Estimating Monopoly Power with Economic Profits

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ABSTRACT

Monopolization and attempted monopolization are prohibited by section 2 of the Sherman Act. Proving the existence of monopoly power, however, is problematic. While direct evidence is sometimes available, more often courts rely on indirect—and sometimes highly disputed—evidence that firms have obtained or attempted to obtain monopoly power. Courts have been reluctant to rely on direct evidence of supra-normal profits as proof of monopoly power because (1) accounting profits as recorded in firms’ financial statements differ from economic profits, which fundamentally determine whether a firm as monopoly power, and (2) short-term, supra-normal profits are not, by themselves, evidence that a firm has monopoly power.

However, using a discounted cash flow approach for firm valuation (the Miller-Modigliani “MM” framework) a dynamic estimate of a firm’s economic profits over an extended period of time, either for individual lines of business or for the firm as a whole, can be estimated using the firm’s transaction records. When transaction records are unavailable, analysts can reverse-engineer them using the firm’s financial accounting records. The implications of this analysis are important: rather than relying on indirect methods of estimating the degree of a firm’s market power in a given market (with such heavily litigated issues as market definition and the characterization of high market shares and barriers to entry), analysts can instead examine a firm’s financial records to generate direct evidence of monopoly power in one or more of the markets in which that firm competes. Since an analysis based on the MM framework permits a reverse engineering of accounting records to determine market power, and since both private and publicly traded companies maintain such records in the ordinary course of business, determining whether a firm has a monopoly power in a given market will be more reliable and straightforward.
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I. INTRODUCTION

Monopolization and attempted monopolization are prohibited by section 2 of the Sherman Act. Proving the existence of monopoly power, however, is problematic. While direct evidence is sometimes available, more often courts rely on indirect—and sometimes highly disputed—evidence that firms have obtained or attempted to obtain monopoly power. Courts have been reluctant to rely on direct evidence of supra-normal profits as proof of monopoly power because (1) accounting profits as recorded in firms’ financial statements differ from economic profits, which fundamentally determine whether a firm as monopoly power and (2) short-term, supra-normal profits are not, by themselves, evidence that a firm has monopoly power.

However, using a discounted cash flow approach for firm valuation, based on the Nobel Prize-winning work of economists Merton Miller and Franco Modigliani,¹ a dynamic estimate of a firm’s economic profits over an extended period of time, either for individual lines of business or for the firm as a whole, can be estimated using the firm’s transaction records. When transaction records are unavailable, analysts can reverse-engineer them using the firm’s financial accounting records. Analysis of a firm’s economic profits in specific product lines or geographic markets may require access to more detailed transaction records than are contained in its publicly available financial accounting records. The implications of this analysis are important—a firm’s transaction records, obtained either directly or indirectly through its financial accounting records, generally can be used to determine whether the firm has monopoly power in some or all of the markets in which it competes.

II. MONOPOLY POWER UNDER THE SHERMAN ACT

Section 2 of the Sherman Act prohibits monopolization and attempted monopolization. Monopolization requires proof that a defendant (1) possesses monopoly power and (2) acquired, enhanced, or maintained that power through the use of exclusionary conduct.\(^2\) We show that the Modigliani-Miller ("MM") framework can be used to determine whether a firm has monopoly power in some or all of the markets it serves. However, the MM framework by itself does not determine whether such monopoly power was acquired, enhanced, or maintained through the use of exclusionary conduct. Attempted monopolization requires proof that the defendant (1) engaged in exclusionary conduct, (2) with the intent to monopolize, and (3) that there is a "dangerous probability" of success.\(^3\)

A brief review of the harm to consumers and competition caused by monopolies is useful. As a standard against which to measure the performance of actual markets, economists use the model of a perfectly competitive market. In such an idealized market, the price of a product or service equals its marginal cost, i.e., the change in total cost that occurs when the firm increases its output by a small amount.\(^4\) As a firm’s market power increases, it can profitably charge a price above its marginal cost of producing that product or service. A monopolist reduces its output, thereby causing the market price to increase to the point that its marginal revenue equals its marginal cost. Because the quantity demanded by consumers falls with the rise in price, consumers pay a price above the monopolist’s marginal cost. This creates both a deadweight welfare loss and a transfer of money from buyers to the monopolist, thereby “diminishing the


\(^4\) See, e.g., H. Herbert, ECONOMICS AND FEDERAL ANTITRUST LAW § 1.1 (1985); at §1.12.
consumer surplus that would exist at competitive price levels." Monopolistic markets, therefore, are inefficient: some buyers chose not to purchase a product they would have purchased at the competitive price, while other buyers reduce their purchases compared to the quantities they would have purchased at the competitive price.

“Monopoly power can be proven by direct evidence of the actual exercise of control over prices in the relevant market and/or the actual exclusion of competition from the relevant market” Courts traditionally have established whether a firm has monopoly power by analyzing (1) whether the firm has a large share of the relevant market and (2) the presence of barriers to entry that would permit the firm to exercise market power for a sustained period. The first step of the test is to determine whether the firm has a high market share, generally not less than 50% of the relevant market. Defining a “relevant market” is a highly litigated issue, as are the issues of determining the economic significance of the firm’s market share and whether barriers to entry are sufficient to enable the firm to exercise monopoly power.

Rather than relying on indirect, market structure methods to determine whether a firm has monopoly power, one can analyze that issue directly. However, relying simply on direct evidence from accounting measures of profit is problematic. First, accounting profits and economic profits are very different creatures. Accounting practices, for example, do not consider costs in a way that economists find meaningful, since the records do not reflect the true economic cost of producing goods or services. Depreciation rules, for example, reflect social policy underlying the tax code rather than any real estimate of the reduction in the value of assets.

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6 ABA Section of Antitrust Law, 1 ANTITRUST LAW DEVELOPMENTS (SIXTH) 229 (2006).
over time. That is why, for example, a vehicle can be depreciated over three years, when it may have considerable residual value. The very methods of factoring depreciation can manipulate the outcome, as with accelerated depreciation rules. The replacement cost of assets is reported in ways that reflect a conservative bias rather than calculated in a way that would reflect realistic replacement cost estimates. Similarly, accounting calculations rarely include any estimate of opportunity cost, without which calculations regarding the cost and market value of assets is incomplete. Finally, accounting records fail to reflect the risk inherent in an investment, so that the calculations are unhelpful in determining market power. A highly risky investment that generates a high accounting return may, in fact, reflect only a modest economic rate of return.

Not surprisingly, courts have been skeptical regarding proposals to accept accounting profits as evidence of monopoly power. As Judge Posner wrote, “not only do measured rates of return reflect accounting conventions more than they do real profits (or losses), as an economist would understand these terms, but there is not even a good economic theory that associates monopoly power with a high [accounting] rate of return.” However, some courts have found that the ability of a firm to earn supra-normal profits for a sustained period constitutes evidence of monopoly power.

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8 Modified Accelerated Cost Recovery System (MACRS) GDS Property Classes Table.
9 Modified Accelerated Cost Recovery System (MACRS), enacted with the Tax Reform Act of 1986 (TRA-86) permits a front-loading of depreciation. Rather than “straight-line” depreciation, businesses are permitted to take greater depreciation deductions early in the life of an asset.
10 Blue Cross & Blue Shield United of Wisc. v. Marshfield Clinic, 65 F.3d 1406, 1412 (7th Cir. 1995) (references omitted). See also United States v. Empire Gas Corp., 537 F.2d 296, 307 (8th Cir. 1976) (in an attempt to monopolize case, no dangerous probability of obtaining monopoly power despite “fairly high rate of profit for the past ten years” combined with a 50% market share).
11 Borden, Inc. v. FTC, 674 F.2d 498, 511-12 (6th Cir. 1982) (supra-normal profits supported a finding of monopoly power), vacated and remanded for entry of consent judgment, 461 U.S. 940 (1983); Baker’s Aid v. Hussmann Foodservice Co., 730 F. Supp. 1209, 1219 (E.D.N.Y. 1990) (“Evidence that an entity is able to earn above normal profits may be an indication that such entity has market power.”). See also Data General Corp. v. Grumman Sys. Support Corp., 36 F.3d 1147, 1182 n.60 (1st Cir. 1994); Colorado Inter. Gas Co. v. Natural Gas Pipeline Co. of Am., 885 F.2d 683, 695 (10th Cir. 1989) (supra-competitive prices are evidence of monopoly power), Direct Media Corp. v. Camden Tel. and Tel. Co., 989 F.Supp. 1211 (S.D. Ga. 1997) (profit levels are relevant in determining whether
There is similar skepticism regarding the use of price-cost margins (i.e., price less marginal cost, divided by price, known as the Lerner index) as evidence of monopoly power. First, it is unclear how much of a margin of price over cost would be sufficient to demonstrate monopoly power. “Since monopolistically competitive firms have some market power in the sense that price exceeds marginal cost, presumably the deviation . . . should be significant if it is to expose the firm to antitrust scrutiny. But no consensus exists in the courts or among economists as to how large this deviation should be.”\(^{12}\) Even if there were agreement on the level of the price-cost margin that would demonstrate monopoly power, any individual price-cost margin measured at some point in time does not provide evidence regarding the ability of the firm to exercise monopoly power for a sustained period. Monopoly power, however, is proven only when the ability to obtain supra-normal profits exists over a sustained period of time; short-run profits are insufficient. Monopoly power is defined as “the ability to ‘(1) to price substantially above the competitive level and (2) to persist in doing so for a significant period without erosion by new entry or expansion.’”\(^{13}\) More useful in establishing monopoly power would be a price-cost margin based on a measure of a firm’s long-term marginal cost.\(^{14}\) Such information, including calculations that incorporate adjustments for the risk of an investment, is difficult to obtain.\(^{15}\)

\(^{12}\) Dennis W. Carlton, *Does Antitrust Need to be Modernized?*, J. ECON. PERSP. 155, 164 (Summer 2007).

\(^{13}\) AD/SAT v. Associated Press, 181 F.3d 2216, 227 (2d Cir. 1999) (citing 2A Phillip E. Areeda et al., *ANTITRUST LAW* ¶ 50, at 90 (2d ed. 2002) (emphasis original)).


III. HISTORICAL APPROACHES TO MEASURING MARKET POWER AND THEIR LIMITATIONS

An analysis of the problem of estimating market power requires a review of the historical approaches to the problem: the SCP and NEIO paradigms.

A. Structure- Conduct- Performance

The structure-conduct-performance ("SCP") paradigm, developed in the 1950s,\(^{16}\) posits that market structure (e.g., market shares, market concentration, and barriers to entry) has a causal relationship with the conduct of firms in a market and, hence, the performance of that market (e.g., observed price-cost margins). In particular, an increase in the level of market concentration is predicted to lead to reduction in the level of competition among firms, and hence to higher prices.\(^{17}\) The SCP paradigm underlies the U.S. Department of Justice and Federal Trade Commission HORIZONTAL MERGER GUIDELINES.\(^{18}\)

B. NEIO

NEIO, or the “new empirical industrial organization,” evolved in the 1980s as a departure from the previous SCP paradigm. Under the NEIO paradigm, models were designed to create “an empirical framework from which to analyze the behavior of specific industries.”\(^{19}\) The premise was that the way firms act and react to one another, and act and react to other non-firm behavior (for example governmental regulation), explained an industry’s dynamics, including market concentration. Using game theory, analysts created dynamic models that sought to explain a firm’s behavior within an industry. “Game theory—more specifically noncooperative game

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\(^{19}\) Jaison R. Abel, Entry into Regulated Monopoly Markets: The Development of a Competitive Fringe in the Local Telephone Industry, 45 J. LAW & ECON 289, 292.
theory—can be a useful tool for investigating a comprehensive model of competitive advantage in that it demonstrates the linkages between resources, competitive moves and responses, and advantage.”

Use of the NEIO framework rests on the assumption that a firm’s economic rate of return and price-cost margin cannot be accurately measured.

C. The MM Framework

Significant advances have been made in financial economics that enable estimates of a firm’s economic profits to be made using data on its cash flows, thus avoiding the pitfalls associated with accounting measures of profitability. The framework for estimating a firm’s economic profits derives from the work of Modigliani and Miller. They developed a discounted cash flow approach for firm valuation based on the firm’s invested capital, cash flow, and cost of capital. Their approach was then adapted for real-world applications; important contributors include presents a theory of why firms would chose economic profits as a criterion in order to incent managers to choose efficient investment levels.

A firm has market power if it earns positive economic profits (or a positive economic rate of return) over a non-transitory period of time. Economists have for many decades attempted to measure a firm’s market power directly with a variety of profitability measures, including

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accounting rates of return, internal rates of return, Tobin’s $q$, and the profits-sales ratio (or census price-cost margin). These measures of a firm’s profits have been thoroughly critiqued in the literature.\textsuperscript{25} The Miller and Modigliani-based economic profit framework offers an alternative approach for measuring a firm’s market power that avoids the pitfalls of these previous techniques.

In an important sense, the MM framework for estimating a firm’s economic profits has passed the market test. A survey of Fortune 500 firms published in 1997 found that 40 percent used the framework to make investment and compensation decisions based on estimates of economic profits,\textsuperscript{26} while a survey of Fortune 1,000 firms published in 2002 found that 54 percent used the framework.\textsuperscript{27} The percentage of both large and small firms using the framework is likely higher today.\textsuperscript{28} As a consequence, decisions involving investments of hundreds of billions of dollars annually, as well as the compensation of senior executives at these companies, are now made using the MM framework. Empirical evidence suggests that firms adopting the framework earn higher after-tax operating profits.\textsuperscript{29}

Economic profit models, as they are commonly applied in corporate finance, evaluate economic profits by comparing invested capital to operating cash flows and the cost of capital.

\textsuperscript{25} For a critique of accounting rates of return, see Franklin Fisher and John McGowan, \textit{The Misuse of Accounting Rates of Return to Infer Monopoly Profits}, 73 \textsc{American Economic Review} 82 (1983); for internal rates of return, see Stephen Martin, \textsc{Advanced Industrial Organization}, Maldin, MA: Blackwell Publishers, Chapter 6 (2002); for Tobin’s $q$, see Henry McFarland, \textit{Evaluating $q$ as an Alternative to the Rate of Return in Measuring Profitability}, 70 \textsc{Review of Economics and Statistics} 614 (1988); and for profits-sales ratios, see Franklin Fisher, \textit{On the Misuse of the Profits-Sales Ratio to Infer Monopoly Power}, 18 \textsc{Rand Journal of Economics} 384 (1987).

\textsuperscript{26} John Balkcom, Christopher Ittner, and David Larcker, \textit{Strategic Performance Measurement: Lessons Learned and Future Directions}, 1 \textsc{Journal of Strategic Performance Measurement} 22 (1997).


\textsuperscript{28} Patricia Gray, \textit{What’s Your Magic Number}, 16 \textsc{Fortune Small Business} 72 (2006).

\textsuperscript{29} Harley Ryan and Emery Trahan, \textit{Corporate Financial Control Mechanisms and Firm Performance: The Case of Value-Based Management Systems}, 34 \textsc{Journal of Business Finance and Accounting} 111 (2007) (defining “after-tax operating profits” as net operating profits less adjusted taxes).
Since economic profits earned on invested capital can be caused or offset by factors other than market power, additional adjustments are required to evaluate the source of observed economic profits. For example, returns on invested capital and economic profits are affected by such factors as changes in the value of operating assets that are unrelated to the operations or market power of the firm. These returns reflect the change in market value of assets held and are unrelated to the exercise of market power, yet will be a source of economic profits or losses. A firm may earn sustained economic profits when some of its assets are undervalued over a long period of time, creating the illusion of market power. This problem can be eliminated by marking these assets to their market values.

Conversely, acquisitions can distort the measurement of economic profits and will often understate the economic profits that would reflect the acquired company’s market power. The acquiring firm’s investment in an acquired company will likely be at a value that equates the price paid to the present value of expected economic profits discounted by the acquiring firms’ cost of capital. In this case, the acquiring firm will earn a normal profit on the assets of the acquired firm, regardless of the degree of market power possessed by the acquired firm. Adjustments that must be made to account for sources of economic profit and loss not associated with the exercise of market power.

Firms are a combination of projects, and a firm may have significant market power in business segment and no market power in other business segments. Information regarding business segments of firms can be hidden in its consolidated financial statements and sufficient information to evaluate the economic profits of different business segments may not be disclosed in the firm’s Securities and Exchange Commission (“SEC”) filings for reporting companies. To the extent that the summarized transactions in the SEC forms disclose information on the
operating segments of a company, the lines of business can be disaggregated and evaluated separately. Detailed transaction data kept by firms in their normal course of business will typically contain enough detail for segments to be disaggregated. However, even with detailed transaction data a study of a firm’s economic profits in specific, highly localized markets, e.g., local retail markets, can be problematic.

IV. A MODEL FOR ESTIMATING ECONOMIC PROFITS AND RETURNS

The economic profits earned by a firm can be measured by comparing investments in its operations to the cash flows generated by those operations. Information on the cash invested in operations, as well as the cash flow from operations, is contained in the transaction records of a company. The complete set of a company’s transaction records is available to insiders but not to the public. (In the case of litigation, non-public transaction records are available on a confidential basis to experts.) Financial accounting measures of performance found a firm’s financial statements are an inadequate measure of economic performance. Still, the financial accounting records themselves are derived from an accurate measure of the firm’s transactions and cash flows. Since these publicly available accounting statements are based on a consolidation of the company’s transaction records, information on the firm’s cash flows can be derived by adjusting and re-categorizing publicly available financial accounting records. The firm’s investments in its operations and returns on these investments can be estimated.

A firm’s economic profits equal its operating returns minus the opportunity cost of the capital employed. Similarly, a firm’s economic return equals its operating returns divided by the amount of capital employed, minus the opportunity cost expressed as a percentage. The process

of estimating economic profits starts with the definition of economic capital, which dictates the framework for calculating both the opportunity cost of capital and operating returns. Although the terms capital, operating profits, and economic profits are elemental concepts in financial economics, they have been used in so many disparate ways as to be meaningless outside of their context. *Capital* as the dollar value of all resources devoted to some productive end; *operating profits* are defined as revenues less operating expenses and taxes from this same productive end; *economic profits* are defined as the difference between operating profits and the opportunity cost of the capital employed; and *economic returns* are defined as economic profits divided by the amount of operating capital employed.

A. *Deriving Economic Profits and Returns from Discounted Cash Flows*

The discounted cash flow approach, based on the net present value (“NPV”) method in discussions of capital budgeting, is well known.\(^{31}\) In this method, when valuing any project, the stream of cash receipts generated by the project, plus any residual or scrap value at the termination of the project, minus the stream of cash outlays for labor, materials, repairs, and capital additions are discounted by the corresponding cost of capital (see Appendix A). In other words, the economic profit equals the project’s total profit minus the opportunity cost of the capital available at the beginning of the period. When using market prices to estimate implied expected economic profits, there is no need to estimate the cost of capital, future investments, future depreciation, tax rates, or operating income. The market price is known and the estimate

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\(^{31}\) The standard NPV definition differs slightly from our definition in that it only considers net working capital, removing the opportunity cost, if any, of such things as accounts payable from the weighted average cost of capital calculation and including this cost as an operating expense. This is a practical short cut that will usually have no material effect on the outcome, however there may be circumstances where non-interest bearing, short-term liabilities are a significant source of capital or where they have a material cost. In such cases, NPV definition of capital needs to be modified as done here.
of the present value of expected economic profits requires only an estimate of invested capital (see Appendix B).

B. Calculating Economic Returns

Economic returns can be affected by variables other than those that reflect economic productivity. Appreciation of a company’s real estate holdings, for example, could cause an increase in economic returns despite having nothing to do with the company’s economic productivity or market power. Thus, economic returns for a single period can be affected by estimates of economic depreciation, random events, and cyclical variations unrelated to a firm’s ability to earn economic profits through the exercise of market power. Economic returns earned over the life of a project are not affected by estimates of economic depreciation, but are still subject to the effects of cyclical or random factors. The effect of cyclical variation is mitigated over longer time periods as the firm moves through the entire business cycle. The effects of random events are also mitigated over longer time periods. For a mathematical analysis of these points, see Appendix C.

V. Estimating a Firm’s Economic Profits and Returns

A firm’s transaction records contain an accurate record of its investments, revenues, expenses, cash flows, and financing activities. By categorizing these transactions from the perspective of the firm’s economic profits, rather than its accounting profits, the firm’s investment in operations and returns on these investments can be determined. Financial accounting records are derived from the firm’s transaction records, but are prepared and categorized according to financial accounting rules. By understanding the rules used to prepare the firm’s financial accounting statements, the financial accounting can be reverse engineered to
provide reasonable estimates of the original transactions. This approach differs from prior attempts to use accounting measures to measure economic performance.

The failure to apply a rigorous framework for use in estimating economic profits and returns from a firm’s transaction records or, if unavailable, its public accounting information has had two important ramifications. First, the NEIO approach was developed in part in response to the criticisms of Fisher and McGowan regarding the use of accounting profits to measure a firm’s market or monopoly power. Second, the empirical results of studies that utilized accounting measures of profitability are compromised by measurement error. For example, Waring, despite acknowledging Fisher and McGowan’s critique, uses accounting profits to study the persistence of firm-specific returns.32 Waring defines a firm’s total return as its after-tax income plus interest, divided by assets. From an economic perspective, there are many problems with this definition. For example, as discussed in detail below, a firm’s balance sheet assets differ substantially from its operating capital. Also, a firm’s after-tax income incorrectly includes payments to suppliers of capital. Moreover, the tax provision in a firm’s income statement may not be a good estimate of taxes actually paid. Finally, tax deductible interest payments reduce the firm’s tax liability, but they are unrelated to the firm’s operating returns.

A. Adjusting Financial Accounting Records to Recover Historical Transaction Records

The economic performance of a company cannot be directly measured from its financial accounting statements. These statements are not intended to measure economic performance. The transaction information found in financial accounting statements disclosed on the forms filed

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with the SEC by publicly traded companies is a consolidated subset of their transaction data. Rules that determine the format, required disclosures, and categorization of accounting information are set by the Financial Accounting Standards Board (“FASB”), which does not have measuring economic performance as its objective. Financial accounting rules are not fixed and static. They differ across countries, industries, and time periods and should not be directly compared without adjustments to reflect these differences. Also, since financial accounting measures are prepared according to a set of rules designed to promote standardization, they differ from economic measures in several important ways. Financial accounting records:

- are concerned with recording assets owned by the corporation—not those used in operations;
- are based on the matching principle, where revenues, expenses, and liabilities are matched to the time period in which they occurred—not when the cash was paid or received, as required by the MM framework and as reflected in the company’s transaction records before accounting adjustments;
- record assets at historical cost—not current market values; and
- have an intentional conservative bias.

A firm’s financial accounting records are derived from its historical transaction records. Many of the differences between reported financial accounting profits and economic profits result from the financial accounting rules for categorization of transactions. These differences in large part can be eliminated by re-categorizing or adjusting financial accounting information.

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33 Forms that contain financial information that firms file with the SEC include Form 10K, the annual report which is audited, and Form 10Q, the unaudited quarterly reports.

34 FASB is the designated, non-governmental organization tasked with establishing standards for financial accounting and reporting. These standards govern the preparation of financial reports. FASB is officially recognized as authoritative by the SEC. FASB’s mission is to “[i]mprove the usefulness of financial reporting by focusing on the primary characteristics of relevance and reliability and on the qualities of comparability and consistency” (FASB, 2007).
Assets on the balance sheet differ from economic capital in several respects. Assets included on a firm’s balance sheet show only those assets owned by the company—not those used in the operations of the company. Assets need to be adjusted by excluding owned assets that are not part of economic capital, and by adding assets used in the company’s operations but that are not owned by the firm. Also, accounting’s conservative bias keeps certain economic assets off financial accounting balance sheets, and adjustments need to be made to include these missing assets when estimating invested economic capital. The principle of conservatism in accounting, sometimes called the “prudence principle,” implicitly assumes it is better to understate than to overstate assets or revenues, and that it is better to overstate than to understate liabilities or expenses. When faced with a situation where judgment is required, prudent accountants will tend to introduce a conservative bias that understates assets and revenues, while overstating liabilities and expenses. For example, a portion of funds spent on research and development (“R&D”) during the year can be considered to be an investment—not an expense. The conservative judgment required by accounting principles, however, will often cause the entire R&D expenditure to be categorized as an expense. Expensing R&D conservatively understates the firm’s true assets (since the R&D assets will not appear on the firm’s balance sheet), while at the same time overstates its expenses. Finally, the customary capital budgeting definition of capital assumes “net working capital.” If this definition is used, non-interest bearing current liabilities, such as accounts payable, are also excluded.

35 These non-operating assets include such things as excess cash, minority interests in other companies, construction in progress, and unused real estate.

36 Firms commonly obtain the use of capital through operating leases, which do not appear on the balance sheet as assets.

37 Accountants reduce capital by expensing items that are not true economic expenses. Funds spent on R&D, for example, represent an investment since they are made with the expectation of providing a return in future years. Treating these R&D investments as an expense is conservative in the sense that the true investment is understated. When these expenses are offset by a long-term liability on the balance sheet, no adjustment is required. When these expenses are accompanied by the reduction in value of an asset, the value of the reduced asset needs to be restored.
It is important to distinguish a firm’s investments from its operating costs using the criterion that any expenditure that yields benefits lasting more than one period constitutes an investment—not an operating cost. For such investments, the portion of the expenditure that yields a benefit lasting beyond the end of the period has been capitalized. Expenditures on R&D and advertising are clearly investments, but so too can be a firm’s expenditures in acquiring and training its employees.

Accounting earnings differ fundamentally from economic profits. Accounting earnings are calculated from the point of view of the stockholder and include a mixture operating, non-operating, and conservative rule based revenues and expenses. Also, some payments to suppliers of capital are considered as expenses when accounting rules are used. To adjust accounting earnings to the point of view of the firm rather than equity holders (residual claimants on the firm), payments to suppliers of capital that were expensed are added to earnings. Non-operating earnings are subtracted on an after-tax basis since the marginal tax liability would not have occurred in the absence of the non-operating income. Non-economic expenses resulting from accounting’s conservative bias are also added back to the accounting earnings.

A related approach to measuring firm’s economic profits was employed by Gugler, Mueller, and Yurtoglu in a study evaluating a sample of approximately 19,000 companies from 61 countries. Their study analyzed how a country’s corporate governance regulations affected the ratio of a company’s economic returns to its cost of capital. Returns were estimated on new investments by employing the relationship described in equation 12, and estimating changes in

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38 Payments to suppliers of capital include such items as interest, lease payments, and preferred dividends. Interest and the interest portion of lease payments are added back on an after-tax basis.

39 These expenses include research and development, warranty reserves, LIFO reserves, and deferred taxes.

investments by adjusting accounting information and re-categorizing and capitalizing R&D and advertising expenses. Since the study was conducted to measure management performance—not to estimate market power—the authors used a definition of capital that adjusted for the conservative accounting bias, but they did not distinguish between capital owned by a company versus capital used in operations. The study also did not distinguish between a firm’s financial investments and its investments in operations.

B. Addressing Well-Known Problems with Using Accounting Information to Measure Economic Performance

Transaction-based economic profit analysis differs fundamentally from an analysis of accounting returns. Nevertheless, the MM framework does rely to a certain extent on accounting records, so problems identified with using such records should be addressed. There is a well-known set of problems associated with efforts to use accounting information to calculate a firm’s economic rate of return.\footnote{Dennis Carlton and Jeffrey Perloff (2005), MODERN INDUSTRIAL ORGANIZATION, Boston, MA: Pearson Addison Wesley, 4\textsuperscript{th} ed., ch. 8 (2005); see also, Jeffrey Church and Roger Ware, INDUSTRIAL ORGANIZATION: A STRATEGIC APPROACH, Boston, MA: Irwin McGraw-Hill, ch.12 (2000).} The MM system of analysis, however, can accommodate such difficulties, as discussed infra.

Valuing Capital. Capital based on accounting definitions differs from the economic definition in that accounting definitions are based on a conservative definition of capital that excludes certain intangible assets and values capital at depreciated, historical cost. Economists value a firm’s operating capital at replacement cost. The book value of assets on the financial accounting balance sheet is not a measure of a firm’s economic capital, and it cannot be used as a base for estimating economic profits or returns.

With the MM framework, capital is measured by the amount of cash invested in operations, and the opportunity cost is the cost of acquiring funds in the capital market. There are
two issues associated with valuing economic capital using transaction information derived from accounting statements. First, there are differences between the definition of economic capital and the assets found on financial accounting statements. Second, changes in the value of operating assets can cause the value of economic capital to differ from the accounting value. Recategorizing transactions eliminates issues of definition; the appropriate adjustments for changes in valuation will depend on the objective of the analysis. For example, to measure the economic profits derived from market power, it would be appropriate to mark assets to market if the change in market value of the assets were unrelated to the market power of the firm, but inappropriate to mark them to market if they were. As a practical matter, not all assets have efficient secondary markets where replacement assets are available so as to provide pricing information to establish replacement costs. There can be cases where depreciated book value is the best available source of unbiased information. Also, some assets such as technology or specialized software may not have the same value outside of the context of the firm’s operations, so the market value of such assets would understate the value of the invested capital to the firm. In this case, depreciated investment would provide a more useful measure of market value.

Treatment of Depreciation. Transaction information provides an accurate measure of an asset’s value at the time of acquisition. Consistent with the accounting treatment, such values

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42 Consider a firm that has no market power but owns real estate that has appreciated dramatically in value. The firm’s initial investment of, say, $100 now has a market value of $500. Suppose the firm earns a ten percent return on its real estate investment, yielding $50 per year. It then earns an economic profit from the point of view of the firm’s initial investment of $40 per year if its cost of capital equals 10%. Adjusting the capital to reflect the current market value to isolate the effects of market power would show an economic profit of $0. The normal profit on the true value of the capital is $50 ($500 X 10%), and the economic profit is total profits ($50) less normal profits ($50) = $0.

43 Consider a firm that has market power based on owned real estate that has appreciated due to its unique zoning status that keeps out potential competitors. The firm’s initial investment of $100 now has a market value of $500. The firm also earns $50 per year and also earns an economic profit from the point of view of the firm’s initial investment of $40 per year if its cost of capital is 10%. Adjusting the capital to reflect the current market value would show economic profits of $0, when in fact the firm was earning economic profits that were capitalized in the value of the real estate.
need to be adjusted over time to account for the use of assets with finite economic lives. Accounting depreciation for an asset is typically based on its market value at time of purchase, an estimate of its economic life, and the estimated salvage value at the end of that economic life. Accounting depreciation rates are based on the expected life of the depreciating assets and are not intended to have a particular bias to under- or overestimate the interim values of the asset. All things being equal, the use of accounting depreciation provides an unbiased estimate of economic depreciation, which equals the change in market value of the asset. Depreciation assumptions do not affect cash flow, but will affect the economic profit measured for a particular time period, as well as other measures such as the IRR.

For a single project, depreciation does not affect the present or future value of the economic profits, nor does it affect the realized return $\rho$ over the life of the project. Distortions caused by incorrect estimates of depreciation in the measurement of economic profits for a firm comprised of multiple projects occur because the time period measured does not include the entire duration of all projects. Here the beginning and ending values of overlapping projects that differ from market values can distort results. Also, economic returns for any single time period will be distorted by incorrect depreciation assumptions. Generally the longer the time period evaluated the smaller the potential errors from incorrect depreciation estimates.

Distortions in economic rates of return for a particular time period can be eliminated by marking the assets to market (when appropriate) at the beginning and end of the time period. The change in market value is itself a return for the time period. To the extent that changes in market values of the assets are caused by factors unrelated to the operations of the firm, the invested capital should be adjusted without altering the operating returns when the objective is to measure market power.
**Investments in Intangible Assets.** Expenditures on such R&D, advertising, training, and customer acquisition often create benefits in future time periods, which from a financial economic point of view is an investment. The conservative bias of accounting rules causes these investments to be categorized as expenses, distorting the rate of return by underestimating both the capital base and its return. To estimate economic profits and returns, the expenditures on intangible assets that have been expensed are reclassified as investments and subsequently depreciated. Capitalized values of intangible assets are then depreciated over time in much the same way, with the same issues, as other wasting assets.

**Rates of Return May Not be Properly Adjusted for Risk.** In the MM framework, operating returns are reduced by the weighted average cost of capital (“WACC”), which represents the opportunity cost of the funds invested by the firm. The WACC is risk adjusted and takes into account both the credit risk of the firm’s debt and the systematic risk of the firm’s equity. Failure to adjust properly for risk occurs when the opportunity cost of capital is applied without a clear and consistent definition of sources of capital.

**Rates of Return May Not Correctly Take Into Account Debt.** Researchers often use the rate of return to stock holders as a measure of the firm’s profitability. Stockholders have a residual claim on the firm’s cash flows, and the return to the stockholder is what remains after all other claims have been paid. Cash flows to equity are affected by the capital structure of the company and are not comparable from company to company since capital structure is a characteristic of a firm—not an industry. It is almost never appropriate to use return on equity as a measure of performance of a company. The MM framework calculates returns to the enterprise and operating cash flows are not affected by the firm’s capital structure.
**Inflation Adjustment.** The accounting practice of recording and carrying assets at historical costs means that the value of the assets on the books does not reflect the changing value of the currency in which it is recorded. Since the opportunity cost of the capital is considered in nominal terms, the effect of inflation is implicitly considered when normal returns are estimated.

**Capitalization of Monopoly Profits.** Acquisition prices of companies can be thought of as the present value of expected cash flows of the target company. If such cash flows include the expectation of monopoly profits, then the monopoly profits of the target company will have been capitalized when put on the acquiring company’s books. From the point of view of the acquiring company, the investment is expected to earn only a normal return despite the monopoly profits earned by the target. To the extent goodwill booked in the transaction represents the difference between the market value of the acquired company’s invested capital and the price paid for the acquired company, that goodwill reflects an estimate of the present value of economic profits at the time of the transaction. When evaluating the economic profits earned by the acquired firm after the acquisition, it is appropriate to use the market value of the acquired assets as the measure of invested capital—not the amount paid by the acquiring firm.

**Pre-Tax Rates of Return May have been Calculated Rather the Appropriate After-Tax Return.** Calculating after-tax returns on operations from accounting records is complicated by three factors requiring adjustments.

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44 Consider a company that has invested capital of $100 and is expected to have a single cash flow of $1,100 in one year. The market value of such a firm would be $1,000 if the cost of capital were 10%. The present value of the expected economic profit would be $900 ($1,000 - $100). The acquiring company would expect to earn only a normal profit on its $1,000 investment. The target company would receive the present value of the expected monopoly profits. The accounting rate of return on the acquiring company’s $1,000 investment will reflect a normal return, masking the market power of the acquired company.
(1) The tax provision shown in the income statement is based on taxes due under the financial accounting rules, and differs from taxes actually paid. Differences between the tax provision and taxes actually paid will show up as changes in various deferred tax accounts. Actual taxes paid can be obtained by adjusting tax provisions by the appropriate changes in deferred taxes.

(2) Certain forms of financing such as debt create tax deductions unrelated to after-tax operating cash flows. Reductions in taxes due to finance-related tax deductions are added to the taxes paid when calculating after-tax operating profits. After-tax operating profits are not affected by the capital structure of the firm.

(3) Non-operating assets of the firm can create taxable income or tax deductible losses unrelated to after-tax operating cash flows. Non-operating income is adjusted on an after-tax basis.

*Ex Ante vs. Ex Post Profitability.* Accounting information is a record of the past and will necessarily reflect the ex post results of the firm. The ex post results for any single time period can be affected by factors that are unrelated to the market power of the firm. For example if there is a single project that required a $10 investment and has a 50% chance of returning $5 and a 50% chance of returning $15. Ex ante the expected profit is zero (a $10 investment and a $10 return) however, ex post the profits will be either a negative $5 or a positive $5. Under these conditions the ex ante estimate is a better indication of the market power of the firm than of the project than either of the ex post outcomes. Such an approach is not practical given the difficulties of assigning probabilities to potential outcomes and does not consider the risk or opportunity costs of the capital employed in the project. The MM framework is developed from a capital budgeting framework based on ex ante estimates. Still, it can be applied to either an ex
The ex post analysis uses the accounting transaction records and adjusts them for the opportunity cost of capital. Although ex post results for any single project or time period can be affected by factors unrelated to market power, measuring multiple projects over multiple time frames will diversify away much of the distortion caused by random external factors. The ex ante analysis can be based on profitability estimates, or for firms with established markets for their securities, expected economic profits implied by market prices can be derived from a comparison of market value of the firm to its invested capital as described in eq. 12.

**Ricardian Rents vs. Economic Profits.** The conservative accounting practice of carrying assets at the lower of historical cost or market value can result in an illusion of economic profits in the absence of market power. For example, a firm with real estate carried on its books at a fraction of its market value could appear to earn economic profits when there are none if accounting measures of return on assets are employed. These economic profits are corrected when the assets are marked to market.

**VI. Conclusions**

The Miller-Modigliani framework can be used to provide direct evidence of the ability of a firm to exercise monopoly power in a given market. Rather than relying on indirect methods of estimating the degree of a firm’s market power in a given market (with such heavily litigated issues as market definition and the characterization of high market shares and barriers to entry), analysts can instead examine a firm’s financial records to generate direct evidence of monopoly power in one or more of the markets in which that firm competes. Since an analysis based on the MM framework permits a reverse engineering of accounting records to determine market power, and since both private and publicly traded companies maintain such records in the ordinary
course of business, determining whether a firm has a monopoly power in a given market will be more reliable and straightforward.
Appendix A

Consider a single project with a finite life ending in period $T$, with all cash flows occurring at the end of each period $t$, where $0 \leq t \leq T$. Let $\Pi_t$ be the project’s total profit at period $t$ which is calculated by subtracting economic depreciation out of operating cash flow. Let $R_t$ be the cash flow at period $t$. Let $I_{t-1}$ be the capital value measured at the end of period $t-1$ and beginning of period $t$. Suppose there is no new investment in any period. Let $\Delta I_t$ be the capital depreciation or appreciation occurred during period $t$, \( \Delta I_t < 0 \) in case of capital depreciation. The capital value at the end of period $t$ can be measured as $I_t = \Delta I_t + I_{t-1}$. The relationship among $\Pi_t$, $R_t$, and $\Delta I_t$ can be expressed by the following equation:

$$\Pi_t = R_t + \Delta I_t. \quad (1)$$

Let $k$ be the rate of the opportunity cost of capital, which is assumed to be constant over time. Let $\pi_t$ be the project’s economic profit at time $t$. Then:

$$\pi_t = \Pi_t - kI_{t-1}. \quad (2)$$

Depreciation here means economic depreciation or the loss in market value during the period, which is typically estimated in the financial accounting statements and has no effect on cash flow. Conversely, depreciation for tax purposes shelters taxes and affects both operating profits and cash flow but is unrelated to economic depreciation. In a typical capital budgeting NPV calculation, there would be no new investment, only depreciation for tax purposes. Tax depreciation does not need to be estimated, since in the absence of future changes in tax law, the depreciation schedule is known at $t_0$.\[45\]
Appendix B

Equations (1) and (2) together imply:

\[ \pi_t = R_t - kI_{t-1} + \Delta I_t. \]  

(3)

From (3), it is evident that the periodic economic profit depends on \( \Delta I_t \), which is susceptible to errors in estimates of economic depreciation. Nevertheless, the present value of the project’s total economic profit does not depend on accounting measures or estimates of economic depreciation. This is because the present value of a project’s total economic profit equals the project’s NPV. To see this, consider the traditional definition of NPV:

\[ \text{NPV} = \sum_{t=0}^{T} \frac{R_t}{(1+k)^t} + \frac{I_T}{(1+k)^T} - I_0. \]  

(4)

Equation (4) states that the NPV equals the discounted cash flow minus the initial investment at the beginning of the project, plus any residual or scrap value at the termination of the project. Substituting (1) into equation (4) gives:

\[ \text{NPV} = \sum_{t=1}^{T} \frac{I_t - (I_t - I_{t-1})}{(1+k)^t} + \frac{I_T}{(1+k)^T} - I_0. \]  

(5)

Using the definition of economic profit in (2) to transform (5) gives:
\[ NPV = \sum_{i=1}^{T} \frac{\pi_i}{(1+k)^i} - \sum_{i=1}^{T} \frac{I_i}{(1+k)^i} + \sum_{i=1}^{T} \frac{I_{i-1}}{(1+k)^{i-1}} + \frac{I_T}{(1+k)^T} - I_0. \]  

(6)

The second, third, fourth, and fifth terms in (6) together cancel out. Thus,

\[ NPV = \sum_{i=1}^{T} \frac{\pi_i}{(1+k)^i}. \]  

(7)

Since the NPV defined in (5) is independent of estimates of economic depreciation, (7) states that the present value of the project’s total economic profit equals the NPV and, hence, is independent of depreciation estimates. In other words, estimates of economic depreciation or appreciation rates will not affect the present value of the project’s total economic profit.

Now, suppose that more investments are made during the project’s life. Specifically, let \( \Delta^* I_t \geq 0 \) be the additional amount of capital invested at the beginning of period \( t \). Suppose that the additional investment earns a normal profit during period \( t \). Then the total profit at period \( t \) changes:

\[ \Pi_t = R_t + \Delta I_t + k\Delta^* I_t. \]  

(8)

However, the economic profit at period \( t \) remains unchanged:

\[ \pi_t = \Pi_t - kI_{t-1} - k\Delta^* I_t = R_t + \Delta I_t - kI_{t-1}. \]  

(9)
Thus, the present value of the project’s total economic profit does not depend on additional capital investments that earn a normal profit. While periodic economic profit is susceptible to errors in estimating economic depreciation, from (7) and (9), it can be concluded that the present value of the project’s total economic profits is not affected by either depreciation rates or additional capital investments that earn only a normal profit.

This same approach can be applied to value the operations the firm, which may be thought of as a portfolio of projects.\textsuperscript{46} The difference between a contemplated project and an existing firm is that in the firm there will be existing investments, and the term \( I_0 \) no longer represents an investment required to achieve the assumed cash flows, but rather than value of existing investments at period zero. Again, assume that there is no additional investment in any period and let \( V_0 \) be the value of the firm at period zero, then:

\[
V_0 = \sum_{t=1}^{T} \frac{R_t}{(1 + k)^t} + \frac{I_T}{(1 + k)^T},
\]

which is the discounted cash flow plus discounted residual capital in the last period.\textsuperscript{47} Substituting the definition of cash flow and economic profit in (1) and (2) to (10) gives:

\[
V_0 = \sum_{t=1}^{T} \frac{\pi_t}{(1 + k)^t} + I_0.
\]


\textsuperscript{47} In the project NPV example, \( I_T \) represents the scrap value of assets employed at the termination of the project which is analogous to the liquidation value of a firm that ceases to be a going concern at time \( T \). If the firm is assumed to continue as a going concern after time \( T \), \( I_T \) represents the amount of capital at the beginning of time period \( T \), given that the firm earns only normal profits in subsequent time periods.
The present value of a firm therefore equals its existing investment in economic capital plus the present value of total expected economic profits. Unlike the single project example, \( I_0 \) does not represent transactions that occur at period zero, presumably at market values, but rather the value of investments made in previous time periods. The evaluation of a portfolio of projects can be distorted by changes in market values and accounting entries not based on transactions made in previous periods, such as depreciation and reserves. Thus, the present value of total expected economic profits is not independent of accounting measures and requires additional adjustments before it can be used as an indicator of market power. Issues related to estimates of economic depreciation are mitigated when project information is disaggregated and the time frame studied is extended.

The same approach can be used to evaluate the expected economic profits implied by the observed market values of a firm’s publicly traded securities, assuming the capital market is efficient. Substituting the observed market value \( M \) of all securities of a firm for \( V_0 \) and subtracting \( I_0 \) yields:

\[
M_0 - I_0 = \sum_{t=1}^{T} \frac{\pi_t}{(1+k)^t}, \tag{12}
\]

When using market prices to estimate implied expected economic profits, there is no need to estimate the cost of capital, future investments, future depreciation, tax rates, or operating income. The market price is known and the estimate of the present value of expected economic profits requires only an estimate of invested capital.
Economic return on capital in period $r_t$ equals economic profits for period $t$ divided by the beginning invested capital at $t - 1$.

$$r_t = \frac{\pi_t}{I_{t-1}}$$  \hspace{1cm} (13)
Appendix C

The realized return on a project, $\rho$, can be calculated by comparing its future value measured at termination to its initial investment.\(^{48}\)

$$\rho = \left(\frac{(I_0 + \text{NPV})(1+k)^T}{I_0}\right)^{1/T} - 1 \quad (14)$$

Although the return over the life of a simple project, where a single investment is followed by a finite series of positive cash flows, can be expressed accurately, no single rate can accurately measure the return of a portfolio of projects. A portfolio of projects spanning different time periods will have multiple investments with multiple returns. These multiple returns cannot be expressed as a single number without compromising accuracy. Typically attempts to express portfolio returns as a single number fall into one of three approaches: dollar weighted, arithmetic average, and time weighted (geometric average), none of which provides an accurate measure of $\rho$.

Fisher and McGowan (1983) define the economic rate of return on an investment as the discount rate that equates the present value of its expected net revenue stream to its initial outlay. This definition is the same as the dollar-weighted method, which is also referred to as the internal rate of return (“IRR”). While this method does consider the changing amount of invested

\(^{48}\) As used herein, the term “realized return” to represent the actual return earned over a period of time expressed as an annualized rate. For example, a $100 investment that is worth $121 after two years would have a 21% return over the two-year time period. Expressed as an annualized return $1.21^{1/2} = 1.10$. In this case the realized return would equal 10%. The realized return equates the initial investment with the present value of an actual or expected terminal value.
capital over time, it is not actually a measure of return.\textsuperscript{49} Moreover, when the IRR is used in situations where $\rho$ can be calculated, the IRR will always overstate $\rho$ when there is an economic profit and understate $\rho$ when there is an economic loss.

Arithmetic returns are valid for a single time period only, and an arithmetic average of returns over multiple time periods cannot be used as an estimate of $\rho$.\textsuperscript{50} The time weighted rate of return (geometric average) will equal $\rho$ in our single project example,\textsuperscript{51} but does not give additional weight to time periods with larger amounts of invested capital. Of the three alternatives, the time-weighted return usually provides the most accurate estimate of $\rho$. Alternatively, the present or future value of economic profits expressed in dollars can be measured without the problems associated with expressing returns. Such an approach is useful when evaluating a single firm or assessing damages, but is less useful when making comparisons among firms. Accounting rates of return, such as return on assets (the ratio of accounting profits to book value of assets) differ from economic measures of return, and except for a correlation with economic returns in some instances, are irrelevant.

\textsuperscript{49} The IRR is sometime used (incorrectly) as if it were a rate of return. Despite its name, the IRR is not a measure of return but rather a discount rate that yields a net present value of zero to a series of cash flows $0 = \sum CF_t / (1+ IRR)^t - I_0$. Multiple solutions to the IRR calculation are possible (Descartes’ rule of signs), including real and imaginary numbers. The IRR implicitly assumes that all cash flows are reinvested at the IRR even though there is no relationship between the reinvestment rate and the IRR.

\textsuperscript{50} For example, an investment of $100 that grows to $200 in the first year, but then declines in value back to $100 after the second year would have a realized return of zero or $\rho = 0\%$. The return for the first year would be 100\% ($100 to $200) and the return for the second year would be a negative 50\% ($200 to $100); the arithmetic average would be 25\% (100\% + -50\%)/2.

\textsuperscript{51} For example, an investment of $100 that grows to $200 in the first year, but then declines in value back to $100 after the second year would have a realized return of zero or $\rho = 0\%$. The return for the first year would be 100\% ($100 to $200) and the return for the second year would be a negative 50\% ($200 to $100); the geometric average would also be 0\% \((1+100\%) \ast (1 + -50\%))^{\frac{1}{2}}\).