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An Introduction to Game Theory: Applications in Environmental Economics and Public Choice with Mathematical Appendix

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Abstract:

Game Theory is a mathematical technique developed to study choice under conditions of strategic interaction (Zupan, 1998). In the sections that follow the concept of a game is defined. The Nash Equilibrium is introduced and several applications are given in the areas of agriculture and public choice. Topics in game theory are introduced in the context of the work of Elinor Ostrom, including repeated games, grim trigger, and trembling hand trigger strategies. A mathematical appendix follows which introduces notation for games and motivation for the proof of the existence of a Nash Equilibrium.

When someone else's choices impact you, it helps to have some way to anticipate their behavior. Game Theory provides the tools for doing so (Nicholson, 2002). Game Theory is a mathematical technique developed to study choice under conditions of strategic interaction (Zupan, 1998). It allows for the analysis of interdependent situations.

As portrayed in the movie "A Beautiful Mind," the mathematician John Nash makes the comment that "Smith was wrong." In the movie, Nash was contending that a different analysis was in order in an interdependent environment. In such an environment acting in what appears to be in one's own self interest may not lead to a state of efficiency or pareto optimality (a situation where no person's well being can be improved without harming another) as depicted by traditional microeconomic equilibrium analysis.

In game theory, a **game** is a decision-making situation with interdependent behavior between two or more individuals (Harris,1999). The individuals involved in making the decisions are the **players**. The set of possible choices made by the players are **strategies**. The outcomes of choices and strategies played are **payoffs**. Payoffs are often stated as levels of utility, income, profits, or some other stated objective particular to the game. A general assumption in game theory is that players seek the highest payoff attainable, preferring more utility to less (Nicholson, 2002).

When a decision maker takes into account how other players will respond to his choices, a utility maximizing strategy may be found. It may allow one to predict in advance the actions, responses, and counter responses of others and then choose optimal strategies (Harris, 1999). Such optimal strategies that leave players with no incentive to change their behavior are **equilibrium strategies**.

Example: Cooperate or Defect (chart adapted from The International Encyclopedia of the Social Sciences, 2nd Edition)

Suppose players can choose to cooperate, or defect. The players, strategies, and payoffs are represented below.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

In this game, suppose player 1 chooses a strategy (picks a row). Their payoff is depicted by the first number in each cell. Player 2 will choose a strategy in return (picking a column). Player 2's payoff is indicated by the second number in each cell.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

The 2nd number in each cell = possible payoffs for player 2

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

The 1st number in each cell = possible payoffs for player 1

As depicted below, If player 1 chooses to cooperate, then and the best strategy for player 2 is to defect. The payoff of defecting for player 2 is highest (payoff = 3) when player 1 employs a strategy to cooperate.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,**3**)
	Defect	(3,0)	(1,1)

On the other hand, if player 1 chooses to defect, the best strategy for player 2 also is to defect.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,**1**)

As depicted below, if player 2 picks a strategy first and they choose to cooperate, the best strategy for player 1 is to defect since the payoff of defecting for player 1 is highest (payoff = 3) when player 2 chooses to cooperate.

		Player 2	
Player 1		Cooperate	Defect
	Cooperate	(2,2)	(0,3)
	Defect	(**3**,0)	(1,1)

As shown below, if player 2 chooses to defect, then the best strategy for player 1 is to defect.

		Player 2	
Player 1		Cooperate	Defect
	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(**1**,1)

So, no matter how we play the game, the best strategy for player 2 (no matter what player 1 chooses to do) is to defect. Likewise, no matter what player 2 chooses to do, the best strategy for player 1 is to defect. Both players have a dominant strategy to defect. This represents an equilibrium strategy of {defect, defect}.

		Player 2	
Player 1		Cooperate	Defect
	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	**(1,1)**

This outcome is also described as a prisoner's dilemma or a **Nash Equilibrium.** *In a Nash equilibrium each player's choice is the best choice possible taking into consideration the choice of the other players* (Zupan, 1998). This concept was generalized by the mathematician John Nash in 1951 in his paper "Equilibrium Points in n-Person Games." This is the type of outcome that Nash was referring to in the bar room example in the movie *A Beautiful Mind*.

It's easy to see that if the players would cooperate, they could both be made better off because the strategy {cooperate,cooperate} yields payoffs (2,2) which are much higher than the Nash Equilibrium strategy's payoff of (1,1).

ESCAPING THE NASH EQUILIBRIUM

It turns out that in many situations there are possibilities of escaping a Nash Equilibrium. As discussed before a Nash Equilibrium that characterizes the prisoner's dilemma is an example of rational self interested behavior leading to a less than socially optimal outcome. However, recall in the movie *A Beautiful Mind*, Nash actually proposed an escape. He stated that in the dating game example, if each player would go for the second choice date they would all be better off.

Just as competitive market forces elicit cooperation by coordinating behavior through price mechanisms, so too must players in a game find some means of coordinating their behavior if they wish to escape the sub-optimal Nash Equilibrium. After discussing a few game theoretic concepts we will look at some applications and explore how individuals may coordinate their behavior to escape a Nash Equilibrium.

ADDITIONAL CONCEPTS

Multiple Period Games

Multiple period games are games that are played more than once, or more than one time period. If we could imagine playing the prisoner's dilemma game multiple times we would have a multi-period game. If games are played perpetually they are referred to **infinite games** (Harris, 1999).

Punishment Schemes

Punishment schemes are used to elicit cooperation or enforcement of agreements. **Tit-for-tat** punishment mechanisms are schemes in which if one player fails to cooperate, the other player will refuse to cooperate in the next period.

In the game presented above, suppose both players wanted to cooperate and not defect. If it turned out that player 2 cheated (defected), then in the next period player 1 would refuse to cooperate. If the game is played repeatedly, player 2 would learn that if he sticks to the deal both players would be better off. In this way punishment schemes in multi-period games can elicit cooperation, allowing an escape from a Nash Equilibrium. This may not be possible in the single period games that we looked at before.

Trigger Strategy

In *infinitely* repeated games a trigger strategy involves a promise to play the optimal strategy {cooperate, cooperate} as long as the other players comply (Nicholson, 2002).

Grim Trigger Strategy

This is a trigger strategy that involves punishment for many periods if the other player does not cooperate. In other words if player 2 defects when he should cooperate, player 1 will not offer player 2 the chance to cooperate again for a long time. As a result both players will be confined to a N.E. for many periods or perpetually (Harris, 1999).

Trembling Hand Trigger Strategy

This is a trigger strategy that allows for mistakes. Suppose in the first instance player 1 does not realize that player 2 is willing to cooperate. Instead of player 1 resorting to a long period of punishment as in the *grim trigger strategy*, player 1 allows player 2 a second chance to cooperate. It may be the case that instead of playing the *grim trigger strategy*, player 1 may invoke a single period *tit-for-tat* punishment scheme in hopes to elicit cooperation in later periods.

Folk Theorems

Folk theorems result from the conclusion that players can escape the outcome of a Nash Equilibrium if games are played repeatedly, or are infinite period games (Nicholson,2002).

In general, folk theorems state that players will find it in their best interest to maintain trigger strategies in infinitely repeated games.

APPLICATIONS:

A Single Period Game for Lobbying/Rent Seeking

In the article 'Bootleggers and Biotechs' from the 2003 Summer issue of *Regulation* Henry Miller and Gregory Conko state the following:

"From the claims of opponents of the new biotechnology, it would be easy to conclude that the biotech Industry has vigorously fought government efforts to regulate its products. But in fact, the industry has been anything but a consistent opponent of extensive, and even unnecessary, regulation. It has lobbied for protectionism of various sorts — including public policy that makes regulatory costs excessive –…Since the 1980s, large biotech companies like Monsanto, Ciba-Geigy (now Syngenta), and Pioneer Hi-Bred International (now owned by DuPont), along with their trade associations, have actively and aggressively lobbied in favor of certain major regulatory or legislative initiatives that often are more restrictive even than those sought by regulators themselves."

We can depict the choice to lobby as a game as shown below.

		Monsanto	
		Don't Lobby	Lobby
DuPont	Don't Lobby	(200,200)	(0,300)
	Lobby	(300,0)	(100,100)

In this game, both DuPont and Monsanto have a dominant strategy to lobby for increased government regulation of their industry. It makes sense that if Dupont chooses to lobby, and Monsanto does not, new regulations sought by Dupont could put Monsanto at a competitive disadvantage leading to lower payoffs for Monsanto. Perhaps if both companies lobby for increased regulation, both can carve out some sort of competitive advantage while also protecting themselves to some extent from the lobbying efforts of others. However, there are opportunity costs to lobbying. Expending resources to gain advantages through government intervention is what economists refer to as *rent* seeking. It may be the case that if both would abstain from lobbying, those resources could be devoted to R&D efforts that would actually yield greater returns, improved marketing to consumers, or other benefits leading to higher payoffs for both players.

In another article from Regulation (see Lemieux, 2004) public choice economist Dennis Mueller is quoted:

"The larger the state and the more benefits it can confer, the more rent-seeking will occur. The entire federal budget...can be viewed as a gigantic rent up for grabs for those who can exert the most political muscle."

In The Wealth of Nations, Adam Smith argues:

"People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices. It is impossible indeed to prevent such meetings, by any law which either could be executed, or would be consistent with liberty and justice. But though the law cannot hinder people of the same trade from sometimes assembling together, it ought to do nothing to facilitate such assemblies; much less to render them necessary."

Considering these observations, one method of escaping the Nash Equilibrium strategy of rent seeking would involve limiting the power of the federal government to grant such favors through regulatory handouts, bailouts, and subsidies.

<u>A Single Period Model for Pasture Management and Environmental Degradation</u>

In the 1968 Science article the Tragedy of the Commons, Garrett Hardin proposes a dilemma referred to since as *the tragedy of the commons*. This situation can be depicted as a game with a Nash Equilibrium solution that results in overgrazing.

"Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another; and another... But this is the conclusion reached by each and every rational herdsman sharing a commons. Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit--in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom in a commons brings ruin to all."

		Rancher 2	
		Conserve	Overgraze
Rancher 1	Conserve	(20,20)	(0,30)
	Overgraze	(30,0)	(10,10)

In the game depicted above, we can see that both ranchers have a dominant strategy to overgraze, leading to a Nash equilibrium strategy. {overgraze,overgraze}. Hardin goes on to explain that the tragedy of the commons could also be extended to include other environmental issues like pollution and overpopulation. Hardin concludes that one approach to this dilemma may be the assignment of property rights, but that that strategy may not apply in all cases due to technological limitations:

"The tragedy of the commons as a food basket is averted by private property, or something formally like it. But the air and waters surrounding us cannot readily be fenced, and so the tragedy of the commons as a cesspool must be prevented by different means, by coercive laws or taxing devices that make it cheaper for the polluter to treat his pollutants than to discharge them untreated. "

However, research by Elinor Ostrom has taken exception to the typical conclusion that commons problems must be solved by private property, punitive taxes, or regulation, and has won the Nobel Prize in Economics for her work related to governing the commons. In a 1998 article published in the American Political Science Review, she states:

"The Tragedy of the Commons is based on an assumption that rational individuals are helplessly trapped in social dilemmas from which they cannot extract themselves without inducement or sanctions applied from the outside. Many policies based on this assumption have been subject to major failure and have exacerbated the very problems they were intended to ameliorate."

Ostrom argued that individuals can solve the problems posed by social dilemmas (often represented by a Nash Equilibrium) without government intervention and even in cases where transactions costs or technology limit the effectiveness of private property. She proposes that *'reciprocity, reputation, and trust can help to overcome the strong temptations of short run self interest.'* How so? Ostrom argues that if we look beyond single period games and consider the outcomes from repeated games, that tit-for-tat strategies can elicit the social norms and reciprocity necessary to avoid the *'dominant strategies of one-shot and finitely repeated games that yield suboptimal outcomes.'*

One advantage of this game-theoretic solution to social dilemmas is that, consistent with our view of the price mechanism, it allows society to take advantage of 'local knowledge' in crafting a solution:

"Extensive research on how individuals have governed and managed common-pool resources has documented the incredible diversity of rules designed and enforced by participants themselves to change the structure of the underlying social-dilemma. The particular rules adopted by participants vary radically to reflect the local circumstances and the cultural repertoire of acceptable and known rules used generally in a region."

Ostrom concludes her article with implications for policy that are quite different than Hardin:

"If one sees individuals as helpless, then the state is the essential external authority that must solve social dilemmas for everyone. If, however, one assumes individuals can draw on heuristics and norms to solve some problems and create new structural arrangements to solve others, then the image of what a national government might do is somewhat different…National governments are too small to govern the global commons and too big to handle smaller scale problems."

Appendix: Mathematical Notation and Concepts for Games

Mathematical notation makes it possible to describe games in a very compact way. By describing games and their components mathematically it is possible to exploit mathematical theorems that can be used to prove the existence of equilibrium strategies.

Games are represented by players {I}, strategies {S}, and payoffs{U(S)}. If we define:

I = i players
S_i = strategies played by player i
Ui = utility derived from each strategy played

Then any game can be represented symbolically as:

$\Gamma = [\ I,\ \{S_i\},\{U_i(.)\}]$

Nash Equilibrium

A strategy profile s ε S constitutes a Nash Equilibrium of game $\Gamma = [\ I,\ \{\ S_i\ \},\{U_i(.)\}]$ if for every i,

$U_i(s_i,\ s_{-i}) >= U_i(\ s'_i,s_{-i})$ for all s'_i , s_i, s_{-i} ε S

That is the strategy s_i gives the greatest utility (as opposed to some other strategy s'_i) given the opponent plays s_{-i}. *It is the best choice possible given the choice of the other players.*

Best Response Correspondence

$b_i : s_{-i} \rightarrow s_i$.

The best response correspondence maps the opponent's strategy $s_{-i.}$ to the best response s_i

The strategy profile s is a N.E. of the game $\Gamma = [\ I,\ \{\ S_i\ \},\{U_i(.)\}]$ if and only if s_i ε $b_i\,(s_{-i})$. In other words, if a strategy is not the best response to a given opponent's strategy i.e. if it is not part of the mapping b(.), then it can't be a N.E.

Properties of Sets and Functions

Convexity: A set is convex if it contains *all* the line segments connecting any pair of its points.

Concavity: A set is concave if it *does not* contain all of the line segments connecting any pair of its points.

Closed: A set X is closed if every point outside X has a neighborhood disjoint from X

Bounded: A set is bounded if it is contained inside some ball of finite radius.

Compactness: Compact sets are closed and bounded.

Function: Maps each point in set X to exactly one other point

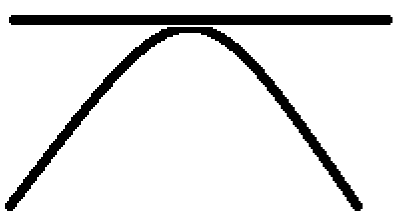
Correspondence: A mapping that maps each point in set X to one or more points

Concave Function: A concave function lies entirely below any tangent

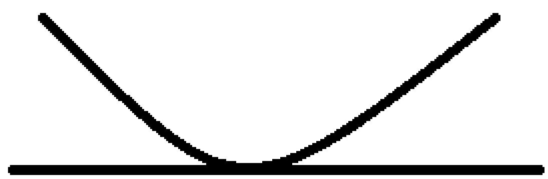
Convex Function: A convex function lies entirely above any tangent

Quasiconcave Function: For a given arc AB, all points are ≥ A

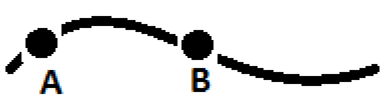
Quasiconvex Function: For a given arc AB, all points are ≤ B



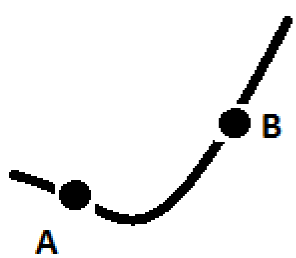
Concave Function



Convex Function



Quasiconcave Function



Quasiconvex Function

Kakutani's Fixed Point Theorem: Every correspondence that maps a compact convex subset of a locally convex space into itself with a closed graph and convex nonempty images has a fixed point.

<u>Existence of Nash Equilibria</u>

The conditions for the existence of Nash Equilibrium hinge on the form of the strategy sets $\{S_i\}$ and the best response correspondence b(.)

A N.E. exists in game $\Gamma = [\, I, \{S_i\},\{U_i(.)\}]$ if

(i) S is a compact convex subset of R^n and $u_i(s)$ is continuous and quaisiconcave
(ii) b(.) is a correspondence that maps S to itself with a closed graph and nonempty convex images.
(iii) By Kakutani's theorem a fixed point therefore exists for this correspondence

This is the basic argument made by Nash in Equilibrium Points in n-person Games:

'The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we-see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if Pi, P2, ... and Qi, Q2, ... Qn, ... are sequences of points in the product space where Q. -n Q, P n P and Q,, counters P,, then Q counters P.Since the graph is closed and since the-image of each point under the mapping is convex, we infer from Kakutani's theorem' that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point."

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