) - \left(av_Y(X) - \max\left\{ \sum_{j \in Y} w_j^*, v(Y) \right\} \right),$$

we must have

$$av_i(i \cup X^* \backslash Y^*) - \left(av_{Y^*}(X^*) - \max\left\{ \sum_{j \in Y^*} w_j^*, v(Y^*) \right\} \right) - \pi_i^{\min} > 0.$$

This means that the agents in X^* could exclude a subset $Y^* \subset X^*$ of agents and replace them with i, thus making themselves better off together with agent i. Because such a mutually profitable deviation cannot happen if the value distribution is consistent with competition,[38] a contradiction is obtained and hence we have that $\pi_i^{\min} \geq s_i^*$.

We conclude the proof by demonstrating that $\pi_i^{\max} \leq s_{-i}^*$. Letting $s_{-i}^* < w_{-i}^*$ to avoid triviality, the proof parallels the one showing that $\pi_i^{\max} \leq w_{-i}^*$ in Proposition 1.

Appendix C. Proof of Proposition 3

From the generalized competitive intensity, we have that for each $i \in N$, $S \subseteq N \backslash \{i\}$ and $\pi \in C$ that

$$\pi_i \geq v(S \cup i) - \sum_{j \in S} \pi_j,$$

and therefore in the particular case where $S = C_i$ we have that

$$\pi_i \geq v(C_i \cup i) - \sum_{j \in S} \pi_j$$

$$\geq v(C_i \cup i) - \min\left\{ av_{C_i}(N), \sum_{C_i} av_j(N) \right\} = \underline{w}_i.$$

Moreover, we also have, for any $S \subseteq N$,

$$\sum_{j \in S} \pi_j \leq av_S(N).$$

Thus, for every $i \in N$ and $S \subset N \backslash \{i\}$

$$\pi_i \leq V - \sum_{j \in N \backslash \{i\}} \pi_j \leq V - v(N \backslash S \cup i) - \sum_{j \in S} \pi_j \leq av_{S \cup i}(N) - \sum_{S \backslash \{i\}} \underline{w}_j.$$

Therefore, we also have that $\pi_i \leq \bar{w}_i$. Hence, for each $i \in N$ and $\pi \in C$, we have that $\bar{w}_i \geq \pi_i \geq \underline{w}_i$. Since in addition both the elements of DCI and C are efficient, it follows that $C \subseteq DCI$. Moreover, $\underline{w}_i \geq w_i^*$ since the set over which C_i is obtained contains the set used to obtain w_i^*. Since residuals are obtained using the respective competitive intensities, it also follows that $w_{-i}^* \geq \bar{w}_i$.

Appendix D. Proof of Proposition 4

We have shown $\bar{w}_i \geq \pi_i^{\max} \geq \pi_i^{\min} \geq \underline{w}_i$ for every $i \in N$. It follows that if $\pi^\phi \in C$, then those agents in its front and end receive, respectively, the upper and lower bounds of their competitive intervals. Moreover, since $C \subseteq DCI$, if the boundaries of the DCI lie in the C, it must be that $DCI = C$.

Appendix E. Proof of Proposition 5

Suppose that $DCI = C$, then for all $\phi \in N^\phi$ we have that $\pi^\phi \in C$ and therefore $v(S) \leq \sum_{i \in S} \pi_i^\phi$ for every $S \subseteq N$. We show that in this case, (25) is satisfied. Denote the set of first elements of a given ϕ by $N \backslash S$ and the last by S. There are two possibilities: (i) all of the elements of $N \backslash S$ receive their generalized residuals or (ii) not all elements of $N \backslash S$ receive their generalized residuals. In case (i) we have that

$$v(S) \leq \sum_{i \in S} \pi_i^\phi = V - \sum_{i \in N \backslash S} \pi_i^\phi = V - \sum_{i \in N \backslash S} \bar{w}_i$$

while in case (ii) we have that each player in S receives only the value of their individual generalized competitive intensity; that is,

$$v(S) \leq \sum_{i \in S} \pi_i^\phi = \sum_{i \in S} \underline{w}.$$

If we combine the two cases, we obtain (25). Conversely, assume that (25) holds for each $S \subseteq N$; then, since the core is convex, we only need to show that in each ordering ϕ the associated greedy residual vector $\pi^\phi \in C$. Thus,

$$v(S) \leq \max\left\{ \sum_{i \in S} \underline{w}_i, V - \sum_{i \in N \backslash S} \bar{w}_i \right\}$$

$$\leq \max\left\{ \sum_{i \in S} \pi_i^\phi, V - \sum_{i \in N \backslash S} \pi_i^\phi \right\} = \sum_{i \in S} \pi_i^\phi.$$

So, $\pi^\phi \in C$ and therefore the converse is also verified.

Endnotes

[1] Gans and Ryall (2016) provide a recent review of this line of work.

[2] We say "the" because a suitable answer to this question is needed to answer others, such as: Why are there persistent differences in firm performance?; Is firm performance the result of management choice or other factors?; What capabilities must a firm cultivate in a given situation to maximize performance?; and so on.

[3] As we discuss in greater detail in Section 2, the analytical implications of added value are, themselves, implied by the logic of the core (indeed, Brandenburger and Stuart 1996 already suggest the usefulness of the core for strategy theory, p. 22).

[4] If wide, then supracompetitive sources of value capture are a major determinant of firm performance. If narrow, then control of such sources is of little effect. Note that interest in the performance implications of competitive indeterminacy is recent, particular to the field of strategy, and a direct result of formally derived, theoretical insights. For an early empirical application, see Chatain and Mindruta (2017).

[5] Under the latter conception, extreme competitive intensity describes situations in which the interval determined by competition collapses to a single point: competition fully determines profit—high, low, or otherwise.

[6] This is similar in spirit to the way unilateral deviations are used in the concept of Nash equilibrium.

[7] In the following narrative, we generally refer to "the firm" as the agent of interest, in keeping with the usual focus in strategy. Readers should keep in mind, however, that the logic presented applies to *all* agents in the market, not just firms.

[8] The transactions required to produce $v(G)$ are "arms-length" in the sense that they are undertaken only if each member of G receives a share of $v(G)$ sufficient to induce it to engage with G and not some other group. In particular, central coordination or collusion is not implied.

[9] Superadditivity requires, for all $G, G' \subseteq N$ such that $G \cap G' = \varnothing$, $v(G \cup G') \geq v(G) + v(G')$: adding more agents to a group does not destroy value. This restricts attention to markets in which negative externalities can be avoided via an appropriate organization of activities.

[10] Coalitional game theory dates back to von Neumann and Morgenstern (1944). Far earlier, Edgeworth (1881) developed the idea that the arm's-length alternatives available to agents in a market influences not only the amount of value produced but also how that value is distributed.

[11] In empirical settings, $v(i)$ is the value of agent i's "outside option" in the specific sense of the value i could *appropriate* were it to engage in the next-best economic activity exclusive of the agents in N_{-i}. Normalization is useful in theoretical applications because $\pi_i > 0$ means i captures economic value. Empirically, nominal v (e.g., aggregate revenue minus costs) and π_i (e.g., cash flow) are preferable as they correspond more directly to the data one observes in the real world. Then, the economic value captured by agent i is $\pi_i - v(i)$.

[12] Brandenburger and Stuart (1996) are the first to apply CGT to strategy and argue in favor of added value as a strategic goal ("added-value strategies"). Economists use the term "agent marginal product" for the same concept, which was at the heart of the career-spanning research agenda by economists Makowski and Ostroy. Their work (e.g., Makowski 1980, Ostroy 1980) is acknowledged by Brandenburger and Stuart (1996, p. 23) as having had an important influence on their own (interested readers are referred to the excellent survey by Makowski and Ostroy 2001). Notable strategy contributions that also focus on added value include Adner and Zemsky (2006), Chatain and Zemsky (2007), Adegbesan (2008), and Chatain and Zemsky (2011).

[13] This assumes a nonempty core; see the following discussion.

[14] MacDonald and Ryall (2004) introduce core analysis to strategy, analyzing the power of competition to guarantee value capture. Since then, a number of papers have appeared that also use the core; for example, Stuart (2005, 2016), Brandenburger and Stuart (2007), Ryall and Sorenson (2007), and MacDonald and Ryall (2017).

[15] An actual theoretical concern is that the core may be empty; that is, given (N, v), there may be no π that satisfy (1) and (3). The conditions for core existence are well known outside of economics (see Bondareva 1962) and always amount to V being sufficient to satisfy (3). Stuart (1997), also publishing outside of strategy, shows that this is not an issue in a wide range of market settings.

[16] We thank R. Casadesus-Masanell for helpfully doing the math and sharing this fact.

[17] Though, this concern must be tempered. The actual number of constraints in (3) that are binding for an agent's interval may be small (as little as two), and these may be salient to those they affect. Additionally, consistent with theories of learning, agents may "grope their way" into the core over time (see, e.g., Fudenberg and Levine 1998).

[18] See Adegbesan and Higgins (2011), Chatain (2011), Bennett (2013), Jia et al. (2012), Obloj and Capron (2011), and Grennan (2014).

[19] For example, Gans et al. (2008) and Gans and Ryall (2016).

[20] To be sure, buyers want to grab as much value as possible. The distinction being made is between haggling (i.e., trying to persuade others to part with value within their competitive interval) and competition (i.e., being able to make a take-it-or-leave-it demand as a result of a credible alternative).

[21] Note that $\mathscr{P}^*$ is a disjoint collection of subsets of N that require at least two members to produce value. Agents who do not transact should be collected into a zero-value, "no-transaction" group. This latter possibility, which requires rounding $n/2$ up, is offset by the fact that X_i^* is not included in the computation of w_i^*.

[22] Notice that these definitions embody the spirit of immunity to unilateral deviations associated with the noncooperative game concept of Nash equilibrium.

[23] Proposition 1 holds for all value partitions. Therefore, our notion of competitive intensity can be strengthened by considering not only $\mathscr{P}^*$, but all maximal value partitions; that is, all maximal $\mathscr{P} \in \mathbf{P}$.

[24] When computing competitive intensities, the agent's outside option must be factored into its lower bound (e.g., $\pi_i^{\min} \geq 0$ when v is normalized). This is automatic when there is at least one value network in i's competitive periphery (since $\pi_i^{\min} \geq av_i(X^*) \geq 0$). In the trivial case in which the market consists of only one value network, we must consider i's added value to the empty group (i.e., $av_i(\varnothing) = v(i) = 0$ when v is normalized). Thus, $m \geq 2$.

[25] The authors thank an anonymous referee for suggesting this example (Stuart 2016 makes a similar connection to this paper).

[26] Finally, we note that iterating this logic may lead to sharper estimates of the effect of competition on appropriation. That is, one can pursue the following process for agent i: at step 0, compute $s_i^*(0)$ according to (17); at step 1, set $w_i^*(1) = s_i^*(0)$, compute $s_i^*(1)$ according to (17) using $w_i^*(1)$; continue with this procedure. Since this sequence is monotonically increasing and bounded above by $\pi_i^{\min}$ of the core, it has a limit that can be used to define an iterated measure of competitive intensity for i and for i's partners. The preceding algorithm provides a tighter bound of an agent's competitive interval, yet convergence to the core interval is not guaranteed. That said, in some fairly general settings (such as the differentiated product markets model we explore in Section 5), one round of iteration is all that is required to completely pin down the agents' core intervals.

[27] Our notion of greedy residual vector is inspired by the well-known concept of a "greedy vector." The *greedy vector associated with an ordering* $\phi \in \Phi$ is constructed by paying each agent her added value to the group consisting of the players that precede her in ϕ. The convex hull of the greedy vectors in Φ is called the *Weber set*. Greedy vectors have been used to characterize several solution concepts for coalitional games. For example, the Weber set coincides with the core when a coalitional game is convex. The Shapley value can be obtained by averaging over the set of greedy vectors.

[28] Our concept of generalized competitive intensity is also related to the notion of *core covers* (CC) (Tijs and Lipperts 1982). While we omit the proposition due to space limitations, it can be shown that $GCI \subseteq CC$.

[29] It is worth noting that our results in Section 5.2 do not depend on the assumption that disutility is linear in the distance between a buyer and a firm. All that is needed is increasing disutility in the distance between them.

[30] Keep in mind that the firms are asymmetric, so the implications of, for example, (1, 2) and (2, 1) are, likewise, asymmetric.

[31] The proofs to Lemmas 1 and 2 are included in the online supplement.

[32] The presence of b's excess capacity is a source of competition for a's buyers, which helps to nail down a's competitive interval.

[33] When $(q_a, q_b) = (2, 2)$, there does exist a distribution of value consistent with competition in which π_a is zero and π_b is small yet positive, as shown in the online supplement.

[34] This distinction is made as early as MacDonald and Ryall (2004). Empirical findings by Bennett (2013), Grennan (2014) suggest that persuasive resources can, indeed, play a central role in performance advantages.

[35] The change in $\pi_a^{\max}$ is a result of our assumption $t < (2l_b - l_a)$.

[36] Because adding a to the value network $\{b, 2, 3\}$ creates this much additional value by switching buyer 2 to a.

[37] Indeed, when firms are able to commit to capacity levels prior to engaging in market activities involving value creation and capture, excess capacity never arises (proof available on request).

[38] In terms of costs and benefits of such a deviation, note that the agents in Y^* obtain no less than $\max\{\sum_{j \in Y^*} w_j^*, v(Y^*)\}$ out of the value

$av_{Y^*}(X^*)$ they bring in to X^*, so excluding them from X^* costs no more than

$$av_{Y^*}(X^*) - \max\left\{\sum_{j \in Y^*} w_j^*, v(Y^*)\right\},$$

whereas the opportunity cost of having i join $X^* \backslash Y^*$ is $\pi_i^{\min}$. So the benefit of replacing Y^* with i for the subset of agents $X^* \backslash Y^*$, which equals $av_i(i \cup X^* \backslash Y^*)$, always exceeds the opportunity costs of such an action, which cannot happen if $\pi_i^{\min}$ is a payoff consistent with competition.

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