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Legal Applications of Modern Finance

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LEGAL APPLICATIONS OF MODERN FINANCE

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Summary:

While scholars and practitioners have applied economics to law successfully for decades, there has been almost no similar application of modern finance. Courts have used the central concept of classical finance, time value of money, for many years, but their use is still unsophisticated.

This article details two ways to apply modern finance to law. This article first describes a method of improving courts’ time value of money calculations, by using a systematically complete four factor analysis to determine the appropriate discount rate. This article then describes a method of calculating future damages that uses market price of risk, based on a simplified version of the Black-Scholes options valuation formula and the Capital Asset Pricing Model, to account for the uncertainty inherent in the future growth of the damages. This article uses a persistent challenge in future damages calculation, the new business rule, as a simple example of using market price of risk to compensate for future uncertainty in damages calculations.

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   D. Fact finders’ choices are, most charitably, naïve
      1. Often set by court rule or statute
      2. Usually arbitrary, or at best, outdated
      3. Market interest rates change greatly over time
         a. Cf. now to, e.g., the early 1980s
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      e. From where we get numbers
         i. m is the historical growth rate in profits of similar
            businesses
         ii. s is the historical volatility in the growth of m
         iii. MPR
            (1) Estimate using the CAPM
            (2) CAPM is a linear regression on broad stock market
                return
                (a) CAPM formula for MPR is \( \lambda = \rho / \sigma (\mu - f) \)
                   (i) \( \lambda \) is MPR
                   (ii) \( \rho \) is historical correlation between
                        profit growth and stock market
                        return
                   (iii) \( \mu \) is the historical return on the stock
                        market
                   (iv) \( \sigma \) is historical volatility of stock
                        market return
                   (v) \( f \) is the risk free rate of interest,
                       estimated by U.S. Treasury bills
                (b) Substitute for variables and calculate
                (c) First use CAPM to calculate MPR
                   (i) E.g., if historical volatility of market
                       return is 20%
                   (ii) Market excess return is five percent
                   (iii) And the correlation between profit
                        and market return is 0.3
                   (iv) Then the formula gives us 7.5% as
                        MPR
(d) Next step: use MPR to calculate risk neutral profit growth rate
   (i) If, e.g., historical profit growth is 15%
   (ii) With 30% volatility in that growth
   (iii) Then future risk neutral growth rate is 12.75%

(e) Next: calculate PV
   (i) Gross up profits at 12.75%
   (ii) Then discount them at risk free rate
   (iii) Comes to about $60 million
   (iv) Vs. unadjusted calculation
   (v) Would not always favor defendant; based on facts

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Text:

I. Introduction

The influence of economic analysis of law is decades old and well known.² Applying economic theory to the law, the legal system, and litigation in modern America is one of the major legal academic advances in recent years.³ Finance, having itself undergone revolutionary advances in recent decades, also provides a great body of theory and knowledge relevant to law; scholars, policymakers and practitioners have, however, mostly ignored the potential useful applications modern finance presents to our legal system.

This article rectifies that by providing two detailed examples of how quantitative finance can improve the solutions to current practical litigation problems.

II. Thesis

A. Modern finance theory has many valuable and practical applications to legal practice

Like economics, modern (or quantitative) finance has the potential for providing many theoretical applications to law; these applications have valuable practical consequences for litigation and the legal system.

B. Two detailed applications to support subsection II(A)

To provide evidence for the proposition in the previous paragraph, this article describes two detailed and new applications of modern financial theory to common litigation problems. The first application assists the fact finder’s choice of a proper discount rate with which to reduce future damages to present value. The second shows how to use market price of risk to compensate for the uncertainty regarding the post-litigation growth rate of any damages element.

III. Roadmap

This article begins by reviewing the economic analysis of law movement, then defines and differentiates economics and finance. The article then explains and differentiates classical finance from modern, or so called quantitative or analytical finance. This article next reviews the most basic theory of classical finance, time value of money. Then the article introduces a few of the fundamental concepts in modern finance, and explains the Black Scholes formula and capital asset pricing model (“CAPM”) in more detail, as the article uses those in its second application of modern finance to law. Next, the article explains its first application of modern finance to law: improving interest (especially discount) rate calculations in litigation (improving on traditional time value of money concepts). The article then explains the second application: using market price of risk to improve future damages calculations (applying the Black Scholes and CAPM formulas). The article then concludes.

IV. Financial analysis of law

The modern economic analysis of law movement began in the 1960s with the publication of two influential articles.⁴ Since then, researchers have famously revolutionized American law by using economic analysis.⁵ The economic analysis of law movement has been very influential, and is still in ascendance.⁶ Many academic taxonomists classify finance as a subdivision of economics (e.g., the University of California, Los Angeles, and the University of Chicago both include their undergraduate finance courses in their economics departments).⁷ Most economic analysis of law to date does not, however, derive from finance theory,⁸ but from other aspects of economics such as game theory, market structure, and cost analysis.⁹

A. vs. economic analysis

I. Law and economics movement

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It is perhaps because of the relative novelty of modern finance that commentators have not yet attempted to apply its quantitative precepts to our legal system. While economic analysis of law as an academic field and legal doctrine is relatively new, Anglo-American courts have for centuries used economics in developing and shaping our law.\textsuperscript{10} The United States and its legal forebear, England, have in modern times had essentially capitalist economies and materialist cultures.\textsuperscript{11} Legal systems reflect a culture’s basic values, and the U.S. legal system has effectively incorporated the capitalistic and materialistic precepts of our society;\textsuperscript{12} this incorporation has led courts to use economic theory before the beginning of the economic analysis of law movement in the 1960s.\textsuperscript{13} Examples abound, from the prosaic (courts using free market prices as conversion damages\textsuperscript{14}) to the philosophical (the intellectual property clause of the United States Constitution famously chooses a market based reward to encourage authors and inventors to create intangibles that benefit society\textsuperscript{15}).

2. Definitions of economics and finance

One dictionary defines economics as “a social science concerned chiefly with description and analysis of the production, distribution, and consumption of goods and services.”\textsuperscript{16} A standard academic description is the study of “the way resources are allocated among alternative uses to satisfy human wants.”\textsuperscript{17} The same dictionary defines finance as “the science or study of the management of funds.”\textsuperscript{18} A more academic definition is “the relations among assets, time, and risks; and the allocation across time of assets.”\textsuperscript{19}

Finance is a narrower field of study than is economics, and the provenance of most finance theory is much more recent. While most intellectuals date the study of economics to antiquity,\textsuperscript{20} and the birth of modern economics to Adam Smith’s publication of \textit{The Wealth of Nations} in 1776, scholars believe modern quantitative finance to have been born in the 1950s with the publication of two famous articles.\textsuperscript{21}

3. Economic analysis has not yet applied modern finance to law

With limited exceptions that this article notes below,\textsuperscript{22} the economic analysis of law movement has not yet applied modern finance to law.

B. Classical vs. modern finance

\textsuperscript{11} See generally, e.g., \textit{Adam Smith, The Wealth of Nations} (Random House Publishing Group 2000) (1776).
\textsuperscript{12} See, e.g., examples in footnotes 14 and 15.
\textsuperscript{14} DAN B. DOBBS, LAW OF REMEDIES: DAMAGES, EQUITY, RESTITUTION \S 5.13(1) (2d ed. 1993).
\textsuperscript{15} U.S. Const. art. 1, \S 8, cl. 8 (1787).
\textsuperscript{16} MERRIAM-WEBSTER’S COLLEGIATE DICTIONARY 394 (11th ed. 2004).
\textsuperscript{17} EDWIN MANSFIELD, ECONOMICS: PRINCIPLES, PROBLEMS, DECISIONS 6 (4th ed. 1983).
\textsuperscript{18} MERRIAM-WEBSTER’S COLLEGIATE DICTIONARY 469 (11th ed. 2004).
\textsuperscript{19} See generally, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 341 (5th ed. 2002).
\textsuperscript{22} Subsection IV(B)(3)(b).
1. History of classical

The simplest finance is almost as old as human history. The practice of lending money and charging interest goes back to ancient times.\textsuperscript{23}

2. Time value of money

Perhaps the oldest financial concept, and certainly the one with which courts and lawyers currently feel most comfortable, is the time value of money (“TVM”).\textsuperscript{24} A dollar today is always worth more than a dollar tomorrow.\textsuperscript{25} That is because a person with a dollar today may invest that dollar, with no realistic risk of capital loss, and by tomorrow will have, quite literally, more than one dollar.\textsuperscript{26} If you have a dollar today, you can invest it immediately by depositing it in a government insured bank account. If the bank’s saving rate is, e.g., three percent, then in one year you will have $1.03. One dollar today is therefore worth $1.03 one year from now; one dollar one year from now, being less than $1.03, is therefore worth less than the dollar today.\textsuperscript{27}

The TVM theory is therefore not part of modern finance. As the most familiar and basic financial concept, however, it deserves a quick review here before the description of truly modern theories (many of which have their bases in TVM). TVM is also a familiar concept to most lawyers and other educated people today, because of TVM’s frequent practical use in consumer loans, for such things as automobiles and residences. This article’s suggested method for calculating future damages continues to use TVM to the same extent courts currently use TVM in those calculations; a quick review of TVM is therefore helpful here.

TVM focuses on the different values a sum of money has now as opposed to its value at some other definite point or points in time, whether in the past or the future. TVM calls today’s value the “present value,” or PV, and the value at the specified date to come the “future value,” or FV. Whenever an investor knows the applicable interest rate for the period between now and the future date, the investor can calculate the PV if the investor knows the FV, and vice versa. The investor can also, correspondingly, calculate the applicable interest rate (called the discount rate) if the investor knows the FV and PV. TVM easily handles long-term complex loans and investments, such as those with periodic payments and receipts.\textsuperscript{28}

The arithmetic of arriving at a future value, or “grossing up,” is:

$$FV = PV \times (1 + r)^T,$$

\textsuperscript{24} See generally, e.g., Henry N. Butler & Christopher R. Drahozal, \textit{Economic Analysis for Lawyers} (2d ed. 2006).
\textsuperscript{27} See, e.g., Richard A. Brealey & Stewart C. Myers, \textit{Principles of Corporate Finance} 14 (7th ed. 2003).
\textsuperscript{28} See generally, e.g., Richard A. Brealey & Stewart C. Myers, \textit{Principles of Corporate Finance} 118 --150 (7th ed. 2003).
where $r$ is the applicable annual interest rate and $T$ is the investment’s time period, expressed in years. Elementary algebra results in the corresponding formula for calculating present value, or “discounting”:

$$PV = FV \div (1 + r)^T.$$  
Advanced modern finance, including asset pricing, still makes extensive use of TVM, although often in much more complex ways.

a. Many fields widely apply TVM

A vast number of academic and practical fields use TVM, including actuarial science, taxation, accounting, retirement planning, insurance, the lease or purchase decision, the make or buy decision, banking, employment, mergers and acquisitions, industrial engineering, education (e.g. prepaid tuition plans), consumer economics, etc.

b. Many longstanding legal applications

Our legal system is no stranger to TVM. The system has used, with varying degrees of success, the TVM principle for centuries. A few examples follow.

i. Discounting judgments

The classic legal application of TVM is when trial courts use the method to reduce judgments representing future damages to present value. This is the discounted cash flow method, what financial analysts call “capitalizing an income stream.” If the fact finder finds the defendant liable for, e.g., annual medical expenses of $10,000 for the next 30 years, the fact finder will reduce each of the 30 cash flows to present value with a two step process. First, the fact finder will divide $10,000 by $(1+r)^n$ 30 times; the initial time the fact finder uses one for $n$, each subsequent time the fact finder increases $n$ by one. The fact finder will then sum the resulting 30 values. The fact finder uses its choice of the appropriate discount rate as the constant $r$. Table One below illustrates this example assuming that the fact finder has chosen five percent as the appropriate annualized interest rate. Note well how, even at this modest interest rate, the TMV discounting reduces the final judgment to barely half of $30 \times 10,000.00$. 

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29 E.g., RICHARD A. BREALY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE 35 (7th ed. 2003).
30 See generally, e.g., RICHARD A. BREALY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE (7th ed. 2003).
32 E.g., Goddard v. Foster, 84 U.S. 123 (U.S. 1873), Slacum v. Pomery, 10 U.S. 221 (1810).
33 E.g., RICHARD A. BREALY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE 64 (7th ed. 2003).
34 HENRY N. BUTLER & CHRISTOPHER R. DRAHOZAL, ECONOMIC ANALYSIS FOR LAWYERS 448 (2d ed. 2006).
### TABLE ONE

<table>
<thead>
<tr>
<th>Year</th>
<th>Future value</th>
<th>Present value</th>
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</thead>
<tbody>
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<td>$8,227.02</td>
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<td><strong>Totals</strong></td>
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<td><strong>$153,724.51</strong></td>
</tr>
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</table>

**ii. Interest accruing on unpaid judgments**

In addition to discounting, courts also do the reverse with TVM: gross up past amounts to PV. The classic example is the interest that accrues on unpaid damages judgments. Most jurisdictions provide that from the time the trial court enters judgment until the time the
defendant pays the judgment in full, interest accrues on the judgment amount, thus “grossing up” the original amount to a larger future value.\textsuperscript{35} For example, if a court awards the plaintiff a judgment against a defendant of $50,000.00, and the jurisdiction’s interest rate on unpaid judgments is eight percent annually, to satisfy the judgment by paying at a given time, the defendant must pay the plaintiff increasing amounts, as Table Two below illustrates.

**TABLE TWO**

<table>
<thead>
<tr>
<th>Years from judgment date</th>
<th>Present value</th>
<th>Future value</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>$ 50,000.00</td>
<td>$ 50,000.00</td>
</tr>
<tr>
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<td>$ 55,125.00</td>
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<td>$ 57,881.25</td>
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<td>4</td>
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<td>$ 50,000.00</td>
<td>$ 63,814.08</td>
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<td>6</td>
<td>$ 50,000.00</td>
<td>$ 67,004.78</td>
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<td>$ 70,355.02</td>
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<td>8</td>
<td>$ 50,000.00</td>
<td>$ 73,872.77</td>
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<td>$ 77,566.41</td>
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<td>12</td>
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<td>13</td>
<td>$ 50,000.00</td>
<td>$ 94,282.46</td>
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<tr>
<td>14</td>
<td>$ 50,000.00</td>
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<tr>
<td>15</td>
<td>$ 50,000.00</td>
<td>$103,946.41</td>
</tr>
</tbody>
</table>

iii. Etc.

There are countless examples of legal use of TVM, including valuation for mergers and acquisitions, equitable distribution of property, estate planning, secured claims in bankruptcy, CERCLA, settlements, etc.\textsuperscript{36}

3. Modern, a.k.a. quantitative or analytic

Beginning in the mid twentieth century, finance moved beyond TVM and developed new techniques that are considerably more sophisticated. This article will first briefly mention three well known analytic finance theories as examples (the efficient capital markets hypothesis, modern portfolio theory, and arbitrage pricing theory), then will give a more in depth look at two others this article uses later in its second legal applications (the Black-Scholes formula and the capital asset pricing model).


\textsuperscript{36} See generally, e.g., HENRY N. BUTLER & CHRISTOPHER R. DRAHOZAL, ECONOMIC ANALYSIS FOR LAWYERS ch. 8 (2d ed. 2006).
a. Examples of influential modern finance theories

Well known examples of modern analytic finance include the efficient capital markets hypothesis (“ECMH”), which postulates that no publicly traded stock is either overvalued or undervalued. As modern electronic stock markets are extremely liquid, international, nonstop, have a great many participants, and distribute relevant information about each stock issuer, any given stock’s market price almost instantly reflects all relevant publically available information about the stock and its issuer.

Another famous example is modern portfolio theory (“MPT”), which recognizes that investors, being rational, will diversify their investments to minimize the specific risk of any one investment. Any particular company’s stock, for example, is subject to many individualized risks that go with the company’s industry, management, individual markets, etc. By simultaneously investing in other companies, investors reduce the idiosyncratic risk that accompanies each particular investment. (For example, if ABCCorp’s president dies suddenly, that event could have a disastrous effect on ABCCorp’s stock, but is unlikely to most other companies’ stock value. Indeed, the death could have a salutary effect on ABCCorp’s competitors’ stock prices.)

Arbitrage pricing theory (“APT”) states that one can put a value on assets that have no readily available liquid market by assembling hypothetical “replicating portfolios” of other assets, easier to value, that have roughly the same expected future cash flows as the target asset.

These three theories are famous and much used (including some legal uses described below), but not as strictly relevant to this article as the next two; this article therefore describes the next two in more detail.

i. Black Scholes Merton options pricing formula

When used in finance, “derivative” is short for “derivative security.” A derivative security is a financial instrument the value of which depends upon the value of another financial asset (called the “underlying” asset). Derivatives contrast with “primitive securities,” such as underlying stocks or bonds, in which payments depend only upon the financial status of the security’s issuer. Derivatives, on the other hand, typically yield returns that depend on factors beyond the issuer and that may relate to other assets’ prices.

37 Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 341 (5th ed. 2002).
38 Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 342 (5th ed. 2002).
39 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 208 – 239 (5th ed. 2002).
40 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 208 – 239 (5th ed. 2002).
41 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 208 – 239 (5th ed. 2002).
43 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 257 – 412 (5th ed. 2002).
44 Subsection IV(B)(3)(b).
45 See generally Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 985 (5th ed. 2002).
46 See generally Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 985 (5th ed. 2002).
47 See generally Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 985 (5th ed. 2002).
The best known derivative is the option. An option is a contract that gives one party (called the **optionee**) the right, but not the obligation, to buy or sell a specific financial asset on or before a specific date, for a specific pre-agreed price, from the other party (called the **writer** of the option or the **optionor**).\(^48\) Options are financial options or real options, depending upon whether the underlying asset the option requires the optionor to buy or sell is a real asset (e.g. land), or a financial asset (e.g. corporate common stock).\(^49\)

A financial option is an option to buy or sell a financial asset; these are the most typical type of option encountered, and the type regularly traded on public options exchanges.\(^50\) The underlying assets for financial options are usually bonds (investors often call these options **interest rate options**) or stocks.\(^51\)

Anyone can write (i.e. issue) an option to buy or sell a share of publicly traded corporate stock. (When the corporation itself writes such a purchase option, the option is a specific kind, called a **warrant**.)\(^52\) Investors refer to publicly traded options of this kind as **calls**, if they are options to buy, and as **puts** if they are options to sell, the stock.\(^53\)

In 1973, Fischer Black and Myron Scholes derived a formula to value stock options.\(^54\) They realized that the change in the value of a call is a function of the interacting changes in all the factors relevant to the call’s value. They knew that the relevant factors were the price of the underlying stock, the risk free interest rate, the amount of time remaining until the call expires, and the historical variance in the stock’s price. They created a partial differential equation containing all these variables, and solved it for the price of the call.\(^55\)

The result was the famous Black-Scholes formula for valuing financial derivatives. One can approach understanding the formula, without advanced skill in mathematics, by knowing that the formula uses the stock’s current price and historical price variance to calculate risk-adjusted probabilities that the call will expire in the money. If the call is very likely to expire in the money, the call’s value is approximately the stock’s price less the present value of the exercise price. If the call is very unlikely to expire in the money, the call’s value approaches zero.\(^56\)

Consider a profound implication of the formula. *Even though no one can know the future, and even though it is very possible that a particular option may never actually be in the money at any point in the option’s life, Black-Scholes gives a positive dollar value (even if very small) to any unexpired option.*\(^57\) That is a powerful analytic tool, and this article applies a simplified version of the formula to the problem of future damages. The method works because

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50 E.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 6 (5th ed. 2003).

51 E.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 151 (5th ed. 2003).

52 E.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 714 (5th ed. 2003).

53 JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 6, 163 (5th ed. 2003).


56 See ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 710 (5th ed. 2002).

57 E.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 246 (5th ed. 2003).
the problem is the same: trying to put a current market value on future cash flows, the amounts of which are frankly unknowable (and could very well be zero).58

ii. The capital asset pricing model

A major step in modern quantitative finance came in 1964 -- 1966,59 with the publication of three seminal articles.60 These articles described the capital asset pricing model, or “CAPM.”

The insight that underlies the CAPM is that all investors, being rational and wanting to maximize expected return and minimize risk, will choose efficiently diversified portfolios, as the text above61 describes. All efficiently diversified portfolios will have approximately the same (i.e. the maximum practical) diversification as the entire market of all possible investments. The value of any particular investment, when considering MPT, is therefore purely a function of the correlation of that investment’s expected return with that of the entire stock market as a whole (the “broad”) market.62

If a particular investment, say XYZ Inc. common stock, goes up in price at the same time and in the same proportion as the broad market, there is really no reason for a well diversified investor to buy that stock. If, on the other hand, ABC stock goes up when the broad market goes down, ABC is a particularly valuable stock, because ABC stockholders have hedged for broad market declines.63

The CAPM at its essence is a simple statistical linear regression: the formula regresses the excess return (the additional return beyond the return on the risk free asset) of a particular investment on the equity risk premium of the entire stock market.64 The formula is:

\[
r_i - r_f = \alpha + \beta (r_m - r_f) + \varepsilon\]

The variables stand for:

\( r_i \) = the rate of return on the analyzed investment (e.g., any particular stock)

\( r_f \) = the rate of return on the risk free asset, and

\( r_m \) = the rate of return on the entire stock market.65

58 See generally, e.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 234 – 256 (5th ed. 2003).
59 ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 263 (5th ed. 2002).
61 Subsection IV(B)(3)(a).
64 ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 297 (5th ed. 2002).
65 See generally ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 293 -- 312 (5th ed. 2002).
The left-hand side of the equation is what financial analysts call the particular stock’s excess return. On the right-hand side of the equation, “\( r_m - r_f \)" is the equity risk premium, i.e. the broad market’s excess return.\(^{67}\)

To analyze a stock using the CAPM, one acquires the stock’s historical prices, the broad market’s historical price performance over the same time period, and the value of the risk free interest rate for the same period (these data are readily available to investors and analysts). \(^{68}\) Using common spreadsheet software, the investor quickly and easily calculates the excess return on the subject stock and the market by subtracting out the risk free return rate from both; then the investor regresses the stock’s excess return on the market’s excess return.\(^{69}\)

The regression returns values for the Greek letters in the equation above. Epsilon ("\( \varepsilon \)") is the error term; it shows the portion of the subject stock’s excess return for which the correlation of the stock’s excess return with the equity risk premium cannot account. Epsilon is therefore the portion of the stock’s return attributable to the firm’s idiosyncratic risk. Alpha ("\( \alpha \)") represents the stock’s return in excess of that attributable to changes in the entire market and the firm’s idiosyncratic risk.\(^{70}\)

Beta ("\( \beta \)"), most famously, is the portion of the stock’s return attributable to price changes in the stock market as a whole. Beta can theoretically be any number, positive or negative, although it is usually between zero and two. Beta expresses the correlation between the price changes of the individual analyzed stock and the price movements of the entire market. If XYZ’s stock has a beta of one, for example, that means that if the broad market went up 10%, XYZ stock also went up 10%. If the beta is -1, a 10% increase in the broad market resulted in a 10% drop by XYZ. A beta of -2 would mean a drop of 20%, a beta of +0.5 would mean an increase of 5%, etc.\(^{71}\)

The CAPM predicts that, at market equilibrium, alpha and epsilon will generally be insignificant enough to ignore,\(^{72}\) and that rational investors therefore should ultimately care only about the investment’s beta, because high beta stocks have higher than average returns, while low beta stocks have lower systemic risk.\(^{73}\)

The method of this article uses a simplified version of the Black-Scholes formula to estimate the market price of risk (“MPR”) for a future damages element (e.g., post trial lost wages, profits, medical expenses, etc.). This method then uses a simplified version of the CAPM to adjust for the future uncertainty in the growth of the relevant element.

b. Legal applications to date

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\(^{67}\) Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 294 (5th ed. 2002).

\(^{68}\) Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 302 – 305 (5th ed. 2002).

\(^{69}\) See Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 304 (5th ed. 2002).

\(^{70}\) See, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 297 (5th ed. 2002).

\(^{71}\) Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 265 (5th ed. 2002).

\(^{72}\) E.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 295 (5th ed. 2002).

As the text above\cite{sectionI} states, legal applications of modern finance to date are sparse at best. Courts’ one major embrace of modern finance is the use of the ECMH in litigation alleging fraud in the purchase or sale of publicly traded securities. The particular subtype of litigation in which courts and lawyers frequently use the ECMH is class actions arising under rule 10b-5\cite{17CFR24010b-5} pursuant to section 10(b)\cite{15USCA78j(b)} of the federal Securities Exchange Act of 1934.\cite{E.g.,Basic,Inc.v.Levinson,485U.S.224,246–247(1988)}


While no publication has ever cited the article the previous paragraph mentions (at least as currently according to Shepard’s online service), many other commentators on 10b-5 and similar litigation analyze the law using similar quantitative financial theories.\cite{E.g.,JanineSHiller&StephenPFerris,UseofEconomicAnalysisinFraudontheMarketCases,38CLEV.ST.L.REV.535,549(1990),CODIFICATIONOFACCOUNTINGSTANDARDSANDPROCEDURES,StatementofFinancialAccountingStandardsNo.123(Am.Inst.ofCertifiedPub.Accountants2004),UNIF.PRUDENTINVESTORACT§3(1994)}

Litigants and SEC reporting companies also regularly use the Black-Scholes formula to value options, generally without much discussion. For reporting companies, the Financial Accounting Standards Board requires Black-Scholes or a similar valuation method.\cite{CODIFICATIONOFACCOUNTINGSTANDARDSANDPROCEDURES,StatementofFinancialAccountingStandardsNo.123(Am.Inst.ofCertifiedPub.Accountants2004),UNIF.PRUDENTINVESTORACT§3(1994)} Courts normally accept Black-Scholes valuations as well.\cite{CODIFICATIONOFACCOUNTINGSTANDARDSANDPROCEDURES,StatementofFinancialAccountingStandardsNo.123(Am.Inst.ofCertifiedPub.Accountants2004),UNIF.PRUDENTINVESTORACT§3(1994)}

The Uniform Prudent Investor Act, which the National Conference of Commissioners on Uniform State laws and the American Legal Institute adopted in 1994, requires fiduciaries to invest reasonably in the context of a diversified portfolio, an implied nod to MPT.\cite{CODIFICATIONOFACCOUNTINGSTANDARDSANDPROCEDURES,StatementofFinancialAccountingStandardsNo.123(Am.Inst.ofCertifiedPub.Accountants2004),UNIF.PRUDENTINVESTORACT§3(1994)}

\textbf{c.} No publication suggests extending application beyond these subjects

Courts and commentators to date thus endorse the use of modern quantitative finance theory in 10b-5 and some financial fiduciary cases. They do not, however, go beyond these two relatively narrow substantive areas of civil law in their suggested or approved use of finance theory. This article extends the use of finance theory to any case involving future damages.

\textsuperscript{74}Section I.
\textsuperscript{75}17 C.F.R. § 240.10b-5.
\textsuperscript{76}15 U.S.C.A. § 78j(b).
\textsuperscript{85}UNIF. PRUDENT INVESTOR ACT §3 (1994).
This article provides two examples illustrating that modern finance has valuable practical applications to law. Both of these relate to calculating future damages more accurately. The first application improves TVM calculation by aiding courts and litigants in choosing better discount rates, which are crucial in calculating damages accurately. Inaccurate damages are both unjust and economically inefficient. The second application uses a modification of Black-Scholes and the CAPM to compensate for the uncertainty of future damages.

C. Transition: from review of finance to first legal application

This section of this article gave a basic review of TVM, the Black-Scholes formula, and the CAPM to familiarize the reader with these concepts, which the article applies in the next two sections to litigation. The first legal application is modernizing TVM calculations, and the second uses Black-Scholes and the CAPM to improve the calculation of future damages.

V. Modernizing TVM calculations in litigation

This article’s first application of modern finance to improve our legal system is the modernization of TVM calculations in litigation.

A. Present and future value calculations are of cash flows

Our legal system has a doctrine known as the “single recovery” rule. This means that American courts have not historically granted conditional judgments or judgments payable in installments. Instead, plaintiff receives one single lump-sum judgment for all damages, which includes past, present, and future (i.e. pre-trial and post-trial) damages.

One way to solve the problem of future damages would be for the fact finder to “structure” the judgment, making the defendant’s payments contingent in amount upon the actual outcome of the variables unknowable at the time of trial. American courts have rejected this solution, perhaps because of the practical difficulties such judgments entail. Drafters of uniform statutory codes have suggested legislative implementation of this alternative, but legislatures have been slow to accept the broad recommendations. A majority of states has experimented legislatively with structured settlements and period payments of judgments in a variety of actions. One obvious problem with structured settlements is that most tort plaintiff’s
lawyers receive payment from the proceeds of the judgment or settlement payment, and cannot afford to wait for years or decades to recoup their fees and litigation expenses.95

Most courts will allow the parties to agree voluntarily to a settlement that incorporates contingent future payments.96 Courts will even enter these structured settlements as judgments.97 Courts will not, in general, force structured judgments on unwilling parties, however; courts instead revert to the single recovery rule’s lump sum judgments.98 Despite the ability of structured settlements to solve (or at least reduce) the future damages problem, these settlements have never been popular among litigants (outside of a few fields such as divorce, workers’ compensation, etc.).99 In most civil litigation, therefore, the court, the parties, and the lawyers have to manage with the single recovery rule, and do the TMV calculation to discount the damages to PV.

B. Very sensitive to interest (“discount”) rate changes

Present value calculations are extremely sensitive to variations in the discount rate used.100 The higher the discount rate is, the lower is the PV.101 At five percent annual compound interest, for example, $1,000 grows to $1,648 in ten years; at ten percent, the principal grows to $2,718 in that time, much more than double the total interest at the lower rate ($1,718 minus $648 is $1070). At one percent, the interest alone is merely $105; at 20%, the interest is $6,387.

C. Choice of discount rate therefore very important

This sensitivity makes the selection of the correct discount rate extremely important, as even minor changes or errors in the discount rate applied have a significant effect on the PV the fact finder calculates.

D. Fact finders’ choices are, most charitably, naive

I. Often set by court rule or statute

Many jurisdictions have a so-called “legal” rate of interest, which local courts often use for a variety of purposes, including discounting future cash flows to a present value judgment.102 (Other uses for the same rate can include calculating pre- and post- judgment interest on unpaid judgments.)103 Often the legislature establishes the rate (e.g. in Arizona there is a statutory

discount rate\textsuperscript{104}), which may be roughly in the range of market rates when the legislature acts, but can become wildly out of synch as time passes and market rates change. Congress has established a slightly more enlightened rate that consists of a U.S. Treasury bill rate (which fluctuates with the market) plus a premium\textsuperscript{105}. Sometimes courts establish the rate by rule or decision.\textsuperscript{106} Courts generally use the same rate for various purposes, even though the rate is often irrelevant for many purposes.\textsuperscript{107}

2. Usually arbitrary, or at best, outdated

Courts or legislatures often pick round numbers (e.g. ten percent)\textsuperscript{108} as the legal rate. Their choices are essentially arbitrary; they usually use a rate without any investigation into its appropriateness for any of the uses courts make of it. In California currently, for example, the legal interest rate is ten percent,\textsuperscript{109} ridiculously high at a time when worldwide market interest rates have been at historic lows (near, if not below, zero in some cases) for almost a decade.\textsuperscript{110}

3. Market interest rates change greatly over time

a. Cf. now to, e.g., the early 1980s

At the time of this article’s writing, market interest rates have been at historic lows for almost a decade.\textsuperscript{111} Public policymakers have intentionally increased the money supply (which lowers rates \textit{ceteris paribus}) since 2001, when the so-called “dot com” market bubble burst, because of the expected recession from that event and the later terrorist attacks that have become known as 9/11.\textsuperscript{112} Policymakers have continued to increase the money supply, in order to keep market interest rates low and the economy inflated, throughout the subsequent housing price bubble and its inevitable bursting, the stock market crash of 2008, and the resulting second recession of the decade.\textsuperscript{113}

The short term U.S. Treasury bill rate is currently about 0.07\% annually.\textsuperscript{114} In the early 1980s, however, policymakers sharply reduced the money supply to fight the persistent inflation that the Vietnam War started in the 1970s.\textsuperscript{115} The rate peaked at about 16\% in late 1981,

\textsuperscript{104}ARIZ. REV. STAT. § 12–589(A).
\textsuperscript{106}See, e.g., Cavman v. Quality Control Parking, Inc., 696 S.W.2d 549 (Tex. 1985).
\textsuperscript{107}DAN B. DOBBS, LAW OF REMEDIES: DAMAGES – EQUITY – RESTITUTION § 8.5(3) (2d ed. 1993).
\textsuperscript{108}E.g., CAL. CODE CIV. PROC. § 685.010, ARIZ. REV. STAT. § 44-1201, HAW. REV. STAT. § 478-2, N.H. REV. STAT. § 336-1, TENN. CODE ANN. §§ 47-14-103, 47-14-121, 47-14-123.
\textsuperscript{109}CAL. CONST. art. XV, § 1(2), CAL. CODE CIV. PROC. § 685.010.
\textsuperscript{110}Jean-Pierre Daunhine, Member of the Governing Board of the Swiss National Bank, Speech at the Money Market Event, A World of Low Interest Rates (22 March 2012), http://www.bis.org/review/r120323c.pdf?frames=0 (last checked 29 July 2012).
\textsuperscript{112}E.g., FREDDIE S. MISHKIN, THE ECONOMICS OF MONEY, BANKING, AND FINANCIAL MARKETS 404 (7th ed. 2004).
approximately 228 times what it is now. This shows the vast range one particular market interest rate can have in a historically short period (less than 31 years).

b. At same time, different rates for different investments

At any given time, there is a range of market interest rates, which can be quite wide. As the text below explains in detail, the 28 day U.S. Treasury bill is generally the lowest market rate, because it is the shortest term and most risk free. Depending on the factors described below that compose a specific market interest rate at any time, other rates can range quite high. For example, the rates on personal unsecured credit cards, often among the highest rates, are as high as around 30% currently.

4. Fact finders should choose financially appropriate rate for facts

As explained above, choice of discount rate can have profound consequences. FV and PV calculations are extremely sensitive to discount rates. Courts’ choices of the rate to use in any litigation calculation are therefore important to the parties, to courts’ authority and credibility, and to society, because of the aggregate effect these choices have on the macroeconomy.

a. Some courts have adjusted for inflation

General price inflation in the macroeconomy devalues money; if an economy’s price level increases, e.g., two percent over a one year period, at the end of that time it takes $1.02 to purchase something that $1.00 would have purchased a year earlier.

b. A few courts have suggested “natural” core rate net of inflation

Some courts, during and shortly after the severe price inflation of the 1970s, laudably attempted to deal with the constant changes in market interest rates. Their actions, however, were less than well informed. They theorized that as inflation changed, the market interest rate changed, and therefore there exists a so-called “natural,” unchanging “pure” rate of interest. These courts therefore decided that the correct discount rate was the current rate of inflation plus this natural rate (which they estimated at three percent, from historical averages).

5. Existing research on modern calculation methods

Modern finance provides extensive research on choosing correct discount rates for TVM calculations.

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118 Subsection VI(F)(2)(a)(iv).
120 Subsection V(B).
E. Rates are actually result of four variables

Quantitative finance has shown that any actual market interest rate is the result (and essentially the sum) of four variables: expected inflation; credit risk; market risk; and the interaction of the supply of, and demand for, money.  

1. Expected inflation

As the text above explains, inflation devalues future cash flows; lenders therefore, as a matter of course, include expected inflation as a factor in the interest rates they charge. If a lender desires a real (i.e., inflation adjusted) return of, e.g., three percent on a loan, and the lender expects the average inflation rate to be, e.g., two percent during the repayment period, the lender will sum the two rates and charge the borrower about five percent. (The arithmetic is slightly more complex, but a simple sum serves for this article’s purposes).

2. Credit risk

All lenders take credit risk. Credit risk is simply the possibility that the borrower will not repay the loan’s principal, and interest, in full, on time, as agreed. Lenders compensate themselves for credit risk by charging higher risk borrowers (i.e. borrowers lenders believe more likely to default) higher interest rates. The more likely a lender considers a borrower to default, ceteris paribus, the higher the interest rate the lender will charge. Lenders generally make a simple expected value calculation to determine the credit risk component of the loan’s interest rate.

Consider, for example, a hypothetical lender who wants a five percent return on his loan of $100. He can lend to a perfectly safe borrower at five percent interest. What if the borrower, however, is a credit risk? The lender uses the borrower’s credit information to calculate how likely it is that the borrower will default. Suppose the lender concludes that the probability of the borrower defaulting is ten percent. There are therefore two possible outcomes: an 85% chance that the lender will get a five percent return, and a ten percent chance that the lender will get nothing. The lender will thus charge this borrower about 17% interest on the loan, so the lender’s expected value is 85% * $117 + 15% * $0 = $105, which gives the lender his five percent rate of return.

3. Market risk

Market risk is the possibility that interest rates on any particular type of loan will increase during the repayment period of a loan of that type. If a lender makes a ten year loan at five percent annually, for example, because ten percent is the current market interest rate for that type of loan, the lender takes the risk that during the loan’s ten year repayment period, the market rate

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of interest for similar loans will increase. Two years into the repayment term, the market rate may increase to seven percent. The lender cannot take advantage of the increase in market rates, however, because she has agreed to a lower rate for this particular loan for eight more years.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 120 – 140 (7th ed. 2004).}

4. Changing supply of, and demand for, money

Interest rates are the price of money now (hence the term time value of money).\footnote{See generally, e.g., Edwin Mansfield, Economics: Principles, Problems, Decisions 730 – 731 (4th ed. 1983).} Borrowers reasonably expect to have money in the future, but do not have it when they need it, e.g., to start a business or to purchase an education or a residence. Parties that have money currently may not have a need to spend it now, and wish to invest it, making a reasonable return on their investment, for later, when they (or their beneficiaries) may need to spend the money.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 85 – 119 (7th ed. 2004).}

The credit markets match these two complementary parties; the party with money but no consumption need now (the lender) loans the money to the party with no money but a need to consume now (the borrower). The borrower agrees to pay the entire amount borrowed (the principal) back to the lender at a specific time or times in the future, and to pay additional amounts to the lender as the price of the loan (the interest), also at agreed times.\footnote{See generally Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 85 – 119 (7th ed. 2004).}

As in any other free market, the most basic determinant of anything’s price is the interplay of supply for, and demand of, the commodity.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 – 117 (7th ed. 2004).} When lending money, the commodity lenders supply is money right now. Immediate funds are what borrowers need.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 – 117 (7th ed. 2004).} Most people who want to purchase a house, an automobile, an education, etc., usually do not have the funds to pay the seller in full at the time the buyer wishes to purchase the item.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 – 117 (7th ed. 2004).} Lenders meet this demand by supplying the funds now, i.e. at the time the purchaser needs the money, in exchange for interest payments.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 – 117 (7th ed. 2004).} Borrowers ultimately repay more than the nominal principal sum they borrowed, because the borrowers must compensate the lender for providing the money when the borrower needed to spend it, as opposed to when the buyer actually had the requisite funds.\footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 – 117 (7th ed. 2004).}

The supply of current lendable funds varies from time to time, depending upon many factors.\footnote{See generally, e.g., Edwin Mansfield, Economics: Principles, Problems, Decisions 730 – 731 (4th ed. 1983).} The demand for borrowed money also varies, based upon various macroeconomic influences.\footnote{See generally, e.g., Edwin Mansfield, Economics: Principles, Problems, Decisions 730 – 731 (4th ed. 1983).} The interest rate lenders charge borrowers is the price of the current funds.\footnote{See generally, e.g., Edwin Mansfield, Economics: Principles, Problems, Decisions 730 – 731 (4th ed. 1983).} The basic interplay of supply and demand for credit functions is the same as most typical markets,
with a downward sloping demand curve (i.e., borrowers borrow less at higher interest rates), and an upward sloping supply curve (lenders are willing to lend more at higher rates). \footnote{E.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 90 (7th ed. 2004).} \textbf{Graph One} below illustrates this concept. \footnote{E.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 90 (7th ed. 2004).}

\textbf{GRAPH ONE}

In the credit market \textbf{Graph One} describes, at ten percent interest, borrowers will demand $10,000,000 in loans, but lenders will only supply $1,000,000. At, e.g., seven percent, the demand is $7,000,000, but the supply is only $4,000,000. The annual interest rate at which this market is in equilibrium is 5.5\%, at which the supply of and demand for loans is equal ($5,500,000).

\textbf{F.} Courts can and should consider all four factors

Financial analysts consider all four variables for any given transaction, and can estimate the correct total rate by summing the four variables. \footnote{See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 482 -- 530 (5th ed. 2002).} Lenders and borrowers of all types and sizes regularly make these calculations every day; professionals do so also when they calculate TVM. \footnote{See generally, e.g., Frederic S. Mishkin, The Economics of Money, Banking, and Financial Markets 61 -- 140 (7th ed. 2004).} There is no reason why courts cannot or should not do the same thing when determining the correct discount rate.

\textbf{I.} Information easily available
There is a great deal of information relevant to all four factors easily available from various sources, public and private, such as libraries, government agencies, universities, websites, etc.\textsuperscript{144}

2. Extensive financial research on how to evaluate each factor

Academics and practitioners have for decades researched how to choose the best discount (or premium) rate in many various situations, because of the economic, practical, and personal importance of using the best rate, and the commonplace nature of TVM calculations.\textsuperscript{145}

a. Example: where plaintiff can invest judgment proceeds

The easiest example of the choosing the best discount rate in litigation is well known and very helpful analytically, even if it is rarely feasible as a practical matter.\textsuperscript{146} The example involves annuities. If a particular element of damages represents a cash flow stream of lost future income, after the fact finder has decided the annual amount due in the future, the defendant can (at least theoretically, and occasionally practically) purchase an annuity that will produce the desired income stream for the appropriate time. The implied discount rate in that situation is the one that determines the current price of the annuity.\textsuperscript{147}

An annuity is much like a fully amortized loan in reverse. The consumer who purchases the annuity lends the annuity’s purchase price to the selling annuity company. The company repays the “loan” (the purchase price), with interest, in a series of defined amount payments, annually over the annuity’s term. In our analytical model, the specific annual amount the company pays replaces the lost income or additional expense the plaintiff incurs due to the defendant’s tort or breach.\textsuperscript{148}

For example, consider a plaintiff whose damages are lost future wages of $50,000 per year, and is age 40, therefore having a 30 year remaining working life. The defendant can purchase an annuity that pays $50,000 per year each year for 30 years. The price of this annuity will, of course, be the PV of $50,000 for the next 30 years, using the appropriate discount rate for the annuity company. Analysts call the discount rate that makes the price the PV of the cash flow stream the internal rate of return of an annuity.\textsuperscript{149}

b. A more realistic example

For a more typical and complex example, consider a hypothetical tort defendant found liable for plaintiff’s annual lost wages of $50,000 for a 30 year period (the remainder of plaintiff’s working life). In determining the proper discount rate to arrive at the PV lump sum judgment amount, the fact finder can easily consider all four interest rate factors as follows.

\textsuperscript{144} A good place to start is the websites of the Federal Reserve System of the United States at \url{www.FederalReserve.gov} (last checked 29 July 2102).
\textsuperscript{145} See generally, e.g., \textsc{Richard A. Brealey & Stewart C. Myers}, \textit{Principles of Corporate Finance} 12 -- 150 (7th ed. 2003).
\textsuperscript{146} See generally, e.g., \textsc{Daniel W. Hindert, Joseph Julnes Deiner, & Patrick J. Hindert}, \textit{Structured Settlements and Periodic Payment Judgments} § 3.05 (2005).
\textsuperscript{147} \textit{E.g.}, \textsc{Richard A. Brealey & Stewart C. Myers}, \textit{Principles of Corporate Finance} 38 -- 40 (7th ed. 2003).
\textsuperscript{148} \textsc{Richard A. Brealey & Stewart C. Myers}, \textit{Principles of Corporate Finance} 38 -- 40 (7th ed. 2003).
\textsuperscript{149} \textit{See}, e.g., \textsc{Richard A. Brealey & Stewart C. Myers}, \textit{Principles of Corporate Finance} 96 -- 98 (7th ed. 2003).
i. Supply of, and demand for, funds

There is a current liquid and worldwide market for the U.S. Treasury’s inflation protected securities, a.k.a. “TIPS.” The quoted return on these bonds is always in addition to actual inflation; it is what economists call the “real” return, as opposed to the “nominal” return, which inflation devalues,150 as the text above describes.151 At any time, the bonds’ yields therefore reflect the then current pure interaction of supply and demand for 30 year credit. A quick glance at an internet financial page (e.g. Bloomberg) gives this information. The 30 year TIPS bond today yields about 0.3% annually.152

ii. Expected inflation

Fact finders can determine expected inflation just as easily, using the TIPS yield together with the then current 30 year U.S. Treasury bond yield. The yield on these bonds is nominal, not real (i.e., inflation adjusted). This information is easily available to anyone with internet access. As of this writing, the rate is about four percent annually.153 The current 30 year Treasury bond yields 2.5%. Expected inflation over the next 30 years is the difference between the Treasury bond yield (2.5%) and the TIPS bond yield (0.3%), or 2.2%.154 This is because the TIPS yield is in real (inflation adjusted) terms, while the basic Treasury bond yield contains a premium for expected inflation.

iii. Market risk

For litigation purposes, Treasury bond and TIPS yields sufficiently incorporate a market risk premium. This is because the government will not pay the principal on these bonds for a 30 year period. The market prices of these bonds, and the bonds’ resulting yields, therefore reflect the possibility that market interest rates may rise before the bonds mature.155

iv. Credit risk

The fourth and final component of an interest rate is credit risk. Fact finders can easily determine this as well, using the annuity model (even if it is not actually possible for the plaintiff to purchase an actual appropriate annuity with the judgment proceeds). If the plaintiff could purchase an annuity that would provide the precise future cash flows the damages award replaces, the price of the annuity would depend in part on the creditworthiness of the annuity seller.156

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150 See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 421-422 (5th ed. 2002).
151 Subsection V(D)(4).
154 See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 134 (5th ed. 2002).
156 See generally, e.g., DANIEL W. HINDERT, JOSEPH JULNES DEHNER, & PATRICK J. HINDERT, STRUCTURED SETTLEMENTS AND PERIODIC PAYMENT JUDGMENTS § 3.05 (2005).
As the text above explains, an annuity’s price is the sum of all the annual payments, discounted at a certain rate. This rate includes the components above: market risk, expected inflation, the supply of and demand for money, and a credit risk premium. In the judgment scenario, the relevant credit risk is that of the annuity company. If, for example, the annuity company has high credit risk, it is less likely to pay all the annual benefits to the buyer in full and on time. Buyers will therefore pay less, ceteris paribus, for the annuity than they otherwise would, because the risk of nonpayment makes the annuity less valuable. If, alternately, the company has little credit risk, buyers will pay more, ceteris paribus, for the annuity. The annuity buyer will do an expected value analysis just like the lender the text above describes.

The annuity company’s credit risk interest rate premium is therefore the internal rate of return on the annuity, less the other three interest rate factors. If the plaintiff can actually purchase an appropriate annuity, any calculation is unnecessary; the annuity’s price is the damages, because that amount includes all four interest rate factors. In all but the simplest cases, however, it is very unlikely an annuity that perfectly matches the plaintiff’s projected damages cash flow stream exists. That does not mean the annuity analysis is of no use. The fact finder can estimate the appropriate credit risk premium by evaluating evidence of a typical annuity company’s credit risk premium.

It may initially seem incorrect to increase the discount rate (thus reducing the judgment) to reflect the credit risk of the (possibly hypothetical) annuity company. Recall, however, that in contrast to a structured settlement, the preferred single judgment rule shifts the risk that the defendant will become insolvent during the annuity period away from the plaintiff, because the plaintiff receives the present value of the damages immediately in cash. It is therefore appropriate for the plaintiff to compensate the defendant for that assumption of risk with a reduction in the present value of the future damages when the plaintiff receives a lump sum.

v. Composition of factors into final interest rate

The final task for the fact finder in determining the appropriate discount rate for the future damages is to put the four factors together. It is much easier than the detailed analysis above might suggest. For litigation purposes, the appropriate TIPS rate (the 30 year rate in our example) includes three of the four factors: the supply and demand for money, the market risk premium for a 30 year loan (as explained above), and expected inflation.

This is because including expected inflation in the discount rate reduces the plaintiff’s judgment (the higher the discount rate, the less the present value). Unless the future damages cash flows contain inflation premiums (which they generally do not), there is no need to

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157 Subsection V(F)(2)(a).
159 See generally, e.g., DANIEL W. HINDERT, JOSEPH JUILNES DEHNER, & PATRICK J. HINDERT, STRUCTURED SETTLEMENTS AND PERIODIC PAYMENT JUDGMENTS § 3.05 (2005).
160 See generally, e.g., SURESH SUNDARESAN, FIXED INCOME MARKETS AND THEIR DERIVATIVES 629 – 642 (2nd ed. 2003).
161 Subsection V(E)(2).
162 DANIEL W. HINDERT, JOSEPH JUILNES DEHNER, & PATRICK J. HINDERT, STRUCTURED SETTLEMENTS AND PERIODIC PAYMENT JUDGMENTS § 3.05 (2005).
include the expected inflation premium in the discount rate. (If for some reason the future damages cash flows do include increases for expected inflation, the fact finder can simply use the nominal Treasury bond rate instead of the corresponding TIPS rate.)

The appropriate discount rate is therefore either the TIPS rate (if damages unadjusted for expected inflation), or the Treasury bond rate (if damages include inflation) plus the annuity company credit risk premium. In our example, if the annuity company credit risk premium is, e.g., one percent, the court’s discount rate is three percent (TIPS) plus one percent (credit risk), or four percent total. Note the elegant ease with which using the TIPS rate automatically includes the gross up for expected inflation and the (cancelling) discount for it.

G. Transition: from first legal application to second

Now that this article has explained using interest rate analysis to improve TVM calculations, its first application of modern finance to law, the article moves to its second application: improving future damages calculations.

VI. Future damages

A. Introduction

This article’s second application of modern quantitative finance to law is the use of market price of risk to account for the uncertainty inherent in future damages calculations.

B. Analyze damages temporally

Pecuniary damages, whether compensating for past or future losses, replace the plaintiff’s additional expenses or lost income caused by the defendant’s tortious or breaching action or omission. One can construct a two by two matrix of pecuniary damages when analyzing these factors. Pecuniary damages are for either: (1) past or present (i.e., pre-trial or post-trial), and (2) forgone “income” (more accurately personal “gross income,” or “revenue,” for a business, in an accounting sense) or additional expense the plaintiff incurs due to the defendant’s action. Item (1) forms one axis of the matrix, and item (2) forms the matrix’s other axis.

This resulting graph, Table Three below, shows the four possible types of pecuniary damages in this sort of analysis:

<table>
<thead>
<tr>
<th></th>
<th>Pre-trial</th>
<th>Post-trial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lost income</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Additional expense</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

165 See, e.g., James M. Fischer, Understanding Remedies § 9 (2d ed. 2006).
166 See, e.g., James M. Fischer, Understanding Remedies § 9 (2d ed. 2006).
For example, suppose a plaintiff suffers bodily injury due to a defendant’s negligence, and the injury both permanently prevents the plaintiff from earning wages and permanently makes the plaintiff incur additional medical expenses. The plaintiff’s loss of wages, and additional medical expenses, begin at the time of the accident and continues until her death. The wages the plaintiff loses before the judgment are category one, and those lost after the judgment are category two. The plaintiff’s additional medical expenses before the judgment are category three, and those occurring after the judgment are category four.

C. Ease vs. difficulty in calculation

Category three is the easiest of the four for triers of fact to measure. The plaintiff or her counsel typically must (and do) provide the fact finder with invoices and receipts for these items. Category one is more difficult than category three, requiring more than reimbursement of proven expenses. For the pretrial period, data exist as to the plaintiff’s lost revenues. If the plaintiff claims lost wages, for example, due to inability to work as result of the defendant’s negligence (which resulted, say, in plaintiff’s disabling injury), the plaintiff can provide evidence showing the plaintiff’s pre-tort wages, and what similar workers’ wages were during the period post-tort and pretrial. It is still a much easier task than either item in the post-trial column, because the latter are future damages, not past damages.

D. Challenges calculating future damages

Categories two and four illustrate a superhuman task our damages system regularly requires fact finders to perform: the calculation of damages to compensate for losses or expenses that have not yet occurred (and that very possibly never may occur). It is with this problem that modern quantitative finance theory can best assist.

E. Drayton case is a good example

There are presumably countless cases that illustrate this serious and endemic problem with our system of future damages. One major law school casebook provides an example that is perhaps as good as any. The case is the so-called “liquid-plum’r” case, Drayton v. Jiffee Chemical Corp.

“Liquid-Plumr®” is a registered trademark for a household drain opener, now owned by the Clorox Company. In the 1960s, the trademark was “liquid-plum’r,” and the Jiffee Chemical Corp. distributed the corresponding product. Drain openers at that time consisted (and most still do) largely of strongly concentrated, and powerfully caustic, chemicals like lye.

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168 E.g., JAMES M. FISCHER, UNDERSTANDING REMEDIES § 8.1 (2d ed. 2006).
169 DAN B. DOBBS, LAW OF REMEDIES: DAMAGES – EQUITY – RESTITUTION § 3.3(4) (2d ed. 1993).
Today, however, such products come in unbreakable plastic bottles with childproof caps and carry obvious warning labels.\footnote{Drayton v. Jiffee Chem. Corp., 395 F. Supp. 1081, 1093 (N.D. Ohio 1975).} In the 1960s, liquid-plum’r drain opener and similar household products came in glass bottles that shattered easily, had simple twist-off caps, and bore labels describing the contents as “safe.” (The defendant claimed it meant the product was safe for \textit{pipes}.)\footnote{Drayton v. Jiffee Chem. Corp., 413 F. Supp. 834, 836 (N.D. Ohio 1976).}

Theresa “Terri” Drayton was a one-year-old black girl, living with her mother in a poor neighborhood of Cleveland, Ohio, in 1968. Terri suffered severe and widespread chemical burns when she exposed herself to the contents of a bottle of defendant’s brand of drain opener.\footnote{Drayton v. Jiffee Chem. Corp., 395 F. Supp. 1081, 1095 (N.D. Ohio 1975).}


The injuries did not, however, shorten her expected lifespan.\footnote{E.g., 22 AM. JUR. 2D Damages § 345 (2003).}

This case provides excellent examples of the future damages problem. The fact finder had to calculate both the plaintiff’s unknown future expenses, e.g. medical and psychotherapeutic, and lost income that results from the disability defendant caused.

Terri’s injuries made it much less likely she would be able to attend college or work regularly. As of the date of trial, her remaining life expectancy was over 70 years. The unlucky fact finder had to try estimating 70 years of medical and psychotherapeutic expenses, and a working life’s worth of lost or reduced wages.\footnote{Drayton v. Jiffee Chem. Corp., 413 F. Supp. 834 838 – 839; \textit{see also} DAN B. DOBBS, LAW OF REMEDIES: DAMAGES – EQUITY – RESTITUTION § 8.1(3) (2d ed. 1993).}

Beyond these initial difficulties, the fact finder must contend with 70 years’ worth of contingencies. To assess damages properly, the fact finder must award only those damages that the defendant’s negligence caused and that the plaintiff could not avoid.\footnote{Drayton v. Jiffee Chem. Corp., 395 F. Supp. 1081, 1095 (N.D. Ohio 1975).} The fact finder must therefore determine (actually, at best, estimate) what the plaintiff’s actual medical expenses will be for the next 70 years, then subtract from that what the plaintiff’s medical expenses \textit{would have been} without the defendant’s negligence, for the next 70 years. The fact finder must then make the same estimates regarding the plaintiff’s actual psychotherapeutic expenses, and what these expenses would have been.\footnote{Drayton v. Jiffee Chem. Corp., 413 F. Supp. 834, 836 (N.D. Ohio 1976).}

This endeavor, calculating the plaintiff’s additional expense, is obviously difficult to the point of impossibility. It pales in comparison, however, to the fact finder’s consideration of the
plaintiff’s income lost due to the defendant’s negligence. The fact finder again must consider two alternative scenarios, neither of which is knowable at the time of trial: (1) what plaintiff’s future income actually will be, and (2) what plaintiff’s future income would have been had it not been for defendant’s tort or breach. These determinations involve not just the idiosyncratic personal variables regarding the plaintiff, but equally unpredictable macroeconomic factors, such as wage growth, productivity, random chance, etc.

When plaintiff is an adult with an established career, the fact finder usually assumes the plaintiff would have remained in the same career. This simplifying assumption helps greatly; if the defendant’s conduct has rendered the plaintiff totally disabled, the fact finder can attempt to predict what the plaintiff’s wages would be over his remaining work life (still no easy task), and reduce those to present value. When the plaintiff is a child under five years old, as in Drayton, the task becomes exponentially more complex. Other problems in Drayton and similar cases include the plaintiff’s race and sex. How does the fact finder factor in such complexities as lower wages for blacks and women, due at least in part to discrimination, and how these factors will play out in the future?

F. Classic future damages conundrum: the new business rule

A personal injury case like Drayton is a good illustration of the multiplicity of future damages problems in our jurisprudence. Using modern finance to solve all the problems in Drayton, however, would make this article unnecessarily lengthy. To illustrate the method as clearly as possible, this article therefore uses Drayton merely as an example of the problem, and solves a different, simpler, but classic, example of the future damages problem: the so-called “new business rule.” In these cases, there is only one unknown future stream of damages cash flows, the lost profit of the plaintiff’s business. That makes for a simple application of the method this article teaches.

Courts have struggled for decades with damages representing the lost profits of new businesses. If a defendant’s tort or breach caused the failure of a business that had a history of profits, it was easy enough for courts to average the historical profits and project them into the future. When, however, the failed business had little relevant profit history, courts traditionally invoked the uncertainty doctrine to bar such damages entirely. In recent years, many courts have eased the so called “new business rule” (prohibiting recovery of such damages as too uncertain), and allowed plaintiffs to present evidence of similar businesses with established profit histories, as comparables. Those courts are now the majority.

186 E.g., 22 AM. JUR. 2D Damages § 328 (2003).
190 See, e.g., 22 AM. JUR. 2D Damages § 48 (2003).
191 1 ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS §§ 4.1 – 4.3 (2005).
192 1 ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS §§ 4.1 – 4.3 (2005).
193 1 ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS §§ 4.1 – 4.3 (2005).
194 1 ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS §§ 4.1 – 4.3 (2005).
195 1 ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS § 4.3 (2005).
These courts do already apply basic finance, i.e. TVM, to calculate damages representing a new business’s lost profits. The lump sum judgment is present value of all lost future profits. One method courts could use would be to assume that the business’s profits would last indefinitely into the future, and TVM provides a simple formula for calculating the present value of the business’s future lost profits. (Profits, of course, are the business’s revenues for a given time period, conventionally one calendar year, less the business’s expenses for that same year. Expenses are the amount the business spends to generate that year’s revenues.)

The formula is:

$$\text{Present value} = \left(\frac{\text{annual profit}}{\text{annual discount rate}}\right)^{\text{198}}$$

If the comparable business’s average annual profit is, for example, $1,000,000, and the appropriate discount rate is five percent annually, the fact finder’s PV calculation is $1,000,000 / 0.05 = $20,000,000. The fact finder would then award this amount as the plaintiff’s lost profit damages.

Courts generally do not assume the profits would have continued indefinitely; instead, they project the profits out for some definite period according to the expert testimony and related evidence received (e.g., 20 years).

1. Plaintiff will argue that profits would have grown in future

A further complication with new businesses is that their profits tend to start relatively small and grow, often significantly, over time. This is also true of most, perhaps all, future damages elements, such as wages, medical expenses, etc., if for no other reason than general price inflation. Inflation is not, however, the same as economic growth; otherwise all future damages would grow at the same rate, whereas different variables have obviously grown at radically different rates historically. Plaintiffs in this situation generally want their damages to include the lost growth.

Courts’ current approaches to this problem are naïve: courts may, e.g., ignore growth as too uncertain, use the “below market discount rate method,” or assume certain factors, e.g., growth and inflation, (conveniently) cancel each other. Of these approaches, the best is the below market discount rate method. When courts use that method to predict future growth in a damages element, the finder of fact calculates the growth in, e.g., plaintiff’s wages by estimating

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196 E.g., 22 AM. JUR. 2D DAMAGES § 483 (2003).
197 I ROBERT L. DUNN, RECOVERY OF DAMAGES FOR LOST PROFITS § 4.1 – 4.3 (2005).
198 RICHARD A. BREALEY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE 37 (7th ed. 2003).
199 See generally, e.g., JAMES M. FISCHER, UNDERSTANDING REMEDIES § 9.3 (2d ed. 2006).
200 See generally, e.g., JAMES M. FISCHER, UNDERSTANDING REMEDIES § 9.3 (2d ed. 2006).
205 See, e.g., JAMES M. FISCHER, UNDERSTANDING REMEDIES § 9.3 (2d ed. 2006).
the all factors that would contribute to that growth except for inflation, then discounts by the real interest rate.\textsuperscript{207} The U.S. Supreme Court has approved of this method, in Jones & Laughlin Steel Corp. v. Pfeifer,\textsuperscript{208} and the federal courts’ use of it is widespread,\textsuperscript{209} but academic criticism persists.\textsuperscript{210} The supreme court wisely used Pfeifer to reject the total offset approach (which conveniently holds that the growth rate of all damages is exactly the same as the discount rate),\textsuperscript{211} and also showed skepticism about the existence of an unchanging risk free rate of interest.\textsuperscript{212}

2. Applying modern finance to solve the problem

Modern finance provides an excellent solution for the new business problem and similar future damages conundrums. In the context of the new business rule, the conundrum simplifies to possibilities that the new business might have be very profitable, or not profitable at all, or anywhere in between. Similar future damages problems are the same: wages, or medical costs, might grow quickly, slowly, or not at all in the future. There is simply no way to know for sure at what rate any factor will grow without being able to see the future.

Recall, however, that the Black-Scholes formula gives a present positive dollar value to any unexercised option;\textsuperscript{213} from a financial perspective, our future damages problem is exactly the same. Depending upon the price of the underlying security (which itself depends on countless microeconomic and macroeconomic factors), the option may become extremely valuable, slightly valuable, worthless, or anywhere in that continuum, but that is impossible to know at the present. Future damages are unknowable in the same way. An implication of Black-Scholes, and a generalization of the CAPM, known as market price of risk, works as well in law as it does in finance. Using MPR involves risk neutral pricing theory, another major development in modern quantitative finance, explained immediately below.

a. Risk neutral pricing theory

i. Posits world in which everyone is risk neutral

Risk neutral pricing theory simplifies financial calculations by eliminating the difficult challenge of choosing the appropriate discount rate. The theory posits an imaginary world in which everyone is risk neutral. In that world, there is only one discount rate: the risk neutral rate of return on investment.\textsuperscript{214}

ii. Definition of risk neutrality and related terms

\textsuperscript{207} JAMES M. FISCHER, UNDERSTANDING REMEDIES § 9.3 (2d ed. 2006).
\textsuperscript{208} Jones & Laughlin Steel Corp. v. Pfeifer, 462 U.S. 523, 545 (1983).
\textsuperscript{209} E.g., Hull by Hull v. United States, 971 F.2d 1499, 1511 –1512 (10th Cir. 1992), Scott v. United States, 884 F.2d 1280, 1286 –1287 (9th Cir. 1989).
\textsuperscript{210} See generally, e.g., JAMES M. FISCHER, UNDERSTANDING REMEDIES § 9.3 (2d ed. 2006).
\textsuperscript{213} Subsection IV(B)(3)(a)(ii).
\textsuperscript{214} E.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 205 (5th ed. 2003).
What do economists and financiers mean by risk neutral? To be risk neutral simply means to be willing to pay any amount up to the expected value of any venture. A simple example of the author’s own invention is the best way to explain. Suppose someone presents you with the opportunity to win $100,000 in cash. There is no trick or catch; all you do is choose a piece of paper from an opaque glass jar. There are ten pieces of paper in the jar; nine contain the word “lose,” and the tenth says “win.” If you pick the piece that says “win,” you immediately receive $100,000 in cash (tax free). If you pick a slip that says “lose,” you walk away with nothing.

The question, of course, is: what is the maximum amount you would pay to play this game? You can write a check, use your credit card, borrow money with which to play, run to the automatic teller machine, etc. Think about it. Whatever your initial response is, consider that amount plus a penny until you get to the point where you would not go one cent higher. There is no right or wrong answer, and it can be fun to play this game with a group of people, collecting the amounts each is willing to pay, and their explanations as to why they chose that amount.

A typical set of results usually includes a few who would pay something close to zero, some will pay up to $1,000, some up to $5,000, etc. Note that one can arrange these people on a relative continuum of what economists call risk aversion: the less any player is willing to pay for the chance, the more risk averse that player is in relation to other players. Players willing to pay more are less risk averse (or more risk loving) than others. All of these players (those who would pay $1, $1,000, and $5,000) are, however, as an absolute matter, risk averse. The payment amount for this game that is neither risk averse nor risk loving is the expected value of the game: a ten percent chance of winning times the $100,000 proceeds of winning, or $10,000. That is the risk neutral amount (any amount below $10,000 is risk averse in an absolute sense, and any amount above $1,000 is absolutely risk loving).

iii. Use single discount rate for all future cash flows

The essence of risk neutral pricing theory states that as long as one restates all cash flows in a TVM calculation to what they would be in the hypothetical risk neutral world, the correct discount rate is the risk neutral rate of return on investment. This rate is the same for all investments, because everyone in this hypothetical world is risk neutral. We can use the real world estimate of the risk free interest rate as the risk neutral rate.

iv. Estimate risk neutral rate with risk free rate

The risk free rate does not exist; it is a hypothetical construct. It is the interest rate that reflects solely the supply of and demand for money, with no added premium for credit risk.

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215 E.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 157 – 161 (5th ed. 2002).
216 The author based this on his own experience teaching business courses to law students over several years.
217 See, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 943 (5th ed. 2002).
218 E.g., John C. Hull, Options, Futures, and Other Derivatives 204 (5th ed. 2003).
219 E.g., John C. Hull, Options, Futures, and Other Derivatives 245 (5th ed. 2003).
220 E.g., John C. Hull, Options, Futures, and Other Derivatives 663 (5th ed. 2003).
221 E.g., John C. Hull, Options, Futures, and Other Derivatives 45 (5th ed. 2003).
market risk, or expected inflation.\textsuperscript{224} In the real world, of course, as the text above explains,\textsuperscript{225} any loan contains all four rate components. Analysts can estimate the risk free rate, however, by considering loans that have almost no credit risk, because the borrower is extremely solvent and historically creditworthy.\textsuperscript{226} To reduce expected inflation and market risk as much as is possible, analysts consider extremely short term loans; i.e. loans that are due so quickly after the lender makes the loan, there is not enough time to allow for significant inflation or market interest rate changes.\textsuperscript{227}

As described above,\textsuperscript{228} analysts therefore generally use the 28 day United States Treasury bill rate as their estimate of the risk free rate. The federal government has never defaulted on a loan in over 200 years, and its taxing power along with the size of the U.S. economy makes the U.S. very solvent (at least theoretically).\textsuperscript{229} These facts minimize credit risk. The very short term due date, less than one average month, on these U.S. Treasury bills minimizes both expected inflation and market risk.

b. Future growth in a risk neutral world

The formula for expected future growth rate of any economic variable in a risk neutral world is $r = m - \lambda s$.\textsuperscript{230}

i. What formula means in lay terms

Courts that consider a damages element’s growth use its historical growth rate as the best estimate of its future growth rate.\textsuperscript{231} Everyone knows, however, that past performance does not usually indicate future performance, even roughly. Examples abound. A recent one is the spurt in U.S. housing prices in the late 1990s and early 2000s; during those years, prices grew at a much faster rate than past growth predicted.\textsuperscript{232} Courts can significantly improve their estimates of future growth rate by modifying that rate to adjust for this future uncertainty by using the MPR formula.

ii. For what variables stand

In the formula, the dependent variable, $r$, for which we solve, stands for the expected future growth rate. The dependent variables, $m$ and $s$, respectively stand for the historical growth rate of the variable and the volatility of the variable’s historical growth rate (measured by the variable’s standard deviation). The constant, $\lambda$, is the MPR for the future growth rate.\textsuperscript{233}

\textsuperscript{224} E.g., John C. Hull, Options, Futures, and Other Derivatives 711 (5th ed. 2003).
\textsuperscript{225} Subsection V(E).
\textsuperscript{226} E.g., John C. Hull, Options, Futures, and Other Derivatives 45 (5th ed. 2003).
\textsuperscript{227} E.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 186 (5th ed. 2002).
\textsuperscript{229} See, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 186 (5th ed. 2002).
\textsuperscript{230} The author based this example extensively on a similar example in John C. Hull, Options, Futures, and Other Derivatives, 484 – 486, 661 – 667 (5th ed. 2003).
\textsuperscript{233} John C. Hull, Options, Futures, and Other Derivatives 485 (5th ed. 2003).
iii. Change all other variables to risk neutral state

Risk neutral pricing theory therefore allows lawyers to calculate a risk adjusted growth rate for future damages variables. If the fact finder uses the risk free rate as the discount rate in its calculations, the present value of the judgment adjusts for the future uncertainty of the cash flows the damages represent, as long as the fact finder also revalues the cash flows themselves for risk neutrality.  

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1. Formula adjusts actual historical growth rate to risk neutral growth rate

The formula does this by adjusting the actual historical growth rate to the growth rate in a risk neutral world.  

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Financial analysts measure risk (which is nothing more than future uncertainty) with past volatility.  

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They believe the best estimate of future variance is the past variance (or more precisely, the standard deviation, the variance’s square root).  

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d. By adjusting for market price of risk

i. MPR

MPR expresses a value that investors as a whole put on some financial variable (e.g. wages, prices, business profits), based on that variable’s past volatility and correlation with changes in the value of the broad stock market.  

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The MPR is the value the market gives to the volatility in the growth rate of any financial variable.  

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Investors, being risk averse, value any given growth rate higher if the rate is less volatile historically.  

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For example, if the overall historical rate of return on two investments (e.g. two particular company’s stocks) has been, say, five percent annually over the past eighty years, investors will prefer (i.e., pay more for) the stock that has the lower volatility in its growth rate.  

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For example, if both stock A and stock B have average annual growth rates of five percent, but stock A’s annual return is exactly five percent for each year, while stock B’s return is ten percent and zero percent in alternating years, A’s volatility is lower. This means that A is less risky than B, and investors will prefer stock A to stock B, ceteris paribus. Not all volatility is the same; the more closely an investment’s growth correlates with the value of the stock market, the greater the investment’s MPR.  

242

ii. Subtracts product of historical volatility and MPR

238 See, e.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 484 – 486 (5th ed. 2003).
239 See, e.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 665 (5th ed. 2003).
241 See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 137 (5th ed. 2002).
242 E.g. JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 665 (5th ed. 2003).
The formula multiplies the variable’s MPR times its volatility, then subtracts that amount from the variable’s historical growth rate. As explained above, modern courts already use the historical growth rate of a damages variable (lost profit, lost wages, etc.) to predict its future growth. Using MPR improves that estimate of future growth by adjusting the historical growth for the growth’s future uncertainty, based on how the markets value this uncertainty. This is a small and easy additional step for trial courts that provides a great leap forward in the economic accuracy of damages calculations.

If, for example, the growth rate of similar businesses has been extremely volatile, using the MPR formula to adjust historical growth will, ceteris paribus, reduce the expected future growth (because past growth has been widely varied, i.e., uncertain). If past growth has been consistent (not volatile, as reflected by low variance of historic growth), then the MPR formula (ceteris paribus) will increase the expected future growth rate (to reflect its increased certainty).

If past growth is highly correlated, ceteris paribus, to growth in the general market for assets (as estimated by the broad stock market), the formula lowers expected future growth, because the future growth is not as valuable (as diversified investors can easily replicate the variable’s future growth with other investments). If, on the other hand, the variable’s past growth in value is less correlated with general asset growth (as represented by the broad market’s value growth), the formula increases the historical growth rate because the measured variable is a valuable investment diversification.

e. From where we get numbers

Parties to litigation already offer most of this information into evidence in lost future profits cases; the litigants get the information from publicly available sources (government publications and private research) and often have expert witnesses present it to the fact finder. There are firms dedicated to forensic accounting economics that regularly provide evidence on the following factors in many cases.

i. \( m \) is the historical growth rate in profits of similar businesses

As stated above, courts have for many years used the historical growth rate in the profits of similar local businesses as their sole estimate of the future growth rate of a new business’s lost profits. Using the method of this article, courts will continue to use that rate; it is the variable \( m \) in the formula.

ii. \( s \) is the historical volatility in growth of \( m \)

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243 Subsection V(F)(2)(b)(i).
244 See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 269 (5th ed. 2002).
245 See generally, e.g., JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 661 – 665 (5th ed. 2003).
246 See generally, e.g., DANIEL W. HINDERT, JOSEPH JULNES DEHNEN, & PATRICK J. HINDERT, STRUCTURED SETTLEMENTS AND PERIODIC PAYMENT JUDGMENTS ch. 12 (2005).
248 Subsection V(F)(2)(b)(i).
Courts and litigants have traditionally ignored the volatility in profits’ historical growth rates when estimating future growth. Courts would therefore use, e.g., 20% as the expected future growth rate, regardless of whether the historical volatility of that growth were two percent or 80%. Using that historical volatility to compensate for future uncertainty is the central improvement in calculating future damages this article suggests.

iii. MPR

The only remaining unknown to determine in the MPR itself.

(1) Estimate using the CAPM

Financial analysts use the CAPM to estimate the market price of the risk inherent in investing in any asset.

(2) CAPM is a linear regression on broad stock market return

Linear regression is a simple statistical technique taught in introductory college mathematics courses. Regression, which modern computer spreadsheet programs perform quickly and easily, compares changes in one variable with changes in another variable, and returns the correlation between the changes. The CAPM is just a linear regression, in which the variables compared are the investment return of one particular stock and the investment return of the entire stock market.

(a) CAPM formula for MPR is \( \lambda = \rho / \sigma (\mu - f) \)

The simplified CAPM formula for to determine MPR is \( \lambda = \rho / \sigma (\mu - f) \). The variables stand for:

(i) \( \lambda \) is MPR

\( \lambda \) is the value for which we are solving.

(ii) \( \rho \) is historical correlation between profit growth and stock market return

For our purposes, it is not necessary to perform a full regression on the data; a simple correlation analysis suffices. This information is easily available from libraries, databases, government websites and publications, etc. The correlation coefficient is a number that represents the relationship between historical changes in the growth rate of similar local businesses’ profits and the historical growth rate of the broad stock market.

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250 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 967 – 970 (5th ed. 2002).
251 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 967 – 970 (5th ed. 2002).
252 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 297 (5th ed. 2002).
253 John C. Hull, Options, Futures, and Other Derivatives 665 (5th ed. 2003).
254 John C. Hull, Options, Futures, and Other Derivatives 665 (5th ed. 2003).
255 E.g., http://www.census.gov/compendia/statab/ (last checked 27 July 2012).
256 See generally, e.g., Zvi Bodie, Alex Kane, & Alan J. Marcus, Investments 965 – 967 (5th ed. 2002).
If, for example, the stock market grew an average of ten percent per year over the period considered (e.g., 20 years), and the similar local businesses’ profits grew an average of 20% per year over the same period, the correlation is two (20% divided by ten percent). If instead the local businesses’ profits shrank five percent annually on average over the period, the correlation is -0.5 (negative five percent divided by ten percent). As the text above explains, a financial variable’s degree of correlation with the broad market significantly affects the variable’s value.

(iii) \( \mu \) is the historical return on the stock market

This is the average annual rate of past growth in the value of the stock market. Financiers and analysts use this as a proxy for the value growth rate of all the financial assets in the macroeconomy.

(iv) \( \sigma \) is historical volatility of stock market return

This is the standard deviation in the historical growth of \( \mu \), above.

(v) \( f \) is the risk free rate of interest, estimated by U.S. Treasury bills

As the text above regarding interest rates explains, financial analysts use the yield for current short term U.S. Treasury bills as their estimate of the theoretical risk free interest rate.

(b) Substitute for variables and calculate

Once litigants have presented their evidence and arguments regarding the correct numbers to use for the various variables in the formula, the fact finder can choose what it believes are the best numbers for each, then can substitute those values into the formulas to determine the risk (i.e., uncertainty) adjusted expected future annual growth rate of the business’s profits.

(c) First use CAPM to calculate MPR

The first step is to use the simplified CAPM formula to calculate the MPR for the similar businesses’ profits.

(i) If historical volatility of stock market return is, e.g., 20%.

The historical volatility of the stock market return is the standard deviation of the annual return for the period considered. (As explained above, it is mathematically it is the square root

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257 Subsection VI(F)(2)(d)(2).
259 See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 294 (5th ed. 2002).
261 Subsection VI(F)(2)(a)(iv).
262 Subsection VI(F)(2)(c).
of the average squared annual deviation from the mean annual return for the period.) For example, a typical volatility might be 20%. This is the variable $\mu$ in the formula.

(ii) Stock market excess return is 5%

The “excess” growth rate of any financial variable, as explained above,\(^{263}\) is the variable’s nominal growth rate minus the risk free rate. If, for example, at the time of litigation, the risk free rate is three percent and the stock market’s average annual growth rate is eight percent, then the stock market’s excess growth rate (i.e., $\mu$ minus $f$) is five percent.

(iii) And the correlation between profit growth and market is 0.3

This is the correlation coefficient between the similar businesses’ annual growth rate and the stock market’s annual growth rate over the period considered.\(^{264}\) Longer periods are better, as they contain more data (i.e., longer periods give a larger sample size, which statistically reduces uncertainty).\(^{265}\) This number is $\rho$ in the formula.

(iv) Then the formula gives us 7.5% as MPR

\[
(0.3/0.2) \times 0.05 = 0.075 = 7.5\% = \lambda
\]

Those three values (correlation, excess return, and volatility) are all that a fact finder needs to calculate MPR. By plugging the values into the MPR formula, the fact finder concludes that the MPR for similar local businesses’ profits is 7.5%.

(d) Next step: use MPR to calculate risk neutral profit growth rate

Now that the fact finder knows the relevant MPR, the fact finder can use the second formula to adjust similar businesses’ historical profit growth by those profits’ MPR, predicting the risk neutral future growth by adjusting for uncertainty.

(i) If average historical annual growth rate of profits is 15%

Assume that historical data evidence shows that similar businesses grew over the relevant period at an annual rate of 15%. Courts have traditionally done only this part of the analysis, and simply used this figure (15% in this example) to estimate future growth, without adjusting for future uncertainty.\(^{266}\) This number is $m$ in the second formula.

(ii) With 30% volatility in that growth

Any historical mean (average) has its standard deviation (variation). If similar businesses’ profits grew at an average of 15% over the past twenty years, the growth could have been steady (i.e. in a narrow range of, say, 13 to 17%) or volatile, changing greatly from one year

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\(^{263}\) Subsection IV(B)(3)(a)(ii).

\(^{264}\) E.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 166 (5th ed. 2002).

\(^{265}\) See, e.g., ZVI BODIE, ALEX KANE, & ALAN J. MARCUS, INVESTMENTS 962 (5th ed. 2002).

to the next (e.g. a wide range of annual amounts over the period, say 0% to 30%). As explained above, the standard deviation is the square root of the average squared deviation of each annual growth figure from the average annual growth. For our example, say historical data show the standard deviation of the similar businesses’ profit growth over the relevant period to be 30%.

(iii) Then future risk neutral growth rate is 12.75%

\[ 0.15 - 7.5\% \times 0.3 = 12.75\% = r \]

That is all the information a fact finder needs to calculate the risk neutral (i.e. uncertainty adjusted) expected future growth rate for the plaintiff’s failed business. The rate is the actual historical growth rate of comparable business (15% in this example) minus the historical volatility of the growth rate (30%) times the MPR for the profit growth (7.5%). The arithmetic result is 12.75%, significantly lower on these (hypothetical but realistic) numbers than the unadjusted historical growth rate.

(e) Next calculate PV

The rest of the calculation is something courts have done for decades when determining future damages (including in lost profits cases). The only change this article proposes is to the rates the courts will use. Instead of the unadjusted historical profit growth rate, the fact finder will use that rate as adjusted by the MPR for the historical growth rate’s volatility; then the fact finder will use the risk free rate as the discount rate, instead of whatever other discount rate the courts normally would have used.

(i) Gross up profits at 12.75%

The fact finder takes the estimated current profits and grosses it up for each year in some definite future period (e.g., 20 years, not as a perpetuity). If the current profit is, say, one million dollars, the fact finder will multiply that amount times the calculated gross interest rate (i.e., the interest rate plus one, e.g. 1.05 for a net interest rate of five percent). In this case, the growth rate is 12.75%, so for the first future year the fact finder’s calculation is $1,000,000 \times 1.1275^1 = $1,127,500. For the second year, the calculation is $1,000,000 \times 1.1275^2 = $1,271,256. For the third year, the calculation is $1,000,000 \times 1.1275^3 = $1,433,341, and so on. For the final year, the calculation is $1,000,000 \times 1.1275^{20} = $11,023,792.

(ii) Then discount them at risk free rate

After the fact finder has grossed up the current year’s lost profits with the risk neutral growth rate it calculated, the fact finder discounts those future predicted cash flows at the risk free rate. In our example, that rate is three percent annually. The calculation for the first cash flow is therefore $1,127,500 \div 1.03^1 = $1,094,660. This is the (risk neutral) PV of the business’s profit one year from now (in a risk neutral world). For the second future year’s lost profits, the

267 Subsection VI(F)(2)(c).
268 See generally, e.g., Jones & Laughlin Steel Corp. v Pfeifer, 462 U.S. 423 (1983).
PV discount calculation is $1,271,256 / 1.03^2 = $1,198,280, and so on. The final future year’s lost profit calculation is $11,023,792 / 1.03^{20} = $6,103,606. See Table Four below, a spreadsheet containing all the calculations.

### TABLE FOUR

<table>
<thead>
<tr>
<th>Year</th>
<th>Present value</th>
<th>Growth at 12.75%</th>
<th>Discounted at 3%</th>
<th>Growth at 9.75%</th>
<th>Growth at 15%</th>
<th>Discounted at 10%</th>
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<td>$1,000,000.00</td>
<td>$1,000,000.00</td>
<td>$1,000,000.00</td>
<td>$1,000,000.00</td>
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<td>$1,321,945.61</td>
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<td><strong>$89,641,768.64</strong></td>
<td><strong>$60,018,627.40</strong></td>
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<td><strong>$118,810,120.01</strong></td>
<td><strong>$33,953,974.41</strong></td>
</tr>
</tbody>
</table>

(iii) Comes to about $60 million

The final judgment amount, which represents the present value (at the risk free discount rate) of the plaintiff business’s lost future profits (with expected growth at the uncertainty adjusted risk neutral rate), is about 60 million dollars on these numbers.

It is also possible to simplify the calculation further, by merely grossing up the amounts by the difference in the gross up and discount rates. In our example the difference is 12.75% - 3.00% = 9.75%. While the PV does not come to precisely the same amount, it is very close (about 104% of the more precise amount). Some courts have expressed that simplification is worth the loss of precision in future damages calculations, not only to make it easier on the fact finder, but also because simply calculated amounts are less uncertain.269

(iv) Vs. unadjusted calculations

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Note how the award is triple the amount courts would currently award on these facts, using a perpetuity formula ($20,000,000), and how it is almost double the amount using the unadjusted growth rate (15%) and the ten percent legal rate as the discount rate.

(v) Would not always favor defendant; depends on facts

The outcome in this particular example does not mean that using the uncertainty adjusted method this article describes would always result in an outcome much more favorable to the plaintiff (and conversely unfavorable to the defendant). It is purely a function of the various inputs this example uses, which are realistic, but far from fixed in any sense. The method this article teaches is, however, vastly economically superior, and as damages are an extremely common and economics based remedy, improving damages calculations will improve the economy and therefore society as well.

The reason why this example turns out to favor the plaintiff significantly more than the old fashioned method of not adjusting the growth rate is because the profit growth rate of the new business does not correlate much with the broad market growth. This makes good logical sense; if the historical growth in profits were highly correlated with the stock market’s historical return (say, a correlation of more than 0.7), the MPR of the profit growth would be high.

This is because the business would be a poorer investment; its profit growth is highly correlated with the stock market. It is easier for hypothetical investors merely to buy stocks; the business is not a good diversification opportunity. In the actual case, the business is a good investment because its profit growth rate is not easy for investors to duplicate merely by buying the broad market. The new business is therefore one that is economically efficient, one which society wants to encourage. The defendant in our example, by its tort or breach, terminated this desirable new business.

G. Many other potential applications

Litigants and courts can use the MPR adjustment for future uncertainty in many future damages calculations besides the problem of businesses’ profits.

I. E.g. torts

As the text above describes, tort litigation is rife with future damages problems. A permanently injured plaintiff will likely have additional expenses (e.g. medical) and lost income (e.g. wages) far into the future and long beyond the time of trial. Courts have traditionally dealt with these damages items in much the same way they have with business profits, by either denying them as uncertain, or predicting future growth with past growth alone.

a. Future (lost) wages

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270 See generally, e.g., DAN B. DOBBS, LAW OF REMEDIES: DAMAGES – EQUITY – RESTITUTION § 3.1 (2d ed. 1993).
271 Subsection VI(E).
If a plaintiff earns wages at the time of her injury, it is usually easy enough for her to prove her lost current wages. Wages, however, just like profits, grow over time. Courts can estimate future growth as a continuation of average past growth, but courts will come to predictions that are more accurate if the courts value and factor in the uncertainty in future growth by using the MPR process this article describes.

b. Future (additional) medical payments

As the text above regarding the Drayton case demonstrated, permanently injured plaintiffs also face future medical expenses they would not have incurred but for the injury. Litigants can use the MPR formula to adjust for the uncertainty in the growth rates of these expenses as well.

2. Contracts

Many typical elements of contract breach damages, such as lost revenues, additional incurred expenses, etc., are tied to growth in financial variables as much as profits, medical expenses, and lost wages. This article’s method works just as well for those damages elements.

VII. Conclusion

Quantitative finance, like economics before it, provides many practical legal applications. In particular, there are two initial simple yet detailed applications of modern finance to law: the consideration of all four interest rate factors in calculating discount rates for judgments, and the use of MPR to account for the uncertainty inherent in future damages calculations.

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272 Subsection VI(E).