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Research Article

Segregation and Integration: A Study of the Behaviors of Investors with Extended Value Functions

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This paper extends prospect theory, mental accounting, and the hedonic editing model by developing an analytical theory to explain the behavior of investors with extended value functions in segregating or integrating multiple outcomes when evaluating mental accounting.

1. Introduction and Literature Review

1.1. Prospect Theory and Mental Accounting

A central tenet within economics is that individuals maximize their expected utilities [1] in which all outcomes are assumed to be integrated with current wealth. Kahneman and Tversky [2] propose prospect theory to reflect the subjective desirability of different decision outcomes and to provide possible explanations for behavior of investors who maximize over value functions instead of utility functions.

Let \( \mathbb{R} \) be the set of extended real numbers and \( \Omega = [a, b] \subset \mathbb{R} \) in which \( a < 0 \) and \( b > 0 \). Rather than defining over levels of wealth, the value function \( v \) is defined over gains and losses relative to a reference point (status quo) \( x_o \in \Omega \) with \( a < x_o < b \), satisfying

\[
(-1)^i v^{(i)}(x) \leq 0 \quad \text{for any } x \in [x_o, b], \quad \text{and } v^{(i)}(x) \geq 0 \quad \text{for any } x \in [a, x_o), \quad i = 1, 2, \quad (1.1)
\]

where \( v^{(i)}(x) \) is the \( i \)th derivative of \( v \).

The value function is a psychophysical function to reflect the anticipated happiness or sadness associated with each potential decision outcome. Without loss of generality, we
assume the status quo to be zero. Thus, we refer to positive outcomes as gains and negative outcomes as losses. In this situation, investors with the value functions \( v \) are risk averse for gains but risk seeking for losses. Since the value function is concave in the positive domain and convex for the negative domain, it shows declining sensitivity in both gains and losses. Kahneman [3] comments that evaluating an object from a reference point of “having” (“not having”) implies a negative (positive) change of “giving something up” (“getting something”) upon relinquishing (receiving) the object.

Many functions have been proposed as value functions; see, for example, Stott [4]. Kahneman and Tversky [2] first propose the following value function:

\[
v(x) = \begin{cases} 
  x^\gamma, & \text{if } x \geq 0, \; \gamma_C \in (0,1), \\
  -\lambda(-x)^\gamma, & \text{if } x < 0, \; \lambda > 1, \; \gamma_L \in (0,1).
\end{cases} \tag{1.2}
\]

Al-Nowaihi et al. [5] show that under preference for homogeneity and loss aversion, the value function \( v \) will have a power form with identical powers \( \gamma \equiv \gamma_C = \gamma_L \) for gains and losses. Tversky and Kahneman [6] estimate the parameters and identify \( \gamma = 0.88 \) and \( \lambda = 2.25 \) as median values whereas Abdellaoui [7] estimates a power value function varying in the range \( \gamma \in (0.2, 0.9) \).

The parameter \( \lambda \geq 1 \) in (1.2) describes the degree of loss aversion and \( \gamma_C \) and \( \gamma_L \in (0,1) \) measure the degree of diminishing sensitivity. Nonetheless, Levy and Wiener [8], M. Levy and H. Levy [9, 10], Wong, and Chan [11] and others suggest extending the value function in (1.2) without restricting \( \lambda \) to be greater than one. In this paper, we first study the behavior of investors who possess the traditional value functions in which \( \lambda > 1 \). We then examine the behavior of investors with the extended value functions to include \( 0 < \lambda \leq 1 \). We call the agent a loss averter or say that s/he is loss averse if \( \lambda > 1 \), loss tolerant if \( 0 < \lambda < 1 \), and loss neutral if \( \lambda = 1 \).

The value function used in prospect theory measures a single event. A question arises when it is used to evaluate multiple events aggregately or separately. To answer this question, Thaler [12] introduces the concept of mental accounting in which investors frame their financial decisions and evaluate elementary outcomes of their investments jointly. Mental accounting is the cognitive processes illustrated by the preceding anecdotes to organize, evaluate, and keep track of financial activities [13, 14]. Studying mental accounting enhances our understanding of the psychology of choice involving multiple events belonging together in a single mental account or separately in different mental accounts because mental accounting rules violate the economic notion of fungibility [15].

### 1.2. Editing Processes and Hedonic Editing Model

The concept of mental accounting can be used to develop specific models for investors’ behavior. Thaler [12] states the hedonic editing hypothesis to test the concept of mentally integrating and segregating two events before they are evaluated so as to code outcomes to make value maximizers as happy as possible.

The two events, \( x \) and \( y \) with the subjective value, \( v(x, y) \), of their combined events, \( (x, y) \), are said to be mentally integrated, given by \( v(x, y) = v(x+y) \), if they are combined before being subjectively evaluated. On the other hand, two events are said to be mentally segregated, given by \( v(x, y) = v(x) + v(y) \), if they are separately evaluated before being combined. For a joint outcome \( (x, y) \), people integrate outcomes when integrated evaluation is more desirable.
than separate evaluations, that is, $v(x + y) > v(x) + v(y)$, and segregate outcomes when segregation yields higher value, that is, $v(x + y) < v(x) + v(y)$.

The hedonic editing hypothesis mainly characterizes value maximizers who mentally segregate or integrate outcomes to be more desirable for the following cases: (1) pure gains, involving two positive events; (2) pure losses, involving two negative events; (3) mixed gains, involving a large gain and a small loss; and (4) mixed losses, involving a small gain and a large loss. In this paper, we study one more case: (5) “tie,” involving equal amounts of gain and loss. The hedonic editing model suggests that individuals should segregate gains and integrate losses because the value function exhibits diminishing sensitivity as the magnitude of a gain or a loss becomes greater. The model also hypothesizes that individuals prefer integrating losses and gains when the gain is bigger than the loss. In addition, diminishing sensitivity of the value function implies that it is preferable to segregate a small gain with big loss, known as a “silver lining principle.”

There are many studies that test the hedonic editing hypothesis. For example, Kahneman and Tversky [2] study the isolation effect and find that segregation rather than integration of prior outcomes leads to risk aversion in the gain domain and risk seeking in the loss domain. Thaler and Johnson [16] find that, consistent with hedonic editing, subjects believed that it was better to separate two financial gains on different days but, contrary to hedonic editing, subjects also believed that it was better to separate two financial losses on different days. Benartzi and Thaler [17] show that the gamble is rejected for segregated evaluation and it is accepted in aggregated evaluation. Gneezy and Potters [18] and Thaler et al. [19] find that the sequence of risky gambles was considered more attractive if just the combined return of three consecutive draws was reported, but not each single outcome. Lim [20] discovers that investors are more likely to bundle sales of stocks that are trading below their purchase price (“losers”) on the same day that sales of stocks are trading above their purchase price (“winners”).

2. Theory

To examine whether Thaler’s [12] hedonic editing model is valid, we first state the following theorem in which $x$ and $y$ have the same signs.

**Theorem 2.1.** Let $v$ be an extended value function defined in (1.1). For any $x, y \in \Omega$,

1. if $x, y \geq 0$, then $v(x + y) \leq v(x) + v(y)$,
2. if $x, y \leq 0$, then $v(x + y) \geq v(x) + v(y)$.

The proof of Theorem 2.1 is straightforward. Readers could easily obtain the proof by modifying the proof of the Petrovic theorem; see Petrovic [21]. We next examine the validity of the hedonic editing model by studying the situation in which $x$ and $y$ are of different signs and different magnitudes as shown in the following theorem.

**Theorem 2.2.** Let $v$ be an extended value function defined in (1.2) with $y \equiv y_G = y_L$, and $x$ and $y \in \Omega$ with $y < 0 < x$ and $x + y \neq 0$. Then, there exist $\lambda, \bar{\lambda} \in \Omega$ such that

1. if $\lambda > \bar{\lambda}$, then $v(x + y) > v(x) + v(y)$ (integration is preferred),
2. if $\lambda = \bar{\lambda}$, then $v(x + y) = v(x) + v(y)$ (neutrality),
3. if $\lambda < \bar{\lambda}$, then $v(x + y) < v(x) + v(y)$ (segregation is preferred).
In addition, if \( x + y > 0 \), then \( 0 < \lambda < 1 \) with \( \lambda(x, y) = [x^\gamma - (x+y)^\gamma] / (-y)^\gamma \), and if \( x + y < 0 \), then \( \lambda > 1 \) with \( \lambda(x, y) = x^\gamma / [(x+y)^\gamma - (x-y)^\gamma] \).

The proof of Theorem 2.2 is in the appendix. We illustrate different preferences for mixed gains and mixed losses stated in Theorem 2.2 by the following example.

**Example 2.3.** Consider \( v \) to be an extended value function defined in (1.2) with \( \gamma_G = \gamma_L = 0.88 \).

We first let \( x = 40 \) and \( y = -6 \) such that \( x + y > 0 \). When \( \lambda \equiv 0.707, v(x + y) = v(34) = v(x) + v(y) = v(40) + v(-6) = 22.26 \), and thus, the property of neutrality holds. On the other hand, when \( \lambda = 2.25, v(x + y) = v(34) = 22.26 > v(x) + v(y) = v(40) + v(-6) = 14.85 \), and thus, integration is preferred. At last, when \( \lambda = 0.4 \), we have \( v(x + y) = v(34) = 22.26 < v(x) + v(y) = v(40) + v(-6) = 23.75 \), and hence, segregation is preferred in this circumstance.

We then let \( x = -40 \) and \( y = 6 \) such that \( x + y < 0 \). When \( \lambda = 1.41, v(x + y) = v(-34) = v(x) + v(y) = v(-40) + v(6) = -31.75 \), and thus, the property of neutrality holds. On the other hand, when \( \lambda = 2.25, v(x + y) = v(-34) = -50.105 > v(x) + v(y) = v(6) + v(-40) = -52.97 \), and thus, integration is preferred. At last, when \( \lambda = 0.4 \), we have \( v(x + y) = v(-34) = -8.908 < v(x) + v(y) = v(6) + v(-40) = -5.438 \), and hence, segregation is preferred in this circumstance.

So far, the literature mainly studies the properties of investors’ preference on mixed gains and mixed losses. To complete the theory, in this paper we also study the property of investors’ preferences for a “tie,” in which the amount of gains and losses is equal as shown in the following theorem.

**Theorem 2.4.** Let \( v \) be an extended value function defined in (1.2) with \( \gamma_G = \gamma_L \), and \( x, y \in \Omega \), with \( y < 0 < x \) and \( x + y = 0 \). Then,

1. if \( \lambda > 1 \), then \( u(x + y) > u(x) + u(y) \) (integration is preferred),
2. if \( \lambda = 1 \), then \( u(x + y) = u(x) + u(y) \) (neutrality),
3. if \( \lambda < 1 \), then \( u(x + y) < u(x) + u(y) \) (segregation is preferred).

The proof of Theorem 2.4 is in the appendix. We illustrate Theorem 2.4 by the following example.

**Example 2.5.** Let \( v \) be an extended value function defined in (1.2) with \( \gamma_G = \gamma_L = 0.88 \).

Consider \( x = 10 \) and \( y = -10 \) so that \( x + y = 0 \); we examine the following situations:

1. if \( \lambda = 1 \), then \( u(x + y) = u(0) = u(x) + u(y) = 0 \),
2. if \( \lambda = 2 > 1 \), then \( u(x + y) = u(0) = 0 > u(x) + u(y) = -7.56 \),
3. if \( \lambda = 1/2 < 1 \), then \( u(x + y) = u(0) = 0 < u(x) + u(y) = 3.79 \).

We summarize the findings from Theorems 2.1 to 2.4 as follows:

1. if \( x, y > 0 \), then value maximizers who are loss averse, loss tolerant, or loss neutral will prefer to segregate,
2. if \( x, y < 0 \), then investors who are loss averse, loss tolerant, or loss neutral will prefer to integrate,
3. if \( y < 0 < x \) with \( x + y > 0 \), then loss averters and investors who are loss neutral will prefer to integrate, whereas investors who are loss tolerant will sometimes prefer
to integrate, sometimes prefer to segregate, and will be neutral between integration and segregation in other circumstances,

(4) if $y < 0 < x$ with $x + y < 0$, then investors who are loss neutral or loss tolerant will prefer to segregate whereas loss averters will sometimes prefer to segregate, sometimes prefer to integrate, and will be neutral between integration and segregation in other circumstances,

(5) if $y < 0 < x$ with $x + y = 0$, then loss averters will prefer to segregate, investors who are loss tolerant will prefer to integrate, and investors who are loss neutral will be neutral between integration and segregation.

3. Concluding Remarks

This article develops an analytical theory to explain the mental accounting of multiple outcomes for investors with extended value functions. Theorem 2.1 supports the hedonic editing hypothesis that predicts that value will be maximized by either separating two gains or combining two losses, not only for investors with traditional value functions but also for investors with extended value functions. However, the hedonic editing model hypothesizes that individuals prefer integrating losses and gains when the gain is bigger than the loss and vice versa. Nonetheless, Theorems 2.2 and 2.4 show that this statement and the “silver lining principle” are only partially correct. Theorems 2.2 and 2.4 show that the preference of integration, neutrality, and segregation in the situations of mixed gains, mixed losses, and “tie” depends on both the relative curvature and steepness of the value functions for gains and losses. The theory developed in this paper shows that there is a turning point, say, $\lambda$ ($\lambda = 1$ in the situation of “tie”), such that value maximizers prefer to integrate when $\lambda > \lambda$, prefer to segregate when $\lambda < \lambda$, and are neutral when $\lambda = \lambda$.

Many works study the hedonic editing hypothesis; see, for example, Linville and Fischer [22] and Lim [20]; some develop theories to explain the hedonic editing model; see, for example, Odean [23] and Langer and Weber [24]; some examine how mental accounting of multiple outcomes affects the behavior of market participants in various contexts in finance; see, for example, Loughran and Ritter [25] and Ljungqvist and Wilhelm [26]; and some provide experimental evidence for the hedonic editing model to make reliable predictions of individual behavior; see, for example, Loewenstein et al. [27] and Van Boven et al. [28]. Further study could apply the theory developed in this paper to explain the behavior of investors in diversification ([29–31] Egozcue and Wong 2009), under- and overreaction [32, 33], and some other well-known financial phenomena or financial anomalies [34–36] and to model investment risk [37–39]. Further research could also extend the theory developed in this paper to study the preference of risk averters and risk seekers [40–43], investors with a reverse S-shaped utility function [11, 44], or other behavior [45, 46]. Further research could also incorporate advanced econometrics [47, 48] and a Bayesian approach [33, 37] to measure the behavior of value maximizers.

Appendix

Proof of Theorem 2.2. We prove only the situation in which $x + y > 0$ here. The situation in which $x + y < 0$ could be obtained similarly. Let $y \equiv y_G = y_L$. Since $x + y > 0$ and $y < 0 < x$, we have $v(x + y) = (x + y)^\gamma$, $v(x) = x^\gamma$, and $v(y) = -\lambda(-y)^\gamma$. Proving the assertion of the theorem...
is equivalent to proving that if \( \lambda < (\rightarrow) \lambda \), then \( (x + y)^T < (\rightarrow) x^T - \lambda (y)^T \). In order to achieve the objective, we define

\[
T(\lambda) = (x + y)^T - x^T + \lambda (-y)^T. \tag{A.1}
\]

First, as \( 0 < x + y < x \), \( T(0) = (x + y)^T - x^T < 0 \). In addition, because \( (-y)^T > 0 \), \( T'(\lambda) > 0 \), implying that \( T(\lambda) \) is a strictly increasing unbound linear function of \( \lambda \). Thus, there exists \( \lambda \), say \( \bar{\lambda} > 0 \) such that \( T(\bar{\lambda}) = 0 \) and \( T(\lambda) > (\lambda)0 \) whenever \( \lambda > (\lambda)0 \).

One could easily compute the value of \( \bar{\lambda} \) by solving \( T(\bar{\lambda}) = 0 \) in (A.1). Now, we turn to prove that \( 0 < \bar{\lambda} < 1 \). The first inequality is trivial. Proving the second inequality is equivalent to proving that

\[
x^T - (-y)^T < (x + y)^T. \tag{A.2}
\]

The conditions \( y < 0 < x \) and \( x + y > 0 \) together imply that \( x > -y > 0 \), and thus, \( -x/y > 1 \). Let \( t = -x/y \), then inequality in (A.2) becomes \( t^T - 1 < (t - 1)^T \). Consider the function \( H(t) = t^T - 1 - (t - 1)^T \); we thus have \( H(1) = \gamma^T - \gamma(t - 1)^T < 0 \) because \( t > 1 \) and \( \gamma < 1 \). Together with the fact that \( H(1) = 0 \), we obtain \( H(t) \leq 0 \) and thereby the assertion of the theorem follows.

Proof of Theorem 2.4. As \( x + y = 0 \), we have \( v(x + y) = 0 \), \( v(x) = x^T \), and \( v(y) = -\lambda(x)^T \). Thereafter, we define \( T(\lambda) = v(x + y) - v(x) - v(y) = (\lambda - 1)x^T \). As \( x^T \geq 0 \), if \( \lambda \geq (\leq)0 \), then \( T(\lambda) \geq (\leq)0 \), and thereby the assertion of the theorem follows.

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