General equilibrium model of arbitrage trade and real exchange rate persistence

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Martin Berka
Massey University*

Abstract

Heterogeneity of marginal shipping costs leads to persistent and volatile deviations in real exchange rate. In a two-country, three-good endowment general equilibrium model, arbitrage firms use transportation technology which depends positively on distance and physical mass of goods. The model exhibits endogenous tradability, non-linearity of law of one price deviations and trade-inducing and suppressing substitution effects due to heterogeneity in trade costs. When endowments follow an AR(1) process that matches comovement of US and EU GDPs and the aggregate trade costs consume 1.7% of GDP, real exchange rate persistence matches the data. A model with quadratic adjustment costs also induces substantial real exchange rate volatility.

Keywords: Arbitrage trade, heterogeneity, real exchange rate, persistence, volatility

JEL Classification: F3, F41

*School of Economics and Finance, Private Bag 102 904, Auckland, New Zealand, Tel: +649-414-0800 ext. 9474, Fax: +649-441-8177, Email: m.berka@massey.ac.nz
1 Introduction

This paper explains how persistence and volatility of real exchange rate deviations arise as a result of heterogeneous shipping costs in a dynamic general equilibrium framework with arbitrage trade. In a two-country three-good endowment model with identical households, arbitrage trading firms chose trade volumes in response to profitable arbitrage opportunities. Because the marginal shipping costs are heterogeneous (motivated by the heterogeneity of physical characteristics important in shipment) a country-specific shock may lead to trade in some goods but not in others. Moreover, heterogeneity leads to substitution effects between traded and non-traded goods in each country. This substitution can induce or suppress trade and has a measurable influence on the dynamic properties of the real exchange rate. A careful calibration of the model matches persistence of the real exchange rate in the data and, when adjustment costs are added, also generates volatility in real exchange rate deviations.

The concept of purchasing power parity (PPP) maintains that national price levels should be equal when expressed in the units of a common currency (Cassel 1918). Translated into observables, it states that the real exchange rate should be constant. The central puzzle in the international business cycle literature is that fluctuations in the real exchange rate are very large and persistent. Traditional attempts to address this puzzle based on the Harrod-Balassa-Samuelson objection to PPP (Balassa 1961) were found to be empirically unwarranted for developed countries (e.g., Engel 1999). In particular, many empirical studies document large, volatile and persistent deviations in prices of traded goods across countries. Several avenues have been explored to motivate the deviations in prices of traded goods from parity. Betts and Devereux (2000) and Bergin and Feenstra (2001) find that pricing to market with segmented markets and nominal rigidities creates volatile deviations in the real exchange rate. A year-long price stickiness combined with a low degree of intertemporal elasticity of substitution and consumption - leisure separable preferences generates sufficient volatility but insufficient persistence in the real exchange rate (Devereux 1997, Chari, Kehoe and

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1Harrod-Balassa-Samuelson (HBS) objection is based on the relative price of traded and non-traded goods. Engel (1999) shows that in the U.S. data, no more than 2% of the variation in the real exchange rate can be attributed to the fluctuations in the relative price of non-traded to traded goods. HBS proposition holds better for emerging and developing economies, and at lower frequencies. See, i.a., Choudhri & Khan (2004)
McGrattan 2008). A distribution costs approach (e.g., Corsetti and Dedola 2005, Burstein, Neves and Rebello 2003) justifies wedges between the prices of tradable goods by very large costs to product distribution (up to 60% of product price) in order to match the volatility of the real exchange rate. Differences in preferences across countries have also been used to rationalize deviations from the law of one price (e.g., Lapham and Vigneault 2001) but rely on volatile and highly persistent shocks to preference substitution parameters to match the observed fluctuations in the prices of traded goods. Finally, models of the costs of arbitrage trade were so far unsuccessful in generating sufficiently persistent law of one price deviations (e.g., Obstfeld and Rogoff 2000, Dumas 1992, Ohanian and Stockman 1997, Canjels, Prakash-Canjels and Taylor 2004, Sercu, Uppal and van Hulle 1995).

Recent evidence (e.g., O’Connel and Wei 2002, Crucini, Telmer and Zachariadis 2005) shows that law of one price deviations behave in a threshold non-linear and heterogeneous way. Obstfeld and Taylor (1997) find that threshold estimates for sectoral RERs are significantly related to exchange rate volatility and city distances, a result which holds also at an international level and at various frequencies (Zussman 2002). Imbs et al. (2003) confirm this at a sectoral level. Berka (2009) finds that, at the level of individual goods, heterogeneity of marginal transport costs, proxied by price-to-weight ratios, explains a large part of the variation in thresholds and conditional half-lives of price differences. Prices of heavier or more voluminous goods deviate further before becoming mean reverting, suggesting that shipping costs are important in explaining heterogeneous behaviour of law of one price deviations.

The two general equilibrium models presented in this paper show how heterogeneity of shipping costs explains persistence and volatility in deviations of good prices – and the real exchange rate – from parity. Three goods which only differ by their marginal shipping costs (physical weight) are traded for arbitrage purposes. Arbitrage trading firms decide

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2 The border effect literature tries to understand the vastly higher density of trade flows when two equidistant locations are separated by a border. This phenomenon also includes a very high cross-border price volatility of identical products, and is therefore closely related to literature on real exchange rates (Engel and Rogers 1996, and Jenkins and Rogers 1995).

3 Hummels (1999) documents that shipping costs depend on weight or volume of the transported goods.

4 Because the purpose of this paper is to explain price differences and not trade volume, relevance of this modelling approach does not require existence of a large amount of arbitrage trade. A threat of arbitrage is sufficient in controlling price differences. Arbitrage trade can be thought of as a limiting case of specialized production and trade of substitutable goods and offers a simple way of introducing shipping costs into the model.
on the timing and magnitude of trade to maximize their profits by comparing marginal revenues (proportional to the size of the price difference and trade volume) with arbitrage costs (proportional to shipping distance and a heterogeneous good friction). In the second model, arbitrage costs also include quadratic adjustment costs in the change of trade volume. This makes large changes in the volume of trade more than proportionately costly due to needed costly adjustments in legal contracts, infrastructure, as well as the establishment of new (or changes of the existing) business relationships and distribution networks. Firms then optimally smooth the trade volume leading to more volatile price differentials.

Equilibrium in both models has three notable characteristics. First, the tradability of goods is determined endogenously by the endowment shock and the physical characteristics of all products. Second, price differences exhibit threshold non-linearity. In the linear model, the symmetric threshold equals the marginal trade cost. Third, the size of law of one price deviation depends on physical characteristics of all products and their endowments. General equilibrium effects due to substitution among traded and non-traded goods in each country can induce or suppress trade, thus influencing the real exchange rate distribution. Logarithm of the real exchange rate exhibits a string-type nonlinearity\(^5\). For large deviations from parity, thresholds of all RER components are crossed, yielding a stronger mean-reverting tendency and a larger arbitrage trade volume. Real exchange rate persistence declines in the volatility of the endowment shock process and increases in the persistence of the endowment shocks and in the trade friction. Volatility of the real exchange rate increases in all three of the above factors (it is concave in shock volatility).

A careful calibration of the first model matches the persistence of real exchange rate found in the data, while producing meaningful persistence and co-movements along various important dimensions. However, size of the transport friction limits RER volatility when trade is instantaneous. The quadratic adjustment cost model yields a dynamic and highly non-linear model which retains the core features while improving results. It goes a long way towards matching both RER persistence and volatility while being very close to the data along other dimensions.

\(^{5}\)This is an empirical regularity, documented by Taylor, Peel and Sarno 2001, Kilian and Taylor 2003 who show that smooth-threshold AR models provide a better empirical description of the data.
The rest of the paper is structured as follows. Sections 2 and 3 discuss models with linear heterogeneous shipping costs and quadratic adjustment costs, respectively. Section 4 analyzes stochastic properties of the real exchange rate and section 5 discusses parameter calibration. Section 6 analyzes persistence, comovement and volatility of the real exchange rate and other variables. Section 7 concludes.

2 General equilibrium model of arbitrage trade

The two-country world consists of households and arbitrage trading firms. Each country is endowed with positive amounts of three tradable goods which differ in their physical characteristics.

2.1 Households

A representative household at Home chooses its consumption path to maximize instantaneous CES utility function subject to a resource budget constraint:

$$\max_{C_{1t}, C_{2t}, C_{3t}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\theta} \left[ \gamma_1^{\frac{1}{\theta}} C_{1t}^{1-\frac{1}{\theta}} + \gamma_2^{\frac{1}{\theta}} C_{2t}^{1-\frac{1}{\theta}} + \gamma_3^{\frac{1}{\theta}} C_{3t}^{1-\frac{1}{\theta}} \right]^{\frac{\theta}{1-\theta}} \right\}$$

s.t. $$p_{1t} C_{1t} + p_{2t} C_{2t} + C_{3t} = p_{1t} Y_{1t} + p_{2t} Y_{2t} + Y_{3t} + \frac{1}{2} AP_t \quad (1)$$

given $AP_t$ and $Y_{it}$, $i = \{1, 2\}$, where $Y_{it}$ is an endowment of good $i$ at time $t$, $\sum_{i=1}^{3} \gamma_i = 1$, $\theta > 1$, $p_{it}$ ($i = 1, 2$) is the relative price of goods $i$ to good 3 and $AP_t$ is the amount of current-period arbitrage profits transferred to the household from a firm, assuming an equal splitting rule between households at home and abroad. The first order conditions for this problem imply the usual demand functions:

$$C_{1t} = \gamma_1 p_{1t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}}, \quad C_{2t} = \gamma_2 p_{2t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}}, \quad C_{3t} = \gamma_3 \frac{Y_t}{P_t^{1-\theta}} \quad (2)$$

where $Y_t$ is home country’s real GDP expressed in the units of good 3 ($Y_t = p_{1t} Y_{1t} + p_{2t} Y_{2t} + Y_{3t} + \frac{1}{2} AP_t$) and $P_t$ is a composite price index $P_t = (\gamma_1 p_{1t}^{-\theta} + \gamma_2 p_{2t}^{-\theta} + \gamma_3)^{\frac{1}{1-\theta}}$. Preferences of households at Home and Abroad are identical, with prices abroad denoted by an asterix.
2.2 Arbitrage trading firms

There is a representative arbitrage trading firm in each country. It chooses the time and amount traded of each good, taking into account the transportation costs.

\[
\max_{N_1, N_2} \Pi_t = \max_{N_1, N_2} \sum_{t=1}^{\infty} \beta^t AP_t = \max_{N_1, N_2} \sum_{t=1}^{\infty} \beta^t \left[ \sum_{i=1}^{3} (p_{it}^* - p_{it})N_{it} - T(N_{1t}, N_{2t}) \right]
\]  

where \( N_{it} \) is the amount of trade in good \( i \) (\( N > 0 \) implies exports from Home to Abroad) and \( T(N_{1t}, N_{2t}) \) is the cost function of the arbitrage trading firm. An arbitrage firm has to purchase \( T(N_{1t}, N_{2t}) \) units of good 3 to trade \( \{N_{1t}, N_{2t}\} \). It is assumed that good 3 has a zero trade friction, implying that the law of one price always holds\(^6\). The cost function is linear in the heterogeneous trade friction \( t_i \):

\[
T(N_{1t}, N_{2t}) = (t_1|N_{1t}| + t_2|N_{2t}|) = (aw_1|N_{1t}| + aw_2|N_{2t}|)
\]

where \( t_i \) is assumed to be a linear function of the weight of a good \( i \) \( w_i \) and a positive constant homogeneous component of the shipping cost \( a\(^7\). The first order conditions approximately yield:

\[
I(N)(p_{it}^* - p_i) = aw_i \text{ iff } |N_i| > 0, \quad I(N)(p_{it}^* - p_i) < aw_i \text{ iff } N_i = 0, \quad \forall i \in \{1, 2\}
\]

where \( I(N) \) is an indicator function, such that \( I(N)= 1 \) when \( N \geq 0 \), \( I(N)= -1 \) otherwise.

Trade occurs when the marginal revenue of arbitrage (left-hand side of (4)) exceeds the marginal cost (right-hand side (4)). Trade leads to price convergence, and stops when all profit opportunities are eliminated and absolute value of price difference equals marginal

\(^6\)The assumption of zero trade friction is innocuous. A positive friction for each good would make the computation more complicated but would not change the results qualitatively. Parameters \( t_1 \) and \( t_2 \) can be thought of as trade frictions of goods 1 and 2 relative to the trade friction of good 3.

\(^7\)\( a \) can be thought of as a per-kilogram fraction of good 3 which is used when a good is transported between Home and Abroad. For the sake of simplicity and expositional clarity, insurance costs, costs of setting up distribution networks, and other costs are ignored in this specification.
trade cost. FOCs hold with inequality only in autarky. It is intuitive to rewrite (4) as:

\[ -a \leq \frac{MRA \text{ per kg}}{p^*_i - p_i} \leq \frac{MCA \text{ per kg}}{a} \quad i = 1, 2 \]

The middle part of this inequality captures the marginal revenue of arbitrage per kilogram of good \( i \) (MRA) and the outside parts represent the marginal arbitrage cost per kilogram of good \( i \) (MCA). While MCA is identical across goods, MRA is not. Goods that are relatively heavier (or for another reason have a larger marginal shipping cost) need a larger price difference in order for MRA to exceed MCA. Thus, maximum law of one price deviation for each good proportional to its weight:

\[ \text{LOPD} \leq t_i, \quad \forall i \in \{1, 2\} \quad (5) \]

This leads to heterogeneous filtering. Consider an endowment shock \( x \) which leads to an identical law of one price deviation for goods 1 and 2. The value of \( x \) can be divided into three subsets in terms of its effect on the price deviations. \( x \in [0, x^*_1) \) results in autarky because the law of one price deviations for goods 1 and 2 are in a no-trade region \((|\text{LOPD}_i| < t_i \iff MR_i < MC_i \quad i = 1, 2)\). For \( x \in [x^*_1, x^*_2) \), only the good with a smaller trade friction (thereafter good 1) is traded because autarky price difference exceeds \( t_1 \) but not \( t_2 \): \(|\text{LOPD}_1| > t_1 \iff MR_1 > MC_1, \quad |\text{LOPD}_2| < t_2 \iff MR_2 < MC_2\). For \( x \in [x^*_2, \infty) \), all goods are traded as respective autarky price differences exceed \( t_i \) \((|\text{LOPD}_i| > t_i \iff MR_i > MC_i \quad i = 1, 2)\).

### 2.3 Market clearing

Three goods markets clear at home as well as abroad. The direction of trade in goods 1 and 2 depends on the size and sign of the initial deviation from a law of one price, as determined by the endowments of these goods \((N_i = -N^*_i)\). The market clearing conditions can then be written as:

\[ C_i + N_i = Y_i, \quad C^*_i - N_i = Y^*_i, \quad \forall i \in \{1, 2\} \quad (6) \]
\[ C_3 + N_3 + \frac{1}{2} T(N_1, N_2) = Y_3, \quad C_3^* - N_3 + \frac{1}{2} T(N_1, N_2) = Y_3^* \]  

(7)

2.4 Equilibrium

The equilibrium is a set of prices and quantities \( \{p_1, p_1^*, p_2, p_2^*, C_1, C_1^*, C_2, C_2^*, C_3, C_3^*, N_1, N_2, N_3\} \) such that the households maximize their utility (equation (1)), arbitrage trading firms maximize their profits (eq. (3)) and markets clear (eqs. (6) - (7)).

2.4.1 Frictionless trade

Without transportation costs \((t_i = 0)\), profit maximization problem faced by the firm implies that law of one price holds for all goods \((p_i^* = p_i, \ i \in \{1, 2\})\). The equilibrium relative prices then depend on the world endowments and the preference parameters:

\[
\frac{p_i}{p_j} = \frac{p_i^*}{p_j^*} = \left[ \frac{Y_i^W}{Y_j^W} \gamma_i \right]^\frac{1}{\theta} \text{ \forall } i,\]

(8)

where \(Y_i^W \equiv Y_i + Y_i^*\). The equilibrium consumption levels are:

\[
C_1 = Y_1 \left( \frac{\gamma_1}{\gamma_2} \right)^\frac{1}{\theta} \left( \frac{Y_1^W}{Y_1^W} \right)^\frac{1}{\theta} + \frac{Y_2}{Y_1}, \quad C_2 = Y_2 \frac{Y_2^W}{Y_1^W} \left( \frac{\gamma_1}{\gamma_2} \right)^\frac{1}{\theta} \left( \frac{Y_2^W}{Y_1^W} \right)^\frac{1}{\theta} + \frac{Y_2}{Y_1^W}
\]

and similarly for \(C_1^*\) and \(C_2^*\). \(C_i = Y_i, \ C_i^* = Y_i^*\) iff \(\frac{Y_1}{Y_2} = \frac{Y_1^*}{Y_2^*}\). Country which is endowed with a relatively larger amount of good \(i\) will export good \(i\) and import good \(j\) – a standard comparative advantage argument.

2.4.2 Equilibrium with positive trade frictions \(t_2 > t_1 > 0\)

With positive trade frictions and \(Y_1 = Y_2\), three cases can arise. In Case 1, endowments are such that \(LOPD_i < MC_i\) in autarky (i.e., (5) holds with inequality) and no goods are traded. In Case 2, the endowments imply autarky prices which exceed the marginal cost of arbitrage for one good but not the other. Consequently, trade occurs in one good but not the other ((5) holds with equality for 1 and inequality for 2). Finally, in Case 3 the endowments...
imply autarky prices such that the law of one price exceeds MC, ∀i ∈ {1, 2}, both goods are traded. I summarize the equilibrium in all three cases.

**Case 1: No trade in goods 1 & 2** The equilibrium conditions are:

\[
\gamma_i p_i^{* - \theta} Y = Y_i, \quad \gamma_i p_i^{* - \theta} Y^* = Y^*_i, \quad \forall i \in \{1, 2\}
\]

\[
\gamma_3 \left( \frac{Y}{P_{1-\theta}} + \frac{Y^*}{P_{1-\theta}} \right) = Y_3 + Y^*_3
\]

where \(Y = p_1 Y_1 + p_2 Y_2 + Y_3\), \(Y^* = p_1^* Y_1^* + p_2 Y_2^* + Y_3^*\), \(P = (\gamma_1 p_1^{1-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}\) and \(P^* = (\gamma_1 p_1^{11-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}\). Walras’ law implies that the system can be uniquely solved for prices \(\{p_1, p_2, p_1^*, p_2^*\}\), which recursively define other equilibrium values.

**Case 2: No trade in good j** In this case, \(N_j = 0\) and the equilibrium is characterized by:

\[
\gamma_i (p_i^* - I(N_i)(t_i)^{-\theta} Y = Y_i = Y_i^*
\]

\[
\gamma_j p_j^{* - \theta} Y = Y_j \quad \text{and} \quad \gamma_j p_j^{* - \theta} Y^* = Y_j^*
\]

\[
\gamma_3 \left( \frac{Y}{P_{1-\theta}} + \frac{Y^*}{P_{1-\theta}} + t_i \left[ Y_i - \gamma_i (p_i^* - I(N_i) t_i)^{-\theta} Y = Y_3 + Y^*_3 \right.\right]
\]

where \(Y = (p_1^* - I(N_1) t_1) Y_1 + p_2 Y_2 + Y_3\), \(P = (\gamma_1(p_1^* - I(N_1) t_1)^{1-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}\) and \(I(.)\) is the indicator function defined in (4). Walras’ law implies that this system uniquely determines \(\{p_1^*, p_2^*, p_j\}\) and consequently all other equilibrium values as functions of preferences, endowments, and the trade friction \(t_i\).

**Case 3: All goods traded** Here, equilibrium prices solve the following reduced system:

\[
(p_i^* - I(N_i)(t_i))^{-\theta} Y = Y_i + p_i^{* - \theta} Y^* = \frac{1}{\gamma_i} (Y_i + Y_i^*), \quad \forall i \in \{1, 2\}
\]

\[
\gamma_3 \left( \frac{Y}{P_{1-\theta}} + \frac{Y^*}{P_{1-\theta}} + t_1 \left[ Y_1 - \gamma_1 (p_1^* - I(N_1) t_1)^{-\theta} Y = \frac{1}{\gamma_i} (Y_i + Y_i^*), \quad \forall i \in \{1, 2\} \right.\right]
\]

\[
\gamma_3 \left( \frac{Y}{P_{1-\theta}} + \frac{Y^*}{P_{1-\theta}} + t_2 \left[ Y_2 - \gamma_2 (p_2^* - I(N_2) t_2)^{-\theta} Y = Y_3 + Y^*_3 \right.\right]
\]

where \(Y = (p_1^* - I(N_1) t_1) Y_1 + (p_2^* - I(N_2) t_2) Y_2 + Y_3\) and \(P = (\gamma_1(p_1^* - I(N_1) t_1)^{1-\theta} + \gamma_2(p_2 - I(N_2) t_2)^{1-\theta} + \gamma_3)^{1/(1-\theta)}\). Walras’ law reduces the above system into two equations.
that solve uniquely for \( \{p^*_1, p^*_2\} \) and implicitly all other variables as functions of endowments, preferences, and the trade frictions.

### 2.4.3 Properties of the equilibrium

When shocks to endowments are identical across sectors, goods with larger trade friction have, on average, larger and more volatile LOPDs. Larger endowment differences increase LOPD but only to the point where arbitrage takes place; they are neutral afterwards. Moreover, trade in good \( i \) is a sufficient, but not a necessary condition for the adjustment of prices of good \( i \).

Trade frictions \( t_1 \) and \( t_2 \) affect equilibrium prices and allocations in all three cases: directly in cases 2 and 3 and indirectly in cases 1 and 2 by defining endowments for which the autarky solutions apply. In case 2, *prices of non-traded good at home and abroad are affected by the price convergence in traded good*. This general equilibrium effect is caused by consumers in exporting country substituting away from the traded good 1 (whose price rises due to a shrinking domestic supply) into non-traded good 2, while the consumers in the importing country move away from the non-traded and into the traded good. Consequently, law of one price deviation for the non-traded good 2 is smaller when good 1 is traded than it would have been if good 1 was not traded. Figure 1 below shows this effect as a smaller slope of \( \text{LOPD}_2 \) in case 2 than in case 1 (note that good 2 is not traded in both cases). When endowment shocks are country- or sector-specific, this substitution effect can induce or suppress trade and affects the dynamic properties of the real exchange rate.

Figure 1 plots the equilibrium law of one price deviations against endowment differences. Keeping the endowments Abroad fixed, Home endowments of goods 1 and 2 vary by the same amount, leading to changes in \( p_1 \) and \( p_2 \). In case 1, price differences are smaller than marginal costs of trade. In case 2, trade occurs for good 1 but good 2 remains non-traded. When Home exports good 1, \( p_1 \) rises and \( p^*_1 \) declines until \( p^*_1 - p_1 = t_1 \). Therefore, graph of \( \text{LOPD}_1 \) has a threshold in case 2. As the demand for non-traded good rises in exporting and declines in importing country (due to the aforementioned substitution effect), \( \text{LOPD}_2 \) increases in the endowment difference at a lower rate than when good 1 is not traded. Consequently, slope of

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8This is the simplest way to perturb the model to illustrate the three aforementioned cases.
$\text{LOPD}_2$ is lower in case 2 than in case 1. To the extent that goods are substitutable, trade in one sector lowers the law of one price deviations in non-traded sector\(^9\). Finally, when the endowment differences induce trade in the second goods, equilibrium $\text{LOPD}_2$ reaches a threshold.

### 3 Arbitrage trade model with adjustment costs to trade

The second model has an identical endowment and preference setting. However, trade costs also include quadratic adjustment costs in the change of trade volume. Changes in the trade volume require hiring of labour resources, adjustment in the distribution system and possibly investment in new (or a changes of the existing) trade infrastructure. Larger swings in trade volume are therefore more-than-proportionately costly. The arbitrage firms’ problem is now:

$$\max_{N_{1t},N_{2t}} A\Pi_t = \max_{N_{1t},N_{2t}} \sum_{j=t}^{\infty} \beta^{(j-1)} A P_j$$

$$= \max_{N_{1t},N_{2t}} \sum_{j=t}^{\infty} \beta^{(j-1)} \left[ \sum_{i=1}^{2} (p_{ij}^* - p_{ij}) N_{ij} - T(N_{1j}, N_{2j}) \right] \quad (9)$$

s.t. $T(N_{1t}, N_{2t}) = t_1 |N_{1t}| + t_2 |N_{2t}| + c_1 \Delta N_{1t}^2 + c_2 \Delta N_{2t}^2 \quad (10)$

The firm has to purchase $T(N_{1t}, N_{2t})$ units of good 3 to trade $\{N_{1t}, N_{2t}\}$. The total cost consists of a shipping cost and an adjustment cost. The shipping cost is identical to that in the first model: $t_i = a w_i$, $i = 1, 2$ where $w_i$ is the weight of good $i$ and $a$ is a constant. The adjustment cost is quadratic in the change of trade volume\(^10\).

Due to its non-differentiability at 0, I use a smooth approximation $G(.)$ to the absolute value function that allows a continuous mapping between the first order conditions and the objective function. Let $I(N_{i,t}) \equiv dG(.)$ denote the first order derivative of a "smooth" absolute value function. $I(N_{i,t})$ can be thought of as an approximation to the indicator function: $I(N_{i,t}) = 1$ when $N_{i,t} > 0$, $I(N_{i,t}) = -1$ when $N_{i,t} < 0$ and $I(N_{i,t}) = 0$ when

\(^9\)For example, trade in shaving machines would reduce law of one price deviation in barber services.

\(^{10}\)A quadratic adjustment cost function provides a reduced form which captures firm’s gradual response in a smoother way than the assumption of a pre-determined volume of shipment (i.a., Ravn & Mazzenga 2004).
\( N_{i,t} = 0 \) (see Appendix A). The first order optimality conditions then yield:

\[
0 = \left\{ \left( p^*_{i,t} - p_{i,t} \right) - \frac{\partial T(t)}{\partial N_{i,t}} - \beta E_t \frac{\partial T(t+1)}{\partial N_{i,t}} \right\}
\]

\[
0 = p^*_{i,t} - p_{i,t} - \left[ t_i I(N_{i,t}) + 2c_i (N_{i,t} - N_{i,t-1}) \right] - \beta E_t \left[-2c_i(N_{i,t+1} - N_{i,t}) \right]
\]

Rearranging, we get:

\[
\frac{1}{2c_i} \left[ p^*_{i,t} - p_{i,t} - t_i I(N_{i,t}) \right] = -\beta E_t N_{i,t+1} + (1 + \beta) N_{i,t} - N_{i,t-1}
\]

\[
= \left[-\beta + (1 + \beta)B - B^2\right] E_t N_{i,t+1} \forall i
\]

where B is a backshift operator. The quadratic form on the right hand side has one stationary and one non-stationary root and can be rewritten as: 

\[-(B - 1)(B - \beta)\].

The first order conditions for the firm then are:

\[
-\frac{1}{2c_i} \left[ p^*_{i,t} - p_{i,t} - t_i I(N_{i,t}) \right] = (1 - B^{-1})(1 - \beta B^{-1}) E_t N_{i,t-1} \forall i
\]

Expanding the stable eigenvalue forward and the unstable backward, a forward-looking form of the FOC can be written as:

\[
N_{i,t} = N_{i,t-1} + \frac{1}{2c_i} E_t \sum_{j=0}^{\infty} \beta^j \left( p^*_{i,t+j} - p_{i,t+j} - t_i I(N_{i,t+j}) \right) \forall i
\]

The optimal amount of trade in good \( i \) in period \( t \) depends positively on the volume of trade in the last period and on the expected future path of price differences in excess of the trade friction. Firms care about the future path of LOPDs because they prefer to smooth their trade pattern over time. The size of expected price difference in excess of trade friction in period \( t + j \) increase trade in all periods after \( t \). The expected future direction of trade \( E_t I(N_{i,t+j}) \) is also important: an expectation of a change in the direction of trade lowers trade volume today.
3.1 Equilibrium

The equilibrium is a set of prices and quantities \( \{ p_{1,t}, p_{1,t}^*, p_{2,t}, p_{2,t}^*, C_{1,t}, C_{1,t}^*, C_{2,t}, C_{2,t}^*, C_{3,t}, C_{3,t}^*, N_{1,t}, N_{2,t}, N_{3,t} \}_{t=0}^{\infty} \) such that the representative household maximizes its utility subject to budget constraint arbitrage trading firms maximize their profits (equation (11) for both goods) and all markets clear (equations (6) - (7)). It can be simplified into a 4-by-4 system in \( \{ p_{1,t}, p_{2,t}, p_{1,t}^*, p_{2,t}^* \} \):

\[
\Delta Y_{i,t} - \gamma_i \left[ p_{i,t}^{1-\theta} \frac{Y_t}{P_{1-\theta}^t} - p_{i,t-1}^{1-\theta} \frac{Y_{t-1}}{P_{1-\theta}^{t-1}} \right] = \frac{1}{2c_4} E_t \sum_{j=0}^{\infty} \beta^j \left[ p_{i,t+j}^* - p_{i,t+j} - t_i I(N_{i,t+j}) \right], \; i \in \{1, 2\} \tag{12}
\]

\[
\gamma_i p_{i,t}^{1-\theta} \frac{Y_t}{P_{1-\theta}^t} + \gamma_i p_{i,t}^{1-\theta} \frac{Y_{i,t}^*}{P_{1-\theta}^t} = Y_{i,t} + Y_{i,t}^*, \; i \in \{1, 2\} \tag{13}
\]

where \( Y_t = p_{1,t} Y_{1,t} + p_{2,t} Y_{2,t} + Y_{3,t} + \frac{1}{2} AP_t \), \( P_t = (\gamma_1 p_{1,t}^{1-\theta} + \gamma_2 p_{2,t}^{1-\theta} + \gamma_3) \frac{1}{i} \) and \( AP_t \) are the contemporaneous arbitrage profits. For goods 1 and 2, equations (12) and (13) represent the intertemporal and intratemporal equilibrium conditions, respectively.

3.1.1 Intuition

Two pieces of intuition about the influence of adjustment costs can be built by considering a one-period partial equilibrium version of the model. First, firm chooses a finite trade volume with adjustment costs while it would chose an infinite trade volume in their absence. Second, price deviations can exceed shipping costs in equilibrium. Conversely, trade may occur when price difference does not exceed shipping costs\(^{11}\).

In a one-period version of the model with one good and a positive trade friction \( t \), first order condition implies: \( p^* - p - I(N) = 2c(N - N_{-1}) \) where \( N_{-1} \) is the last period’s trade volume. With \( c > 0 \) and \( |p^* - p| > t \), firm chooses a finite volume of trade that depends positively on \( p^* - p \) and last period’s trade volume \( N_{-1} \), and negatively on the cost parameters \( t \) and \( c \). Figure 2 compares the profit functions between linear and simplified QAC models when \( t = 0.2 \) and \( N_{-1} = 0 \). In the upper segment, autarkic price Abroad is 30% higher than at Home and trade takes place. The trade volume in a simple QAC model is finite because the profit function is parabolic (with a kink). In the lower segment, \( p^* \) is 15% below \( p \), and

\(^{11}\)Note that the quadratic adjustment cost model nests the linear shipping cost model. When \( c = 0 \), (12) is identical to (4).
trade does not take place.

Because trade is the only source of price adjustment, a smaller trade volume requires a smaller price adjustment. By lowering trade volume, adjustment costs lead to law of one price deviations in excess of the threshold $t$ in equilibrium. This is despite the fact that, in terms of price differences, quadratic adjustment cost model creates the same no-trade region as the linear model.

When $N_{-1} \neq 0$, the relationship between $N$ and LOPD is qualitatively unchanged as long as the good remains traded and $I(N)$ does not change. But the range of autarkic values of LOPD decreases in $N_{-1}$ (left-hand panel of Figure 3). When $N_{-1} \neq 0$, costly trade deceleration can imply positive trade volume even though $|p^* - p| < t$ as the firm strikes balance between contemporaneously loss-making trade and costs of trade deceleration. Therefore, profits can be negative in equilibrium when $c > 0$ (the right-hand panel of figure 3). Larger values of $|N_{-1}|$ require smaller $|p^* - p|$ to optimally induce trade.

The tendency for price differences to exceed marginal shipping cost also translates to the full version of the model: law of one price deviations increase in the endowment difference even when both goods are traded (the increasing thresholds can be seen in figure 8). As in the linear shipping cost model, three cases exist, and the influence of the substitution effect is visible in the change of the slope of $LOPD_2$ after good 1 becomes traded. Trade volume depends negatively on frictions $c$ and $t$. The adjustment costs force firms to spread trade in more steps of smaller magnitude: length of adjustment time depends positively on $c$ and $dY_i$. The general equilibrium effect in which price of a non-traded good is affected by trade in another good is also present: LOPD$_2$ is smaller than it would have been if good 1 was not traded. As expected, trade volume depends negatively on frictions $c$ and $t$, and that the adjustment costs force firms to smooth the trade changes in more steps of smaller magnitude (the length of adjustment time increases in $c$ and $dY_i$).

3.2 Solution method

Due to a high degree of non-linearity, the model is solved numerically. First, to limit the time span for adjustment, I assume a steady state equilibrium to which countries converge following a shock, and a number of time periods $T$ available for the adjustment. Conditional
on $T$, the model is solved using method of relaxation by Boucekkine (1995) in which a finite-period approximation $\hat{f}(\cdot)_{t=1:T} = 0$ to the system $f(\cdot)_{t=1:\infty} = 0$ is solved by stacking all equations for all time periods into one large system $F(\cdot) \equiv [\hat{f}(\cdot)_{t=1} ... \hat{f}(\cdot)_{t=T}]' = 0$ which is then solved numerically. Second, in order to compute the Jacobian of the stacked system $F(\cdot)$ in one step, it is necessary to select a functional form for $I(N_{i,t})$. This selection is described in detail in Appendix A. Third, to facilitate the numerical solver in finding an equilibrium, (12) is replaced with their simpler forms (14) and (15) which do not include an infinite forward-looking sum. This step facilitates convergence because an error in $p_{it}$ by the numerical solver only affects the $4(t-1):4(t+1)$ partition of the Jacobian, not all $(4T)^2$ values it would otherwise.

\[
\frac{1}{2c_1}(p^*_1 - p_{1,t} - t_1 I(N_{1,t})) = (1 + \beta)Y_{1,t} - \beta Y_{1,t+1} - Y_{1,t-1} + \gamma_1 p_{1,t-1} Y_{t-1} = - (1 + \beta)\gamma_1 p_{1,t} - \frac{Y_t}{p_{1,t-1}} + \beta \gamma_1 p_{1,t+1} Y_{t+1} = 0 \tag{14}
\]

\[
\frac{1}{2c_2}(p^*_2 - p_{2,t} - t_2 I(N_{2,t})) = (1 + \beta)Y_{2,t} - \beta Y_{2,t+1} - Y_{2,t-1} + \gamma_2 p_{2,t-1} Y_{t-1} = - (1 + \beta)\gamma_2 p_{2,t} - \frac{Y_t}{p_{2,t-1}} + \gamma_2 \beta p_{2,t+1} Y_{t+1} = 0 \tag{15}
\]

A system $\hat{f}(\cdot)_{t=i}$, part of the large stacked system $F(\cdot)$, consists of equations (14), (15) and (13). Period $T + 1$ values found in the inter-temporal Euler equations of $\hat{f}(\cdot)_{T}$ are set to steady-state equilibrium values associated with a full adjustment to the shock. Finally, values of $I(N_{i,t})$ in the approximate solution obtained above are replaced with 1, -1, or 0 and system $F(\cdot)$ is solved again to ensure that the approximation is valid.

### 4 Real exchange rate

This section explains the behaviour of the real exchange rate in the model for a range of parameter values when endowments are stochastic. Logarithm of the real exchange rate from the model is a weighted average of the three law of one price deviations\footnote{This is the method of constructing of RER in the empirical literature and is therefore appropriate when matching model’s results to the data. Each country $j$’s CPI is a geometric average of goods and services with weights corresponding to the consumption shares. Hence, $\log(CPI_j) = \gamma_1 p_{1,t} + \gamma_2 p_{2,t} + \gamma_3$. When $\gamma_i$ is the same in both countries, the RER result follows.}: $\log(RER) = \frac{1}{2c_1}(p^*_1 - p_{1,t} - t_1 I(N_{1,t})) = (1 + \beta)Y_{1,t} - \beta Y_{1,t+1} - Y_{1,t-1} + \gamma_1 p_{1,t-1} Y_{t-1}$
γ_1 \log(LOP\,D_1) + γ_2 \log(LOP\,D_2).\) At first, endowments Abroad are fixed while at Home they follow an AR(1) process: \(Y_{i,t} = αY_{i,t-1} + (1 - α)\bar{Y} + u_t\) \(i = 1, 2\) \((u_t \sim N(0, σ^2))\). Because RER depends on the endowments of traded and non-traded goods relative to other countries (see equation (8))¹³, the assumption that only one country is subject to the shocks and that both sectors receive the same shock is relaxed in sections 4.2 and 4.3, respectively.

### 4.1 Real exchange rate in a linear shipping cost model

Persistence of the real exchange rate in the linear model depends on three factors: *positively on the shipping costs*, *positively on the heterogeneity of the shipping costs* as measured by the ratio of \(t_2/t_1\) (greater heterogeneity increases the range of operation of the aforementioned general equilibrium effect), and *negatively on the volatility of endowment shocks* because smaller shocks are more likely to lead to initial price differences below the threshold.

Conditional on the trade friction, persistence of the real exchange rate is positively related to the persistence of the endowment shock process as measured by \(α\) (Table 1). For \(α ≤ 0.9\), half lives of convergence do not exceed 6 time periods. Half life increases sharply in \(α\) for values near 1, to about 11 when \(α = 0.95\), and up to 933 time periods when \(α = 0.99\). Variance of shocks decreases half life because it increases the likelihood of triggering arbitrage and consequently price convergence.

Volatility of the real exchange rate increases both in shock persistence \(α\) and shock volatility \(σ\) (Table 1). Endowment shocks increase LOPD volatility as long as at least one good is not traded. When both goods are traded, additional shock volatility is neutral because the additional price differences are arbitraged away. Higher \(α\) leads to longer-lived LOPDs, thus increasing their volatility, ceteris paribus. This is especially visible when \(σ\) is small so that most shocks leave LOPDs below their thresholds. RER volatility then exceeds \(σ\). Conversely, high \(σ\)s only have a marginal effect on std(RER).

Shipping costs increase persistence of real exchange rate for any given \(α\) and \(σ\) because a larger endowment shock is needed for arbitrage trade to occur. Moreover, *heterogeneity* of the shipping costs increases persistence and volatility. This effect is caused by the aforementioned substitution from traded into non-traded goods (section 2.4.3), resulting in a positive co-

¹³This is a standard result in multi-sector endowment models of real exchange rate determination.
movement of the LOPDs in case 2 which add to the persistence of the RER. Consequently, this effect is a possible source of the "aggregation bias" discovered by Imbs et al. (2005). This yields the increasing loci of persistence and volatility in $t_2/t_1$ (Table 2). The effect is stronger at higher values of $\alpha$.

4.2 Country-specific shocks

Now let the endowments vary in both countries, assuming they follow a similar AR(1) process:

\[
\dot{Y}_{i,t} = \alpha \dot{Y}_{i,t-1} + (1-\alpha) \dot{Y} + \dot{u}_t \quad \text{for} \quad i = 1, 2 \quad \text{where} \quad \dot{Y}_{i,t} = [Y_{i,t}, Y^*_{i,t}]', \quad \dot{u}_t = [u_t, u^*_t]' \quad \text{and} \quad \dot{u}_t \sim N(0, \dot{\Omega})
\]

where

\[
\dot{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}.
\]

The left panel of Figure 4 shows that RER persistence increases in the correlation coefficient $\eta$ ($\eta \equiv \frac{\dot{\gamma}}{\dot{\sigma}^2}$) while volatility decreases in $\eta$. Negatively correlated shocks lead to relatively larger LOPDs and larger average RER while positively correlated shocks lead to relatively smaller LOPDs and smaller average RER. With more mass of the RER distribution near the mean when $\eta > 0$, RER deviations do not change much from one period to another, leading to a more persistent and less volatile RER. When $\eta < 0$, RER distribution has a relatively larger proportion of the mass in its tails (near the thresholds). Repeated draws from this distribution lead to a process with less persistence (deviations differ from mean more often) and a higher volatility. The monotonicity of average LOPDs as $\eta$ increases leads to monotonicity in persistence as well as volatility when shocks are country-specific.

The aforementioned substitution effect from a traded into a non-traded good (case 1\textsuperscript{14}) affects the size of the RER and therefore its persistence and volatility. As trade lowers $|LOPD_T|$ to arbitrage threshold, $|LOPD_{NT}|$ also declines. Thus, substitution effect lowers the average $|RER|$.\textsuperscript{15} Because the proportion of case 1-trades (only one good traded) in all trades increases in $\eta$ when shocks are country-specific, influence of the substitution effect on the RER also increases in $\eta$. RER persistence is up to 8% higher and volatility up to 9% lower as a result of the substitution from traded into non-traded goods.

\textsuperscript{14}In case 2 when both goods are traded, substitution effect does not have a measureable effect on ex-post price deviations, only on the volume of trade.

\textsuperscript{15}When $\alpha = 0.88$, difference in the average $|RER|$ and $|RER_{noS.E.}|$ increases in $\eta$ from 3 to 9%.

16
4.3 Sector-specific shocks

Now assume that the endowments differ across sectors. For simplicity, endowments Abroad are kept constant and Home endowments follow an AR(1) process: $\tilde{Y}_t = \alpha \tilde{Y}_{t-1} + (1 - \alpha)\bar{Y} + \tilde{u}_t$ for $i = 1, 2$ where $\tilde{Y}_{i,t} = [Y_{1,t} Y_{2,t}]'$, $\tilde{u}_t = [u_{1,t} u_{2,t}]'$ and $\tilde{u}_t \sim N(0, \tilde{\Omega})$ where $\tilde{\Omega} = \begin{pmatrix} \sigma_2^2 & \gamma \\ \gamma & \sigma_1^2 \end{pmatrix}$. The right panel of Figure 4 shows the asymmetric U-shaped relationship between RER persistence and $\tilde{\eta}$, the correlation coefficient of shocks across sectors. Volatility of RER increases monotonically in $\tilde{\eta}$.

When $\tilde{\eta} < 0$, shocks to sectoral endowments at Home tend to be of opposite signs, leading to opposite signs of LOPDs (and the direction of trade flows) of goods 1 and 2. Such LOPDs partly cancel each other, bringing $|RER|$ closer to zero. When $\tilde{\eta} > 0$, shocks to sectoral endowments at Home tend to have the same sign, leading to LOPDs of identical signs and a larger average $|RER|$. From the definition of the shock process, frequency of $\{\text{sign}(LOPD_1) = \text{sign}(LOPD_2)\}$ increases in $\tilde{\eta}$, which causes $|RER|$ to be increasing in $\tilde{\eta}$ also. This drives the increasing tendency in RER volatility: repetitive draws from a distribution which is more compressed around its mean ($\tilde{\eta} < 0$) lead to a less volatile RER process. As $\tilde{\eta}$ increases, frequency of situations when both goods are traded in the same direction increases, and with it the frequency of $RER$ reaching its threshold ($\gamma_1 t_1 + \gamma_2 t_2$). As the mass of the distribution of $|RER|$ increases around the threshold, RER persistence increases$^{16}$.

The influence of the substitution effect on RER also depends on the signs of sectoral shocks. If the shocks are of opposite signs, the positive correlation between changes in $LOPD_T$ and $LOPD_{NT}$ due to substitution effect leads to trade induction. $LOPD_{NT}$ can be brought to its no-arbitrage threshold and, consequently, become traded. Conversely, if the sectoral shocks are of the same sign, non-traded good is less likely to become traded. This trade suppression effect is also a result of the substitution from a traded to non-traded good in the exporting country$^{17}$. Trade induction can either increase or decrease $|RER|$, depending on which good is not traded. Trade suppression always decreases $|RER|$. As the proportion

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$^{16}$RER persistence is high when $\tilde{\eta}$ is close to -1 because $\{\text{sign}(LOPD_1) = \text{sign}(LOPD_2)\}$ at all times, thus preventing $|RER|$ to exceed $\gamma_2 t_2 - \gamma_1 t_1$.

$^{17}$Equivalently, we can think of the effective degree of substitutability between goods depending on trade: when sectors receive endowment shocks of opposite signs, the ability to substitute between the products is limited by the (induced) trade, and conversely with shocks of identical signs.
of trade suppression increases in $\tilde{\eta}$, so does the downward influence of the substitution effect on $|RER|$ (mean $|RER|$ decreases by 2 to 8% and the standard deviation of RER by 3 to 5%). The shift of the RER distribution away from the RER threshold due to substitution effect leads to a decline in RER half-life between 1 and 22% (depending on $\tilde{\eta}$).

This exercise highlights that the persistence of RER does not imply equal persistence in its components. Moreover, small RER deviations may be mean reverting because they originate from larger deviations for individual goods of opposite magnitude - an effect which has been empirically documented by Crucini, Telmer and Zachariadis (2005)$^{18}$.

4.4 Real exchange rate in a model with quadratic adjustment costs

Real exchange rate in the quadratic adjustment cost model is necessarily more volatile and more persistent than in a linear shipping cost model because the additional friction reduces profitability of arbitrage following an endowment shock (see section 3.1.1 above).

Let the autoregressive endowment process follow $Y_{i,t} = \alpha Y_{i,t-1} + (1 - \alpha)\bar{Y} + u_t \ i = 1, 2$ where $u_t \sim N(0, \sigma^2)$, assuming that $t \in [1, T]$ and $u_t = 0$ for $t > 1$. This $T$-period simulation is repeated $M$ times$^{19}$. Table 3 reports the means of RER half-life and volatility estimates.

As in the linear cost model, half life of convergence decreases in $\sigma$ and increases in $\alpha$. The convergence speed declines in $\sigma$ at a much lower rate than in the linear shipping cost model. Adjustment costs increase RER half life as well as volatility estimates for any $\alpha$ and $\sigma$ (Table 4). Compared with the linear shipping cost model, RER volatility is less sensitive to $\sigma$. Higher $\sigma$ leads to a higher LOPD volatility which keeps their ratio unchanged. The standard deviation estimates decline in $\alpha$ because of a smoother adjustment in prices imposed by the quadratic adjustment costs.

5 Calibration

Preference parameters are calibrated to the usual values in the literature: weights of the utility function are symmetric ($\gamma_i = \frac{1}{3} \ \forall i$) and the elasticity of substitution $\theta$ assumes the

$^{18}$This effect works in the opposite direction to the "aggregation bias" effect introduced by Imbs et al. (2003).

$^{19}$Because of the limit on the number of periods needed for adjustment, results are not perfectly comparable between the two models. They are less precise in the quadratic adjustment cost model.
standard value 1.5 (see Chari et al. 2008, Chari, Christiano and Kehoe 1994, and McGrattan 1994). Shipping costs are calibrated directly as a tax (a heterogeneous iceberg cost) that disappears in the course of shipment, assuming they exhibit constant returns to scale. In particular, they depend multiplicatively on the distance and the weight of a good. The US and EU are chosen as locations because of their similar size. Distance between their two major ports New York and Hamburg (6000km) is used as the shipping distance (most goods are shipped by sea between Europe and the US).

Shipping cost frictions are calibrated from two sources. In a survey of transportation modes, Runhaar et al. (2001) quote an average price in 2001 for a standard 40’ container on a route Rotterdam – Singapore of NLG 3060 (USD 1220), including a fuel surcharge. They estimate the average load of a 40’ container is 16.25 ton, yielding an average rate of USD 0.0077 per ton per km. Perishable goods such as most of foodstuffs are shipped in chilled containers. In a survey of shipping costs for fish (chilled) containers Brox et al. (1984) survey costs across a range of distances. The implied per ton per km shipping costs is well approximated by a hyperbolic function (Figure 5). At the 6000km, it implies a unit cost of USD 0.11 per ton per km between US and Europe. A dataset of physical weights and average prices in Berka (2009) implies that 24% of the goods require refrigerating for transport. This yields an average shipping cost per ton per km of USD 0.033. An average weight of a good in the dataset is 43kg, and the average price USD 745 (2001 prices). Two weights (20kg and 66kg) picked to match the average weight of a good in that dataset imply per-kg-per-km shipping frictions $t_1 = 0.0054$ and $t_2 = 0.0174$, respectively. That is, about 0.54% of good 1 and 1.74% of good 2 get used in transportation. These cost estimates are conservative compared to the literature.

Calibration of the quadratic adjustment cost parameter $c$ does not appear in the literature.

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20 The CRS assumption is inconsequential, as there is only 1 distance (2 countries). Many authors calibrate the transportation costs using indirect estimates (e.g., Ravn and Mazzegna 2004). The dependance of shipping costs on weight has been established by many (e.g., see Table 7 in Hummels 1999).

21 Harrigan (1993) finds transportation barriers of 20%. Hummels (1999) uses 2-digit SITC data to estimate a transportation costs of 9%. Using 4-digit SITC data, Ravn and Mazzegna (2004) find that the weighted average of transportation costs declined from 6.31% in 1974 to 3.49% in 1994. IMF frequently uses 11% as a rule of thumb for transportation costs. All these are greater than the 1.14% average in my calibration. As can be seen in the right segment of Table 5, a higher calibration of trade costs has only a marginal effect on the results, in the expected directions. RER persistence and volatility increase while the comovement of the consumptions vectors declines due to a lower trade volume.
Consequently, $c$ is calibrated indirectly by letting the model track closely the co-movement of consumption vectors between the two countries, implying $c = 0.4$.

The stochastic endowment process at Home is calibrated to match the detrended quarterly U.S. GDP series from 1973:1 to 1994:4, implying AR(1) coefficient $\alpha = 0.88$. The standard deviation of the shocks is selected so as to reproduce output volatility in the data, 1.82%. Similarly, cross-correlation of the endowment shocks is chosen to reproduce the correlation between output processes is equal to US-EU output correlation: $corr(Y, Y^*) = 0.6$ (see Chari, Kehoe and McGrattan 2008). This calibration makes the model stationary, with no secular trends in any variables. Due to the lack of a trend, the model does not need to be de-trended, and the model-based statistics are comparable with the statistics based on de-trended data.

6 Simulation results

6.1 Simulation results in a linear shipping cost model

To assess quantitative properties of the real exchange rate, a bivariate vector of 10,000 normally distributed shocks is used to generate the stochastic endowment vectors at Home and Abroad\(^{22}\): $\hat{Y}_{i,t} = \alpha \hat{Y}_{i,t-1} + (1 - \alpha)\tilde{Y} + \hat{u}_t$ for $i = 1, 2$ where $\hat{Y}_{i,t} = [Y_{i,t} Y^*_{i,t}]'$, $\hat{u}_t = [u_t u^*_t]$ and $\hat{u}_t \sim N(0, \hat{\Omega})$ where $\hat{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. Table 5 summarizes statistics of interest.

6.1.1 Persistence

Persistence of logarithm of the real exchange rate in the linear model aligns closely with the persistence in the data. Model’s AR(1) estimate $\hat{\alpha} = 0.845$ with a standard error 0.0053 implies a half-life of convergence of about 4.1 quarters (3.7 in the data). Price deviations for good 2 are more persistent than for good 1 (AR(1) slope estimates of 0.86 and 0.77, respectively). The core result of the linear model is that the heterogeneity of marginal shipping costs leads to persistent RER deviations. Persistence of the GDP and consumption in the model matches the data very closely, as well.

\(^{22}\) Qualitative properties of the solution are described in section 2.4. Good 1 is traded more frequently (86% of the time periods) than good 2 (32%) (see Figure 6). Distribution of law of one price deviations in Figure 6 is clearly bimodal, with peaks corresponding to thresholds.
6.1.2 Comovements

The real exchange rate is partially disconnected from the real economy. A sufficiently large endowment difference lowers prices at Home relative to Abroad. This depreciation (increase) in the real exchange rate leads to a $corr(RER, Y) = 0.44$, somewhat higher than in the data. As in Chari, Kehoe and McGrattan 2008 (CKM hereafter), real exchange rate is highly positively correlated with relative consumption vectors (0.9 compared to -0.35 in data and 1 in CKM) as the expenditure-switching motive is not sufficiently strong to decouple the two. Correlation of consumptions between countries is strong (0.6, vs. 0.38 in the data and 0.49 in CKM) because trade instantaneously eliminates endowments differences that lead to arbitrage opportunities. Because net exports are zero in equilibrium, their comovements in the data can not be matched by the model\textsuperscript{23}.

6.1.3 Volatility

With zero net export balance due to instantaneous trade, relative consumption volatility equals that of GDP, which is marginally higher than in the data. Linear shipping cost model does not generate sufficient RER volatility because adjustment to endowment shocks is instantaneous. $|RER|$ has a well-defined maximum, equal to a weighted average of the trade frictions ($= (t_1 + t_2)/3$), limiting volatility\textsuperscript{24}.

6.2 Simulation results in a quadratic adjustment cost model

The additional friction brings countries' consumption sets closer to their endowments and therefore leaves equilibrium prices further away from the parity. Hence, volatility of RER increases while the trade volume stays positive for longer periods of time.

\textsuperscript{23}Substituting zero arbitrage profits into the household budget constraint implies that the value of absorption equals the value of endowments.

\textsuperscript{24}Nominal rigidities could increase volatility of the price aggregates over that of the endowment shocks. However, unlike quadratic adjustment costs, nominal rigidities are orthogonal to the mechanism present in the model.
6.2.1 Persistence and comovements

The real exchange rate persistence remains very close to the data, with an AR(1) coefficient estimate of 0.81 in the baseline calibration. Persistence of consumptions and net exports also nearly matches the data. The comovement of variables in the QAC model is also very close to the data. Correlation of consumptions at Home and Abroad of 0.54 is near the 0.38 in the data. At 0.16, $corr(RER, Y)$ is also much closer to the data (0.08) than in CKM (0.51). Real exchange rate is nearly uncorrelated with net exports in the model (0.02), which is not far from the 0.14 in the data. As is standard in the international business cycle literature, the model fails to capture the negative correlation of RER and the relative consumption vector (-0.35 in the data, vs. 0.96 in the model and 1 in CKM).

6.2.2 Volatility

Quadratic adjustment cost model is successful in generating some volatility of prices relative to GDP. The average standard deviation estimate is 1.34, some way from the 4.4 in the data but a significant improvement on the linear shipping model, and achieved without nominal rigidities. As before, $LOPD_2$ is more volatile than $LOPD_1$ and the distribution of $LOPD$s is bimodal with a larger mass near the thresholds. The bimodality is not as pronounced as in the linear model because thresholds increase in endowment differences (Figure 8). Due to the small amount of trade (which leads to $std(NX)$ below data), economies are more disconnected than in reality. As a result, aggregate consumption is more volatile than in the data.

7 Conclusions and extensions for future research

This paper studied two general equilibrium models in which persistence and volatility of real exchange rate in equilibrium are a result of heterogeneity in shipping costs due to the importance of goods’ physical characteristics in transport. In both models, tradability of a good is endogenously determined by the endowment differences and trade frictions of all goods. Goods with larger trade frictions need a larger deviation from parity to become traded and are therefore traded relatively less frequently. The calibration exercise shows that half
life of real exchange rate deviation can match the estimates observed in the data.

Firms in the second model also pay a quadratic adjustment cost if they change their volume of trade from one period to the next. Changes in trade volume require hiring of labour resources, adjustment in the distribution system and possibly investment in new (or a changes of the existing) trade infrastructure. Adjustment costs are intended to capture the time dimension of shipping and eliminate the unrealistic assumption of an instantaneous adjustment from the linear model. In the dynamic non-linear environment, firms’ aversion to react to endowment shocks by large adjustments in trade volume creates larger real exchange rate deviations. Although without any nominal frictions, the second model explains about third of the observed RER volatility. It also performs very well in bringing comovements of other relevant variables in line with the data. In this sense, heterogeneity in shipping costs is a plausible, empirically relevant candidate explanation for the observed persistence and volatility in real exchange rates.

In both models, heterogeneity generates additional RER persistence because of the effect substitution between traded and non-traded goods has on trade volume and price differences. When consumers substitute away from a rising price of the export good, higher demand for the good which is not traded increase its price. This brings the price difference of the non-traded good further from the marginal shipping cost and lowers the probability that the good becomes traded. To the extent that trade allows adjustment of countries to shocks, the aforementioned effect acts to insulate the economy without any nominal or real rigidities (a converse result is possible depending on the exact type of the endowment shock). Equivalently, we can think of the effective degree of substitution between products as being endogenous to the size of heterogeneous trade frictions.

A Appendix: Approximating the absolute value function

A suitable choice is \( I(N_{i,t}) \equiv dG = \frac{2}{\pi} \arctan(\lambda N_{i,t}) \) where \( \lambda \) is a choice parameter which governs the approximation error. An inverse of a trigonometric function \( \tan(x) \), \( \arctan(x) \) has a range of \([-\pi/2, \pi/2]\) for \( x \in \mathbb{R} \) and is monotonically increasing, continuously differentiable, and has a convenient property that \( \arctan(x) < 0 \) when \( x < 0 \), \( \arctan(x) > 0 \) when \( x > 0 \) and \( \arctan(0) = 0 \). Further, \( \arctan(\lambda x) \) can reach the bounds arbitrarily fast. Premultiplying
it by $2/\pi$ changes its range to [-1,1], creating a "continuous step function". High $\lambda$ lowers the approximation error, as can be seen in Figure 7. A choice of $\lambda = 10^{40}$ makes the approximation error indistinguishable from zero for any feasible stopping criterion of the numerical solver. However, it is misleading to use this approximation to describe the first order conditions of a system with $|N_{i,t}|$ because the absolute value function is not differentiable at 0. Therefore, a smooth approximation $G(N_{i,t})$ to $|N_{i,t}|$ needs to be constructed first, and then differentiated. Conveniently, function

$$G(N_{i,t}) \equiv \int g(N_{it})dN = \frac{2}{\pi} \left[ \lambda N_{i,t} \left( \frac{2}{\pi} \arctan(\lambda N_{i,t}) - 0.5 \log(1 + (\lambda N_{i,t})^2) \right) \right]$$

can be used to arbitrarily closely approximate $|N_{i,t}|$ by a choice of $\lambda$ (see figure (9)).

References


**Tables and Figures**
Table 1: log(RER) Half-lives, standard deviation, and the shock process in a linear model

<table>
<thead>
<tr>
<th>α</th>
<th>0.65</th>
<th>0.7</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ = 0.008</td>
<td>1.65</td>
<td>1.99</td>
<td>2.44</td>
<td>3.12</td>
<td>4.20</td>
<td>6.16</td>
<td>11.14</td>
<td>93.35</td>
</tr>
<tr>
<td>σ = 0.019</td>
<td>1.45</td>
<td>1.72</td>
<td>2.09</td>
<td>2.61</td>
<td>3.36</td>
<td>4.64</td>
<td>7.5</td>
<td>57.69</td>
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<tr>
<td>σ = 0.034</td>
<td>1.26</td>
<td>1.48</td>
<td>1.77</td>
<td>2.15</td>
<td>2.71</td>
<td>3.58</td>
<td>5.53</td>
<td>172.2</td>
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<td>σ = 0.068</td>
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<td>1.23</td>
<td>1.46</td>
<td>1.75</td>
<td>2.17</td>
<td>2.8</td>
<td>4.35</td>
<td>60.8</td>
</tr>
</tbody>
</table>

Each result is based on 10000 simulations of the linear shipping cost model when \( t_1 = 0.02 \) and \( t_2 = 0.04 \). \( α \) is the AR(1) coefficient of the shock process, \( σ \) is the standard deviation as a proportion of mean GDP.

Table 2: Half-lives of log (RER) and the relative trade friction in a linear model

<table>
<thead>
<tr>
<th>( t_2/t_1 )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_2/t_1 = 2 )</td>
<td>1.4</td>
<td>1.9</td>
<td>2.9</td>
<td>5.0</td>
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<tr>
<td>( t_2/t_1 = 4 )</td>
<td>1.4</td>
<td>2.0</td>
<td>3.1</td>
<td>6.1</td>
</tr>
<tr>
<td>( t_2/t_1 = 6 )</td>
<td>1.4</td>
<td>2.0</td>
<td>3.1</td>
<td>6.2</td>
</tr>
<tr>
<td>( t_2/t_1 = 8 )</td>
<td>1.4</td>
<td>2.0</td>
<td>3.1</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Each result is based on 2000 simulations of the linear shipping cost model starting from \( t_1 = 0.02 \) and \( t_2 = 0.04 \). \( α \) is the AR(1) coefficient of the shock process, \( σ \) is the standard deviation as a proportion of the mean GDP.

Table 3: Mean half-lives of log(RER) in a quadratic model

<table>
<thead>
<tr>
<th>c = 0.01</th>
<th>α = 0.7</th>
<th>α = 0.8</th>
<th>α = 0.9</th>
<th>α = 0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ = 0.8%</td>
<td>1.9444</td>
<td>3.12</td>
<td>2.9</td>
<td>5.0</td>
</tr>
<tr>
<td>σ = 1.9%</td>
<td>1.9438</td>
<td>3.11</td>
<td>6.8</td>
<td>–</td>
</tr>
<tr>
<td>σ = 3.4%</td>
<td>1.9436</td>
<td>3.11</td>
<td>6.7</td>
<td>–</td>
</tr>
<tr>
<td>σ = 6.8%</td>
<td>1.9433</td>
<td>3.1</td>
<td>6.6</td>
<td>–</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c = 0.1</th>
<th>α = 0.7</th>
<th>α = 0.8</th>
<th>α = 0.9</th>
<th>α = 0.99</th>
</tr>
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<tr>
<td>σ = 0.8%</td>
<td>1.943</td>
<td>3.105</td>
<td>6.567</td>
<td>68</td>
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<tr>
<td>σ = 1.9%</td>
<td>1.9429</td>
<td>3.1048</td>
<td>6.567</td>
<td>66</td>
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<tr>
<td>σ = 3.4%</td>
<td>1.9428</td>
<td>3.1047</td>
<td>6.515</td>
<td>65</td>
</tr>
<tr>
<td>σ = 6.8%</td>
<td>1.9428</td>
<td>3.09</td>
<td>6.522</td>
<td>63</td>
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</table>

Each result is based on 1000 simulations of the model when \( T = 20, t_1 = 0.0054 \) and \( t_2 = 0.0174 \) (see section 5). \( α \) is the AR(1) coefficient of the shock process.

Table 4: Volatility of log(RER) in a quadratic model

<table>
<thead>
<tr>
<th>[Mean std(IRER)]/[Mean std(GDP)]</th>
<th>α = 0.7</th>
<th>α = 0.8</th>
<th>α = 0.9</th>
<th>α = 0.99</th>
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<tbody>
<tr>
<td>σ = 0.8%</td>
<td>27.62</td>
<td>14.39</td>
<td>12.59</td>
<td>8.49</td>
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<tr>
<td>σ = 1.9%</td>
<td>17.83</td>
<td>14.88</td>
<td>12.35</td>
<td>8.42</td>
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<tr>
<td>σ = 3.4%</td>
<td>17.68</td>
<td>14.52</td>
<td>12.24</td>
<td>8.54</td>
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<tr>
<td>σ = 6.8%</td>
<td>17.07</td>
<td>14.25</td>
<td>12.14</td>
<td>8.75</td>
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</table>

<table>
<thead>
<tr>
<th>[Median std(IRER)]/[Med. std(GDP)]</th>
<th>α = 0.7</th>
<th>α = 0.8</th>
<th>α = 0.9</th>
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</tr>
</thead>
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<tr>
<td>σ = 0.8%</td>
<td>1.951</td>
<td>1.952</td>
<td>1.953</td>
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<td>σ = 1.9%</td>
<td>1.950</td>
<td>1.951</td>
<td>1.952</td>
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<td>1.950</td>
<td>1.950</td>
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<td>σ = 6.8%</td>
<td>1.949</td>
<td>1.949</td>
<td>1.950</td>
<td>2.71</td>
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</tbody>
</table>

Each result is based on 1000 simulations of the model when \( T = 20, t_1 = 0.0054 \) and \( t_2 = 0.0174 \) (see section 5). \( α \) is the AR(1) shock coefficient.

---

28
<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>linear model&lt;sup&gt;1&lt;/sup&gt;</th>
<th>QAC model</th>
<th>CKMcG&lt;sup&gt;3&lt;/sup&gt;</th>
<th>QAC&lt;sup&gt;t=2t_{base}&lt;/sup&gt;</th>
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<tr>
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<tr>
<td><strong>Ex. rates &amp; prices</strong></td>
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<tr>
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<td>0.81</td>
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<tr>
<td>GDP</td>
<td>0.88</td>
<td>0.88*</td>
<td>0.88*</td>
<td>0.62</td>
<td>0.88</td>
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<td>Consumption</td>
<td>0.89</td>
<td>0.88</td>
<td>0.88</td>
<td>0.61</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>Net Exports</td>
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<td>–</td>
<td>0.93</td>
<td>0.72</td>
<td>0.93</td>
<td>0.89</td>
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<td><strong>STD rel. to GDP</strong></td>
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<td><strong>Ex. rates &amp; prices</strong></td>
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<td>RER</td>
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<td>0.001</td>
<td>1.34</td>
<td>4.27</td>
<td>1.53</td>
<td>1.78</td>
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<td><strong>Business cycle stat</strong></td>
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<td></td>
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<tr>
<td>Consumption</td>
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<td>1</td>
<td>1</td>
<td>0.83</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Net Exports</td>
<td>0.11</td>
<td>–</td>
<td>0.002</td>
<td>0.09</td>
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<td>0.002</td>
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<td><strong>Cross-Correlat.</strong></td>
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<tr>
<td>GDPs</td>
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<td>0.6*</td>
<td>0.6*</td>
<td>0.49</td>
<td>0.61*</td>
<td>0.61*</td>
</tr>
<tr>
<td>Consumptions</td>
<td>0.38</td>
<td>0.6</td>
<td>0.54</td>
<td>0.49</td>
<td>0.54</td>
<td>0.53</td>
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<tr>
<td>NX &amp; GDP</td>
<td>-0.41</td>
<td>–</td>
<td>0.08</td>
<td>0.04</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>RER &amp; GDP</td>
<td>0.08</td>
<td>0.44</td>
<td>0.16</td>
<td>0.51</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>RER &amp; NX</td>
<td>0.14</td>
<td>–</td>
<td>0.02</td>
<td>-0.04</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>RER &amp; Relat. C</td>
<td>-0.35</td>
<td>0.903</td>
<td>0.96</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

<sup>1</sup> based on 10,000 simulations of the linear shipping model with parameter calibration as described in section 5.
<sup>2</sup> based on 1,000 simulations (each 30 periods long) of the quadratic adjustment cost model ($c = 0.4$).

* denotes a calibrated value.

Figure 1: Model solution: thresholds of price deviations in linear model
Figure 2: Volume and prices become detached due to adjustment costs ($N_{-1} = 0$)

Figure 3: Trade and profits in partial equilibrium in QAC model for various LOPDs and $N_{-1}$. $c=0.01$, $t=0.2$

Figure 4: RER properties with country- and sector- specific endowment shocks in linear model (shock volatility as a proportion of GDP: $\sigma = 0.034$)

Figure 5: Calibration of per-kg-per-km shipping cost for cooled sea transport
Figure 6: Distribution of trade and price differences. US-EU simulation of the linear model

Distribution of the trade in good 2 and good 1 (red) from the US−EU model. 10000 simulations.

Distribution of the LOPD for good 2 and LOPD for good 1 (red) from the US−EU model. 10000 simulations.

Figure 7: Approximating the indicator function in QAC model

Figure 8: Thresholds in QAC model when c=0.001 and c=0.1
Figure 9: Approximating functions $g(N)$ and $G(N)$ for $\lambda = 10^5$. 

\begin{center}
\includegraphics[width=\textwidth]{figure9}
\end{center}