Managers, Training, and Internal Labor Markets

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Abstract

We propose a theory that emphasizes the role of managers for the production and allocation of human capital in firms. Managers invest time to train junior employees, and acquire information about the juniors’ abilities that is valuable for job assignments. This dual role of managers matters especially in the modern, multidivisional firm, whose internal labor market can be structured in two distinct ways. In a “siloh” or traditional job ladder, junior workers remain in the same division; in a “lattice,” they can also be assigned to another division. The prospect of losing a good worker to another division undermines a manager’s training incentives and may give her an incentive to misrepresent the information she provides about her workers. We show that because of these two agency problems, implementing a lattice to achieve better job assignments also leads to higher wage costs for the firm. As a result, either silos or a lattice can be optimal. Our comparative-statics analysis suggests that the recent trend for firms to implement lattices may be caused by increased product market competition, a tight managerial labor market, skill-biased technological change, and an increasing importance of general human capital.

Keywords: human capital, internal labor markets, multi-divisional firm, intra-firm mobility, training, careers, talent management, war for talent

JEL-codes: D2, D8, L2, M5
1 Introduction

The internal labor markets of modern, multidivisional firms resemble less and less the traditional job ladders studied by Doeringer and Piore (1971) and a vast literature in economics. Over the past 20 years, many firms have established “talent management” systems that aim to match employees to the positions they are best suited for, anywhere in the company. Divisional job ladders are often perceived as constricting “silos,” and consultants praise the virtues of “lattice”-like career paths with cross-divisional mobility. But if the advantages of cross-divisional mobility seem clear, then why were divisional silos, i.e., traditional job ladders, so common in the first place? And why is the trend towards promoting greater cross-divisional mobility a relatively recent one?

We seek to answer these questions, and in the process highlight the overlooked but fundamental role that managers, or bosses, play for both the creation and allocation of human capital in firms. Managers create human capital by training and mentoring their employees, and they hold private information about employees that is important for their efficient allocation to positions. Both roles lead to agency problems at the managerial level that play out very differently without and with cross-divisional mobility of employees. Our paper contributes to the theory of human capital, and extends the theory of internal labor markets to questions that many of today’s large companies grapple with. Let us begin by explaining the dual role of managers, and then turn to its consequences for the organization of an internal labor market.

Managers are involved in the creation of human capital in firms through their efforts to recruit, train, and mentor employees. These efforts are part of day-to-day interaction with employees, and are difficult or impossible to centralize (Whittaker and Marchington, 2003; Perry and Kulik, 2008). Formal training may be overseen by Human Resources departments, but its implementation is often in the hands of employees’ managers. Mentoring can take the form of direct advice to an employee, or of delegating a problem as an opportunity to learn, when in the short run it would be more efficient for the manager to handle the problem.

1“The term silo is a metaphor suggesting a similarity between grain silos that segregate one type of grain from another and the segregated parts of an Organization” (Rosen, 2010).
The costs of developing human capital, therefore, are borne not only by employees themselves and “the firm,” as according to Becker’s (1964) standard distinction, but partly by managers, which creates a moral-hazard problem at the managerial level because training and its outcomes are difficult to measure. The main reward from training, typically, is to have a good employee on the team, which benefits the manager when individual contributions to team production are hard to distinguish; see Alchian and Demsetz (1972).

Managers are in addition involved in the allocation of human capital because in the course of working with their employees, they acquire private information about their abilities. While firms collect information about employees that can be made available to anyone, bosses still have residual private information, especially but not only about their employees’ “soft” skills. Managers therefore provide important input into employees’ job assignments, but may also be inclined to use their information to their own advantage, as Peter Drucker observed 60 years ago: “Nothing does more harm than the too common practice of promoting a poor man to get rid of him, or of denying a good man promotion ‘because we don’t know what we’d do without him’. The promotion system must ... make difficult alike kicking upstairs and hoarding good people” (Drucker, 1954, p.154-155). Private information about employees thus potentially creates an adverse-selection problem.

When internal labor markets are organized as vertical job ladders, or “silos,” these two agency problems are minimal because junior employees remain within their manager’s unit. Incidentally, the historical prevalence of silos may explain why the economics literature has not given much attention to the role of managers as gatekeepers in internal labor markets.

Large firms, however, are increasingly concerned with filling positions with the best people from anywhere in the organization. Since the days of Doeringer and Piore, many firms have become bigger, more multidivisional, and more global (cf. Roberts, 2004). Managerial skills have become less industry-specific (Murphy and Zabojnik, 2004), and returns to talent have become highly convex because of skill-biased technological change and scale economies (Kaplan and Rauh, 2013; Card and Di Nardo, 2002; Gabaix and Landier, 2008). Accordingly, filling top positions with top talent is widely considered necessary to stay competitive in a global economy. Today, “talent management,” originally a buzzword coined by an influential
1998 McKinsey publication (Chambers et al.), has become a self-declared priority of almost any global firm (Cappelli, 2008; Ready and Conger, 2007; Collings and Mellahi, 2009). It involves identifying and developing talented people, and placing them in positions in which they are most productive, including across divisional boundaries (Bryan et al., 2006; Conati and Charan, 2010). Following management scholars and practitioners, we will refer to an internal labor market with cross-divisional mobility as a “lattice.”

With a lattice, both agency problems discussed above have bite, because cross-divisional mobility of employees gives managers a reason to hoard good people and “kick upstairs” weaker ones, and undermines their incentive to train employees. We show that getting a lattice to work requires a redesign of the firm’s incentive system, and leads to overall higher agency costs for the firm compared with silos. The tradeoff between better job assignments for employees and greater agency costs for managers helps to explain both the historical longevity of silos and the difficulties that firms appear to encounter in transitioning to lattices. Our final set of results relates the choice between silos and a lattice to different parameters of our model and suggests connections with several real-world trends.

We model a firm that has two divisions and is headed by a CEO. Each division consists of a manager and a worker. Each division’s output depends on the abilities of the manager and the worker, where we assume that the manager’s ability matters more than the worker’s, consistent with other theories and with evidence (see our literature discussion below). A manager affects her division’s output both directly through production effort, and indirectly by training her worker, in line with the dual role of most real-life managers. Specifically, prior to production, the manager invests training effort to (stochastically) increase her worker’s ability. A worker’s ability is unverifiable, and is observable only to his division manager. The

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2 Conaty and Charan’s 2010 business bestseller “Talent Masters” is devoted to informal and formal ways in which firms can develop and identify talented employees, based on the experience of companies that have taken the lead.

3 See e.g. Cleaver (2012). The term “corporate lattice” (which has the same meaning), is a registered trademark of Deloitte LLP, see e.g. Benko and Anderson (2010). We thank Ricardo Alonso for bringing the term “lattice” to our attention.

4 We use the terms “division”, “CEO” and “manager” purely for convenience; our analysis applies just as well to any adjacent tiers in a larger hierarchy that generally satisfy the assumptions of our model.
division managers are risk-neutral and protected by limited liability. We consider different sets of feasible incentive contracts that are linear functions of both divisions’ outputs but may depend on whether or not a worker is transferred to the other division.\(^5\)

For exogenous reasons, a managerial position may become vacant. The CEO would prefer to fill it with a qualified worker from within the firm, or otherwise from outside. With silos (Section 4), only the worker in the same division is eligible for promotion. Both the moral-hazard problem and the adverse-selection problem described above are minimal: optimal contracts are based on own divisional performance only; each manager reaps the maximal returns from her training effort, knowing that her worker will stay in her unit; and there is no reason to misrepresent her worker’s ability to anyone. A lattice, in contrast, allows the firm to fill a managerial vacancy with a good worker from the other division, which we assume is preferred to hiring a manager from outside (otherwise silos would always be optimal).\(^6\)

Our analysis has two parts: we first show (in Sections 4 and 5) that while a lattice leads to better job assignments than silos, under weak conditions a lattice also leads to a higher wage cost for the firm. The choice between silos and a lattice then depends on balance of these two effects. In particular, silos can be optimal. The second part of our analysis (Section 6) examines how different parameters of the model influence the choice between silos and a lattice.

We begin our analysis with a benchmark result (Section 5.2): Under very restrictive conditions, agency costs are lower with a lattice than with silos, in contrast to our main message. Three conditions need to hold: (i) contracts can be contingent on the transfer of a worker to the other division, (ii) production effort is unimportant, and (iii) the CEO (or an HR department) can observe the workers’ abilities. The optimal incentive contract then consists of a lump-sum referral bonus that is paid if the manager’s (good) worker is

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\(^5\)To exaggerate our emphasis on middle managers, the workers are reduced in our model to non-strategic pawns in a game between their bosses and top management. In Section 8, we step outside of our model to discuss the workers' perspective.

\(^6\)We focus on promotions to another division in order to juxtapose the lattice with traditional job ladders. However, our argument is more general and applies to any instances—including lateral transfers—in which a worker is more productive in a different division.
promoted to the other division. The resulting wage cost for the firm is lower than with silos because a referral bonus directly rewards the event of developing a good worker through training, whereas with silos, training can be rewarded only indirectly through output-based incentive pay. Interpreting cross-divisional transfers as a form of “selective intervention” in the divisions’ affairs, this benchmark result illustrates that—contrary to the arguments of Williamson (1985)—selective intervention need not undermine the division managers’ incentives but can actually strengthen them, provided the managers can be given a stake in the benefits of the intervention.

However, if any of the above conditions fails to hold, then agency costs are higher with a lattice than with silos. This is most obvious if the firm is unable to reward transfers directly and wages can depend only on the divisions’ outputs (Section 5.5). Here, the prospect of losing a good worker to the other division directly undermines training incentives and requires the firm to pay greater output-based rewards to induce the same training effort as in silos. A similar but more subtle case is where contracts can be transfer-contingent but production effort is costly (Section 5.3). Here, too, the firm must reward own-division performance, which limits its ability to concentrate rewards for training on the transfer of a good worker.

Most importantly, though, when managers are privately informed about their workers, then under a weak condition, training is more costly to incentivize with a lattice than with silos, even if bonuses can depend on worker transfers and if production effort is irrelevant (Section 5.4). Simple referral bonuses are ineffective in this case because they would provide an incentive for managers to “kick upstairs” bad workers—a common concern with employee referral bonuses in many companies. Ensuring that a manager is willing to train and relinquish a good worker without being tempted to get rid of a bad worker too requires a combination of bonuses that raises the wage cost above the level of silos. This result echoes a key argument in the recent literature on decisions in firms that centralization is costly if it requires eliciting information from division managers.7

We proceed to examine how different parameters influence the choice between silos and a lattice (Section 6). We first show that the greater the value of divisional output, the

7See Alonso et al. (2008), Rantakari (2009, 2011), Dessein et al. (2010), and Friebel and Raith (2010).
more important it is to get the best people to the top, and for a sufficiently high value, a lattice dominates silos. One can interpret the value of output as the value of reducing costs or raising quality, which in turn has been linked to the degree of competition in product markets. Our subsequent results show that a lattice is more likely to dominate silos the larger the productivity of managers (possibly caused by skill-biased technological change), the more difficult it is to hire managers from outside, and the less division-specific human capital is, possibly reflecting an increase in the importance of general human capital. Together, the results help to explain why firms’ efforts to facilitate cross-divisional mobility are a relatively recent phenomenon, and why “talent management” within firms appears to go hand in hand with a “War for Talent” between firms (Chambers et al., 1998). Greater mobility of employees, in turn, may explain the apparent decline in training by bosses observed by Capelli (2011).

Overall, then, our results show that contrary to the popular impression that silos are a symptom of organizational dysfunction, they provide the best incentives for managers to recruit and train people. Breaking up silos to establish cross-divisional mobility is not simply a matter of asking managers who their good people are; it requires changes to the firm’s incentive system. The associated higher agency costs for the firm are worth incurring only if the value of finding the best employee for each high-level position is large enough.

2 Related Literature

Our paper relates to several strands of the literature. First, two key features of our theory, asymmetric information about workers and the possible “poaching” of good workers, are reminiscent of Waldman (1984). Waldman shows how firms optimally design promotions and wages to minimize poaching of good workers. He argues that firms know more about their workers than competitors, an assumption recently empirically supported by Kahn (2013). With asymmetric learning, promotions are less frequent and associated with larger wage increases than with symmetric learning. Extending the theory, De Varo and Waldman (2012) predict that the signaling value and hence the wage increase upon promotion is lower for highly educated workers, and find supporting evidence in the data of Baker, Gibbs, and
Our theory shares Waldman’s logic that asymmetric learning poses a barrier to poaching good workers, but we focus on asymmetric information and mobility within a firm. Empirically, firms do not learn perfectly about their employees, not even about those with long tenures (Kahn and Lange, 2014), while managers may have private information about their employees. Second, in our theory managerial moral hazard plays a crucial role and leads to distortions even in the absence of asymmetric information. Finally, the firm’s goal is to increase mobility within the firm rather than to reduce mobility across firms, leading to tradeoffs quite different from Waldman’s analysis. In particular, increasing worker mobility requires adjustments to managers’ incentives, rather than the workers’.

Other theories of internal labor markets, too, emphasize learning about workers (for instance, Gibbons and Waldman, 1999b) but assume that the firm does the learning. Our theory, in contrast, posits that bosses usually know more about their workers than do others in the organization.\(^8\)

Second, our theory focuses on managers as agents whose interests may diverge from the firm’s. Fairburn and Malcolmson (1994) and Prendergast and Topel (1996) have looked at biased evaluations by managers, and Carmichael (1985) and Friebel and Raith (2004) study bosses’ potential incentives to hire unthreatening but inferior subordinates. The present paper highlights what we believe to be a more fundamental incentive problem that concerns both the production and allocation of human capital, and leads to the joint determination of both incentive contracts and the organization of internal labor markets.

Third, our assumption that managers are more important than workers builds on theories such as Rosen (1982), Qian (1994) or Garicano (2000). The importance of “bosses” is also emphasized in recent empirical work by Lazear et al. (2012), who estimate that the average boss is 1.75 times as productive as the average worker.

Fourth, we focus on the demand side of internal labor markets, that is, decisions on how to fill vacancies. Slot constraints and “job vacancy chains” (created when filling one vacancy creates new vacancy elsewhere) have been studied in the industrial-relations literature; see

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\(^8\)Kim (2011), too, assumes dispersed information about workers. His model studies optimal contracting to achieve efficient evaluation of workers by peers.
Chase (1991) and Pinfield (1995). In contrast, most economic theories focus on the supply side, that is, decisions on how to allocate given employees to different possible positions. A prominent exception is Demougin and Siow (1994), which also focuses on the demand side but is otherwise very different from our paper. In particular, it does not consider managers’ incentives, which are central to our paper.

Beyond the literature on internal labor markets, our paper overlaps with a literature concerned with the tension between giving managers incentives to pursue the goals of their unit, and encouraging them to coordinate and communicate with top management or other divisions. Pioneering contributions are by Levitt and Snyder (1997) and Athey and Roberts (2001); more recent work includes that of Alonso et al. (2008), Rantakari (2008, 2011), Dessein et al. (2010), Friebel and Raith (2010), and Dessein (2014).

Finally, our results that link the choice between silos and lattice to product and labor market conditions relates to a diverse literature that studies the interaction between the internal organization of firms and their market environment. Theoretical papers include, for instance, Grossman and Helpman (2002), Raith (2003), Alonso et al. (2012), Gibbons et al. (2012), and Legros and Newman (2013). A rapidly growing empirical literature includes the work of Cuñat and Guadalupe (2005, 2009), Bloom and Van Reenen (2007), Rajan and Wulf (2006), Guadalupe and Wulf (2010), and Giroud and Müller (2010, 2011).

3 Model

Our model consists of a firm with two divisions headed by a CEO (male); see the organizational chart in Figure 1. Division $i = A, B$ consists of a manager $M_i$ (female) and a worker $W_i$ (male). The managers are the main strategic players in our model; the workers are not players in a game-theoretic sense (see Section 8 for a discussion of the workers’ perspective, where we step outside of our model). The job titles (CEO, manager) are chosen only for convenience; the model can more generally be interpreted as representing any two adjacent tiers of a multi-tier organization. Wage contracts are designed by the firm. The CEO acts in the firm’s interest in filling vacant manager positions.
3.1 Team production

Output in each division $i$ depends on the productivity of the manager ($q_i^m$) and the worker ($q_i^w$), and manager $M_i$’s production effort $x_i$. Both the manager and the worker can be either “good” or “bad”: $q_i^m, q_i^w \in \{q_g, q_b\}$, where $1 \geq q_g > q_b > 0$. Let $\Delta q = q_g - q_b$. These productivities will be affected by the managers’ training efforts and the CEO’s assignment decisions. The “team productivity” of division $i$ is defined as

$$t_i = \kappa_m q_i^m + \kappa_w q_i^w.$$  \hspace{1cm} (1)

We assume that $\kappa_m \geq \kappa_w$ to reflect the greater importance of the manager for division output. Division $i$’s output $y_i \in [0,1]$ is a random variable belonging to a family of distributions with c.d.f. $F(y,t)$ such that $E[y|t] = t$, and a full support $[0,1]$ for any $t$. The shift parameter $t$ is the division’s productivity $t_i$, times $M_i$’s production effort $x_i \in \{0,1\}$; thus $E[y_i|t_i] = x_i t_i$. An example of such a family of distributions is the Beta distribution with parameters $\alpha$ and $\beta$, with $\beta = 1$ and $\alpha = t/(1-t)$.

Our specification implies that division $i$’s expected output is increasing in the productivity of both $M_i$ and $W_i$, but it is impossible to infer the productivity of $M_i$ or $W_i$ from realized output. It follows that if a manager is compensated based on output, it is in her interest to have a good worker. Second, $\kappa_m \geq \kappa_w$ means that it may be in the firm’s interest to move
a good worker into vacant manager position where he can be more productive.

3.2 Timing

1. The firm appoints managers $M_A$ and $M_B$ (who may be hired from outside or transferred within the firm), offering each manager a contract whose resulting expected utility weakly exceeds her reservation utility. Each manager is good with probability $p_m$ and bad with probability $1 - p_m$; denote the expected productivity of a manager by $q_m = p_mq_g + (1 - p_m)q_b = q_b + p_m\Delta q$. Managers do not know their type when accepting the position, but learn their type afterwards; a manager’s type is therefore best interpreted as reflecting her match with the position (see also Friebel and Raith, 2004).

2. Each manager $M_i$ ($i = A, B$) hires a worker $W_i$ from a pool of ex ante identical agents and invests effort $e_i \in \{0, 1\}$ in training the worker, at cost $\tau e_i$ for $\tau > 0$. As a result, the worker is good with probability $p_i(e_i)$ and bad with probability $1 - p_i(e_i)$, where $p_i(1) = p_h$ and $p_i(0) = p_l < p_h$. Let $\Delta p = p_h - p_l$. As explained below, we assume that it is optimal for the firm to induce training effort $e_i = 1$. Denote the resulting expected productivity of the worker by $q_h = p_hq_g + (1 - p_h)q_b = q_b + p_h\Delta q$.

3. $M_i$ learns the type of $W_i$, which is her private information.

4. With probability $1 - \sigma$, each manager leaves for exogenous reasons (thus $\sigma$ is the probability of staying). The CEO then fills the vacant position by promoting a worker or hiring a manager from outside; see the next subsection.

5. Each manager $M_i$ (who is either the original manager, or if she left, a newly appointed one) invests production effort $x_i \in \{0, 1\}$ at cost $\xi x_i$, for $\xi > 0$.

6. Division $i$’s output $y_i \in [0, 1]$ is realized, which depends on the division’s team productivity and $M_i$’s production effort as described above.

Although training and production effort occur at different stages of the game, there are
no periods and no discounting; our model is technically a static multi-stage game.9

3.3 Vacancies and communication

There are two types of internal labor markets that differ in whether workers can be promoted only within their own division, or across divisions:

1. With “silos,” either the worker from the division with the vacancy is promoted, or a manager is hired from outside. A promoted good worker becomes a good manager with probability $\phi \leq 1$; a bad worker for sure becomes a bad manager.10 A manager hired from outside (through an unmodeled screening process) is good with probability $p_o$; denote the expected productivity of such a manager by $q_o = q_b + p_o \Delta q$. Since the initial manager, who is good with probability $p_m$, may have been hired from outside or promoted from within the company, it is natural to assume $p_m \geq p_o$. A promoted worker is replaced with a new worker from outside, who is is good with probability $p_w$. Lacking the training that was invested in the initial worker, the most natural assumption would be $p_w = p_l$; however, it suffices to assume more generally $p_w \in (0, p_h)$. Denote $q_w = p_w q_g + (1 - p_w) q_b = q_b + p_w \Delta q$.11

2. With a lattice, there is a third option, to promote the worker from the other division. In this case, a promoted good worker is a good manager with probability $\delta \phi$. The additional “cross-divisional” discount factor $\delta \leq 1$ allows for human capital to be partly division-specific. This constitutes an intermediate case between firm-specific and task-specific human capital; see Gibbons and Waldman (2004).

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9We separate the timing of training and execution based on the idea that the benefits of training tend to accrue with greater delay than those of many other managerial actions.

10If $\phi < 1$, a worker might be promoted “beyond his level of competence” — as stated by the “Peter Principle,” see Peter and Hull (1969), Fairburn and Malcomson (2001), and Lazear (2004).

11An alternative would be to assume that the manager trains the replacement worker at cost $\tau$, resulting in a good worker with probability $p_h$. The resulting effects on all incentive constraints would be the same as with our assumption $p_w < p_h$, but the latter specification is analytically much simpler.
Below (Section 3.5), we state parameter constraints that ensure that the CEO will want to fill a vacancy (1) ideally with a good worker from the own division, (2) alternatively, with a good worker from the other division, and otherwise (3) by hiring from outside.\footnote{Because the divisions are symmetric, it is never optimal to transfer a manager laterally to the other division. In contrast, if e.g. division 2 were more important, it might be preferable to transfer a proven \( M_A \) to division B instead of promoting a worker.}

To fill a vacancy optimally, the CEO needs to learn about the workers’ abilities through communication by the division managers. We assume that a departing manager truthfully reveals her own worker’s type because there is no reason to misrepresent it.\footnote{In our model, there is nothing at stake for the departing manager, so truthtelling is weakly optimal. In reality, managers departing by choice likely care about their legacy and therefore have a positive incentive to be honest about suitable successors.} In contrast, a manager who stays with the firm may have an incentive to “hoard” a good worker or “kick upstairs” a bad one. Consistent with casual observation of business practice, we assume that workers are unable to ascertain (or at least prove) their own suitability for a managerial position, see also Friebel and Raith (2004).

In our model, turnover of workers is generated only by manager departures. A bad worker is not fired even after his manager or the CEO has learned his type. That is, any vacancy at the manager or worker level must be filled, but replacing people because they are below average is prohibitively costly, possibly because newly appointed workers acquire task-specific human capital (Gibbons and Waldman, 2004) that is costly to replace. Enriching the model to allow for replacement of bad workers would reduce managers’ incentives to misrepresent a bad worker as a good one, i.e., weaken a truthtelling incentive constraint.

### 3.4 Payoffs and contracts

Managers are risk-neutral and are protected by limited liability; specifically, assume that \( M_i \)’s compensation \( w_i \) must be non-negative. If manager \( M_i \) stays with the firm, her utility is given by her wage, minus the cost of training and production effort:

\[
U_i = w_i - \tau e_i - \xi x_i.
\]

\( \xi \)
A manager accepts to work for the firm if her expected equilibrium utility exceeds her reservation utility. To simplify the analysis, we assume that the reservation utility is low enough such that the limited-liability constraint is binding and the participation constraint is not.

The firm’s profit from division $i$ is given by

$$\pi_i = Ry_i - w_i$$

for some $R > 0$. We assume that division outputs $y_1$ and $y_2$ are verifiable, and consider wage contracts that are linear in both. The linearity of the wage function implies that in computing expected profits and wages, only the expected value of $y_i$ matters; we therefore do not need any assumptions about the distribution of $y_i$ other than $E(y_i|t_i) = x_i t_i$.

We consider two kinds of contracts. First, output-based contracts that cannot be conditioned on anything else:

$$w_i = \alpha + \beta y_i + \gamma y_j$$

for $i \in \{A, B\}, j \neq i$. In this case, neither the workers’ types nor messages about them are verifiable; that is, the managers’ reports about their workers are cheap talk. The CEO makes promotion decisions that are ex-post optimal for the firm, based on the information available to him.\(^{14}\)

Second, we consider contracts that can in addition be based on the event of a transfer of the own worker to the other division:

$$w_i = \begin{cases} 
\alpha_n + \beta_n y_i + \gamma_n y_{j\neq i} & \text{if } W_i \text{ stays in division } i \text{ (is not transferred)} \\
\alpha_t + \beta_t y_i + \gamma_t y_{j\neq i} & \text{if } W_i \text{ is transferred (promoted) to division } j; j \neq i 
\end{cases}$$

with symmetric coefficients $(\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t)$ for each manager. The motivation for this case is that even if neither types nor messages are verifiable, the promotion of a worker to another division is arguably a verifiable event. (A standard argument in the internal labor markets literature is that wages can be tied to positions, and therefore presumably to changes in positions as well.)

Any manager who leaves receives her reservation utility $U$ and creates a vacancy that must be filled. We assume that a replacement manager works under the same incentive

\(^{14}\)If $Ry$ represents revenue or gross profit, it might seem more natural for incentive contracts to be based on $Ry$ than on just $y$. However, for our results this makes no difference. We chose to condition contracts on $y$ because that way, changes in $R$ (which play an important part in Section 6) have no effect on the contract, whereas in the alternative version the bonus parameters would be decreasing in $R$. 

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contract as an incoming manager at stage 1 of the game. In general, however, our results do not depend on the details of how replacement managers are paid.\footnote{Strictly speaking, since our model is static, replacement managers need not be given any training incentives. In an ongoing organization, though, new managers need to train workers too, which is why it is reasonable to assume that they inherit the same wage contracts.}

For our analysis, we can exploit the symmetry of the model to focus on one division only, say division A. To do so while keeping the bookkeeping clean, define division A’s profit, $\pi_A$, as the revenue from output in division A ($Ry_A$), minus the total compensation paid to $M_A$ or her replacement. Thus, $\pi_A$ includes possible payments to $M_A$ (the original manager or a replacement) based on division B’s output, but does not include payments made to $M_B$ based on A’s output.

### 3.5 Parameter conditions

We assume that with both silos and a lattice, it is optimal for the firm to induce both training and production effort; that is, $e_i = x_i = 1$. We refrain from stating the relevant parameter conditions formally; they amount to upper bounds on the effort costs $\tau$ and $\xi$. The analysis then consists of determining the cost-minimizing contract for each form of internal labor market, and to comparing the costs and benefits of inducing high effort between silos and a lattice. Little is lost by making this assumption. Without it, for high enough training effort cost (or low marginal benefit of effort), for instance, the firm would sometimes prefer a lattice with low training effort to silos with high effort. The tradeoffs at work, however, are the same as in our analysis below.

Two parameter conditions ensure that it is optimal to fill vacant manager positions as described above. To state them, we exploit the additive structure of (1) and denote by $t_{xy} = \kappa_m q_x + \kappa_w q_y$ the productivity of a manager-worker team with expected productivities $q^m_i = q_x$ and $q^w_i = q_y$, for $x \in \{g, b, m, o\}$ and $y \in \{g, b, h, w\}$.

The first parameter condition ensures that the firm prefers promoting a good worker from the other division over hiring a manager from outside. A necessary condition is that the division with the vacancy—whose own worker is bad, or else he would have been promoted—
must be better off, which requires
\[ \delta \phi t_{gb} + (1 - \delta \phi) t_{bb} > t_{ob} \]  
(2)

or equivalently, a positive net productivity gain:
\[ y_G \equiv \kappa_m (\delta \phi - p_o) \Delta q > 0. \]  
(3)

Condition (3) is equivalent to \( \delta \phi \geq p_o \), which means that a good worker from the other division has a greater chance of becoming a good manager than does an outside hire.

However, the other division loses a good worker and must hire a new one, which leads to a productivity loss of
\[ y_L = t_{mg} - t_{mw} = \kappa_w (1 - p_w) \Delta q. \]  
(4)

Promoting a good worker from the other division is optimal for the firm, then, if and only if \( y_G \geq y_L \), or
\[ (C1) \quad \kappa_m (\delta \phi - p_o) > \kappa_w (1 - p_w). \]

Because of \( \delta \leq 1 \), promoting a good worker in the same division to fill a vacancy weakly dominates promoting a good worker from the other division.

The second condition specifies when hiring from outside is preferred to promoting a worker. At the very least, an outsider must be preferred to promoting a worker known to be bad. A stronger but plausible condition is that the firm prefers to hire an outsider (with appropriate screening) than promote a worker whom it knows nothing about. Without additional information, the firm’s rational expectation about a worker’s productivity is the equilibrium value \( q_h \). Taking into account that a promoted good worker becomes a good manager only with probability \( \phi \), hiring from outside is preferred if
\[ \kappa_m q_o + \kappa_w q_h > \kappa_m [p_h \phi q_g + (1 - p_h \phi) q_b] + \kappa_w q_w \]
or\[ \kappa_m (p_o - \phi p_h) + \kappa_w (p_h - p_w) > 0, \]
a sufficient condition for which is
\[ (C2) \quad p_o > \phi p_h. \]  
(5)

Below, we assume throughout that (C1) and (C2) hold.
4 Silos

We begin by constructing the firm’s and the managers’ payoffs. A quantity of interest that appears in many results is the expected productivity of a division as of stage 1 of the game, taking into account the possible departure of the manager, and the stochastic outcome of the manager’s training effort. It can be recursively constructed as a weighted average of the resulting productivity if the manager stays, denoted \( t_0(p) \), and if she leaves and must be replaced (hence the subscript ‘r’), \( t_r(p) \), where \( p \in \{p_h,p_l\} \) is the probability of ending up with a good worker depending on the manager’s training effort:

\[
\begin{align*}
    t_S(p) &= \sigma t_0(p) + (1 - \sigma) t_r(p).
\end{align*}
\]

Its components are given by

\[
\begin{align*}
    t_0(p) &= pt_{mg} + (1 - p)t_{nb} = \kappa_m q_m + \kappa_w (q_b + p\Delta q) \quad (6) \\
    t_r(p) &= p[\phi t_{gw} + (1 - \phi)t_{bw}] + (1 - p)t_{ob}. \quad (7)
\end{align*}
\]

The expression for \( t_0(p) \) is simply the productivity of a team whose manager has an expected ability of \( q_m \), while the worker’s productivity depends on the manager’s training effort. The expression for \( t_r(p) \) captures the possible outcomes when a manager leaves and is replaced. If the manager has a good worker, the worker is promoted to manager but becomes a good manager only with probability \( \phi \). A new worker with productivity \( q_w \) is hired from outside. If the worker is bad, then the manager’s position is filled from outside while the worker stays in his position. Abusing notation, we will use \( t_0 = t_0(p_h) \), \( t_r = t_r(p_h) \), and \( t_S = t_S(p_h) \) whenever we focus on an equilibrium that induces high training effort.

With silos, the firm’s expected revenue in division A is simply \( Rt_S \). Contracts conditioned on transfers are irrelevant; so let us consider simple output-based contracts \((\alpha, \beta, \gamma)\). It is immediate, however, that setting \( \alpha = \gamma = 0 \) is optimal: There is no reason to choose \( \gamma \neq 0 \) because there is no interaction between the divisions. The salary \( \alpha \), in turn, has no effect on the manager’s incentives, and because by assumption the managers’ participation constraint is slack, it follows from limited liability that setting \( \alpha = 0 \) is optimal. The firm’s wage cost is therefore simply \( \beta t_S \).
Two incentive constraints need to be satisfied to induce both training and production effort. Managers exert production effort in stage 5 of the game, after any personnel reallocations have taken place. High production effort leads to a payoff for the manager of \( \beta t_t - \xi \), whereas low effort leads to a payoff of 0. The resulting incentive constraint \( \beta t_t - \xi \geq 0 \) is most restrictive for the worst possible team with productivity \( t_{bb} \). For this team, inducing production effort requires

\[
\beta \geq \frac{\xi}{t_{bb}} =: \beta^x.
\]

The same lower bound \( \beta^x \) for the own-division bonus applies to the case of a lattice analyzed in Section 5 below, regardless of contracting assumptions.

At stage 2 of the game, \( M_A \) invests in training her worker. At this point, she knows her own type \( \mu \in \{g, b\} \), but not whether she will stay or leave. If she stays, the expected division output at stage 2 of the game is \( t_\mu(p) = pt_\mu g + (1-p)t_\mu b \) depending on the probability \( p \) of training a good worker. \( M_A \)’s net payoff then is

\[
V_A^S(\mu, e, x) = \sigma[t_\mu(p(e))\beta - \xi x] + (1-\sigma)U - \tau e,
\]

where the two effort costs enter the payoff differently because the training effort \( e \) is invested before, but the production effort \( x \) after, \( M_A \) knows whether she will stay with the firm. The resulting training incentive constraint reduces to \( \sigma t_\mu(p_h)\beta - \tau \geq \sigma t_\mu(p_i)\beta \), which (noting that \( \frac{\partial t_\mu(p)}{\partial p} = \kappa_w \Delta q \) regardless of \( M_A \)’s type) is straightforward to solve for \( \beta \):

**Proposition 1** With silos, the minimal own-division bonus \( \beta \) that induces \( e_i = 1 \) is given by

\[
\beta^S = \frac{\tau}{\sigma \kappa_w \Delta p \Delta q}.
\]

If \( \beta^S \geq \beta^x \), then the firm’s optimal silo contract is given by \( (\alpha, \beta, \gamma) = (0, \beta^S, 0) \).

For all proofs, see the Appendix. The expression for \( \beta^S \) is intuitive. As in any model with binary effort, effort is easier to induce—and thus the optimal \( \beta \) is smaller—the smaller the cost of (training) effort \( \tau \) and the larger the value of effort \( \Delta p \Delta q \). In addition, in our setting, effort is easier to induce the larger is \( \sigma \), because the manager benefits from a good worker only as long as she (the manager) remains with the firm. Finally, effort is easier to induce the greater the importance of the worker, \( \kappa_w \), in the division’s production.
Table 1: Overview of results of Section 5

Let $\xi = t_{bb}\beta^S$; it is the value of $\xi$ where $\beta^x = \beta^S$. As indicated in Proposition 1, we will focus on the case $\beta^S \geq \beta^x$ where incentivizing training is the binding constraint, and therefore consider the range of production effort costs $\xi \in [0, \bar{\xi}]$. For larger values of $\xi$, (8) becomes binding, and incentivizing production effort automatically incentivizes training as well. Even then, however, the choice between silos and a lattice involves a tradeoff, unless the costs of production effort are so high that providing the necessary incentives makes training incentives irrelevant.

5 Lattice

For the lattice, we begin with deriving payoffs and constraints, and then derive a benchmark result that states conditions under which a lattice unambiguously dominates silos because it leads to better job allocations and lower wage costs. Our subsequent results show that relaxing any one of the conditions overturns the benchmark result by implying a higher wage cost with a lattice, creating a tradeoff for the firm. The connections among this section’s results are shown in Table 1.
5.1 Payoffs and effort constraints

With a lattice, the firm wants to promote a worker, say $W_A$, to become manager in the other division if and only if $M_B$ leaves (with probability $1 - \sigma$), $M_A$ stays (with probability $\sigma$, otherwise $W_A$ would be promoted to $M_A$), $W_A$ is good (probability $p_A$) and $W_B$ is bad (with probability $1 - p_B$, otherwise $W_B$ would be promoted). Thus, with probability $\sigma(1 - \sigma)p_A(1 - p_B)$, $W_A$ is promoted to $M_B$, which leads to a net productivity loss of $y_L$ in division A compared with silos, as discussed in Section 3.5. And with probability $\sigma(1 - \sigma)(1 - p_A)p_B$, $W_B$ is promoted to $M_A$, leading to a net productivity gain $y_G$ in division A.

Denote by $\nu = (1 - \sigma)(1 - p_h)$ the probability, say from $M_A$’s perspective, that a vacancy arises for the position of $M_B$ that will not filled by $W_B$ (who is bad with probability $1 - p_h$ if $M_B$ chooses high training effort). Then in equilibrium, when $p_A = p_B = p_h$, both promotion probabilities are equal to $\sigma v p_h$.

The firm’s expected wage cost for division A, $w^L_A$, can be determined by starting from the “silo case” without cross-divisional transfers, and adding and subtracting terms pertaining to a transfer of $W_A$ or $W_B$:

$$w^L_A = \alpha_n + t_S(\beta_n + \gamma_n) + \sigma v p_h \{\alpha_t + t_{mw}\beta_t + [\delta\phi t_{gb} + (1 - \delta\phi)t_{bb}]\gamma_t - \alpha_n - t_{mg}\beta_n - t_{ob}\gamma_n\} + \sigma v p_h (y_G\beta_n - y_L\gamma_n).$$

The first line of (11) corresponds to the silo case, allowing for an other-division bonus $\gamma_n$. The second line is the probability that a good $W_A$ is promoted to a vacant $M_B$ position, times (in {}) the associated wage cost, net of what it would be in the silo case. The third line of (11) covers the case in which $M_A$ leaves and is replaced by $W_B$, in which case the new $M_A$ by assumption inherits the contract $(\alpha_n, \beta_n, \gamma_n)$ and the wage change is determined by the productivity gain $y_G$ in division A and the loss $y_L$ in division B.

The manager’s incentive constraint for production effort is the same as for silos:

$$\beta_n, \beta_t \geq \frac{\xi}{t_{bb}} = \beta^*.$$  

(12)
$M_A$'s training incentive constraint can be derived from her expected payoff at stage 2 of the game, as function of training effort $e$ and assuming that in equilibrium, $p_B = p_h$ and $x_A = x_B = 1$:

$$V_A^L(\mu, e) = \sigma \{ \alpha_n + t_\mu(p(e)) \beta_n + t_S \gamma_n - \xi \} + (1 - \sigma) L - \tau e$$

$$+ \sigma v p(e) \{ \alpha_t + t_{\mu w} \beta_t + [\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}] \gamma_t - \alpha_n - t_{\mu g} \beta_n - t_{\omega f} \gamma_n \}$$

for $\mu \in \{ g, b \}$. \hspace{1cm} (13)

The first line of (13) is the same as (9), with an additional possible bonus $t_S \gamma_n$ based division 2's output. The second line states the probability of a promotion of $W_A$ to $M_B$, times (in {}) the difference in $M_A$'s compensation relative to the silo case. The first three terms in the {}-brackets are payments to $M_A$, where after the transfer of $W_A$ the productivity of division A is $t_{\mu w}$, and that of division B is $\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}$. The last three terms in {} subtract what $M_A$ would be paid if her worker were not transferred, to correct for terms already included in the first line of (13). The resulting training incentive constraint is

$$\text{(TIC) } \frac{\Delta \mu}{\Delta q} \{ \kappa_w \Delta q \beta_n + v [\alpha_t - \alpha_n + t_{\mu w} \beta_t - t_{\mu g} \beta_n] + (\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}) \gamma_t - t_{\omega f} \gamma_n \} \geq \tau.$$  

for $\mu \in \{ g, b \}$. \hspace{1cm} (14)

Condition (TIC) is by symmetry the same for manager $M_B$, and thus describes the condition for $e_A = e_B = 1$ to be a Nash equilibrium of stage 2 of the game. In contrast to the silo case, (TIC) depends on the manager’s own type because of the terms $t_{\mu w} \beta_t - t_{\mu g} \beta_n$. Specifically, if $\beta_n > \beta_t$, (TIC) is more restrictive for $\mu = g$. Intuitively, the difference $\beta_t - \beta_n$ matters more to a good manager who is more likely to attain high output in her own division, which translates into a greater stake in losing a good worker to the other division. If $\beta_t > \beta_n$, (TIC) is more restrictive for $\mu = b$.

### 5.2 A benchmark result

Suppose that the CEO can perfectly observe the workers' abilities; i.e., the division managers are not privately informed. However, the firm must still give the managers incentives to train their workers. Aside from providing a useful benchmark, this case is interesting in its own right because some firms are good at getting to know people with leadership potential (see Section 7).
Denoting $\zeta = (\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t)$, the firm’s contracting problem can be stated as

$$\min_{\zeta} w_L^L(\zeta) \text{ s.t. } \text{TIC}(\mu) \text{ for } \mu \in \{g, b\}, (12), \text{ and } \alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t \geq 0.$$  \hspace{1cm} (15)

A useful benchmark result holds if production effort is irrelevant:

**Proposition 2** Suppose $\xi = 0$. Then, with a lattice and observable worker types, an optimal contract that induces $e_A = e_B = x_A = x_B = 1$ is given by

$$\alpha_t = \frac{\tau}{\sigma \nu \Delta \rho},$$ \hspace{1cm} (16)

and $\alpha_n = \beta_n = \beta_t = \gamma_n = \gamma_t = 0$. The firm’s expected wage bill is smaller than with silos, and therefore a lattice dominates silos.

Proposition 2 states that the optimal contract that incentivizes training consists of a a simple “referral bonus” $\alpha_t > 0$ that is paid, say to $M_A$, conditional on the promotion of $W_A$ to $M_B$ but not conditional on either division’s output.\(^{16}\) The intuition is that in contrast to noisy outputs, the transfer of a good $W_A$ amounts to a noiseless signal about the worker’s good type, in effect making a good worker’s type verifiable. Because both the firm and the managers are risk neutral, it is therefore optimal to make all incentives conditional on this event, irrespective how rare it is ($\alpha_t$ is inversely proportional to the probability of a transfer).

Having a noiseless performance measure eliminates any rent that needs to be paid to the manager, and thus (dramatically) lowers the wage bill. Since a lattice also leads to more productive teams, a lattice unambiguously dominates. While the details of this result are specific to our model, the broader message is very general: if coordinating the affairs of two businesses (or divisions) creates added value, and contracts can be designed to give the managers a stake in that added value, then managerial incentives may become stronger rather than weaker. We will return to this point in Section 5.6.

### 5.3 Costly production effort

In the next three subsections, we show that reversing any of the key assumptions behind Proposition 2 suffices to create a tradeoff between a lattice and silos, in the sense that a lattice

\(^{16}\)This is not the uniquely optimal contract but the simplest one; there exists a profit-equivalent contract that puts all incentive weight on $\gamma_t$ instead of $\alpha_t$.  

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leads to higher productivity but also a higher wage cost than with silos. First, suppose that worker types are still observable, but that production effort is sufficiently costly:

**Proposition 3** Suppose \( \xi = \bar{\xi} \). With a lattice and observable worker types, an optimal contract that induces \( e_A = e_B = x_A = x_B = 1 \) is given by \( \alpha_n = 0 \),

\[
\alpha_t = \frac{1 - p_n}{\sigma \Delta p - \tau},
\]

\( \beta_n = \beta_t = \beta^S \), and \( \gamma_n = \gamma_t = 0 \). The firm’s expected wage bill is higher than with silos.

In contrast to the previous result, the own-division bonuses \( \beta_n \) and \( \beta_t \) must now be at least \( \beta^x \) to induce production effort. As \( \xi \) reaches \( \bar{\xi} \), both \( \beta_n \) and \( \beta_t \) equal \( \beta^x = \beta^S \) (by definition of \( \bar{\xi} \)), and both rewards combined would suffice to induce training in a silo, where a manager is assured of keeping a good worker in her division. In a lattice, however, the manager loses a good worker to the other division with some probability, and gets a new worker who is worse in expected terms. It follows that \( \beta_n = \beta_t = \beta^S \) alone cannot incentivize training effort in a lattice. An additional reward \( \alpha_t > 0 \) is necessary, leading to a wage bill strictly higher than with silos.

### 5.4 Privately informed managers

Now suppose only \( M_i \) knows the productivity of \( W_i \). The CEO can then fill a vacant \( M_B \) position with a good \( W_A \) only if \( M_A \) (assuming she stays) reports the type of her worker truthfully. Drucker’s observation that “the promotion system must ... make difficult alike kicking upstairs and hoarding good people” enters our analysis in the form of truthtelling incentive constraints that must hold for a manager with a good and a bad worker, respectively. As before, a lattice matters to \( M_A \) only if she stays and \( M_B \) leaves, and if \( W_B \) is bad but \( W_A \) good. We can therefore focus on this case in determining the conditions for truthtelling even though \( M_A \) does not know \( W_B \), because irrelevant events simply cancel out on both sides of \( M_A \)’s truthtelling constraints.

If \( M_A \) has a good worker, she will report the worker’s type truthfully if

\[
(TTg) \quad \alpha_t + \beta_t t_{\mu w} + \gamma_t [\delta \phi t_{gb} + (1 - \delta \bar{\phi}) t_{bb}] \geq \alpha_n + \beta_n t_{\mu g} + \gamma_n t_{ob}.
\]
The left-hand side of \((TTg)\) is \(M_A\)'s expected payoff if a good \(W_A\) is promoted to \(M_B\) and a new \(W_A\) is hired. The right-hand side of \((TTg)\) is \(M_A\)'s expected wage if she reports her good worker to be bad and thus “hoards” her worker, in which case the productivity of division A is \(t_{mg}\), whereas in division B a new manager is hired. Like the training incentive constraint (TIC), \((TTg)\) depends on the manager’s type whenever \(\beta_n \neq \beta_t\): If \(\beta_n > \beta_t\), \((TTg)\) is more restrictive for a good manager, who is more likely to produce high output in her own division and hence is less willing to give up a good worker.

If \(M_A\) has a bad worker, she will report the worker’s type truthfully if

\[
(TTb) \quad \alpha_n + \beta_n t_{\mu b} + \gamma_n t_{ob} \geq \alpha_t + \beta_t t_{\mu w} + \gamma_t t_{wb}.
\]  

(19)

The left-hand side of \((TTb)\) is \(M_A\)'s expected wage from keeping her bad worker, while in division B a new \(M_B\) is hired. If, on the other hand, \(M_A\) reports her worker \(W_A\) to be good, then \(W_A\) is promoted to \(M_B\), in which case a new worker is hired in division A, while in division B both the manager and the worker are bad. Condition \((TTb)\), too, depends on the manager’s type: If \(\beta_n > \beta_t\), \((TTb)\) is more restrictive for a bad manager, who has less to lose than a good manager from “kicking upstairs” a bad worker.

By inspection, the contracts of Propositions 2 and 3 both violate \((TTb)\). Intuitively, when the contract pays an own-division output bonus and a simple referral bonus, managers who have a bad worker will take advantage of the referral bonus. This, precisely, is a main concern with referral bonuses in practice. Referral bonuses are common, including for internal referrals (WorldatWork, 2011; HRWorld, 2008), but companies are aware of the perverse incentives they potentially create and put safeguards in place. One is a careful screening process that approximates the observable-types case considered above. Another is to make the payment of a bonus contingent on the referred person’s successful performance in the new position for a specified amount of time, such as six months or a year (Bilski, 2011). This practice to some extent emulates the use of contracts such as those derived next. The new optimization problem (denoting again \(\zeta = (\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t)\)) is

\[
\min_{\zeta} \omega^L_A(\zeta) \text{ s.t. } \text{TIC}(\mu), \ TTg(\mu), \text{ and } TTb(\mu) \text{ for } \mu \in \{g, b\}, \text{ and } \alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t \geq 0.
\]

(20)

**Proposition 4** Consider a lattice with privately informed managers and suppose \(\xi = 0\).
Then one of three contracts is optimal (i.e., a solution to (20)), each of which is characterized by \( \alpha_n > 0, \alpha_t = 0, \gamma_n = 0, \gamma_t > 0 \) and \( \beta_n, \beta_t \in \{0, \beta^S\} \) with \( \beta_n \geq \beta_t \); the expressions are stated in the proof. Regardless of which contract is optimal, if

\[
(C3) \quad v \leq \frac{k_w q_b}{k_m q_g},
\]

then the firm’s wage cost is higher than with silos.

The contracts described in Proposition 4 resemble those of other situations in which an agent must exert effort to produce a good outcome, but is asked to report a (privately observed) bad outcome as well, such as in Levitt and Snyder (1997). Here, an incentive payment \( \gamma_t > 0 \) is offered as reward for training a good worker who is promoted to manager, while a lump-sum payment \( \alpha_n > 0 \) helps satisfy (TTb), to get a manager to report if she has a bad worker. As long as \( v \) is not too large, however, it actually more efficient to satisfy (TTb) through \( \beta_n > 0 \), because \( \beta_n \) has a positive effect on training incentives whereas \( \alpha_n \) has a negative effect. Thus, in two of the contracts mentioned in the proposition, \( \beta_n \) is set to the maximum possible value consistent with (TTg), which happens to be the silo wage \( \beta^S \).

More generally, any contract that satisfies (TIC) and (TTg) with equality must have \( \beta_n = \beta^S \). Intuitively, if (TTg) holds with equality, a manager with a good worker is indifferent between reporting her worker as good and having him promoted to manager, and reporting him as bad, in which case the no-transfer wages \( (\alpha_n, \beta_n, \gamma_n) \) apply. In turn, incentivizing training effort when the worker always stays in the division is exactly the silo situation and leads to \( \beta_n = \beta^S \) (recall that \( \alpha_n \) and \( \gamma_n \) have no effect on effort incentives in the silo case).

The wage cost must be at least as large as in the silo case when (TTg) is binding, because the manager receives the silo wage \( \beta^S \) if her worker is not transferred, and receives the same amount (in expected terms) if her worker is transferred. In addition, however, \( \alpha_n > 0 \) is still required to satisfy (TTb), which leads to a wage bill strictly higher than with silos. As the proof shows, the wage cost is also higher if (TTg) is not binding and the optimal contract has \( \beta_n = \beta_t = 0 \) (the third of the three possible contracts). This case arises if \( q_b \) is small and \( \beta_n > 0 \) fails to prevent a bad manager from reporting a bad worker as a good one. If
Before moving on, let us report briefly what happens when instead of transfer-contingent contracts, message-contingent contracts are feasible, of the form \( w_i = \alpha_{i\theta} + \beta_{i\theta} y_i + \gamma_{i\theta} y_j \) for reports \( \hat{\theta} \in \{ g, b \} \) about the worker’s type (our analysis is available upon request). With silos, message-contingent contracts cannot improve over the simple contract of Proposition 1, because it is impossible to meaningfully separate a manager with a good or a bad worker, respectively, when the reports have no allocational consequences. With a lattice, separation is possible. The wage cost is again higher than with silos, which already follows from Proposition 4 because message-contingent contracts are conditioned on coarser information, and are hence strictly less powerful than transfer-contingent contracts.

### 5.5 Simple output-based contracts

The contracts derived above may not be easy to implement. For instance, in the case of observable worker types, the referral bonus \( \alpha_i \) according to Proposition 2 may be very large because it is inversely proportional to the probability of a vacancy in another division, cf. \((16)\). In reality, though, very large referral bonuses are rare. Typical amounts (ranging from several hundred to several thousand dollars) offer adequate incentives for getting rank-and-file employees to refer co-workers, but are likely insufficient for getting a boss to refer a good employee of hers. Why do firms not just pay more, then?

One obstacle is that the impact of a manager’s ability on her division’s performance is spread out in time and difficult to measure, a problem that goes beyond our static setting. Indeed, the delay between employees’ actions and measurable outcomes is one of the reasons why internal labor markets exist in the first place (see Milgrom and Roberts, 1992, pp. 363-364). In a dynamic setting, if \( W_A \) is promoted to \( M_B \) at time period \( \tau \), the referring manager \( M_A \) would need to receive the bonuses \( \beta_t \) and \( \gamma_t \) during some window \([\tau + m, \tau + n] \). Determining the appropriate window may be difficult: rewards spread out over a long time might fail to incentive managers whose time horizons are shorter, whereas paying high rewards during a short window might either create perverse incentives for managers, or might create incentives for the firm to renege on its obligations by manipulating the event.
of a transfer. For instance, if a manager refers a good employee for a vacant position, hoping to receive a high reward, top management could claim that the employee is unqualified but transfer him later, ostensibly for reasons unrelated to the manager’s referral.

Whatever the reasons, suppose that conditioning contracts on transfers is infeasible, and consider contracts that depend on the divisions’ outputs only: \( w_i = \alpha + \beta y_i + \gamma y_j \). The expression for \( \pi_A^L(p_A, p_B) \) in (11) simplifies to

\[
\pi_A^L(p_h, p_h) = \alpha + [t_S + \sigma \nu p_h(y_G - y_L)](\beta + \gamma),
\]

and \( M_A \)'s training incentive constraint simplifies to

\[
\sigma \Delta p \{(\kappa_w \Delta q - \nu y_L)\beta + \nu y_G \gamma \} \geq \tau.
\]

Like Proposition 4, the next result relies on a weak assumption that places an upper bound on \( v \). Economically, it implies that the firm cares sufficiently about the quality of workers in their position as workers, and not mainly about how valuable a worker would be if promoted to manager in a different division.

**Proposition 5** Consider a lattice with observable worker types, assume that only output-based contracts are feasible, and assume that

\[(C3') \quad v \leq \frac{\kappa_w}{\kappa_m + \kappa_w}.
\]

Then the optimal contract that induces \( e_A = e_B = x_A = x_B = 1 \) is given by \( \alpha = \gamma = 0 \) and

\[
\beta = \frac{\tau}{\sigma \kappa_w \Delta p \Delta q [1 - v(1 - p_w)]} > \beta^S.
\]

Intuitively, both \( \beta \) and \( \gamma \) provide incentives for training, but as intuition would suggest, it is less costly to incentivize training through \( \beta \) provided that \( (C3') \) holds, i.e., if the probability of a relevant vacancy is not too large. The expression for \( \beta \) equals \( \beta^S \) divided by the term in \( [] \)-brackets, which raises \( \beta \) above \( \beta^S \). This increase in the necessary bonus reflects the
prospect of losing a good $W_A$ to division B, while because of $\gamma = 0$, $M_A$ does not have a stake in the associated gain for the firm.\footnote{With risk-averse managers (and unlimited liability) and linear contracts, $\gamma$ would be positive due to the Informativeness principle. At least for $\sigma$ high enough, however, the wage bill would still be higher with a lattice than with silos because $\gamma$ is paid as unconditional bonus, whereas the firm realizes the gain $y_G$ only if a worker is transferred.}

Proposition 5 establishes a third reason that can overturn the benchmark result of Proposition 2: even if there is no adverse-selection problem and if production effort does not matter, wage costs are higher with a lattice than with silos if contracts cannot (sufficiently) reward managers for the benefits of a lattice.

If the managers have private information about their workers, the truthtelling constraint (TTg) from Section 5.4 becomes

$$\alpha + \beta t_{mw} + \gamma[\delta \phi t_{gb} + (1 - \delta \phi)q_b] \geq \alpha + \beta t_{mg} + \gamma t_{ob},$$

which can be expressed more simply as $\gamma y_G \geq \beta y_L$. This leads to a lower bound for $\gamma$ as fraction of $\beta$:

$$\gamma \geq \gamma_g \beta \quad \text{for } \gamma_g = \frac{y_L}{y_G} = \frac{\kappa_w (1 - p_w)}{\kappa_m (\delta \phi - p_o)}.$$  \hfill (23)

Notice that $\gamma_g < 1$ per condition (C1). For a manager with a bad worker, the constraint (TTb) from Section 5.4 becomes

$$\beta t_{mb} + \gamma t_{ob} > \beta t_{mw} + \gamma t_{bb},$$

which, too, requires $\gamma$ to be a minimal fraction of $\beta$:

$$\gamma \geq \gamma_b \beta \quad \text{for } \gamma_b = \frac{t_{mw} - t_{mb}}{t_{ob} - q_b} = \frac{\kappa_w p_w}{\kappa_m p_o},$$  \hfill (24)

with $\gamma_b < 1$ per condition (C2). Thus, with simple linear contracts, both truthtelling constraints can be satisfied by giving a manager a large enough stake in the other division. From Proposition 5, the only reason to choose $\gamma > 0$ is to induce truthtelling. Therefore, it is optimal to set $\gamma = \max\{\gamma_g, \gamma_b\} \beta$.

It is straightforward to verify that $\gamma_g \geq \gamma_b$ if and only if $p_o \geq \delta \phi p_w$, which in turn is implied by (C2). Thus, if the firm prefers an outside manager over an internally promoted
worker of unknown quality, it follows that preventing managers from hoarding good workers (and forcing the firm to hire from outside) is a more restrictive constraint than preventing managers from kicking upstairs bad workers.

**Proposition 6** Consider a lattice with privately informed managers, assume that only output-based contracts are feasible, and assume that \((C3')\) holds. Then the optimal contract that induces high training and production effort is given by \(\beta = \beta^S\) and \(\gamma = \gamma_g \beta^S\). The firm’s expected wage bill is higher than with silos.

Since \(\gamma_g \geq \gamma_b\), \((TTg)\) is binding with an optimal contract, but \((TTb)\) is not. The result \(\beta = \beta^S\) then follows because all training incentives come from \(\beta\), and none from \(\gamma\): per Proposition 1, \(\beta^S\) is the own-division bonus necessary to induce training effort if \(M_A\) can hold on to a good \(W_A\). In the setting considered here, paying \(\gamma < \gamma_g \beta\) would have no effect on training incentives because \(M_A\) would hoard a good \(W_A\). Paying \(\gamma = \gamma_g \beta\) makes \(M_A\) indifferent between hoarding and relinquishing a good worker without affecting training incentives. At \(\beta = \beta^S\) and \(\gamma > 0\), the firm’s wage bill is trivially larger than with silos.

To conclude, if wages can depend only on outputs, it is best to incentivize training through the own-division bonus \(\beta\). With observable types, the necessary bonus is larger than with silos because managers stand to lose a good worker *without* any direct compensation. With privately informed managers, both the no-hoarding and the no-kicking-upstairs truth-telling constraints require a minimal other-division bonus \(\gamma\). Compared to the case of observable types, wage costs are higher still, and thus a lattice costlier to implement, because of the additional adverse-selection problem.

### 5.6 “Selective intervention” in internal labor markets

Let us interpret our results in the context of Williamson’s (1985) “selective-intervention puzzle”. Even without any other synergies between divisions A and B, selectively transferring employees between divisions is a form of value-increasing “selective intervention,” much like other forms of resource reallocation.\(^\text{18}\) Williamson (1985), Tadelis and Williamson (2012),

\(^\text{18}\)General Electric is a good example of a conglomerate held together significantly by synergies resulting form an actively managed corporate internal labor market; see Linebaugh (2012).
and others have argued that intervention from the top tends to undermine division managers’ incentives, either directly (as in Aghion and Tirole, 1997) or because of deliberate provision of weaker incentives. As Proposition 2 shows, however, this argument is incomplete and relies on (possibly overly restrictive) assumptions about contracting constraints. If division managers’ incentives can be sufficiently tied to the value-increasing interventions, then incentives can in fact be strengthened, in which case integration is unambiguously optimal.

Our subsequent results illustrate why this result may not hold. The most obvious reason is that the contracting space may in fact be restricted, possibly for reasons discussed in Section 5.5. In that case, moral hazard in training alone can lead to higher costs of a lattice (Proposition 5) because managers bear the costs of losing a good worker with little or no stake in the benefit.

Costly production effort (Proposition 3) has a similar effect: even when transfer-contingent contracts are feasible, having to incentivize own-division performance limits the firm’s ability to target incentives at the event of a worker transfer, again leading to higher costs of a lattice if production effort is sufficiently important. Thus, additional incentive constraints can have the same effect as outright restrictions on the contracting space.

Proposition 4, finally, reinforces a key argument of Friebel and Raith (2010): when selective intervention relies on information that resides with the division managers, then the costs of establishing truthful upward communication are genuine costs of integration. Here, this result holds even when transfer-contingent contracts are feasible, because private information introduces a two-sided adverse-selection problem with (so to speak) upward and downward truthtelling incentive constraints that are both binding.

A practically important implication of our results is that a lattice cannot be implemented simply by changing decision rights over personnel allocations without changing the incentive system as well: a silo-like incentive contract with \((\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t) = (0, \beta^S, 0, 0, \beta^S, 0)\) would both undermine managers’ training effort and create incentives for them to misrepresent information about their people. Costly changes in the incentive system are necessary to get a lattice to work at all.
6 Silos or Lattice?

What forces drive a firm’s choice between silos and a lattice? What motivates firms to change their internal labor markets from vertical job ladders to ones that facilitate cross-divisional mobility of employees? As it turns out, several parameters of our model have an unambiguous effect on this choice. All results stated in this section hold for both transfer-contingent and for simple output-based contracts.

Proposition 7 There exists $\hat{R}$ such that a lattice with private information is preferred over silos if and only if $R \geq \hat{R}$.

The proof of the result is simple. Expected output is greater with a lattice than with silos (per (C1)), and the difference in output is multiplied by $R$. The expected wage cost, meanwhile, is higher with a lattice according to Propositions 4 and 6, but the difference does not depend on $R$. It follows that there is a critical value for $R$ above which a lattice is more profitable, and below which silos are more profitable.

Intuitively, not just output but also the difference in the value of having a good employee in the position of a manager or worker is scaled by $R$. Thus, when employees in higher positions are more important ($\kappa_m > \kappa_w$), then a higher value of output implies not just a higher value of having good employees in general, but a higher value of moving the best people to the top.

We have so far left open how to interpret the divisional output $y_i$ and its value $R$. For instance, one can think of $y_i$ not as actual output but as the outcome of efforts to innovate, improve quality, or reduce costs. A growing body of theoretical and empirical work suggests that the marginal value of such efforts is greater in more competitive product markets. The intuition is that in more competitive markets, firms’ demand functions are more elastic, which magnifies the effects of cost or quality advantages or disadvantages, and therefore raises the value of investing to “stay ahead of the competition.” This effect can be counterbalanced by a negative effect of competition on profits and hence the value of investments. However, when market structure is endogenously determined by free entry, the first effect tends to dominate (Raith, 2003). Results that establish a positive relation between product substitutability or market size and the value of investments can be found in Symeonidis (2000, Property
Proposition 7 thus suggests a link between product market competition and the organization of internal labor markets. Intuitively, when increasing quality or lowering costs is a (hierarchical) team production process in which higher-level employees are more important, then greater competition increases the value of moving talented employees into higher-level positions. Practitioners, too, have suggested a link between competition and the value of searching for talent internally:

“As global markets become more dynamic and competitive, companies will need to deploy talent even more flexible across broader swaths of the organization. Since management must develop and execute value-creating initiatives so quickly, talent is becoming more critical to corporate performance” (Bryan et al. 2006).

Our next result shows how the choice between silos and a lattice depends on the external market for managers:

**Proposition 8** The difference in profit between a lattice with private information, and silos, \( \pi^{LP} - \pi^S \), is decreasing in \( p_o \).

In words, a tighter external labor market for managers (smaller \( p_o \)) favors the adoption of a lattice organization internally. Intuitively, while expected output is increasing in \( p_o \) for both silos and a lattice, the effect is smaller for a lattice, and hence the profit difference decreasing in \( p_o \), because with a lattice there is less hiring from the external market for managers. As shown in the proof, the effects of \( p_o \) on training and truth-telling incentives either cancel out or reinforce the direct effect on the firm’s profit.

Proposition 8 thus establishes a link between the “war for talent”—businesses’ perception that recruiting good people externally has become harder—and the increase in firms’ attention towards “talent management”. Tighter labor markets, too, have been linked to greater product market competition; for a theoretical analysis see Gersbach and Schmutzler.
Proposition 7 and 8 thus jointly support the argument that increased product market competition, greater competition for talented people, and the ongoing reorganization of internal labor markets are all causally linked.\textsuperscript{19}

**Proposition 9** *The difference in profit between a lattice with private information, and silos, $\pi^{LP} - \pi^{S}$, is increasing in $\kappa_{m}$ with simple output-based contracts, and is increasing in $\kappa_{m}$ with transfer-contingent contracts if contracts (1) or (3) of Proposition 4 are optimal, and may be increasing or decreasing in $\kappa_{m}$ if contract (2) is optimal.*

To understand the result, note that by assumption, it is preferable to promote a good worker from the other division than to hire from outside, and the difference in productivity is scaled by $\kappa_{m}$. It follows that the more important managers are for production, the more important it is to get the best people into managerial positions, which raises the profitability of a lattice relative to silos. In addition, a higher $\kappa_{m}$ generally relaxes all incentive constraints. An exception occurs when contract (2) of Proposition 4 is optimal, in which case a higher $\kappa_{m}$ can tighten (TTb) and require an increase in $\alpha_{n}$, which can dominate the other effects.

Proposition 9 suggests that firms’ recent efforts to establish greater cross-divisional mobility of employees is partly caused by a skill-biased technological change that has increased the returns to talent in higher managerial positions; see Kaplan and Rauh (2013). Scale and “superstar” effects, partly a result of skill-biased technological change, thus not only have profound effects on managerial labor markets but likely have effects on the internal organization of firms as well.

**Proposition 10** *The difference in profit between a lattice with private information and silos, $\pi^{LP} - \pi^{S}$, is increasing in $\delta$ and $\phi$.*

In words, both greater “human capital similarity” between divisions, and a greater upward transferability of skills, favor cross-divisional mobility. The net output gain (proportional...)

\textsuperscript{19}The causality need not be unidirectional from product market competition to external and internal labor markets. Greater mobility of managers, whatever the reason (see e.g. Murphy and Zabojnik 2004), could also lead to greater product market competition, thus reinforcing the causality we have emphasized. This could be the case, for instance, if the returns to superior managerial talent are not fully appropriated by the managers but in part also by (heterogeneous) firms hiring them.
tional to \( y_G - y_L \) is obviously increasing in \( \delta \), but is increasing in \( \phi \) too because there are more frequent promotions with a lattice. Moreover, increases in \( \delta \) or \( \phi \) relax both (TIC) and (TTg) without tightening (TTb), which results in lower wages, cf. Propositions 4 and 6.

Murphy and Zabojnik (2004) have documented a broad shift in the skill mix of executives from firm-specific to general human capital, which they argue has been responsible for both increased mobility of executives and higher compensation. As Proposition 10 suggests, the same shift in skill mix may be a driver of firms’ reorganization of their internal labor markets.

A striking example of the link between division similarity and cross-divisional mobility is the case of erstwhile Newell Co. (today Newell Rubbermaid Co.). Newell’s divisions produce picture frames, paintbrushes, curtain rods and countless other mundane products, and sell them to mass merchandisers such as Wal-Mart. Although the products are technologically unrelated, the business model of each division is the same, namely to be a “no-problem” supplier to its much larger customers (Montgomery, 1999). As Montgomery argues, the synergies holding the corporation together revolve around the exchange of expertise among division managers and headquarters. To support this strategy, division managers frequently move from one division to another. The ease with which managers can move laterally is closely linked to the similarity of the divisions’ operations (high \( \delta \) in terms of our model).

The example, however, also illustrates the partial nature of our analysis: in reality, \( \delta \) is endogenously determined by the corporation’s strategic decisions on what lines of business to pursue. Nevertheless, we obtain an interesting link between corporate strategy and internal labor markets, and a potentially testable prediction: other things equal, cross-divisional mobility is more likely in firms whose divisions are more similar.

The result for \( \phi \) implies that greater vertical mobility of employees should also favor cross-divisional mobility. In practice, the same firm may offer different mobility opportunities for different types of jobs. Proposition 10 then suggests that measured across jobs, cross-divisional mobility of employees ought to be positively related to upward mobility. Like other parameters, \( \phi \) is in practice endogenous and depends on managers’ efforts not only to develop employees for their current jobs (as already captured by our model) but for higher-level jobs too. This suggests that across firms or jobs, cross-divisional mobility ought to be positively related to efforts to groom managers for higher-level positions.
7 Alternative Solutions

We confined our formal analysis to the design of incentive contracts in order to highlight two key obstacles to establishing cross-divisional mobility, managers’ training incentives and their private information about employees. In reality, of course, firms’ efforts to reorganize their internal labor markets are more complex than that. In this section, we discuss potential and actual alternative solutions.

Direct monitoring of training effort: All solutions discussed so far provide incentives for training that are based on outcomes. Some firms choose to measure and reward training effort directly, such as instrument maker Agilent, whose top managers receive a third of their compensation based on HR development efforts (Conaty and Charan, 2010, page 160). One way in which to measure training effort is through 360-degree reviews, in which subordinates are asked about their boss’s training efforts. A successful example is mutual fund company MFS Investment Management, where evaluations of portfolio managers and analysts are each compiled from up to 60 evaluation forms submitted by peers, subordinates and superiors (Hall and Lim 2002). Most strikingly, these forms are not aggregated by HR staff but by C-level managers. The very elaborate process makes it feasible to ascertain not only performance “by numbers” but many other softer dimensions such as teamwork and training.

The example of MFS is quite exceptional and illustrates the high costs of direct monitoring. In general, rewarding inputs instead of outcomes is less reliable, may lead to distortions in the allocation of effort, and fails to take advantage of agents’ (here, bosses) private information about how best to allocate their time (Baker, 2002; Raith, 2008).

Job rotation: General Electric, Novartis, SAS Institute and many others have programs in which junior and mid-career managers go through different assignments across functions and divisions (recall also our example of Newell Co. above. The primary objective of job rotation is usually to develop junior employees’ skills and to prepare them for higher-level positions (e.g. Conaty and Charan, 2010, 228-229). One might think that job rotation also solves several problems emphasized in this paper: employees can develop their skills through

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20 We cite MFS as an example of an elaborate subjective evaluation processes. Due its lack of a divisional structure it is not an example of a firm with a lattice-like internal labor market.
the rotation program rather than having to rely on training by their boss; bosses can no longer “hoard” good people; and talented employees become more visible to senior managers in different units, reducing an adverse-selection problem.

Only the last point—greater visibility—is a true advantage, however. Job rotation does not diminish the importance of training by managers; it only spreads the training responsibility across multiple managers. While alleviating the adverse-selection problem, it makes the moral-hazard problem worse: managers have even less incentive to train someone who will leave their unit not with some probability, but for sure!

Another downside of job rotation, though not captured in our model, is that while employees acquire general managerial skills, they may lack the time and motivation to acquire division-(or industry-)specific expertise. After many years of emphasizing general managerial skills, General Electric recently decided to keep senior managers in their divisions longer to help them acquire the expertise needed to compete in their industries (Linebaugh 2012). To conclude this point, job rotation has its advantages but its costs as well, and is therefore a good solution for some firms but not others.

Other leadership development initiatives: Other initiatives, too, simultaneously foster human capital development and generate firm-wide information about talented employees. General Electric’s Management Development Institute in Crotonville, NY, runs leadership courses and “workout” sessions in which high-potential employees are encouraged to publicly challenge their bosses’ views (Martin and Schmidt, 2010). Likewise, Procter & Gamble fosters the development of social networks to make sure that managers can rely on their colleagues’ help when staffing jobs.

Many other firms have adopted similar initiatives, cf. Conaty and Charan (2010) and Benko and Anderson (2010). One could argue that as a result, the extent of private information about top talent may be only small. Efforts for everyone to get to know everyone else are costly, however, and can realistically cover only the top tier of managers, perhaps 100-200 people. It follows that even though private information may not be an issue at the very top of a company, moving down the ranks it will eventually begin to matter (recall our

\[21\] For a classical description about the effects of cross-firm rotation of the French engineering elite from the Ecole Polytechnique, see Crozier (1964).
remark in Section 3 that the tiers in our model may represent any adjacent tiers in a larger hierarchy).

In conclusion, job rotation and other leadership development initiatives have clear benefits but have disadvantages or costs as well. The general argument of our paper remains: achieving cross-divisional mobility requires costly solutions that address the key role that bosses (at least traditionally) play as trains and holders of private information.

8 Cross-divisional Mobility from the Workers’ Perspective

Our analysis has focused squarely on the role of middle managers for the production and allocation of human capital formation in firms, and has treated the workers as “drones.” We have argued that the managers’ role is critical to why silos have been so prevalent throughout business history, and why for many firms transitioning from silos to a lattice is not that easy.

That said, firms’ efforts to improve mobility in internal labor markets are significantly driven by the workers’ interests as well. An early example is the case of Johnson & Johnson, whose divisions have traditionally been very independent. Pearson and Hurstak (1992) describe the negative consequences of J&J’s silo structure from the workers’ perspective: “Many junior executives found it tough to move up when young presidents stood in the way, and tougher still to jump over to a separate company [within J&J].” As early as the 1990s, CEO Ralph Larsen took measures to facilitate cross-company mobility in order to remedy J&J’s “chronic problem of career-pathing.” In line with our theory, however, these measures turned out to be unpopular with many senior managers, given the absence of adjustments to their incentives.

Bryan et al. (2006), too, point out the relevance of workers’ career aspirations:

“Many a frustrated manager has searched in vain for the right person for a particular job, knowing that he or she works somewhere in the company. And many talented people have had the experience of getting stuck in a dead-end corner of the company, never finding the right experiences and challenges to
Aside from the allocational objective explored in this paper, therefore, retaining talent is therefore another reason for firms to transition to a lattice, which again emphasizes the link between the “war for talent” between firms and “talent management” within firms.

Employees’ incentives, too, depend on available career paths. With the better opportunities to advance that a lattice offers, and potentially greater visibility to senior managers, employees have better incentives to invest in their own human capital, especially those required at higher levels.

To sum up, employees are likely to favor a lattice over silos. To get a lattice to work, however, requires the cooperation of managers, which in turn requires significant changes to the firm’s incentive system.

9 Conclusion

Traditional vertical job ladders in firms—the subject of a large economics literature—have recently been giving way to active “talent management” aimed at optimally matching people with positions, which includes efforts to promote cross-divisional mobility of employees. Our paper is the first to examine these efforts from an economics perspective. We have argued that managers play a key role as gatekeepers in internal labor markets. Because managers prefer to have good employees working for them, efforts to increase employees’ cross-divisional mobility undermine managers’ incentives to invest in training, and create incentives to use private information about employees strategically, either by “hoarding” good employees or by “kicking upstairs” bad ones. Our model captures the contractual origins of these agency problems: team production in firms, and an inability to measure training effort or its outcomes directly. Our analysis explored how optimal contractual solutions that implement a lattice (under different contracting assumptions) differ from the simple incentive contracts that are optimal in traditional “silos.”

Our results help explain the historical prevalence of silos (job ladders), and shed light on the challenges faced by companies transitioning to lattice structures. Silos lead to inefficient matches of people to positions, but create relatively good incentives for managers to train
their employees. We have argued that firms’ recent efforts to increase employees’ mobility can be explained by a greater importance of getting the best people to the top, which in turn may have both internal or external causes such as skill-biased technological change, or product or labor market competition.

The most important practical implication of our analysis is that establishing greater (cross-divisional) mobility for junior managers is not simply a matter of opening up new career paths, but in addition requires changes to the incentives provided to higher-level managers or other supporting practices such as job rotation, monitoring of training effort, and other development initiatives. Consistent with our conclusion, the experience of many companies suggests that transitioning to lattice structures is harder than it looks. Meanwhile, commentators observe a decline in managerial training (Capelli, 2012) that may be an unintended consequence of changes in internal labor markets.

Our paper departs from the literature in two main ways, each of which lends itself to further analysis. The first is to study the internal labor markets of a multi-divisional firms, which extends the reach of economic analysis to questions of importance to today’s large companies (Conaty and Charan, 2010). The second departure is our argument that the production and allocation of human capital in firms is not simply in the hands of “the firm” but significantly in the hands of its managers, whose interests may not align with their firm’s. This approach, we hope, advances the agenda proposed by Gibbons (2013) for organizational economics to “focus on what managers actually do,” in the spirit of Cyert and March’s (1963) observation that “managers devote much more time and energy to the problems of managing their coalition than they do to the problems of dealing with the outside world” (p. 205-6).

Appendix: Proofs

Proof of Proposition 1: As noted in the text, (9) leads to the training incentive constraint
\[ \sigma t_{\mu}(p_h)\beta - \tau \geq \sigma t_{\mu}(p_l)\beta \quad \text{or} \quad \sigma \kappa \Delta p \Delta q \beta \geq \tau, \]
which leads to the stated expression for \( \beta^S \). If \( \beta^S \geq \beta^x \), then \( \beta^S \) satisfies both (25) and (8), and thus the optimal contract is \( (\alpha, \beta, \gamma) = (0, \beta^S, 0) \). QED
Proof of Proposition 2: We will derive an optimal contract for the general case $\xi \in [0, \overline{\xi}]$. The proof proceeds in several steps.

1. First, denote by $\psi$ the left-hand side of $M_A$'s training incentive constraint (TIC). For $\psi$ and the firm’s wage cost $w_A$, we have

\[
\frac{\partial \psi}{\partial \beta_n} < \frac{\partial \psi}{\partial \alpha_t} \quad \text{and} \quad \frac{\partial \psi}{\partial \beta_t} < \frac{\partial \psi}{\partial \alpha_t},
\]

where the derivatives in the second inequality are evaluated for $\beta_n = \beta_t \equiv \beta$. From (14) and (11), the right-hand side of (26) is easily computed as $\Delta p/p_h$. For the left-hand side of the first inequality, we have

\[
\frac{\partial \psi}{\partial \beta_n} = \sigma \Delta p(\nu \Delta q - vt_{mg}) < \sigma \Delta p(\nu \Delta q - v\kappa \Delta q) = \sigma \Delta p(1-v)\kappa \Delta q,
\]

where the inequality follows because $t_{mg} \geq t_{bg} = (\kappa_m + \kappa)q_b + \kappa \Delta q$; and

\[
\frac{\partial w_A}{\partial \beta_n} = t_S + \sigma v p h (y_G - t_{mg}) > \sigma t_0 - \sigma v p_h t_{mg} = \sigma(1-v)t_0 + \sigma v(1-p_h)t_{mb} > \sigma(1-v)t_0.
\]

Overall, therefore, we have

\[
\frac{\partial \psi}{\partial \beta_n} < \frac{\sigma \Delta p(1-v)\kappa \Delta q}{\sigma(1-v)t_0} = \Delta p \frac{\kappa \Delta q}{t_0} < \frac{\Delta p}{p_h},
\]

where the last inequality follows from $t_0 > p_h t_{mg} = p_h(\kappa m q_m + \kappa q_g) = p_h \kappa \Delta q$. This proves the first inequality in (26). For the second inequality, we have

\[
\frac{\partial \psi}{\partial \beta} = \sigma \Delta p[\nu \Delta q - v(t_{mu} - t_{mw})] = \sigma \Delta p[1 - v(1 - p_h)]\kappa \Delta q,
\]

and

\[
\frac{\partial w_A}{\partial \beta} = t_S + \sigma v p h (y_G - t_{mg} + t_{mw}) > \sigma t_0 - \sigma v p_h (t_{mg} - t_{mw})
\]

\[
= \sigma[1 - v(1 - p_h)]t_0 + \sigma v(1 - p_h)(t_0 - p_h \kappa \Delta q) > \sigma[1 - v(1 - p_h)]t_0,
\]

and the final step is the same as above.

2. Constraint (12) restricts attention to contracts with $\beta_n, \beta_t \geq \beta^x$. Next, $\alpha_n$ and $\gamma_n$ have no effect on production effort incentives and a negative effect on training incentives, cf. $\psi$ in (14). Therefore, set $\alpha_n = \gamma_n = 0$. Next, (13) and (14) lead to

\[
\frac{\partial w_A}{\partial \gamma_t} = \frac{\partial \psi}{\partial \gamma_t} / \partial \alpha_t = \delta \phi t_{gb} + (1 - \delta \phi)t_{bb}.
\]
It follows that training incentives can be provided at the same expected cost through either $\alpha_t$ or $\gamma_t$. Let us focus on the simpler solution of using $\alpha_t$ rather than $\gamma_t$ (if at all).

3. Next, we show that an optimal contract must satisfy $\beta_t = \beta_n$. To see this, observe that

$$\frac{\partial w^L/\partial \beta_t}{\partial w^L/\partial \alpha_t} = t_{mw} \quad \text{and} \quad \frac{\partial \psi/\partial \beta_t}{\partial \psi/\partial \alpha_t} = t_{\mu w}$$

(27)

If $\beta_t \geq \beta_n$, in which case (14) is (weakly) more restrictive for $\mu = b$, then (27) implies that it is less costly to provide training incentives through $\alpha_t$ than through $\beta_t$; hence an optimal contract must have $\beta_t$ at the lowest possible level, which is $\beta_t = \beta_n$ in order to satisfy both $\beta_t \geq \beta_n$ and (12). Conversely, if $\beta_t < \beta_n$, (14) is (strictly) more restrictive for a $\mu = m$, and (27) implies that is optimal to incentivize training through $\beta_t$ rather than $\alpha_t$, while per step 1, providing incentives through $\alpha_t$ is less costly than providing incentives through $\beta_n$. It follows that it is less costly to incentivize training through $\beta_t$ than through $\beta_n$, in which case a contract with $\beta_t < \beta_n$ cannot be optimal. Overall, it follows that $\beta_t = \beta_n$.

4. According to step 1, it is less costly to incentivize training through $\alpha_t$ than through $\beta_n = \beta_t \equiv \beta$. It follows that a cost-minimizing contract is given by $\alpha_n = \gamma_n = \gamma_t = 0$, $\beta_n = \beta_t = \beta^x$, and $\alpha_t$ chosen to satisfy (14) given the values of the other contract variables. The solution is

$$\alpha_t = \frac{\tau}{\sigma v_{\Delta p}} - \frac{1 - v(1 - p_h)}{v q_b} \frac{\kappa_w}{\kappa_m + \kappa_w} \Delta q \xi. \quad (28)$$

The optimal contract is not unique: a different, cost-equivalent contract is one with $\alpha_t = 0$ and $\gamma_t$ chosen to satisfy (14). Any convex combination of these two corner solutions is optimal too.

5. For silos, the expected wage cost is $t_S \beta$, which using $\beta = \beta^S$ from Proposition 1 equals

$$\frac{t_S}{\sigma \kappa_w \Delta p \Delta q} \tau > \frac{\sigma t_0}{\sigma \kappa_w \Delta p \Delta q} = \frac{\kappa_m q_m + \kappa_w (q_b + p_h \Delta q)}{\kappa_w \Delta p \Delta q} \tau > \frac{p_h}{p_h - p_l} \tau. \quad (29)$$

For a lattice, the wage cost for the case $\xi = 0$ is obtained from (11) for $\alpha_n = \beta_n = \beta_t = \gamma_n = \gamma_t = 0$, and is simply $\sigma v p_h \alpha_t = (p_h/\Delta p)\tau$, which in conjunction with (29) proves that the wage cost is lower with a lattice. QED

**Proof of Proposition 3:** If $\xi = \xi_x$, then by definition $\beta_n = \beta_t = \beta^S$. Solving (28) for $\xi = \bar{\xi} = t_{bb} \beta^S$ leads to $\alpha_t = (1 - p_h)\tau/(\sigma \Delta p)$, as stated. (For larger values of $\xi$, the general expression for $\alpha_t$ becomes negative. In this case, setting $\beta_n = \beta_t = \beta^x$ and all other bonuses
to zero suffices to induce both training and production effort.) To evaluate the wage cost, observe in (11) that if \( \beta_n = \beta_s \), then the first term alone leads to a wage cost equal to the silo wage cost. In addition, the second and third terms in (11) are both strictly positive, because \( \alpha_t > 0 \) and the \( \beta \)-terms reduce to \( \sigma v p_h (y_G - y_L) \beta > 0 \). It follows that the expected wage cost with a lattice is strictly higher than with silos. QED

**Proof of Proposition 4:** The proof proceeds in several steps to derive the characteristics of an optimal contract \( \zeta = (\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t) \).

1. For any optimal contract, \( \text{(TTb)} \) must be binding. Solving the problem without it would lead to the contract of Proposition 2 (or a profit-equivalent contract with \( \gamma_t > 0 \); see step 2 of the proof of Proposition 2). But any such contract, with \( \beta_n = \beta_t = \beta^* \), and \( \alpha_n = \gamma_n = 0 \), would violate \( \text{(TTb)} \).

2. If \( \zeta \) is optimal, then \( \alpha_t = 0 \). Suppose instead that for \( \zeta = (\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t, \gamma_t) \), \( \alpha_t \) is positive. Given that

\[
\frac{\partial w_A}{\partial \alpha_t} = \sigma v p_h \quad \text{and} \quad \frac{\partial w_A}{\partial \gamma_t} = \sigma v p_h [\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}],
\]

the contract \( \zeta' = (\alpha_n, \beta_n, \gamma_n, \alpha_t - \Delta, \beta_t, \gamma_t + \frac{\Delta}{\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}}) \) leads to the same wage cost; similarly, it can be shown that both \( \text{(TIC)} \) and \( \text{(TTg)} \) hold for \( \zeta' \) iff they hold for \( \zeta \). The same substitution, however, relaxes \( \text{(TTb)} \) by reducing the right-hand side by \( \Delta \) times \( \frac{t_{bw}}{\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}} \), which is positive. All constraints are thus satisfied, and it follows that if \( \zeta \) is optimal, then \( \zeta' \) must be optimal too. But because of step 1, \( \zeta' \) cannot be optimal because \( \text{(TTb)} \) is slack. It follows that \( \zeta \) cannot be optimal either, proving by contradiction that \( \alpha_t = 0 \).

3. Next we show that if \( \zeta \) is optimal, then \( \beta_n \geq \beta_t \). Suppose instead that \( \beta_t > \beta_n \). In this case, the most restrictive version of both \( \text{(TIC)} \) and \( \text{(TTg)} \) is for \( \mu = b \), whereas the most restrictive version of \( \text{(TTb)} \) is for \( \mu = g \). In this case,

\[
\frac{\partial \psi / \partial \gamma_t}{\partial \psi / \partial \beta_t} = \frac{\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}}{t_{bw}}.
\]

It follows that the contract \( \zeta' = (\alpha_n, \beta_n, \gamma_n, \alpha_t, \beta_t - \Delta, \gamma_t + \frac{t_{bw}}{\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}} \Delta) \) satisfies \( \text{(TIC)} \) iff \( \zeta \) does. The same holds for \( \text{(TTg)} \). In addition, \( \zeta' \) relaxes \( \text{(TTb)} \) by \( \Delta \) times

\[
t_{gw} - \frac{t_{bb}}{\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}} t_{bw} > 0.
\]
Finally, $\zeta'$ leads to a decrease in the wage cost by $\sigma v p_h(t_m - t_w)$. Thus $\zeta'$ is feasible and dominates $\zeta$. It follows that conditional on $\beta_t \geq \beta_n$, it is optimal to choose $\beta_t$ as small as possible, which leads to $\beta_t = \beta_n$ and thus contradicts $\beta_t > \beta_n$. We thus know that an optimal contract satisfies $\beta_n \geq \beta_t$, which means that for the rest of the proof, the relevant version of both (TIC) and (TTg) is for $\mu = g$, whereas the relevant version of (TTb) is for $\mu = b$.

4. Next, if $\zeta$ is optimal, then $\gamma_n = 0$. Suppose instead that $\gamma_n$ is positive. Then the contract $\zeta' = (\alpha_n + \Delta t_{ob}, \beta_n, \gamma_n - \Delta, \alpha_t, \beta_t, \gamma_t)$ would continue to satisfy (TTb) for $\mu = b$, and (TIC) and (TTg) for $\mu = g$. The wage cost, in turn, changes by $\Delta$ times

$$-\sigma[t_0 - t_{ob}] - (1 - \sigma)[t_r - t_{ob} - \sigma p_h(1 - p_h)y_L].$$

The first term in (30) is negative because $t_0(p_h) > t_m > t_{ob}$, where the first inequality follows from (6) and the second from the assumption $p_m \geq p_o$. For the second term in (30), we have

$$\sigma p_h(1 - p_h)y_L < p_h y_G < p_h \kappa_m(\phi - p_o)\Delta q;$$

and from (7) we have

$$t_r - t_{ob} = p_h[\phi t_{gb} + (1 - \phi)t_{bb} - t_{ob}] = p_h \kappa_m(\phi - p_o)\Delta q,$$

which implies that the second term in (30) is positive too. It follows that $\zeta'$ is feasible and dominates $\zeta$. Therefore $\zeta$ with $\gamma_n > 0$ cannot be optimal.

5. We have shown so far that any optimal contract satisfies: $\beta_n \geq \beta_t$, (TIC) binding for $m = g$, (TTb) binding for $m = b$, and $\alpha_t = \gamma_n = 0$. Solving (TIC) and (TTb) for $\alpha_n$ and $\gamma_t$ (with $\alpha_t = \gamma_n = 0$) leads to

$$\alpha_n = \frac{t_{bb}}{\sigma v \delta \phi \kappa_m \Delta p \Delta q} \tau - \left[ \frac{(1 - v)\kappa_w - v\kappa_m}{v \delta \phi \kappa_m} + 1 \right] t_{bb} \beta_n$$

and

$$\gamma_t = \frac{1}{\sigma v \delta \phi \kappa_m \Delta p \Delta q} \tau - \left[ \frac{1 - \delta \phi}{\delta \phi} t_{bb} - \kappa_w p_w \Delta q \right] \beta_t.$$

Any optimal contract must satisfy (31).

6. Restricting attention to corner solutions, it is optimal to choose $\beta_n$ either as small or as large as possible, subject to constraints. A lower bound to $\beta_n$ is given by $\beta^x$; an upper bound
is given by (TTg). If (TTg) is binding, then \( \beta_n = \beta^S \): Substituting the right-hand side of (TTg) for \( \alpha_t + \beta_t \mu \gamma + \gamma_t[\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}] \) in (14) cancels the entire \([\cdot]\)-term in (14) and leaves the same effort incentive constraint as for the silo case. Therefore, \( \beta_n = \beta^S \). For any optimal contract, we therefore have \( \beta_n \in \{\beta^x, \beta^S\} \), which in combination with \( \beta_n \geq \beta_t \) leads to exactly three contracts: (1) \( \beta_n = \beta_t = \beta^S \), (2) \( \beta_n = \beta^S, \beta_t = \beta^x \), and (3) \( \beta_n = \beta_t = \beta^x \), with \( \alpha_n \) and \( \gamma_t \) given by (31) and \( \alpha_t = \gamma_n = 0 \). Each can be optimal for different parameter values. For the first two contracts and the case \( \xi = 0 \) and hence \( \beta^x = 0 \), the expressions for \( \alpha_n \) and \( \gamma_t \) obtained from (31) are as follows, and are all positive:

1. \( \beta_n = \beta_t = \beta^S \): 
   \[
   \alpha_n = \frac{\kappa_m(q_h + \delta \phi q_h \Delta q) + \kappa_w q_h \tau}{\sigma \delta \phi \kappa_m \Delta p \Delta q}, \quad \gamma_t = \frac{1}{\sigma \delta \phi \kappa_m \Delta p \Delta q}. 
   \]

2. \( \beta_n = \beta^S, \beta_t = 0 \): 
   \[
   \alpha_n = \frac{(1 - \delta \phi) \kappa_m + \kappa_w \tau}{\sigma \delta \phi \kappa_m \kappa_w \Delta p \Delta q}, \quad \gamma_t = \frac{\kappa_m + \kappa_w}{\sigma \delta \phi \kappa_m \kappa_w \Delta p \Delta q}. 
   \]

7. It remains to evaluate the wage cost. From (11), using \( \gamma_n = \alpha_t = 0 \), the expected wage cost with a lattice is

\[
\alpha_n + t_S \beta_n + \sigma \phi_h \{t_{mw} \beta_t + [\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}] \gamma_t \} - \alpha_n - t_{mg} \beta_n + y_G \beta_n \}.
\]

For contracts (1) and (2) of step 6, (TTg) is binding and \( \beta_n = \beta^S \). In this case, the term \( t_S \beta_n \) in (32) equals the wage cost with silos. With a lattice, therefore, the wage cost differs by the expression

\[
\Delta w = \alpha_n + \sigma \phi_h \{t_{mw} \beta_t + [\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}] \gamma_t \} - \alpha_n - t_{mg} \beta_n + y_G \beta_n \}.
\]

From (TTg) for \( \mu = g \), we have

\[
\alpha_n = t_{gw} \beta_t + [\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}] \gamma_t - t_{gg} \beta_n.
\]

Substituting for \( \alpha_n \) in the second term of \( \Delta w \), we obtain

\[
\Delta w = \alpha_n + \sigma \phi_h [(1 - p_m)(\beta_n - \beta_t) + (\delta \phi - p_o) \beta_n] \kappa_m \Delta q,
\]

which is positive because \( \beta_n \geq \beta_t \) from step 3, and \( \delta \phi > p_o \) from (C1). Since \( \alpha_n > 0 \), it follows that \( \Delta w > 0 \).

For contract (3), wage costs are lowest if \( \xi = 0 \) and hence \( \beta_n = \beta_t = 0 \). In this case, the wage cost difference compared with silos is

\[
\Delta w = (1 - \sigma \phi_h) \alpha_n + \sigma \phi_h [\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}] \gamma_t - \beta^S t_S
\]

\[
= [t_{bb} + \sigma \phi_h \delta \phi (t_{gb} - t_{bb})] \gamma_t - \beta^S t_S,
\]

\[\text{43} \]
where the second equality follows from $\alpha_n = t_{bb}\gamma_t$ according to (31). Because $\delta \phi$ appears in the denominator of $\gamma_t$, the first term in (33) is decreasing in $\delta \phi$. So we have (using (31) and (10))

$$[t_{bb} + \sigma v p h \delta \phi(t_{gb} - t_{bb})] \gamma_t \geq [t_{bb} + \sigma v p h(t_{gb} - t_{bb})] \frac{1}{\sigma v \kappa_m \Delta p \Delta q} \tau \geq t_{bb} \frac{\kappa_w}{v \kappa_m} \beta^S,$$

whereas $t_S$ in the second term of (33) cannot exceed $t_{gg}$. It follows that a sufficient condition for $\Delta w > 0$ is

$$t_{bb} \frac{\kappa_w}{v \kappa_m} > t_{gg},$$

which is equivalent to condition (C3) stated in the proposition. QED

**Proof of Proposition 5:** If $\xi = 0$, there is only the training incentive constraint $\psi > 0$ to satisfy, and an optimal corner solution will have either $\beta > 0$ and $\gamma = 0$, or $\gamma > 0$ and $\beta = 0$. The first case obtains if and only if

$$\frac{\partial \psi/\partial \beta}{\partial w_A/\partial \beta} > \frac{\partial \psi/\partial \gamma}{\partial w_A/\partial \gamma}. \quad (34)$$

Since $\partial w_A/\partial \beta = \partial w_A/\partial \gamma$ (cf. 21), (34) holds if and only if $\partial \psi/\partial \beta > \partial \psi/\partial \gamma$, or

$$\kappa_w \Delta q - v y_L > v y_G.$$

The left-hand side is $\kappa_w \Delta q - v y_L = \kappa_w [1 - v(1 - p_o)] \Delta q > \kappa_w (1 - v) \Delta q$, and the right hand side is $v y_G = v \kappa_m (\delta \phi - p_o) \Delta q < v \kappa_m \Delta q$. Under condition (C3'), we have $(1 - v) \kappa_w \geq v \kappa_m$, and therefore $\partial \psi/\partial \beta > \partial \psi/\partial \gamma$. It follows that training is least costly to incentivize with $\beta > 0$ and $\gamma = 0$, and solving (22) for this case leads to the expression for $\beta$ stated in the proposition. QED

**Proof of Proposition 6:** As argued in the text, under (C3'), the cost-minimizing value of $\gamma$ that satisfies both (TTg) and (TTb) is $\gamma = \min \{\gamma_g, \gamma_b\} = \gamma_g$ because of (C2). In this case, $\gamma = \gamma_g \beta$ as stated. Substituting this expression for $\gamma$ in (22) and solving for $\beta$ leads to $\beta = \beta^S$. QED

**Proof of Proposition 8:** For transfer-contingent contracts, according to Proposition 4, $\beta_n = \beta^S$ and $\gamma_n = 0$, like in the silo case. The profit difference therefore equals (see (11))

$$\Delta \pi = -\alpha_n + \sigma v p h \{(R - \beta_n) t_{mw} - [\delta \phi t_{gb} + (1 - \delta \phi) t_{bb}] \gamma_t + \alpha_n - (R - \beta_n) t_{mg} + (R - \beta_n) y_G \} \quad (35)$$
This expression is decreasing in $p_o$ via $y_G$. Moreover, none of the wages stated in Proposition 4 depends on $p_o$, which completes the proof.

For output-based contracts, using $\pi^S = (R - \beta^S)t_S$, (21), and $\beta = \beta^S$ according to Proposition 6, we have

$$
\Delta \pi = [t_S + \sigma v p_h(y_G - y_L)](R - \beta - \gamma) - t_S(R - \beta)
$$

(36)

$$
= \sigma v p_h(y_G - y_L)(R - \beta) - [t_S + \sigma v p_h(y_G - y_L)]\gamma
$$

(37)

$$
= \sigma v p_h(y_G - y_L)(R - \beta - \gamma) - t_S\gamma,
$$

(38)

where the first term in (37) is decreasing in $p_o$ via $y_G$, and the second term is decreasing as well because both factors of the product are increasing in $p_o$ via $\gamma$, and $\frac{\partial}{\partial p_o}[t_S + \sigma v p_h(y_G - y_L)] = v\kappa_m(1 - \sigma p_h)\Delta q > 0$. QED

Proof of Proposition 9: For transfer-contingent contracts, there are four effects of $\kappa_m$ on $\Delta \pi$ in (35): first, in the expression in {}, the last term $(R - \beta_n)y_G$ is increasing in $\kappa_m$ via $y_G$. Second, the derivative of $(R - \beta_t)t_{mw} - (R - \beta_n)t_{mg}$ with respect to $\kappa_m$ is $(\beta_n - \beta_t)q_m \geq 0$ in all three versions of the optimal contract according to Proposition 4. Third, for $x \in \{\kappa_w, \kappa_m + \kappa_w, \kappa_w/v\}$ depending on the version of the optimal contract, we have

$$
-[\delta \phi t_{gb} + (1 - \delta \phi)t_{bb}]\gamma_t = -[t_{bb} + \delta \phi \kappa_m \Delta q] \frac{x}{\delta \phi \kappa_m} \beta^S = - \left[ \frac{(\kappa_m + \kappa_w)q_b x}{\delta \phi \kappa_m} + x\Delta q \right] \beta^S, \quad (39)
$$

which is increasing in $\kappa_m$. The fourth effect is on $-(1 - \sigma v p_h)\alpha_n$ which is positive for contracts (1) and (3) because $\partial \alpha_n/\partial \kappa_m < 0$. For contract (2), however, $\partial \alpha_n/\partial \kappa_m < 0$ holds only if $\kappa_m^2(1 - \delta \phi) < \kappa_w^2$, which may or may not hold. If $\partial \alpha_n/\partial \kappa_m > 0$, then $\partial \Delta \pi/\partial \kappa_m < 0$ is possible.

For output-based contracts, the term $\sigma v p_h(y_G - y_L)(R - \beta - \gamma)$ in (38) is increasing in $\kappa_m$ because $y_G$ is increasing and $\gamma$ is decreasing in $\kappa_m$, and all other variables are unaffected. The derivative of the remaining part, $-t_S\gamma$, is positive because $t_S$ has the form $x\kappa_m + y\kappa_w$, while $\gamma$ has the form $z/\kappa_m$. Therefore $t_S\gamma = (xz + yz\kappa_w/\kappa_m)$ is decreasing in $\kappa_m$. QED

Proof of Proposition 10: For transfer-contingent contracts, there are three effects of $\delta$ and $\phi$ on the profit difference, cf. (35): the first is a positive effect equal to $\sigma v p_h(R - \beta_n)\partial y_G/\partial (\delta \phi) = \sigma v p_h(R - \beta_n)(t_{gb} - q_b)$. The second is a profit increase due to a decrease in
\( \alpha_n \) in all three versions of the optimal contract. The third is through the term

\[
-\left[ \delta \phi t_{gb} + (1 - \delta \phi) t_{bb} \right] \gamma_t = -\left[ t_{bb} + \delta \phi k_m \Delta q \right] \frac{x}{\delta \phi k_m} \beta^S = - \left[ \frac{t_{bb} x}{\delta \phi k_m} + x \Delta q \right] \beta^S,
\]

for \( x \in \{ \kappa_w, \kappa_m + \kappa_w, \kappa_w/v \} \) depending on the version of the optimal contract. Either way, this term is increasing in \( \delta \) and \( \phi \).

For output-based contracts, the first term in (38) is increasing in \( \delta \) and \( \phi \) via \( y_G \), provided that a lattice is profitable at all \( (R - \beta - \gamma > 0) \). The second term, \(- t_S \gamma\), is clearly increasing in \( \delta \) because \( t_S \) does not depend on it while \( \gamma \) is decreasing in \( \delta \) through the effect on \( \gamma_g \). For \( \phi \), it takes the form \(- t_S \gamma = -(a + b \phi)c/(d \phi - e)\) for positive constants \( a \) through \( e \), and the derivative is \( c(ad + be)/(e - d \phi)^2 > 0 \). QED

### 10 References


Baker, George P., Michael Gibbs, and Bengt Holmström (1994)


Gibbons, Robert (2013): “Cyert and March (1963) at Fifty: A Perspective from Organizational Economics”, prepared for NBER Working Group in Organizational Economics


Hall, Brian, and Jonathan Lim (2002): “Massachusetts Financial Services” HBS Case 9-902-132


Martin, Stephen (2009): “Microfoundations for the Linear Demand Product Differentiation Model, with Applications”, working paper, Purdue University.


Williamson, Oliver (1985), The Economic Institutions of Capitalism
