Joint Lattice of Reconstructability Analysis and Bayesian Network General Graphs

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Introduction

- This presentation provides an overview of the integration of structures considered in Reconstructability Analysis (RA) and those considered in Bayesian Networks (BN) into a joint lattice of probabilistic graphical models.
- The work builds on the RA work of Klir (1985), Krippendorff (1986), and Zwick (2001), and the BN work of Pearl (1985, 1987, 1988, 2000), Verma (1990), Heckerman (1994), Chickering (1995), Andersson (1997), and others.
- The joint RA-BN lattice of general graphs expands the set of general graphs with unique independence structures beyond what was previously available by either RA alone or BN alone, thus allowing for representations of complex systems which are:
  I. more accurate relative to data and/or
  II. simpler and thus more comprehensible and more generalizable than would be possible by modeling only with RA or only with BN.

Reconstructability Analysis

- Reconstructability Analysis (RA) is a data modeling approach developed in the systems community (Ashby, 1964; Klir, 1976, 1985, 1986; Conant, 1981, 1988; Krippendorff, 1981, 1986; Broekstra, 1979; Cavallo, 1979; Zwick, 2001, 2004; and others) that combines *graph theory and information theory*.
- RA graphs are undirected and can be cyclic or acyclic.
- RA is designed especially for nominal variables, but continuous variables can be accommodated if their values are discretized.
- Graph theory specifies the structure of the model (the set of relations between the variables); information theory uses the data to characterize the strength and the precise nature of the relations.
- Data applied to a graph structure yields a probabilistic graphical model of the data.
- Applications include time-series analysis, classification, decomposition, compression, pattern recognition, prediction, control, and decision analysis (Zwick, 2004).

RA Lattice of General Graphs

- Figure 1 shows the lattice of four variable RA general graphs (adapted from Klir, 1985; Krippendorff, 1986) representing all four variable RA graphs with unique independence structures.
- Bolded general graphs in Figure 1 are acyclic whereas non-bolded general graphs have cycles.
- A general graph represents a unique independence structure which disregards all possible ways that variables could be labeled. Specific graphs include variable labels.
- For three variables, there are 5 general graphs and 9 specific graphs, four variables, there are 20 general graphs and 114 specific graphs.

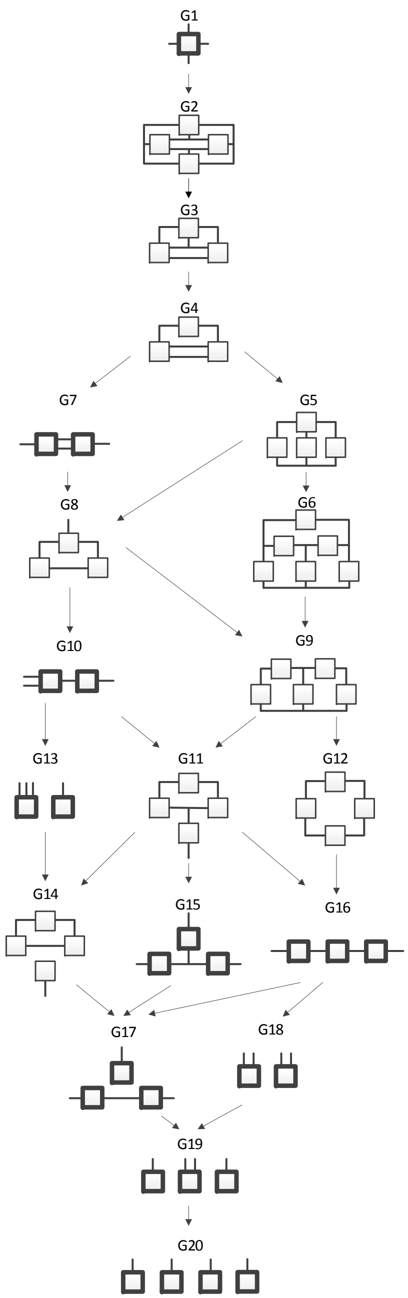

Figure 1 RA Lattice of General Graphs

Bayesian Networks

- A Bayesian Network (BN) is another graphical modeling approach for data modeling that is closely related to RA. Bayesian networks combine graph theory and probability theory. Where BN and RA overlap, they are equivalent, but they have features absent in the other methodology.
- BNs have origins in the type of path model originally described by Wright (1921, 1934), but it was not until the 1980s that BNs were more formally established (Pearl, 1985, 1987, 1988; Neapolitan, 1989).
- BNs are represented by a single type of graph structure; a directed acyclic graph, which is a subset of chain graphs, also known as block recursive models (Lauritzen, 1996).
- A key to BNs is the "V-structure" also known as colliding edges. For example A -> B <- C where edges from both A and C collide on B. This structure and resulting probability expression, p(A)p(C)p(B|AC), are not found in RA.

BN Lattice of General Graphs

- Figure 2 shows the four variable lattice of BN general graphs.
- There are 20 BN general graphs in the lattice, i.e., 20 unique general independence structures for four variable BNs.
- Solid squares represent variables, edges are represented by directed arrows from one square to another.
- The dashed lines with arrows from one general graph to another represent the hierarchy of general graphs, with parent graphs being above child graphs.
- Child graphs result from the deletion of one edge from the parent graph.
- For a three variable BN lattice, there are 5 general graphs and 11 specific graphs; for four variables there are 20 general graphs and 185 specific graphs (Andersson 1997).

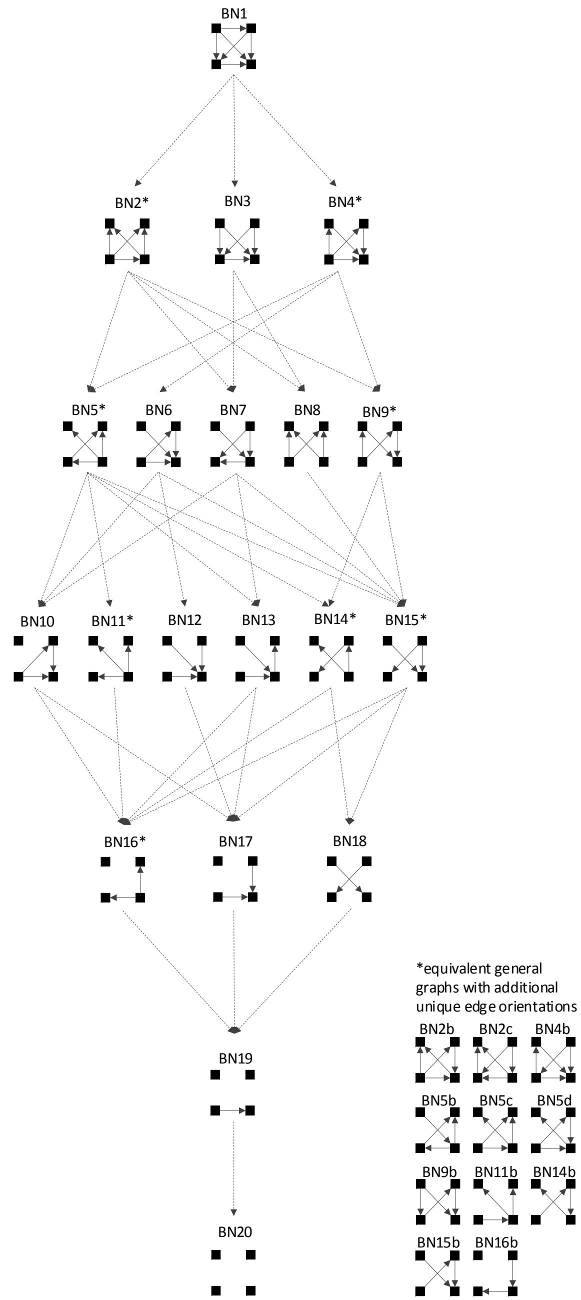

Figure 2 BN Lattice of General Graphs

Rho Lattice

- The four variable Rho (ρ) lattice of Figure 3 (adapted from Klir, 1985, p. 237) is a simplification of the RA lattice of general graphs of Figure 1.
- The Rho lattice represents all possible undirected relations between four variables, and is general enough to map both RA general graphs and BN general graphs.
- Solid dots represent variables; lines connecting dots represent relations between variables.
- The graph ρ1 represents maximal connectedness, or interdependence, between variables, and the graph ρ11 represents independence among all variables.
- Graphs in-between ρ1 and ρ11 represent a mix of dependence and independence among variables.
- Each RA or BN graph corresponds to one, and only one, of the eleven Rho graphs.

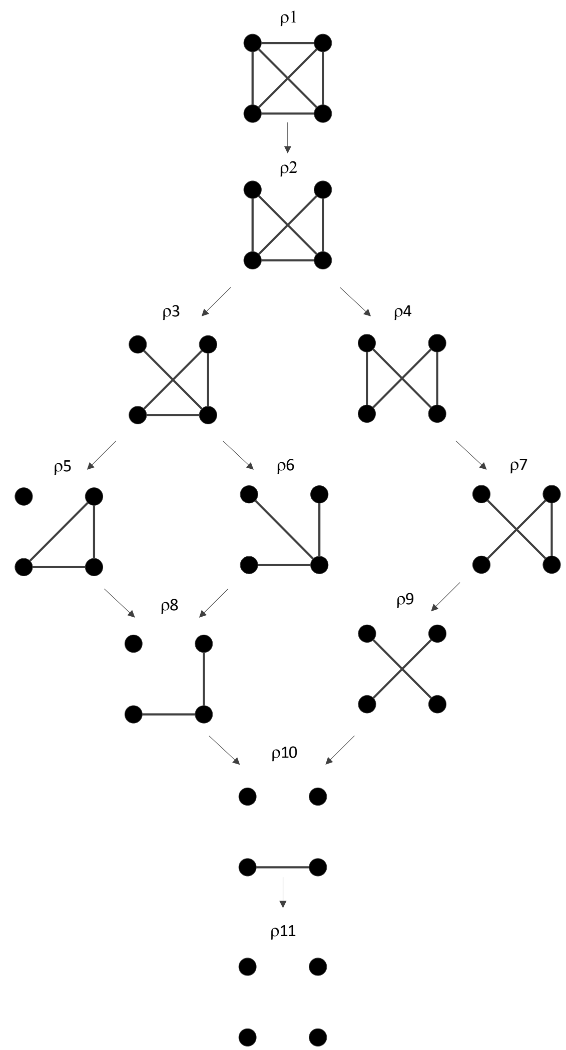

Figure 3 Rho Lattice

Equivalent RA and BN general graphs, example

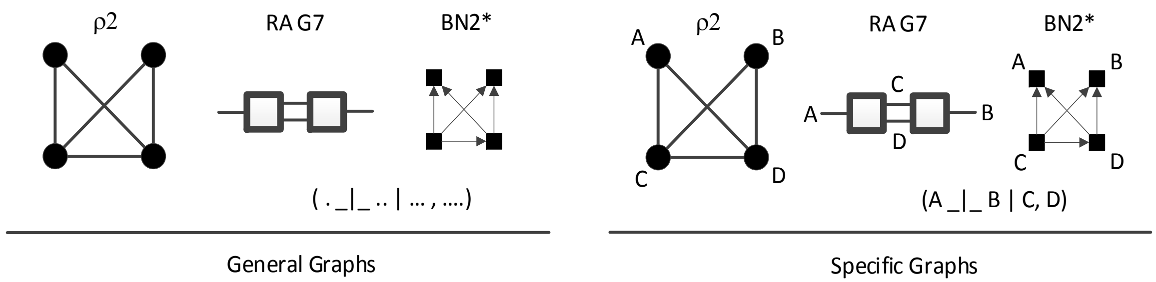

Figure 4 Equivalent RA and BN example

- Figure 4 shows an example of equivalent RA and BN graphs, namely G7 and BN2*, respectively.
- Labeled variables in G7 results in independencies (A _|_ B | C, D) - assigning labels to BN2* yields the same independencies and thus the same general and specific graph.

Equivalent RA and BN general graphs

Table 1. Rho, RA and BN equivalent graphs

Rho graph	RA general graph	BN general graph	Specific Graph Example (RA notation)	Independencies
ρ1	G1	BN1	ABCD	no independencies
ρ2	G7	BN2*	ACD:BCD	(A _|_ B | C, D)
ρ3	G10	BN5*	BCD:AD	(A _|_ B, C | D)
ρ5	G13	BN10	BCD:A	(A _|_ B, C, D)
ρ6	G15	BN11*	AD:BD:CD	(A _|_ B, C | D), (B _|_C | D)
ρ7	G16	BN14*	AD:BC:BD	(A _|_ B | D), (C _|_ A, D | B)
ρ8	G17	BN16*	BD:CD:A	(B _|_ C | D), (A_|_B, C, D)
ρ9	G18	BN18	AD:BC	(A, D _|_ B, C)
ρ10	G19	BN19	CD:A:B	(B _|_ C, D), (A _|_ B, C, D)
ρ11	G20	BN20	A:B:C:D	(A_|_B, C, D), (B_|_ C, D), (C_|_D)

- Table 1 shows the list of all equivalent Rho, RA and BN four variable general graphs, an example of their specific graph notation, and their independences.
- These specific graph examples align with the BN general graphs of Figure 2 assuming labeling of nodes A, B, C, D in the order of top left, top right, bottom left, bottom right.

Joint Lattice of RA and BN General Graphs

- There are 10 RA general graphs, comprising all of the acyclic graphs in the RA lattice that are equivalent to BN general graphs and there are 10 general graphs unique to the RA lattice and 10 general graphs unique to the BN lattice, thus 30 unique general graphs in the joint RA-BN lattice.
- All 10 non-equivalent RA general graphs in the four variable lattice are cyclic and require iteration to generate their probability distributions - BNs are acyclic and have analytic solutions, so there are no BN graphs that are equivalent to these cyclic RA graphs.
- All 10 non-equivalent BN general graphs have a “V-structure” which creates an independence structure not found in any RA general graph.

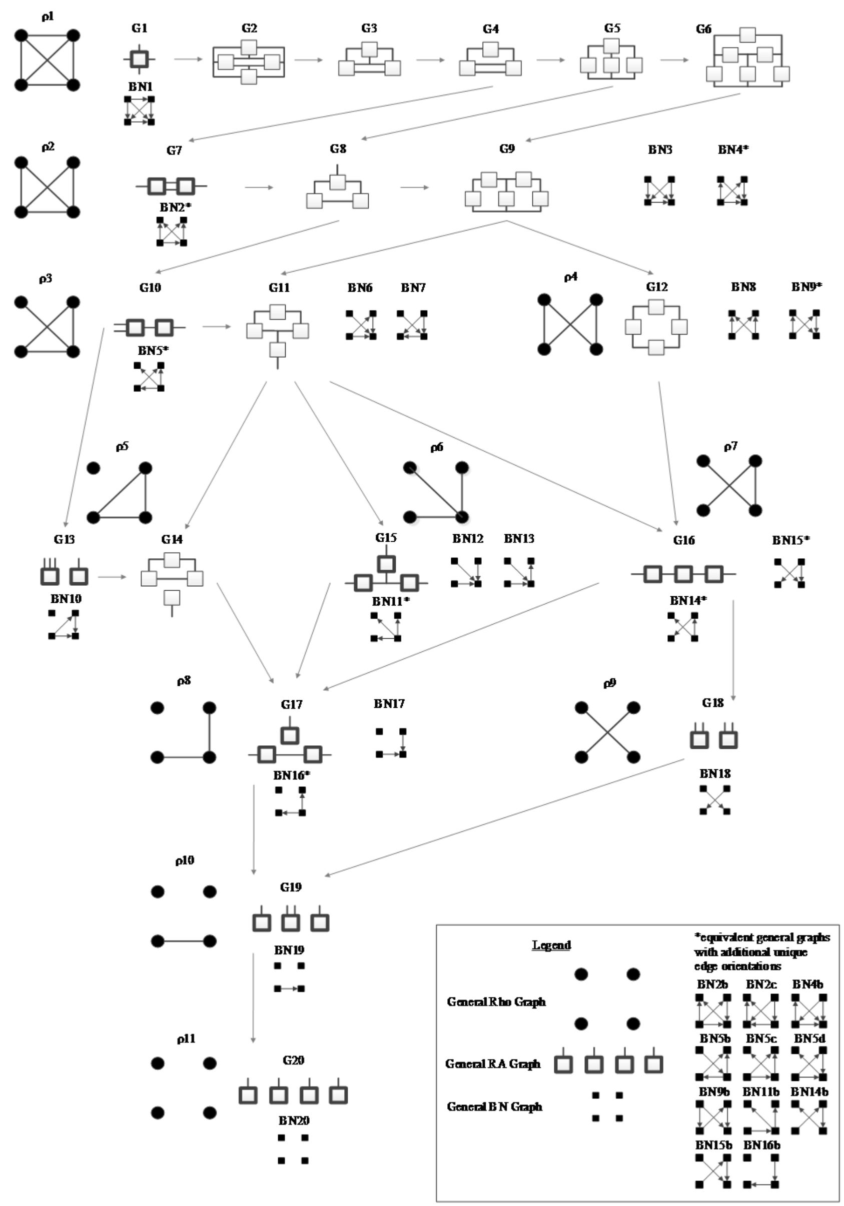

Figure 5 Joint Lattice of RA and BN General Graphs

Summary

- The joint lattice of RA and BN general graphs for four variables increases the number of general graphs with unique independence structures from 20 in the four variable RA lattice and 20 in the four variable BN lattice to 30 in the joint RA-BN lattice, and when variable labels are added, increases the number of unique specific graphs from 114 in the RA lattice and 185 in the BN lattice to 238 in the joint lattice.
- The integration of the two lattices offers a richer and more expansive way to model complex systems leveraging the V-structure unique to BN graphs and the allowability of cycles in RA graphs.
- The joint RA-BN lattice of general graphs expands the set of general graphs with unique independence structures beyond what was previously available by either RA alone or BN alone, allowing for representations of complex systems which are (i) more accurate relative to data and/or (ii) simpler and thus more comprehensible and more generalizable than would be possible by modeling only with RA or only with BN.
- This joint lattice how these two related frameworks – RA and BN – both members of the family of probabilistic graphical modeling methodologies, can be integrated into a unified framework.
- Extension of this work will include designing algorithms to search this joint RA-BN lattice, analysis of RA and BN predictive models in which the IV-DV distinction is made, consideration of “hybrid” RA-BN models, and other topics.

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