General Equilibrium of a Regional Economy with a Financial Sector - Part II: A Simple Behavioral Model

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GENERAL EQUILIBRIUM OF A REGIONAL ECONOMY WITH
A FINANCIAL SECTOR—PART II: A SIMPLE BEHAVIORAL
MODEL

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ABSTRACT. Simple behavioral assumptions are incorporated into an accounting framework
that provides linked budget and balance sheets for sectors of a regional economy. A short-run
Keynesian-type model is developed where quantities rather than prices adjust, and where
regional prices and interest rates are equal to national levels. The analysis highlights the
importance of the financial services sector as an active factor in regional growth. Consumer
deposit and debt preferences, and limitations imposed on credit extension by the financial
services sector can have important effects on the regional economy as evidenced by changes in
the export multiplier.

1. INTRODUCTION

Loanable funds as a determinant of regional output levels and relative sector
expansion is the subject of this paper. The analysis presented here uses the
accounting framework developed in the first part of this article, Part I: An
Accounting Framework Linking Budgets and Balance Sheets, as a basis from
which to build a simple behavioral model. Part I, published in the preceding issue of
this Journal, develops a simple accounting system for a regional economy that
explicitly recognizes both the stocks and flows of a regional economy, and
definitional equations linking the two. These accounts also articulate the institu-
tional detail of a financial sector, yielding a framework that recognizes potentially
important links between financial capital and regional production and consump-
tion.

In this, Part II, the accounting framework is expanded to incorporate
economic behavior of the region’s sectors. A short-run Keynesian-type model is
developed where quantities rather than prices adjust, and where regional price and
interest rate levels are equal to the national level. The model is applied to a
scenario, similar to that analyzed by Harrigan and McGregor (1987) and by Amos
and Wingender (1988), in which suppliers of credit engage in credit rationing in the
loanable funds market. The accounting framework can, of course, be combined
with alternative assumptions such as flexible, market clearing prices.

This analysis highlights the importance of modeling the financial services
sector as an active factor in regional growth. In particular, it is shown that both

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consumer local deposit and debt preferences and limitations imposed on credit
extension by financial institution policy can induce important macroeconomic
effects on the regional economy, as evidenced by changes in the export multiplier.

Much analysis of regional economies takes the form of determining the
multiplicative expansion effect of an increase in regional exports on the regional
economy (for example, Richardson, 1985; Holland, 1988; Kraybill, 1989). Many
useful analyses of regional export multipliers are also presented in Harrigan and
McGregor (1988). A good discussion of recent advances in conceptualizing the
impact of financial structure on the national level is in Gertler (1988), while
analysis of monetary policy on regions is contained in Miller (1978).

In this paper, the effect of an exogenous increase in exports on total regional
income, as well as on the relative sizes of its industry and financial services sectors,
is examined. Different sets of relative parameter values are shown to lead to
qualitatively different export multiplier results. In particular, two types of econo-
 mies are identified: high consumer deposits and consumption rates impact strongly
on currency flows in the first economy type and a (standard) positive export
multiplier results. For the second type, in which low deposit usage and high foreign
investment are key factors in determining currency flows, negative credit market
feedbacks undermine the initial expansionary effect of increased export demand
and a negative multiplier may result.

Section 2 provides a brief overview of the general equilibrium regional model.
The model combines the accounting framework presented in Part I with a few
simple behavioral assumptions. These behavioral assumptions are detailed in
Section 3 of this paper. Formally, the model is a system of 23 simultaneous
equations. Through successive substitution, this model can be reduced to a set of
four linear equations in four variables. Analysis of the model is therefore
unusually straightforward and is accomplished graphically through a simple
four-quadrant diagram. Overall multiplier effects on the region’s economy, as well
as relative gains or losses between sectors of the economy, are analyzed using this
diagram. The diagram is introduced in Section 4 and used in Section 5 to show how
different sets of parameter values can yield qualitatively different multiplier
results. Results are summarized in the concluding Section 6.

2. OVERVIEW OF THE MODEL

This model has four agents: industry, finance, household, and the rest of the
world (ROW), the latter of which includes all area outside of the region. Industry
uses a Leontief production process to produce a composite “real” good which
includes all goods and non-financial services. It sells output to all other agents,
makes investments and pays for inputs which include labor and intermediate
goods, and may earn profits. Industry uses both internal and external financing in
varying proportions for its working capital and fixed investment needs. Industry

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1That is, the equations are linear in the four (remaining) endogenous variables. It is highly
nonlinear in the parameters and several of the exogenous variables.

2For simplicity “rest-of-world” is treated as “rest-of-nation.”
loans can be obtained from both the local financial sector and extraregional financial investors. Working capital is kept with the finance sector as deposits.

The household sector receives two forms of income: labor income (i.e., wages and salaries) and property income (i.e., dividends from industry and finance and interest from bank deposits) and spends it on the local industry good, financial services and imported consumer items. The household sector also keeps deposits with the finance sector and contracts with it for consumer loans, based on available collateral. In modification of the model of Moore, Karaska and Hill (1985), the change in household deposits is linked to income gross of interest receipts. Both the household sector and the region as a whole have variable net savings rates.

The finance sector of the model may be visualized as a set of local banks. The sector accepts deposits, makes two types of loans, consumer and industrial, and holds reserves in fixed proportion to deposits. Following theoretical literature on adverse selection in credit markets (for example, Stiglitz and Weiss, 1981), it is assumed that at higher loan interest rates, higher quality loan applicants are less likely to apply. The interest rate at which bank profits are maximized, therefore, may be below the market clearing rate. If so, banks will turn away potential customers rather than face the increased losses that would occur if they raised the interest rate. Both the local finance sector and extraregional investors are assumed to exhibit this behavior.

The rest of the world (ROW) interacts with the other agents of the model by supplying imports, demanding exports and engaging in interregional financial investment. Just as the trade balance may be positive or negative, so may the net interregional financial flow. The balance of trade need not perfectly offset the change in extraregional financial investment due to the third item of the balance of payments identity, change in currency of the region. Currency in this model is recognized solely as primary deposits. Following previous work (for example, Moore and Hill, 1982), the net change in primary deposits of the region equals the change in bank reserves, or in this model, reserves held by the finance sector. Extraregional investors only lend to industry, not to households.

All accounts of the balance of payments are endogenous in this model. For example, the trade balance may become positive or negative in response to an increase in autonomous export demand. The change in the capital account may be either positive or negative depending on the demand and supply of loanable funds in the region. Unlike the previous theoretical literature mentioned above, careful distinction is made between the change in extraregional financial investment in the region and primary deposits arising from an inflow of currency.

3. SECTOR BEHAVIOR

The purpose of the model developed in this paper is to illustrate how institutional decisions concerning the allocation of financial instruments can affect economic development of a regional economy. Table 1 contains the equations and defines the notation of this model. The first 16 equations, largely accounting identities, are similar to those introduced in Part I of this article. They differ only in that they incorporate behavioral assumptions, concerning, for example, prices, technology, and consumption preferences. By themselves, however, these equa-
tions represent an under-identified system and yield inadequate information as to how the interplay of economic decisions affects the development of the system. A more fully-articulated economic model is developed by including additional behavioral equations relating savings and investment decisions to financial variables.

In this section, each sector's budget, balance sheet and linkage constraints, and the behavioral assumptions embodied in them, are discussed. Then, eight additional equations defining the financial strategy of agents are introduced, and the discussion turns to their economic implications.

Industry's budget constraint is defined by Equations (1) and (2) of Table 1. As stated in Equation (1), industry output \( x_t \) is purchased by the finance sector \( (a_f X_t) \) in fixed proportion to total financial services output; by households \( [y(P + \Omega H)X] \) as a variable portion \( y \) of profits \( PX \) and wages \( ([\Omega H]X) \); by industry for investment purposes \( I_t \); and by the rest of the world at an exogenous level of exports \( E \). As stated in Equation (2), industry uses its receipts in order to purchase financial services, in variable proportion \( a_f \) to its output \( x_t \); wage labor \( (\omega \eta x_t) \) with a fixed wage rate; to pay dividends \( (p \pi x_t) \) at a variable rate, as will be discussed below; to buy imported intermediates in fixed proportion to total output \( (\theta x_t) \); to service financial capital owned by extraregional \( iF \) where \( i \) is the exogenous interest rate). Residual industry revenue is treated as retained earnings \( RE \). Equation (2) implies that the industry sector performs according to a Leontief production function.

The budget constraint of the finance sector is defined by Equations (3) and (4). The services of the finance sector \( (x_f) \) are sold to industry \( (\alpha_f x_t) \) and households \( [e(P + \Omega H)X] \), in variable proportion to sector size. The finance sector produces services by purchasing intermediate goods in fixed proportion to its output level \( (\alpha_f X_f) \), by buying labor at a fixed wage rate \( (\omega \eta x_f) \) and by paying some fixed rate of return to its owners \( (\pi_f x_f) \).

The household sector is composed of both wage earners and entrepreneurs. Equation (5) defines the household budget constraint, with total returns to households \( [\Omega (P + \Omega H)X] \) expended exclusively on local industry output \( [y(P + \Omega H)X] \), local financial services \( [e(P + \Omega H)X] \), external goods \( [\iota(P + \Omega H)X] \) and savings \( x(P + \Omega H)X \). All four of the expenditure coefficients are variable.

Finally, the foreign sector [Equation (6)] sells physical intermediates to industry \( (\theta x_t) \), sells consumer items to the household sector \( [\iota(P + \Omega H)X] \), receives interest income on capital lent to industry \( (iF) \), and buys exports \( E \). The foreign savings term \( SF \) is defined as the regional current account deficit, equal to the value of imports minus exports. [Interconnections between sector budget sheets

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3 Throughout this paper, parameters are denoted by Greek letters with the exception of past period values, composite parameters and the two interest rates. Some values that appeared as parameters in Part I of this paper have become variables here in Part II, hence notation has switched from Greek to Roman. For notational convenience, capital letters are used as aggregate indices for total regional output \( X \), the weighted average unit costs \( \Omega H \), and the weighted average profit rate \( P \) in the SAM. Let \( \Omega H x = \omega \eta x + \omega \eta p x \) denote total labor earnings, and \( PX = p x + \pi x \) denote total profits paid out to households. Then \( \Omega H = (\omega \eta x + \omega \eta p x)/(x + x p) \); \( P = (p x + \pi x)/(x + x p) \); and \( X = x + x p \). See Table 1 for notational definitions.
### TABLE 1: Equations, Variables and Parameters

<table>
<thead>
<tr>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flow Balance Identities—SAM</strong></td>
</tr>
<tr>
<td>(1) ( x_i = a_{q_i} x_I + y(P + \Omega H)X + I_i + E )</td>
</tr>
<tr>
<td>(2) ( x_I = a_p x_I + \omega_1 \eta_1 x_I + p x_I + \theta x_I + RE + iF )</td>
</tr>
<tr>
<td>(3) ( x_f = a_{q_f} x_f + e(P + \Omega H)X )</td>
</tr>
<tr>
<td>(4) ( x_f = a_{q_f} x_f + \omega_1 \eta_1 x_f + \pi_f x_f )</td>
</tr>
<tr>
<td>(5) ( y(P + \Omega H)X + e(P + \Omega H)X + t(P + \Omega H)X + s(P + \Omega H)X = (p_i + \omega_1 \eta_1) x_i + (\pi_f + \omega_f \eta_f) x_f )</td>
</tr>
<tr>
<td>(6) ( SF = \theta x_I + t(P + \Omega H)X + iF - E )</td>
</tr>
<tr>
<td><strong>Wealth Balance Identities—Wealth Matrix</strong></td>
</tr>
<tr>
<td>(7) ( W_t = K_t + Q_t - L_t - F )</td>
</tr>
<tr>
<td>(8) ( Q_h = L_h + L_h + R - Q_h )</td>
</tr>
<tr>
<td>(9) ( W_i = W_i + Q_i - L_h )</td>
</tr>
<tr>
<td>(10) ( W_f = F )</td>
</tr>
<tr>
<td><strong>SAM—Wealth Matrix Linkages</strong></td>
</tr>
<tr>
<td>(11) ( (W_i - W_w) = RE )</td>
</tr>
<tr>
<td>(12) ( I_i = (K_i - K_w) )</td>
</tr>
<tr>
<td>(13) ( a_{q_i} x_i = iL_i - i_d Q_i )</td>
</tr>
<tr>
<td>(14) ( e(P + \Omega H)X = \epsilon L_h - i_d Q_h )</td>
</tr>
<tr>
<td>(15) ( cr = (R - R_c) )</td>
</tr>
<tr>
<td>(16) ( SF - (F - F_s) + cr = 0 )</td>
</tr>
<tr>
<td><strong>Behavioral Equations</strong></td>
</tr>
<tr>
<td>(17) ( I_i = \delta x_i )</td>
</tr>
<tr>
<td>(18) ( RE = \gamma p x_i )</td>
</tr>
<tr>
<td>(19) ( Q_i = \epsilon [\omega_1 \eta_1 x_i + \theta x_i + \tau_i (J_w + I_i)] )</td>
</tr>
<tr>
<td>(20) ( R = \phi (Q_i + Q_h) )</td>
</tr>
<tr>
<td>(21) ( L_h = \beta W_h )</td>
</tr>
<tr>
<td>(22) ( Q_h = (1 - \psi) (P + \Omega H)X + i_d Q_h + Q_{ho} )</td>
</tr>
<tr>
<td>(23) ( (t + y) (P + \Omega H)X = \psi (P + \Omega H)X + (L_h - L_{ho}) - iL_h ) (implied by the other equations of the model)</td>
</tr>
<tr>
<td>(24) ( t/y = \zeta )</td>
</tr>
</tbody>
</table>

#### Definitions of Variables and Parameters

- \( a_{q_i} \): Per unit demand for intermediate inputs (parameter)
- \( a_p \): Per unit use of financial services by industry (variable)
- \( E \): Export demand (parameter)
- \( \delta \): Investment fraction of the value of industry output parameter
- \( \omega_1, \omega_f \): Wage rate for industry and finance, respectively (parameters)
- \( \eta_1, \eta_f \): Industry and finance per unit labor use, respectively (parameters)
- \( p_i, p_f \): Industry (variable) and finance (fixed) dividend payments per unit output, respectively
- \( RE \): Retained earnings (variable)
- \( \gamma \): Fraction of profits that retained earnings comprise (parameter)
- \( X \): Total output (\( X = x_i + x_f \))
- \( x_i \): Total industry receipts (variable)
- \( x_f \): Finance sector output (variable)
- \( i \): Interest rate on loans and extraregional finance charge (parameter)
TABLE 1: Continued

<table>
<thead>
<tr>
<th>Variables and Parameters (continued)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_d$ Rate at which interest payments on deposits are made (parameter)</td>
<td>$F$ Total amount of funds borrowed from extra-regional institutions (variable)</td>
</tr>
<tr>
<td>$SF$ Current account deficit (variable)</td>
<td>$\theta$ Per unit import use coefficient (parameter)</td>
</tr>
<tr>
<td>$y$ Household average propensity to consume the industry good (variable)</td>
<td>$e$ Household average propensity to consume finance (variable)</td>
</tr>
<tr>
<td>$t$ Household average propensity to consume imports (variable)</td>
<td>$s$ Household average propensity to save (variable)</td>
</tr>
<tr>
<td>$K_i$ Physical capital of industry (variable)</td>
<td>$Q_i$ Deposits of industry (compensating balances) (variable)</td>
</tr>
<tr>
<td>$L_i$ Industry loans, contracted in the present period (variable)</td>
<td>$L_{in}$ Industry loans, contracted in past periods (variable)</td>
</tr>
<tr>
<td>$W_h$ Net worth of households (variable)</td>
<td>$R$ Bank reserves (variable)</td>
</tr>
<tr>
<td>$L_h$ Consumer loans (variable)</td>
<td>$Q_h$ Bank deposits owned by households (variable)</td>
</tr>
<tr>
<td>$F_h$ Reserve ratio (parameter)</td>
<td>$W_f$ Assets owned by extraregionals (variable)</td>
</tr>
<tr>
<td>$W_i$ Equity, or net worth of industry (variable)</td>
<td>$\epsilon$ Share of industry receipts of which industry deposits consist (parameter)</td>
</tr>
<tr>
<td>$r_i$ Fraction of loans for investment carried over from last period (parameter)</td>
<td>$I_i$ New investment (variable)</td>
</tr>
<tr>
<td>$I_{in}$ Past period investment (parameter)</td>
<td>$\text{cur}$ Change in currency in the region (variable)</td>
</tr>
<tr>
<td>$\beta$ Fraction of consumer wealth at which consumer loans ($L_h$) are held (parameter)</td>
<td>$\psi$ Fraction of household income immediately used for consumption (parameter)</td>
</tr>
<tr>
<td>$\zeta$ Ratio of household consumption of local industry goods to imports (parameter)</td>
<td></td>
</tr>
</tbody>
</table>

are highlighted in the social accounting matrix (SAM) presented in Table 2, and discussed in more detail in Part I of this article.]}

Equations (7) through (10) define the balance sheets of each sector. All entries enumerated in the balance sheets are endogenous. The equations are discussed in Part I of this article. (The balance sheets can be presented in matrix form, highlighting the interrelationships between sectors. The matrix form, discussed in Part I as a wealth matrix, is presented in Table 3, which is the same as Table 3 in Part I.)

TABLE 2: Social Accounting Matrix

<table>
<thead>
<tr>
<th>Industry</th>
<th>Financial Services</th>
<th>Expenditures and Receipts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>$\alpha_i x_i$</td>
<td>$y(P + \Omega H)X$</td>
<td>$E$</td>
</tr>
<tr>
<td>Financial Services</td>
<td>$\rho x_i$</td>
<td>$e(P + \Omega H)X$</td>
<td></td>
</tr>
<tr>
<td>Household</td>
<td>$(p_i + \omega_i \eta_i) x_i$</td>
<td>$(\pi_i + \omega_i \eta_i) x_i$</td>
<td></td>
</tr>
<tr>
<td>Rest of World</td>
<td>$\delta x_i + iF$</td>
<td>$t(P + \Omega H)X$</td>
<td></td>
</tr>
<tr>
<td>Savings</td>
<td>$RE$</td>
<td>$s(P + \Omega H)X$</td>
<td>$SF$</td>
</tr>
</tbody>
</table>
TABLE 3: Wealth Matrix

<table>
<thead>
<tr>
<th>Industry</th>
<th>Financial Services</th>
<th>Liabilities and Assets</th>
<th>Industry</th>
<th>Financial Services</th>
<th>Liabilities and Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q,</td>
<td></td>
<td>Household</td>
<td>L₀</td>
<td></td>
</tr>
<tr>
<td>Financial Services</td>
<td>Lₜ</td>
<td></td>
<td>Rest of World</td>
<td>Qₜ</td>
<td></td>
</tr>
<tr>
<td>Household</td>
<td>Wₜ</td>
<td></td>
<td>Capital</td>
<td>Kₜ</td>
<td>R</td>
</tr>
<tr>
<td>Rest of World</td>
<td>F</td>
<td></td>
<td></td>
<td>W₀</td>
<td>Wₜ</td>
</tr>
<tr>
<td>Wealth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The linkages are presented in Equations (11) through (22); the first six are similar to those presented in Part I. Since prices and interest rates are exogenous, there are no capital gains, hence Equation (11) is a valid relationship between the change in net worth of industry and retained earnings. The relationship defined by Equation (12) between investment and the change in physical capital is likewise valid, with an additional caveat of abstraction from depreciation. Also for simplicity, cash in circulation is not recognized, allowing the relationship between net currency inflow to the region and the change in bank reserves to simplify into the relationship specified by Equation (15). The balance of payments identity, (16), states that the net trade deficit minus the change in investment by extraregionals into the region, plus the flow of currency into the region, equals zero.

Equations (13) and (14) define financial services in terms of loanable funds activity, abstracting from all speculative elements of financial behavior. Industry use of financial services is defined as the difference between its current local debt servicing requirement (iLₜ) and interest payment on deposits (iₜQₜ). Household use of financial services assumes an analogous form (iLₜ - iₜQₜ).

In order to concentrate on the key implications of financial strategy as it pertains to the impact of loanable funds on regional development, a broad spectrum of motivational detail is not addressed in this model. Agents may base their decisions on inter-temporal issues, risk concerns, or a variety of other considerations. In this simple model, however, agents are grouped into sectors, and it is assumed that the resulting behavior of sectors can be summarized in a small number of equations that incorporate the implications of the financial strategies of agents. Because the interest rates are exogenous the model is essentially static, and the additional notational complexity involved in explicitly formulating decisions in terms of net present discounted value would be greater than insights that would be gained by its inclusion.

Three of the additional equations capture assumptions with respect to the savings and investment strategy of the industry sector. Following basic Keynesian theory, investment is described very simply as a fixed fraction of total industry output in Equation (17). In effect, real investment is driven by the need to enhance physical production.

In this model, total profits are divided between dividend payments to households and retained earnings. Retained earnings (RE) are gross business savings, and may be interpreted as representing both new issues of equity as well as
re-investment of profit from operation. The ratio of retained earnings to "dividends," or household profit income from industry is fixed, as defined in Equation (18). Total profits per unit of output \( ((1 + \gamma)p_i) \) are variable, based on the level of financial services demanded.

As defined in Equation (19), industry maintains deposits with the financial service sector equal to a fixed share \((\epsilon)\) of the current working capital needed to cover payments for labor, non-competitive imports, and loan carry-over before product sale takes place. The rate of use of financial services by industry is not constant since it varies with the volume of the industry sector's outstanding loan and deposit volumes. Thus, the intermediate use coefficient \((\alpha_F)\) in Equations (2) and (3) varies indirectly with output, through financial decisions defined in Equations (13) and (19).

The finance sector is described with an additional two equations concerning lending practices and liquidity requirements. Consistent with regulatory requirements, reserves are maintained at a fixed fraction of total deposits. Reserve requirements are defined in Equation (20).

As discussed above, the finance sector is modeled as rationing consumer loans. This rationing is assumed to take the form of a minimum collateral requirement, so that a fixed relationship exists between total consumer loans and household net worth, as in Equation (21).\(^4\) In effect, the household sector as a whole is credit rationed. Individual households, however, are not necessarily credit constrained. Some households may have more income than they want to spend in the current period, some may have less. The first group saves in the form of household deposits \((Q_h)\). Net savings of the household sector is, thus, lowered by the consumption loans extended to the second group.

Lending to industry is not constrained by a collateral requirement imposed by the local finance sector and is competitive in the sense that industry can source financial capital through its own savings and extraregional financing as well as through the local financial sector.

External financing by industry is determined residually, equalling the difference between current total industry demand for financial capital and retained earnings of the period.\(^5\) This result may be derived by rewriting the industry balance sheet Equation (7) in difference form and making note of Equations (11), (12) and (19); total external financing \((L_i + F)\) can thus be seen to equal the difference between retained earnings and the sum of working capital and current period investment. Total demand for financing, reflecting the need for working capital as well as investment, is positively related to output but is inelastic due to the assumption of prior commitments.

The final three behavioral equations describe decisions by households with respect to trade-offs between consumption and savings, and between consumption of local and imported goods. Current household deposits are composed of a fraction

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\(^4\)Alternative interpretations of this behavioral equation are also valid. For example, households might voluntarily limit their use of consumer credit to a given fraction of their total net worth.

\(^5\)In effect, industry chooses its portfolio (the mix of bank loans and equity financing) by managing changes in the composition of stock \((L_i - L_m)\) rather than total stock.
of this period's income, as well as deposits carried over from last period and interest paid on deposits during this period, as described in Equation (22). When households initially receive their income \((P + \Omega H)X\) they immediately consume some fraction of it \((\psi)\) and deposit the other portion \((1 - \psi)\). During the period total household deposits are augmented by the interest payments on these deposits.

Total household consumption of industrial products and imports is achieved by expending a fixed fraction of income, plus new borrowing net of interest payments, as stated in the inter-matrix balance condition, Equation (23). The relationship may be derived from the other equations of the model as follows. Using the total household consumption Equation (5) and household financial services expenditure, Equation (14), to rewrite Equation (22)

\[(22') \quad (t + y)(P + \Omega H)X = \psi(P + \omega H)X - iL_h + Q_h - Q_m - s(P + \Omega H)X\]

As was derived in Equation 9\(^n\) of Part I of this article, the change in household deposits minus household savings equals the change in household loans. Rewriting Equation (22') to incorporate (9\(^n\)) yields Equation (23). Because this derivation is rather involved, I have chosen to include the household real goods consumption Equation (23) in Table 1, even though it is formally redundant, and hence, not required to solve for regional equilibrium.

Equation (24) defines a fixed relationship \((\zeta)\) between the portions of household income used for consumption of imports and locally produced goods.

The industry profit rate \((p_i)\) is endogenous due to the relationship between industry stocks and flows. Because loans and deposits are carried over from the previous period, the value of financial services provided per unit of industry output varies with the amount of output. Net profit, therefore, also varies.

The expenditure on financial services by industry is derived by using Equation (18) to rewrite Equation (2)

\[(2') \quad a_p x_i = x_i[1 - \omega_\eta_i - \theta - (1 + \gamma)p_i] - iF\]

The net cost of financial services to industry is derived by substituting \((Q_i)\) of Equation (19) into Equation (13) and using Equations (11), (12), and (17) to rewrite the first difference form of Equation (7) in order to obtain an expression to substitute for \((L_i)\) of Equation (13)

\[(13') \quad a_p x_i = [\epsilon(i - i_d)(\omega_\eta_i + \theta + \tau \theta) + i(\theta - \gamma p_i)]x_i + B - iF\]

where the exogenous parameter \(B\) is a function of model parameters and variable values of the previous period

\[B = \epsilon \tau(i - i_d)I_w + i(K_w - W_w)\]

We assume industry pays its bills on time, so we can equate these two expressions and, with a little more rewriting, we find

\[(13\') \quad p_i x_i = \frac{x_i[1 - \omega_\eta_i + \theta][1 + \epsilon(i - i_d)] - \theta[i + \epsilon \tau(i - i_d)] - B}{1 + \gamma(1 - i)}\]

Dividing by \((x_i)\) and taking the partial derivative with respect to \(x_i\)

\[(13\''') \quad p_i' = Bx_i^{-2}\]
where the prime refers to the partial derivative with respect to \( (x_i) \). As is clear from its definition, the parameter \( B \) may be either positive or negative.

The net profit rate of industry to stockholders may either increase or decrease with output due to variable external finance requirements. Since from Equation (7) (7')

\[
K_o - W_o = F_o + L_o - Q_o
\]

If industry is a net absorber of finance (i.e., \( F_o + L_o > Q_o \)) to a large enough extent, \( (B) \) will be positive, implying that the net financial burden carried over from the previous period becomes less per unit as output increases, hence the net profit rate increases. Conversely, if deposits are sufficiently greater than outstanding debt, \( B \) will be negative. A negative \( B \) implies the net financial advantage carried over from the previous period becomes less per unit as output increases, and the net profit rate decreases.\(^6\) As will be discussed in Sections 4 and 5, the sign of the industry profit rate derivative plays a role in determining the magnitude of the export multiplier.

4. GRAPHICAL ANALYSIS OF REGIONAL EQUILIBRIUM

To extract implications for economic development from the extensive institutional detail presented in the model's equations, a graphical presentation of regional economic equilibrium is developed in this section. First, the equations used to develop the four-quadrant diagram are derived. Factors influencing the slopes of the lines and how they shift in response to exogenous parameter shocks are then discussed. Comparative static results for an exogenous increase in regional exports are left for the next section.

The diagram uses four variables: the outputs of industry \( (x_i) \) and financial services \( (x_f) \), net currency flows in or out of the region \( (cur) \) and the change in reserves held against deposits \( (R - R_o) \). Relationships between industry output and currency flows in the southeast quadrant (the "IC" locus), between currency flows and reserve change in the southwest quadrant (the "CR" locus), and reserve change and financial services output in the northwest quadrant (the "RF" locus) are combined to derive a relationship between the industry and financial services sector outputs in the northeast quadrant. This rest of world accounts balance or ROW locus describes possible combinations of values for industry and financial services outputs given that the balance of payments condition holds and the accounts of industry and financial services are balanced.

But the ROW accounts balance equation does not assure that the household sector has maintained a balanced budget. A second set of equations must also hold, defining possible combinations of industry and financial services output with balanced household accounts. This household accounts balance or HH locus can be superimposed on the ROW locus of the northwest quadrant. The HH locus does not require the balance of payments condition to hold. The intersection of the two lines of the northeast quadrant represents the only set of variable values which satisfies all equations of the model, and hence is the regional economic equilibrium.

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\(^6\)In this case, an increase in output may lead to the profit rate falling below a value that industry managers and stockholders consider minimally adequate. In response, stockholders could limit output to the level that maintains \( p \), at some minimum acceptable level.
CASE A: Negative relationship between changes in currency inflow and industry output.

CASE B: Positive relationship between changes in currency inflow and industry output.

FIGURE 1: Four Quadrant Diagrams: Case A and Case B.

A convenient feature of the model is that all of the loci are linear in the endogenous variables. Further, the signs of the slopes of all the lines except the IC, and hence ROW loci are unambiguous. The two diagrams of Figure 1 illustrate the two possibilities; the relationship between industry output and currency inflow can be either positive or negative.

A number of logical steps are subsumed within the IC locus in the southeast quadrant. An increase in industry output implies a move toward a trade deficit, since imports increase with industry output and exports are exogenous. A trade deficit can be offset by either (both) an influx of extraregional financing or (and) net currency outflow. The magnitude of the change in net currency flow is influenced by demand for regional reserves. Extraregional financing depends on sector demands for credit and local supply.

More formally, the IC locus was derived by using the equations for the final demand for industry output (1), the balance of payments (16), and the derived wealth-capital identity (implicit in the final row and column of the wealth matrix and derivable from Equations (7) through (10)) to rewrite the balance of trade Equation (6) in terms of industry and financial services outputs and interregional settlements (currency inflow). Currency inflow can then be expressed as a linear function of industry and financial services output and the (variable) profit rate of industry

\[
cur = \frac{-x_t}{i} \left[ \theta + \gamma(1 - \psi) + (1 - i) \left[ -\theta + \frac{(1 - \psi)\alpha_i}{(1 - i_d)(1 + \beta)} \right] 
- \frac{p_t}{i} \left[ \frac{(1 - i)}{(1 + \beta)} \left[ \gamma + \frac{(1 - \psi)}{(1 - i_d)} \right] 
+ \frac{x_t}{i} \left[ \gamma - \frac{(\alpha_i - 1)(1 - \psi)(1 - i)}{(1 - i_d)(1 + \beta)} \right] + \frac{(1 + \xi)E - Z}{i} \right]
\]  

(25)
where

\[ Z = (1 - i) \left[ K_w - \frac{W_w}{(1 + \beta)} - \frac{Q_{wo}}{(1 - i_d)(1 + \beta)} + R_o \right] - F_o \]

Though industry profit is clearly negatively related to currency inflow, the coefficient terms that multiply industry and financial services output are ambiguous. Net currency inflow (\text{cur}) may therefore be either positively or negatively related to industry output in Equation (25) depending on which terms dominate.

Each of the coefficient terms of the variables in Equation (25) is a complicated composite. The analysis can be simplified by rewriting Equation (25) as a linear relationship between two variables. Since financial services output and industry profits can themselves be stated as linear functions of industry output, through algebraic manipulation Equation (25) can be reduced to a linear relationship between currency inflow and industry output.

Specifically, financial services output can be traced through the income flows [Equation (3)] back to the implicit quantities of loans and deposits outstanding [Equations (13) and (14), then (8) and (20)] and from there to the implied current industry output and past period values [Equations (19) and (22)]. Using Equation (24), the currency flow can then be written in terms of industry output and industry profits providing a more convenient expression for the IC locus

\[
\text{cur} = \frac{-x}{i} \left[ \theta + \xi (1 - \theta) + (1 - i) \left( \frac{1 - \psi}{(1 - i_d)(1 + \beta)} \right) \right] \\
- \frac{x_A}{i} \left[ \epsilon \omega, \eta_i + \theta + \tau \theta \right] + \omega, \eta_i \frac{(1 - \psi)}{(1 - i_d)} \\
- \frac{p, x_i}{i} \left[ \frac{(1 - i)}{(1 + \beta)} \gamma + \frac{(1 - \psi)}{(1 - i_d)} \right] + \frac{A(1 - \psi)}{(1 - i_d)} \\
+ \frac{A(e, \tau I_w + Q_w/(1 - i_d))}{i} + \frac{(1 + \xi)E - Z}{i}
\]

(26)

where

\[
A = \left[ -\xi \alpha_d + \frac{(1 - \alpha_d)(1 - \psi)(1 - i)}{(1 - i_d)(1 + \beta)} \right] \\
\times \left[ \frac{i(1 - \phi) - i_d}{1 - (1 - \psi)(i(1 - \phi) - i_d)[(1 - \alpha_d)/(1 - i_d)]} \right]
\]

The parameter \( A \) is a composite of parameters relating financial services to industry output. The parameter \( Z \) is a constant composed of previous period values.

Since industry profit (\( p, x_i \)) is a linear function of \( x_i \) and past period values, as defined in Equation (13'), it can be factored out to yield the linear relationship between currency inflow and industry output graphed in Figure 1. Industry profits are left in Equation (26) to simplify notation.

The sign of the slope depends on a large number of parameters, but three sets of parameters that may determine the character of the economy can be drawn out.
Through the balance of payments and balance of trade, an inflow of currency is negatively related to imports and positively related to foreign investment. Through the wealth accounts foreign investment is negatively related to household deposits and positively related to physical capital, hence household deposits are negatively related to currency inflow (and industry investment positively related) with respect to the interregional accounts. Household deposits, industry investment and importation rates play key roles in analysis of the multiplier.

The coefficient term on $x_i$ in Equation (26) can be either positive or negative. The total coefficient term on $x_i$ will be negative with a small enough investment rate $\delta$ and if the composite coefficient $A$ is positive. The composite $A$ in turn will be positive with a small enough $\alpha_t$. Small values for $\alpha_t, \delta$ are indicative of a relatively high ultimate consumption for locally produced industry goods $[y(P + \Omega H)X]$. Moreover, the slope of Equation (26) is more likely to be negative if the initial deposit rate $(1 - \psi)$ is close to unity which, in combination with high consumption demand implies strong financial intermediation in household activity. Finally, the slope is also likely to be negative if $\zeta$, the ratio of consumer average consumption propensity to import versus consume locally produced goods, and $\theta$, the industry import rate, are large.

If $(1 - \psi)$ were zero or very low, the composite coefficient $A$ would be negative. Combined with a high industry investment rate the total coefficient term on $x_i$ would be positive, hence the IC line would be positively sloped. Currency flow would rise, ceteris peribus, with output.

It is the ambiguous sign of the slope of the IC locus that bifurcates the discussion into two cases, A and B, in what follows. In Case A, a picture emerges of a regional economy that has a high level of financial intermediation, high consumption rate, and high import demands. With a high importation rate and increases in economic activity generating a surfeit of deposits, industry output is multiplied by a negative coefficient, and the slopes of the IC line and the ROW line are negative. In Case B, while local investment demand is strong, import demand and consumer demand for local products are relatively weak. In Case B, industry investment and debt needs dominate current flows, and $x_i$ is multiplied by a positive coefficient. Thus, the slopes of the southeast quadrant line and the ROW line are positive.

As was discussed in Section 3, the derivative of industry profits with respect to industry output may be either positive or negative. If this derivative is positive, the absolute value of the slope of the IC line, as defined in Equation (26), will be increased by a rising profitability effect. If the derivative is negative, however, the absolute value of the slope will be reduced. The following discussion assumes that the effect of the variability of the profit rate is of secondary importance.

An exogenous increase in exports increases the intercept of the IC line but does not affect the slope. Thus, increased exports shift the line downward/rightward if industry output is multiplied by a negative coefficient. If industry output is multiplied by a positive coefficient, however, an increase in exports will shift the southeast quadrant downward/leftward.

The southwest quadrant illustrates the simple CR relationship between currency and reserves of the financial services sector as defined in Equation (15). The line in the southeast quadrant of the diagrams of Figure 1 is thus drawn at a
45-degree angle

\[ \text{cur} = R - R_c \]

The northwest quadrant illustrates the RF relationship between changes in reserves held by the local financial institutions and the net financial profit, or "output" of the financial services sector. Revenues accruing to the financial services sector, Equation (3), can be written in terms of stocks of loans and deposits outstanding, Equations (13) and (14), which are related to the required reserves through the financial services balance sheet, Equation (8). For a given reserve ratio requirement, more reserves imply more local loanable funds, hence more financial services output as defined in Equation (27)

\[ x_f = \frac{(R - R_c)}{\phi} [i(1 - \phi) - i_d] + \frac{R_c}{\phi} [i(1 - \phi) - i_d] \]

An upward sloping RF line is thus drawn in this quadrant. With a zero change in reserves there is positive output, since presumably the current period is not the first period that the sector is in existence. An increase in the required reserve ratio will decrease the net financial profit margin and hence decrease financial services output for a given money base.

The ROW line in the northeast quadrant depicts combinations of \( x_i \) and \( x_f \) that can be derived by graphically tracing around the quadrants of the diagrams of Figure 1. Since the slope of the IC line can be either positive or negative, the ROW line can also be either positive or negative. Also like the IC line, an exogenous change in exports will change the intercept but not the slope of the ROW line.

A second relationship, a household coffee balance or HH line can be drawn in the northeast quadrant relating financial services to industry output. Equation (5) allows substitution of \( s \) and \( e \) (relatively easy to render in terms of sectoral output) in place of \( y \) and \( t \) in the consumption demand in Equation (23). Noting that consumer imports are a fixed fraction of local consumer goods in Equation (24), \( y \) can be derived in terms of household use of financial services [Equation (14)] and savings [equal to the change in household deposits minus the change in household loan volume, as defined in Equation (9)] of Part I of this article. Using the household balance sheet to translate consumer loans into terms of household deposits [Equation (9)] then using Equation (22) to re-write deposits in terms of current household income, a second linear relationship is derived between financial services and industry output only\(^7\)

\[ x_f = \frac{x_i[(1 - \delta)(1 + \zeta) - \omega, C]}{(1 + \zeta)\alpha_f + (1 - \alpha_f)C} - \frac{(Z + E)(1 + \xi)}{(1 + \zeta)\alpha_f + (1 - \alpha_f)C} \]

\(^7\)Equations which are used to determine the ROW accounts balance line but not the household accounts balance line are the two interregional identities [the balance of trade identity, Equation (6), and the balance of payments identity, Equation (14)], the behavioral equation defining currency inflow as equal to the change in reserves, Equation (13), the behavioral reserve ratio requirement, Equation (20), and revenues of financial services identity, Equation (3). The equation used to determine the household accounts balance line but not the ROW accounts balance line is the behavioral equation of the household allocation of its current income, Equation (5).
where
\[ Z = \frac{(1 - i)\beta}{(1 + \beta)} \left[ W_o + \frac{Q_{ho}}{(1 - i_d)} \right] - L_{ho} \]

and
\[ C = \psi + \frac{(1 - \psi)(1 - i)\beta}{(1 - i_d)(1 + \beta)} \]

The parameter \( Z \) is a constant composed of past period values. Industry profits \((p_{ix_i})\) are a linear function of \(x_i\) and past period values as derived in Equation (13'). The parameter \( C \) is composed of a combination of the direct consumption rate from income and the indirect consumption rate through consumer loans. Equation (28) thus relates \( x_i \) to \( x_f \) through household consumption. Since \( C \) in Equation (28) is less than unity and \( \phi \) is likely to be relatively small, coefficient terms of \( x_i \) and \( p_{ix_i} \), in Equation (28) are assumed positive in the following analysis, so that the household accounts balance line will always be positively sloped. As with the ROW line, an increase in exports reduces the intercept (but does not change the slope), shifting the HH line downward/rightward.

If the derivative of the profit rate with respect to industry output is positive, the slope of the household accounts balance line as defined in Equation (28) would be reduced. If the derivative is negative, the slope would be increased.

5. THE EXPORT MULTIPLIER

In this section the export multiplier of this model is analyzed for two qualitatively different regional economies. Case A is a region where currency inflow decreases with an increase in industry output. The region's households have high income and initial (or gross) deposit rate, as well as a strong appetite for imports. Case B is a region where the currency inflow increases with industry output. The region has high industry investment demand but low wage and profit rates, a low initial deposit rate and, ultimately, little demand for imports or local consumption.

The analysis is conducted using Figure 1, in which both the ROW accounts balance locus and the HH accounts balance locus shift in response to an exogenous increase in exports. Conditions and effects of a shift in the ROW locus are discussed first; such a shift is likely to be the determining factor in the qualitative impact of an export shock. Then, modification of the impact by the concurrent shift of the HH locus is considered. Finally, attention turns to how a change in the profit rate may further modify the result.

Case A is illustrated in the left-hand diagram of Figure 2. Holding industry output constant, an exogenous increase in exports shifts the IC locus of the southeast quadrant downward/rightward as more exports imply a larger currency inflow into the region. The inflow of currency leads to a larger money base and expanded financial profits for financial services as the effects are traced through the other quadrants. The ROW locus in the northeast quadrant therefore shifts up, moving along the HH locus as higher local income increases demand for imports.
and capital inflow. The ceteris paribus implication is higher output for both industry and financial services.

Case B is illustrated in the right-hand diagram of Figure 2. As in Case A, holding industry output constant, an increase in exports implies greater currency inflow and a ceteris paribus increase in the relative size of the financial services sector. The IC line shifts downward/leftward with an exogenous increase in exports, and the ROW line shifts upward.

But with industry investment demand strong relative to household deposits, foreign investment rises more than currency inflow and the local financial service sector \( (x_I) \) shrinks. Consumer demand for industry falls relative to exports. The balance of trade moves toward surplus but industry cuts back production. Through the multiplier process, as output falls so do household income and financial services, and the economy moves back along the HH line.

Increasing exports ceteris paribus shifts the HH line downward/rightward. Holding \( x_i \) fixed, an increase in exports implies a decrease in \( x_i \) since less industry product is available to local buyers. In Case A, the result of the ceteris paribus shift of the HH line is a contraction in financial services and expansion of industry output. In contrast, in Case B, the ceteris paribus shift decreases both industry and financial services output, since the ROW line is steeper than the HH line.

As can be seen from comparing Equations (26) and (28), an increase in exports shifts the ROW line more than the HH line, since

\[
\frac{1}{i} > \frac{1}{(1 + \frac{\zeta}{\xi}) \alpha_f + (1 - \alpha_f) C}
\]

The net effect of the export multiplier in Case A, therefore, is an increase in both financial services and industry output, leading in sum to a positive regional export multiplier.
In Case A, if an increase in industry output implies an increase in the industry profit rate, the ROW line would be relatively flat, implying a larger expansion in industry output and smaller contraction in financial services as the HH line shifts. The implication is an increase in the size of the total regional multiplier. If an increase in industry output implies a decrease in the industry profit rate, the result is a reduction in the total effect of the export multiplier by the same reasoning.

In Case B, with the ROW line steeper than the HH line, as depicted in the right-hand graph of Figure 2, the total effect is a decrease in both financial services and industry output. If an increase in industry output implies an increase in the industry profit rate, both the HH line and the ROW line will be relatively flat, implying smaller decreases in sectoral output from the ceteris paribus ROW line shift. The ceteris paribus HH line shift exaggerates the decline in industry output but ameliorates the decline in financial services, with relatively little effect on total regional output. Thus, if the industry profit rate increases with industry output, the total regional export multiplier in Case B will be smaller in absolute terms than if the industry profit rate falls with an increase in industry output.

6. CONCLUSION

The topic of differential growth rates of regional economies has been much studied, but attention has only recently focussed on the role of regional financial markets. This paper analyzes the effects of integrating real product and loanable funds markets in a regional economy.

The model presented in this paper differs from previous work by incorporating full income statements and balance sheet identities for all of the model's agents. Specifically, the model combines a regional social accounting matrix (SAM) with a complete set of balance sheets, represented by a wealth matrix. The two matrices are linked by a series of definitional equations, as well as equations that specify regulatory and behavioral assumptions. While economic interactions as defined by social accounting matrices have been widely used in the economic development literature, SAM's are only just beginning to be applied to regional economics.

The accounting framework is applied to a quantity-adjusting short-run Keynesian model in which the region's price and interest rate levels are fixed at national levels. The model has four agents or sectors: industry, finance, household and rest of world. An articulated description of the institutional structure of the finance sector is incorporated. The industry sector demands external financing from both extraregional investors and the loanable funds market in proportions that vary with respect to both the two sources and industry total expenditures. This characteristic implies a variable profit rate for industry.

Consistent with recent work on imperfect information and adverse selection, the banks are active allocators and rationers of credit. In particular, interest rates do not adjust in response to supply and demand conditions. Instead, the local financial services sector sets minimum collateral standards for household loans, effectively linking the availability of credit to the income stream of the local economy.

Comparative static analysis was conducted using a four-quadrant diagram that depicts regional equilibrium. The diagram exploits two linear relationships
between industry output and financial services that were derived from the model equations. One of these relationships, the ROW accounts balance locus, represents values of sectoral output which satisfy all income and balance sheet accounts for the rest of world (ROW), financial services, and industry, but not households. The other relationship, the HH accounts balance locus, represents values of sector output which are consistent for all accounts of households, financial services and industry, but not ROW. Only at the intersection of the two lines do all the equations of the model hold. While the HH accounts balance locus is unambiguously positively sloped, the slope of the ROW accounts balance locus may be either positive or negative.

The possible implications of credit constraints for the export multiplier are then illustrated by analysis of two possible cases. In Case A, the rate at which real goods are imported and the rate at which households deposit their earnings imply, through the balance of payments and balance of trade conditions, that an increase in industry output would tend to create a ceteris paribus decrease in currency inflow to the region. A negative relationship therefore exists between industry output and financial services with respect to the ROW accounts balance line. An exogenous increase in exports induces an adequate increase in local loanable funds for production and consumption that is matched by the expansion in final demand generated by exports, and both industry output and financial services increase. Case A is hence consistent with development strategies that focus on increasing export demand as an engine for regional growth.

In Case B, because household income and deposit rates are relatively low, an exogenous increase in exports boosts industry investment demand more than household consumption and ultimately draws in a relatively large volume of foreign investment. The foreign investment both substitutes for local finance (hence does not contribute to household income) and ameliorates the shift toward trade surplus by increasing remittances. Through the multiplier, currency inflow is further reduced. In Case B, development strategies to increase income or local financial intermediation in the region would be more effective than strategies to increase export sales.

The model laid out in these pages is a theoretical device designed to help the economist analyze the logical implications of a set of conditions that could hold in a regional economy. It is not intended to be fully general or empirically applicable to any specific region. Important extensions of the model include such obvious traits as price flexibility, fuller articulation of investment decisions, recognition of the speculative element in financial behavior and a larger set of transactions and actors (for example, second-hand physical capital, secondary market instruments, a government sector, alternative business structures). Extensions such as these would raise additional theoretical issues to resolve, for example, the modeling of capital gains and losses.

Empirical implementation of this modeling approach also raises data challenges. Data availability and quality is frequently limited. Flow of funds data, for example, are generally described in terms of agent/transaction rather than the agent/agent format used in this model and are not currently available on a regional
level (though potentially they could be in the U.S. due to the system of Federal Reserve Districts and data collected by the Internal Revenue Service).

The rapid expansion and subsequent contraction of finance and financial institutions of the 1970's and 1980's have highlighted the significance of loanable funds and issues related to their management. Empirical application of models such as the one discussed in this paper are clearly useful steps in the continuing process of bettering our understanding of regional economic activity.

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