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Game Theory, A Foundation for Agricultural Economics

Matt Bogard



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GAME THEORY: A Foundation for Agricultural Economics

By: Matt Bogard

Submitted in partial fulfillment of AGRI 597 Independent Study/Special Problems in Agriculture.

Advisor: Dr. Alvin Bedel, Ph.D. Department of Agriculture.

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PREFACE

Throughout my undergraduate experience topics from Game Theory were scarcely covered. In preparation for this reference I did not find one definitive text that gave intermediate or even basic coverage of Game Theory. I found many intermediate and graduate level texts that devoted individual chapters and sections of chapters on the topic. However, except for the advanced coverage given in Green, none of these chapters consisted of enough material to teach an entire course. My attempt in developing this reference was to provide an outline of details that summarizes major topics in Game Theory as well as a few examples and applications.

The purpose of this course packet is to provide an introduction to Game Theory for upper level undergraduates and graduate students in Agricultural Economics. Game theory is often part of a normal course sequence in microeconomics for first year graduate students in Economics. Traditional microeconomic theory and equilibrium analysis emphasizes rational self interested decision making in a setting of perfect competition, where only the collective actions of many individuals affect equilibrium outcomes. Game Theory focuses on individual decision making among groups of individuals where one person's decision can affect the well being of others.

The first two sections of the packet cover terminology and the structure of games. The third section emphasizes mathematical notation with proofs. This section follows the notation of Green with my personal interpretations and explanations. While this section could be omitted with continuity, I think that becoming used to the notation would be very helpful for graduate students or undergraduates intending to pursue advanced degrees.

The fourth section re-visits the Prisoner's Dilemma and discusses advanced topics that deal with this very popular game. This penultimate section bridges the more abstract topics covered in earlier sections with real world applications discussed in the final section.

I. Introduction to Game Theory and Terminology for Games

When a person's well being depends on the choices made by others, it is helpful for the individual to have some sort of means to anticipate the behavior of others. Game Theory provides the tools for doing so (Nicholson, 2002). Game Theory is a mathematical technique developed to study choice under conditions of strategic interaction (Zupan, 1998). It allows for the application of economic principles to interdependent situations.

As portrayed in the movie "A Beautiful Mind," the mathematician and game theorist John Nash makes the comment that "Smith was wrong." The point being that much of Economic Theory and specifically Microeconomic Theory deals with interactions of large numbers of individuals in an environment of perfect competition. In such an environment decision makers make their decisions to maximize their well being subject to given constraints. They take the behavior of others as given i.e. prices are fixed by the market. The welfare maximizing choice of the decision maker has no effect on the decisions of others. Such choice situations are not interdependent. In the movie, Nash was contending that a different analysis was in order in an interdependent environment. In such environment acting in what appears to be in one's own self interest may not lead to a state of efficiency as depicted by traditional equilibrium analysis.

A **game** is therefore a decision-making situation with interdependent behavior between two or more individuals (Harris,1999). The individuals involved in making the decisions are the **players**. The set of possible choices made by the players are **strategies**. The outcomes of choices and strategies played are **payoffs**. Payoffs are often stated as levels of utility, income, profits, or some other stated objective particular to the game. A general assumption in game theory is that players seek the highest payoff attainable, preferring more utility to less (Nicholson,2002).

When a decision maker takes into account how other players will respond to his choices, a utility maximizing strategy may be found. It may allow one to predict in advance the actions, responses, and counter responses of others and then choose optimal strategies (Harris,1999). Such optimal strategies, which leave players with no incentive to change their behavior are **equilibrium strategies**.

Games All Around Us

A 'game' does not have to be some esoteric mathematical abstraction. We often find ourselves in situations of strategic interdependence in the normal course of our lives. In an auction we must guess how much others are willing to pay for an item, knowing that the bid we make will elicit a possible higher bid from others. If our bid is too high we may end up paying a higher price than necessary to secure the item for purchase. The same could be true in bargaining the price of a home.

Checkers, chess, and tic-tac-toe are known to us as games for entertainment but they are indeed very similar to the games encountered in agricultural economics. Think about the 'strategies' you employ when playing these games. Every move you make is done in reaction to or in anticipation to your opponent's moves. Payoffs are measured in terms of captured pieces in chess and checkers, squares in tic-tac-toe, or ultimately winning or losing. The missing component is equilibrium.

With so many possible strategies in these games it is difficult to imagine a unique equilibrium strategy. It is possible that there may be more than one equilibrium strategy. The concept of equilibrium strategies in game theory can be quite elusive in agricultural economics as well.

Major Components of Games:

- Players
- Strategies
- Payoffs
- Equilibrium/Equilibria

II. General Forms of Games and Equilibria Studied in Game Theory

Economists have generalized several forms of games commonly encountered in situations of strategic interdependence. Several concepts of equilibrium have been formalized as well.

Cooperative games are games in which players can make binding agreements such as contracts, or terms for third party enforcement (Nicholson, 2002). The types of games that I will be concerned with are *non-cooperative games* which do not provide for binding agreements.

Equilibrium Strategies in Non-cooperative Games

Different types of equilibria exist in non-cooperative games. In this first example adapted from Zupan, a dominant strategy equilibrium is illustrated.

Dominant Strategy Equilibrium: Output Decisions

In this game two chemical firms contemplate herbicide output decisions. Their two choices or strategies involve high or low output decisions. The interaction of both firms decisions will impact market prices and profits per dealer per quarter in thousands of dollars.

		Firm B	
		Low	High

Firm A	Low	10,20	9,30
	High	20,17	18,25

(Adapted from Zupan, 1998
p. 360)

Solving the Game

Choices and Payoffs for Each Firm:

Firm A

High Output: 20 and 18

Low Output: 10 and 9

No matter what firm B does, firm A should choose high output since payoffs are higher in all instances of high vs. low output. The strategy of high output is a **dominant strategy for firm A**.

Firm B

High Output 30 and 25

Low Output 20 and 17

No matter what firm A does, firm B should choose high output since payoffs are double in all instances of high vs. low output. The strategy of high output is a **dominant strategy for firm B**.

The equilibrium strategy that solves the game is **{high, high}** for {player A, player B} . An equilibrium strategy like this one in which both players have a dominant strategy is referred to as a **dominant strategy equilibrium** (Zupan, 1998).

Nash Equilibrium: Another Output Game

To illustrate a less restrictive equilibrium concept, the output game will be re-examined with a slight variation.

		Firm B	
		Low	High

Firm A	Low	22,20	9,30
	High	20,17	18,22

(Adapted from Zupan, 1998
p. 362)

Solving the Game

Choices and Payoffs for Each Firm:

Firm A

High Output: 20 and 18

Low Output: 22 and 9

This time firm A's payoffs are less certain. The high or low strategy outcome depends on firm B's output.

Firm B

High Output 30 and 22

Low Output 20 and 17

Regardless of what firm A does, firm B's payoffs are always going to be the greatest with a high output strategy. Firm B has a **dominant strategy** in choosing high output. Since both players do not have a dominant strategy, the game cannot be solved by a dominant strategy equilibrium.

If firm A knows about firm B's dominant strategy then firm A will know that firm B will choose high output. In this case firm A should choose high output for a payoff of 18 vs. 9.

The equilibrium set of strategies that solves the game is **{high, high}**.

As will be explained, this equilibrium strategy meets the specifications of what describes a **Nash Equilibrium**. *In a Nash equilibrium each player's choice is the best choice possible taking into consideration the choice of the other players* (Zupan, 1998). This concept was generalized by the mathematician John Nash in 1951 in his paper "Equilibrium Points in n-Person Games."

In this case, given firm B's dominant high output strategy, the best choice for firm A was high output. For player B, since B had a dominant strategy, B's best choice would be high output given any choice by A. This reciprocal relationship of each player making the best choice given the choice of the other player {high, high} is therefore a **Nash Equilibrium**.

The first output game was a special case of a Nash Equilibrium. Since both players had a dominant strategy of high output each one's choice was the best possible given the choice of the other player. Every dominant strategy equilibrium is therefore a Nash Equilibrium. The dominant strategy equilibrium is a particular case of the more general Nash Equilibrium. The Nash Equilibrium

strategy being more general allows one to solve games that may not have dominant strategy equilibrium as in the second output game.

The Prisoner's Dilemma

A classic example in game theory that involves the dominant strategy equilibrium is the Prisoner's dilemma.

	Player A	
Player B	Confess	Don't Confess
Confess	10,10	15,1
Don't Confess	1,15	2,2

(Zupan, 1998)

In this game two criminals are being interrogated. They must decide between confessing or not confessing. Each prisoner's jail time depends upon the joint actions of both prisoners.

Solving the Game

Striving for the least amount of jail time, each prisoner has a dominant strategy of confessing. The solution is therefore {confess, confess}. Note this is both a dominant strategy equilibrium and a Nash Equilibrium.

It can be seen however that if the prisoners could cooperate and agree not to confess, they could get by with less jail time. This is the type of outcome that Nash was referring to in the bar room example in the movie *A Beautiful Mind* when he stated that Smith was wrong. This is an example of self interested rational behavior not leading to a socially optimal result which conflicts with the ideas of the 18th century economist Adam Smith.

Another observation about the solution of this game is that it is not Pareto Optimal. Pareto Optimality is a condition in which no one can be made better off without making someone else worse off. In the case of the prisoner's dilemma, the only solution that is pareto optimal is {don't confess, don't confess}. Playing any other strategy may improve the well being of one player but diminish that of another.

Mixed Strategies-The Battle of the Sexes

Pure Strategy Game

	Player A	
Player B	Mountains	Seaside
Mountains	2,1	0,0
Seaside	0,0	1,2

(Nicholson,2002)

In this game a husband and wife desire two different vacation possibilities. Both prefer a vacation together vs. apart giving a payoff of $\{0,0\}$ for separate vacations. The only two remaining strategies are therefore both players choosing mountains or seaside; $\{\text{mountains, mountains}\}$ or $\{\text{seaside, seaside}\}$. This game therefore has two Nash Equilibria.

In such cases there is no single equilibrium solution. In life when we find ourselves in these situations we may resort to simply flipping a coin and letting fate decide which restaurant to dine in or which movie to see. The act of coin flipping is in itself a strategy. In this case we are 'randomizing' our strategies by appealing to chance or probability. When players randomize in this way, the 'pure' strategies become mixed strategies. Instead of consciously choosing with 100% certainty mountains or seaside (pure strategy) the choice is now made with a probability of 50% (a mixed strategy) as it is based on a coin flip.

In games with mixed strategies, payoffs are defined differently as well. As stated in the beginning, payoffs are often expressed in terms of utility or income. With mixed strategies an element of uncertainty is introduced and payoffs are expressed in terms of **expected utility** or **expected income**. These are based on the probabilities with which they choose their strategies. Utility functions that give expected utility are referred to as Von Neuman Morgenstern utility functions.

Expected utility can be calculated by **multiplying the payoff from all strategies by the probability that that each strategy will be played.**

As probabilities deal with fractions, these expected utility functions will yield payoffs other than those given in the pure strategy game in the figure above. Since we are no longer limited to the strategies and payoffs of a pure game, mixed games can have an infinite number of strategies and payoffs. Maximizing expected utility across all possible strategies may yield more equilibria than the two Nash equilibria above.

In fact if we rely on a coin flip to choose the probability of selecting mountain and seaside locations we characterize the strategy as $\{.5,.5\}$ which has the expected utility of $\{.75,.75\}$.

We calculate as follows:

r = probability A chooses mountains.
 $(1-r)$ = probability A chooses seaside

s = probability B chooses mountains
 $(1-s)$ = probability B chooses seaside

Expected Utility of Player A: = $p(\text{mountains, mountains}) * \text{utility} + p(\text{mountains, seaside}) * \text{utility}$
+ $p(\text{seaside, mountains}) * \text{utility} + p(\text{seaside, seaside}) * \text{utility}$

$$\begin{aligned} E(U_a) &= rs(2) + r(1-s)(0) + (1-r)s(0) + (1-r)(1-s)(1) \\ &= 1-s+r(3s-1) \\ &= 1-.5+.5(3*.5-1) \\ &= .75 \end{aligned}$$

In the same manner we get $E(U_b) = .75$.

Further calculations of expected utility with different levels of probability yield a new Nash Equilibrium with probabilities $\{2/3, 1/3\}$ (Nicholson, 2002).

III. Mathematical Notation for Games and Strategies

Mathematical notation makes it possible to describe games in a very compact way. By describing games and their components mathematically it is possible to draw on mathematical theorems to prove the existence of equilibria in games. I have added descriptive notes to explain the notation used by Green.

Normal Form Representation

All of the previous games discussed have been represented in 'normal form.' In normal form games are represented by

$I = i$ players

S_i = strategies played by player i

U_i = utility derived from each strategy played

$$\Gamma = [I, \{S_i\}, \{U_i(\cdot)\}]$$

Notation for mixed strategy games:

$I = i$ players

$\Delta(S_i)$ = mixed strategies for player i

U_i = utility derived from each strategy played

given the set of pure strategies $S_i = \{s_1, \dots, s_n\}$ the set of mixed strategies is represented by

$\Delta(S_i) = \{ (\sigma_{1i}, \dots, \sigma_{ni}) \in \mathbb{R} : \sum \sigma_{mi} = 1 \}$ - that is probabilities sum to 1, where σ_{1i} is the probability that player i plays strategy 1 with probability σ_{1i} .

Mixed strategy payoffs are represented as follows:

$\sum [\sigma_1(s_1) * \dots * \sigma_I(s_I)] * U(s) = U(\sigma)$ This is the von Neumann Morgenstern utility function. That is the sum of the probabilities of playing a strategy times the utility given by the strategies.

Normal form for a mixed strategy game is then represented by

$\Gamma = [I, \{\Delta(S_i)\}, \{U_i(\cdot)\}]$ recalling from above
that $U_i(\cdot) = U(\sigma)$ and $\Delta(S_i) = (\sigma_{1i}, \dots, \sigma_{mi})$

Strictly Dominant Strategies

A strategy $s_i \in S_i$ is a strictly dominant strategy for player i in game $\Gamma = [I, \{S_i\}, \{U_i(\cdot)\}]$ if for all s'_i not = s_i we have $U_i(s_i, s_{-i}) > U_i(s'_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Where S_{-i} is the strategy profile for the opposing player.

The notation above explains that a dominant strategy $\{s_i\}$ yields higher utility to the player than any other strategy $\{s'_i\}$ regardless of the strategy played by the opposing player $\{s_{-i}\}$.

Nash Equilibrium

A strategy profile $s = (s_1, \dots, s_I)$ constitutes a Nash Equilibrium of game $\Gamma = [I, \{S_i\}, \{U_i(\cdot)\}]$ if for every i ,

$$U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i}) \text{ for all } s'_i \in S_i$$

That is the strategy s_i gives the greatest utility given the opponent plays s_{-i} . *It is the best choice possible given the choice of the other players.*

Mixed Strategy NE

A mixed strategy profile $\sigma = (\sigma_1, \dots, \sigma_I)$ constitutes a Nash Equilibrium of game $\Gamma = [I, \{\Delta(S_i)\}, \{U_i(\cdot)\}]$ if for every $i = 1, \dots, I$

$$U_i(\sigma_i, \sigma_{-i}) \geq U_i(\sigma'_i, \sigma_{-i}) \text{ for all } \sigma'_i \in \Delta(S_i).$$

As long as no player is able to improve his or her well being by changing to a pure strategy, a N.E. results.

Best Response Correspondence

Let $b_i(\cdot)$ be player i 's best response correspondence $b_i : S_{-i} \rightarrow S_i$. The best response correspondence gives the best response s_i for the opponent's strategy s_{-i} .

The strategy profile s_1, \dots, s_I is a N.E. of the game $\Gamma = [I, \{S_i\}, \{U_i(\cdot)\}]$ if and only if $s_i \in b_i(s_{-i})$

Existence of Nash Equilibria

Following the notation introduced above I will present those conditions necessary for guaranteeing that a Nash Equilibrium is obtainable. The reliance of mathematical notation and theorems becomes clear.

The conditions for the existence of Nash Equilibrium hinge on the form of the strategy sets $\{S_i\}$ and the best response correspondence $b(\cdot)$. The concepts of convexity, continuity, and compactness are also important. I obtained the following descriptions from <http://mathworld.wolfram.com/>.

A set in Euclidean space R^n is **convex** set if it contains *all* the line segments connecting any pair of its points. If the set does not contain all the line segments, it is called **concave**. Compact sets are **closed** and **bounded**. A set is **closed** if every point outside S has a neighborhood disjoint from S . A set in R^n is **bounded** if it is contained inside some ball of finite radius.

Lemma A: If the sets s_1, \dots, s_I are non empty, S_i is compact and convex, and $U(\cdot)$ is continuous and quasiconcave in (s_1, \dots, s_I) , then player i 's best response correspondence $b_i(\cdot)$ is nonempty, convex valued, and upper hemicontinuous.

Kakutani Fixed Point Theorem: Every correspondence that maps a compact convex subset of a locally convex space into itself with a closed graph and convex nonempty images has a fixed point.

Prop 1: A N.E. exists in game $\Gamma = [I, \{S_i\}, \{U_i(\cdot)\}]$ if for all $i = 1, \dots, N$

- a) S_i is a non-empty, convex, and compact subset of some Euclidean space R^m .
- b) $U_i(\cdot)$ is continuous in (s_1, \dots, s_I) and quasiconcave in S_i .

Proof:

- (i) Define the correspondence $b: S \rightarrow S$ by $b(s_1, \dots, s_I) = b_1(s_{-1}) * \dots * b_I(s_{-I})$.
- (ii) $b(\cdot)$ is a correspondence from the non-empty, convex, and compact set $S = S_1 * \dots * S_I$ to itself.
- (iii) By **lemma A** $b(\cdot)$ is a nonempty, convex valued, and upper hemicontinuous correspondence.
- (iv) A fixed point therefore exists for this correspondence: the strategy profile $s \in S$ such that $s \in b(s)$.
- (v) The strategies that constitute this fixed point constitute a N.E because $S_i \in b_i(S_{-i})$ for all $i = 1, \dots, I$. (Green, 1995).

Prop2: Every mixed strategy game $\Gamma = [I, \{\Delta(S_i)\}, \{U_i(\cdot)\}]$ in which the sets S_1, \dots, S_I have a finite number of elements has a mixed strategy N.E.

In short, as long as we assume that players may use mixed strategies i.e they randomize their choice of strategies, a N.E. exists.

Proof:

(i) The game $\Gamma = [I, \{\Delta(S_i)\}, \{U_i(\cdot)\}]$ with strategy sets $\{\Delta(S_i)\}$ and payoff functions $\{U_i(\cdot)\}$ satisfies all the assumptions of **Prop 1** and is a direct corollary of that result.

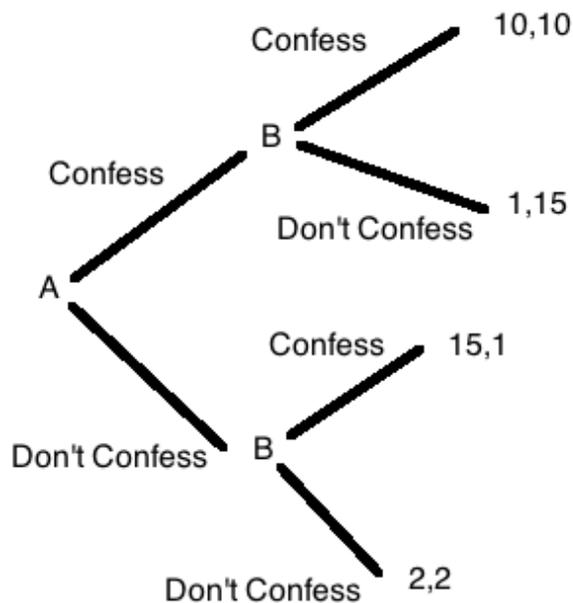
(Green,1995)

IV.Advanced Topics and Extensions

Extensive Game Forms

Games in extensive form are structured like tree diagrams. This structure represents games in a stepwise format. A new branch or *node* represents a decision point in the game. In games with **perfect information** at each node the player is aware of the decision made by his opponent in the previous step. In games with **imperfect information** the player is not aware of his opponent's previous choice.

The prisoner's dilemma can be represented in extensive form as follows:



As can be seen above, games in normal form are represented more compactly than those in extensive form. Games in extensive form differ most in the fact that order of play becomes important.

Nash Equilibrium

Concepts of equilibrium are no different in extensive form games. In the prisoner's dilemma case, it is still apparent from the extensive representation that regardless of what player B does, player A's best strategy is {confess}. The converse is true as well, giving us {confess, confess} for the N.E.

Endgame Reasoning

Endgame reasoning regards the analysis of the final decision in a sequential game. Looking at outcomes at the end of the last branch of a game tree, the best strategy can be determined. This process of determination and reasoning back to earlier decision nodes is referred to as **backward induction** (Harris, 1999).

Sub game Perfect Equilibrium

The determination of this equilibrium involves the use of backward induction to determine the best responses at earlier nodes. Such an equilibrium that results in a Nash Equilibrium can be characterized as a sub game perfect Nash Equilibrium (Harris, 1999).

In other words, backward induction is used to eliminate branches of the decision tree that are not best reply responses. They are **non-credible responses or non-credible threats**. The remaining branches constitute a sub game. The equilibrium in this sub game is referred to as a sub game perfect equilibrium (SPE) (Harris, 1999).

This gives rise to the definition of SPE given by Nicholson:

A Nash Equilibrium in which the strategy choices of each player do not involve non-credible threats.

ESCAPING THE PRISONER'S DILEMMA

It turns out that in many situations there are possibilities of escaping the prisoner's dilemma. As discussed before, the Nash Equilibrium that characterizes the prisoner's dilemma is an example of rational self-interested behavior not leading to a socially optimal outcome. However, recall in the movie *A Beautiful Mind* that after Nash concluded that *Smith was wrong*, he actually proposed an escape from the prisoner's dilemma. He stated that in the dating game example, if each player would go for the second choice date they would all be better off.

This suggestion is what would be termed as cooperation. Just as free markets elicit cooperation through price mechanisms to arrive at a pareto optimal equilibrium, so too must players in a game find some means of coordinating their behavior if they wish to escape the non-optimal equilibrium

Multiple Period Games

Multiple period games are games that are played more than once, or more than one time period. If we could imagine playing the prisoner's dilemma game multiple times (prisoner's are released, paroled and captured multiple times) we would have a multi-period game. If games are played perpetually they are referred to **infinite games** (Harris, 1999).

Punishment Schemes

Punishment schemes are used to elicit cooperation or enforcement of agreements. **Tit-for-tat** punishment mechanisms are schemes in which if one player fails to cooperate the other player will refuse to cooperate in the next period.

In the case of the prisoner's dilemma, suppose both players wanted to cooperate and not confess. If it turned out that player A cheated, then in the next period player B would refuse to cooperate. If the game is played repeatedly, player A would learn that if he sticks to the deal both players could get by with only two years of prison time vs. 15 or 10. In this way punishment schemes in multi-period games can elicit cooperation, allowing an escape from the prisoner's dilemma. This would not be possible in single period games that we looked at before.

Trigger Strategy

In *infinitely* repeated games a trigger strategy involves a promise to play the optimal strategy {not confess, not confess} as long as the other player(s) follow(s) suit (Nicholson, 2002).

Grim Trigger Strategy

This is a trigger strategy that involves punishment for many periods if the other player does not follow suit. In other words if player A confesses when he should cooperate and not confess, player B will not offer player A the chance to cooperate again for a long time. As a result both players will be confined to the N.E. of the prisoner's dilemma for many periods or perpetually (Harris, 1999).

Trembling Hand Trigger Strategy

This is a trigger strategy that allows for mistakes. Suppose in the first instance player A does not realize that player B is willing to cooperate. Instead of player B resorting to a long period of punishment as in the *grim trigger strategy*, player B allows player A second chance to cooperate. It may be the case that instead of playing the *grim trigger strategy*, player B may invoke a single period *tit-for-tat* punishment scheme in hopes to elicit cooperation in later periods.

Folk Theorems

Folk theorems result from the conclusion that players can escape the outcome of a prisoner's dilemma if games are played repeatedly, or are infinite period games (Nicholson, 2002).

In general, folk theorems state that players will find it in their best interest to maintain trigger strategies in infinitely repeated games.

V. Applications in Agricultural Economics

A. Applications from Literature

Competition for Groundwater

At the 2002 Annual Meeting of the American Agricultural Economics Association Nakao presents a paper using game theory to analyze competition for ground water between the border towns of El Paso Texas and Ciudad Juarez, Mexico. The problem occurs with over extraction of water from the shared Hueco Bolson Aquifer. With no cooperation and no exclusive right to the water, one city's use of water has a diminishing effect on the others. This diminishing effect is referred to as an *externality*. (See discussion below). This interdependent relationship allows for water usage to be analyzed using game theory.

The net benefits of water use were described by the following function:

$$U = \sum p_t NB_{i,t} (h_t, w_{i,t})$$

Where U is the utility of water consumption of city (i) and p is the discount rate, h is the pumping cost, and w is the value of water withdrawn in dollars per acre foot.

The diminishing effect of increased water use on pumping cost is given by the function:

$$h_{t+1} = h_t + \alpha (w_{1,t} + w_{2,t}) - \beta$$

Maximizing U subject to h_{t+1} forces both cities to take into consideration the costs imposed on each other from over consumption from the aquifer. Incorporating the constraint of h_{t+1} into the analysis implies that the optimal solution will give the usage rate that is best for El Paso given the consumption of Ciudad Juarez. The reverse will also be true. This symmetry matches that characteristic to a Nash Equilibrium discussed previously.

With the use of a recursive algorithm and dynamic programming using Maple mathematical software a sub-perfect Nash Equilibrium was computed for many different scenarios and discount rates using backwards induction.

Organic Food

In the Journal of Agricultural and Resource Economics, McCluskey uses game theory to analyze markets for organic food. She classifies organic food as a credence good. This is a good that the consumer has no way of verifying its quality even after consumption without complex analysis. The consumer may choose to buy this good because it is fashionable or seems to be the moral or popular thing to do. Therefore the consumer may get the same utility from consumption regardless

of the quality (McCluskey, 2000).

While the consumer must make a choice about the consumption of organic food the producer has to choose among options of producing organic food. Since organic food may be more costly to produce and the consumer is assumed to be indifferent about verification, it is in the best interest of the producer to claim that his food is organic, while actually producing non-organic food. This analysis indicates that the market for organic food will not function properly without some outside coordination. This is especially true if we assume that the consumer is actually interested in genuine organic food products. The paper concludes by proposing a solution in the form of third party labeling and certification.

B. Anecdotal Observations

The advanced topics in the previous sections refer to means by which individuals caught in a situation of strategic interaction attempt to manage the outcomes of a prisoner's dilemma game. It is in these situations that we begin to see how game theory can apply to the real world.

Price Fixing

Instead of engaging in the costly punishment schemes and trigger strategies, players sometimes try to form cooperative agreements. Sometimes contracts and pricing agreements are made between competitors of an industry. Unfortunately this often leads to *price fixing* or violations of federal *antitrust law*. In 1996 Agri-business giant Archer Daniels Midland paid \$100 million in fines due to attempts to fix prices of its lysine and citric acid products.

Externalities-When Folk Theorems Fail

In the 1968 issue of Science, the concept of a *Tragedy of the Commons* was first introduced. It referred to the problems of overgrazing publicly owned pastures. Why would individual owners, say beef producers overgraze publicly owned land? The use of game theory in the context of a Prisoners Dilemma may be helpful in explaining this.

	Player A	
Player B	4 head/acre	5 head/acre
4 head/acre	5000,5000	5500,3000
5 head/acre	2000,5500	3000,3000

Suppose two beef producers are grazing cattle on public land. Suppose there is an optimal stocking rate of 4 head per acre, where anything exceeding this limit would be beyond the grazing tolerance of the forage resulting in lower quality and quantity of forage consumption and rate of gain. Let the payoffs to the players be forage consumption per acre in pounds. Typical of a prisoner's dilemma, a N.E. exists at the stocking rate of {5,5}. This rate is detrimental to both producers. As in previous cases, if they could devise some means of cooperation or communication they may be able to escape the prisoner's dilemma and achieve the optimal solution {4,4}.

The negative effect of overstocking (5 head/acre) on the other party is referred to as an **externality**. Most instances of pollution and resource depletion are analyzed in the context of externalities and the means of internalizing or mitigating their negative effects. Externalities exist when one party does not consider the costs his choices impose on another. Externalities are internalized when both parties are forced to bear the full cost of their behavior. This creates a situation of interdependence in decision making. You may conclude from this example that game theory is often useful in analyzing these situations. It turns out that the prisoner's dilemma is a good model and that the means that we discuss in escaping the prisoner's dilemma are often identical to those proposed for internalizing externalities.

One could treat this like a infinite period game and employ trigger strategies and folk theorems, but it is likely that continued overgrazing would deplete the resource to the extent that the game would become a finite multi-period game. Under conditions of uncertainty every period in this game could be anticipated to be the last period and therefore unravel into a single period prisoner's dilemma.

In cases where games are not infinite, folk theorems will not apply and other means of cooperation need to be employed. One option would be regulation. The government could issue permits for grazing that would limit the stocking rate for each rancher or a per head tax. If the tax is set at the correct amount, each rancher would be forced to consider the costs he is imposing on the other (he must pay for each additional unit of cattle grazing) and opt for a stocking rate of {4,4}.

Another solution would be to assign property rights to grazing. Allowing ranchers to bid against one another for the rights to graze the land would force each one to consider their opportunity costs of overgrazing. If each rancher had exclusive rights to graze the area of land that they bid for, they would have an incentive to maximize yield by maintaining a sustainable stocking rate since doing otherwise would actually decrease their yields and beef production gains.

In his 1960 Journal of Law and Economics article *The Problem of Social Cost* Ronald Coase discussed the approaches of property rights, taxes, and government regulations as mechanisms for internalizing externalities. In the 1967 American Economic Review, Harold Demsetz in his article *Toward a Theory of Property Rights* discusses how Native Americans in the northeast adopted systems of property rights to prevent the externality of over hunting beaver. While neither of these two economists used a game theoretic approach, it can be seen that their solutions would serve as a means of escaping the prisoner's dilemma.

Bt Corn and Regulation

Many students of agriculture would be familiar with Bt corn production. Bt corn is corn that has been genetically modified to produce a crystalline protein that is toxic to corn borer. European corn borer causes substantial losses to corn crops. The use of Bt technology can offer substantial savings to producers as the chart below indicates.

Average Number of Borers/ plant ²	CORN PRICE per BU			
	\$1.50	\$2.00	\$2.50	\$3.00
0.00	(\$4.55)	(\$4.55)	(\$4.55)	(\$4.55)
0.25	(\$1.99)	(\$1.13)	(\$0.27)	\$0.58
0.50	\$0.58	\$2.29	\$4.00	\$5.71
1.00	\$5.71	\$9.13	\$12.55	\$15.97
1.50	\$10.84	\$15.97	\$21.10	\$26.23
2.00	\$15.97	\$22.81	\$29.65	\$36.49

¹This table assumes a yield potential of 144 bu per acre; Bt corn costs \$14 extra per bag; a seeding rate of 26,000/acre; each borer per plant reduces yield by 5%; and Bt corn provides 95% control of corn borers.
²The number of corn borers that would complete development in a non-Bt hybrid.

Source: University of Kentucky College of Agriculture Department of Entomology

Just like in the grazing example, the overuse of Bt technology creates externalities and creates a situation of strategic interdependence. The externality is in the form of corn borer resistance to the Bt toxin. If resistant members of the population thrive then the benefits of the technology are lost. As a result this situation may be analyzed as a prisoner’s dilemma.

The above study concluded that yield advantages of planting Bt corn could be as much as 17 bushels per acre. Using this as a starting point I made the following assumptions for the sake of simplicity. Considering a maximum of 144 bushels per acre yield with 100% of acreage planted in Bt corn, the externality is exhibited in the low 127 bushels that the other producer may incur due to resistance build up. If both planted only 80% (a 20% hold back for conventional corn) of their acreage in Bt corn, resistance would be less, but yield would not be the maximum 144 because the 20% in conventional varieties would bring down the average yield.

While these numbers are hypothetical, it can be seen that without coordination or cooperation both producers would achieve a N.E. of { 100%, 100% }

	Player A	
Player B	80% Bt Acres	100% Bt Acres
80% Bt Acres	140,140	144,127
100% Bt Acres	127,144	130,130

In this case per acreage taxes may achieve the result of cooperation forcing each producer to consider the cost of corn borer resistance in his production plans. The actual solution however has been achieved by means of government regulation. It is required that producers planting Bt corn

varieties must plant at least 20% of their total corn acreage as a *refuge* of non-Bt corn.

Lobbying and Rent Seeking

When the government gets involved in regulating the affairs of business, it creates an incentive for businesses and individuals to lobby for regulations that put them at an advantage. For example if Wal-Mart paid all of its employees \$6.50 per hour while most of its competitors paid their workers the minimum wage of \$5.15 per hour it would be to Wal-Mart's advantage to lobby for an increase in the minimum wage to \$6.00 per hour. This would impose increased labor costs on Wal-Mart's competition and possibly lead to increased profits for Wal-Mart.

In this case it would favor Wal-Mart's competitors to engage in defensive lobbying to prevent an increase in minimum wages or for another regulation that would "level the playing field." These activities are what *Public Choice Economists* refer to as **Rent Seeking**.

Examples of rent seeking in agriculture could be quite numerous. According to Gregory Conko of the Competitive Enterprise institute, agrochemical and biotechnology companies such as Calgene and Monsanto along with the Biotechnology Industry Organization lobbied for a regulatory framework composed of the EPA, USDA, and FDA. It is claimed that companies wanted excessive regulatory requirements to make entry by future competitors too expensive. This would increase their monopoly power.

Other examples would include lobbying for clean air and oxygenate requirements for gasoline. Corn producers and processors would have a strong incentive to support these regulations since ethanol from corn would be a prime substitute for fossil fuel based oxygenates. Likewise oil companies would lobby for a different regulation to offset their loss to the ethanol market.

Just like overgrazing pasture, if everyone engages in rent seeking activities i.e. by lobbying for more regulation of the market place, the net gains in the end can be very questionable. Without some sort of coordination or cooperation this behavior could fit the model of another prisoner's dilemma.

This suggestion poses a dilemma in itself. What if a firm engages in rent seeking as a means to pass a regulation (ex: corn refuge requirements) to escape from a prisoner's dilemma to begin with?

Many economists propose limited government or strict adherence to constitutional constraints on government activities as the only solution for escaping the prisoner's dilemma created by a growing regulatory environment.

Conclusion

Markets function differently when consumer behavior moves away from that characterized by pure competition and becomes interdependent. While perfectly competitive markets are assumed to be coordinated by price mechanisms, situations involving strategic interactions require other mechanisms. The purpose of Game Theory is to analyze these situations to determine how cooperation may or may not be elicited to coordinate consumer behavior.

Game Theory has its own concepts of equilibrium such as the popular Nash Equilibrium. In the case of the Prisoner's Dilemma, the Nash Equilibrium concept does not guarantee an outcome that is pareto optimal. The means of escaping the Prisoner's Dilemma include methods to elicit cooperation and coordination such as punishment schemes, trigger strategies, property rights, and government intervention.

There are many areas of Agriculture that are subject to Game Theory Analysis. Many of these areas include the analysis of externalities and government regulation of the production environment.

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