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CAUSAL APPORTIONMENT: REPLY
TO THE CRITICS

MARIO J. RIZZO and FRANK S. ARNOLD*

In an article appearing in the Columbia Law Review we outlined a theory of liability apportionment based on the idea of relative causal contributions. These contributions were defined in terms of a framework of probabilistic causation. Accordingly, in its simplest form, if A and B simultaneously combine to kill P, we can say that A is twice as important a cause if the probability of A alone killing P was twice that of B alone killing P. Apportionment on the basis of these relative causal contributions means that A bears twice the burden of liability that is borne by B.

David Kaye and Mikel Aickin have recently commented on our system in this Journal. Their criticism, however, is based on a significant misunderstanding of the theory. Indeed almost every problem they have with our apportionment formulas is rooted in a failure to appreciate the nature of probabilistic causation. Our response is accordingly divided into four parts. The first clarifies the nature of probabilistic causation by dispelling

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4 This in part stems from neglect of Rizzo, supra note 2.
our critics’ confusion about our definition of a probabilistic marginal product. The second part shows that a correct understanding of causal independence and interaction leads to precisely the apportionment ratios we originally constructed. The third part demonstrates that the computation of a probabilistic marginal product is free from the ambiguity Kaye and Aickin allege. The final part corrects some misunderstanding about the application of our apportionment system to a special case of chemical interaction.

I. Defining Causes in Isolation

Kaye and Aickin have difficulty understanding the idea of a probabilistic marginal product (PMP), that is, the increment in the probability of a harm attributable to the operation of one cause alone. They argue that we are faced with a dilemma. On the one hand, the PMP’s of causes $A$ and $B$ are independently definable only if $A$ and $B$ have never happened together ($A$ and $B$ are “disjoint”). In this case we can simply compute the PMP of $A$ (or $B$) as the proportion of $A$’s (or $B$’s) alone in which a harm, $H$, also occurred. Unfortunately this excludes situations in which both $A$ and $B$ jointly occur, the very type of case in which we are interested. If, on the other hand, $A$ and $B$ have happened together (they are “conjoint”), then they argue that it is impossible to distinguish the risk created by $A$ (or $B$) alone. In some of those cases in which $A$ occurred, $B$ did as well. To resolve this dilemma, they suggest that the probability that $A$ (or $B$) caused harm be computed as the ratio of the number of times $A$ (or $B$) alone was present to the number of times $H$ was also present.

The problem discussed by Kaye and Aickin exists only on the assumption that a probabilistic causal analysis amounts simply to counting actual frequencies. The mere presence of multiple causes does not mean that the processes generated are physically interactive and, hence, that we cannot ascertain the independent risk produced by each. Two causes can be simultaneously present without being causally interactive. Therefore, causal independence and conjointness are consistent. Independence implies that the probability of $A$ causing $H$ is invariant with respect to the presence or absence of $B$. For example, the probability of heads on a given toss of a coin is unaffected by whether another coin is simultaneously tossed.

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5 This addresses the points made by Kaye & Aickin, supra note 3, at 193–97.
6 The concept of probability underlying our system is that of informed degree of belief. While actual relative frequencies are not irrelevant to a degree-of-belief assessment, they are generally supplemented by other forms of knowledge.
7 In symbols the independence condition is: $P(H_a | A \cap B) = P(H_a | A \cap B^c)$, where $H_a$ is the harm caused by $A$, and $B^c$ is the absence of $B$ or, equivalently, the presence of the “complement” of $B$. An analogous condition can be defined for $B$. 
The PMP’s of A and B (α and β, respectively) are computed on the tentative assumption that A and B are causally independent. Thus in the Venn diagrams displayed by Kaye and Aickin the darkly shaded triple overlap area (cases in which A, B, and H all occur together) should be included in the calculation of α and β. This means that situations of H in which both A and B occurred together are counted twice: once in the PMP of A and again in the PMP of B. This is perfectly correct. On the assumption of causal independence, these cases are those in which A and B each independently caused the relevant harm.

II. DEFINING THE APPORTIONMENT RATIO

Our critics next discuss the apportionment ratios that we construct on the basis of the probabilistic marginal products associated with each cause. Their argument is twofold. First, they claim that a key term (δ) in the apportionment ratio is simply “meaningless.” Second, they argue that if it (δ) is reinterpreted in a way that makes sense, then our apportionment ratio can be shown to be incorrect.

The failure of Kaye and Aickin to appreciate the meaning of δ and a related term, γ, is another consequence of their confusion about physical interaction between causes. Specifically, they do not distinguish between the “accumulated probabilistic effect” of two independent causes and actual physical interaction between causes (“synergism”). Suppose, for example, an individual will die if there is at least one head in a set of two simultaneous but independent tosses of fair coins (this means, of course, that the second “head” is redundant). What is the probability, estimated

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8 See Kaye & Aickin, supra note 3, at 195–97. To the extent that they are used at all, the Venn diagrams should be interpreted as pictorial representations of informed degrees of belief and not as mere counting of actual frequencies. See note 6, supra. Furthermore, on our degree-of-belief interpretation of the Venn diagrams, A (or B) is never pictured as having taken place when H occurs unless a causal relationship between A (or B) and H is claimed. This means that the Venn diagrams merely illustrate what is determined on other grounds rather than provide the basis for probability assessments, as Kaye and Aickin believe.

9 The assumption of causal independence cannot be easily illustrated by the Venn-diagram technique. Causal independence implies that the ratio of H ∩ A ∩ B to A ∩ B is greater than either that of H ∩ B to B or H ∩ A to A. The degree to which it is greater is determined precisely by δ (or in the more general case by γ). On the other hand, if independence is not postulated to underlie the Venn diagrams, they are heuristically misleading. The area of triple intersection would then consist of two very different cases: one in which two independent causes just happen to cause the harm simultaneously, the other in which the two cases physically interact to bring about the result. The first is a part of the independent probabilities of producing harm (α and β), while the second is not and enters the analysis at a later stage. This is the significance of the relationship between γ and δ. See text infra at notes 12–14; and Rizzo & Arnold, supra note 1, at 1410–11.

10 Kaye & Aickin, supra note 3, at 198.

11 Id. at 200.
before the start of the game, that the "targeted" individual will die? It is clearly greater than .5 because he must escape death twice, not once. The accumulated probability that he will die is the sum of: (1) the probability of heads on the first toss ($\alpha$) and tails on the second ($1 - \beta$); (2) the probability of tails on the first ($1 - \alpha$) and heads on the second ($\beta$); and (3) the probability of heads on both tosses ($\alpha \beta$). The probability of death ($\delta$) in this case is:

$$\delta = \alpha(1 - \beta) + \beta(1 - \alpha) + \alpha \beta \quad (1)$$

or .75, for $\alpha$ and $\beta$ equal to 0.5.

The relationship above does not imply that the tosses or causes interact in any physical way to produce the harm. Their relationship is simply notional or, as we said in our original article, they are "two causes acting independently but taken together" conceptually. Thus we construct $\delta$, "the definitional joint probability," as a foil against which we can define true physical interaction. If the actual joint probability ($\gamma$) arising from two causes exceeds the definitional joint probability ($\delta$) arising from independent causes, we have a physical synergism. But if $\gamma$ equals $\delta$, then the only mechanism at work is the phenomenon of having to avoid two separate and unrelated probabilities of death.

Turning now to Kaye and Aickin's specific criticism we can easily see that they miss the mark. Kaye and Aickin incorrectly interpret $\delta$ as the probability that $A$ and $B$ occur given $H$, or $P(A \cap B | H)$. This error stems from their interpreting $A$ and $B$ as outcomes of causes rather than as causes of outcomes, as we intended. Thus Kaye and Aickin interpret $A$ and $B$ as the result (for example, a "one") when each of two dice is thrown. This simple confusion accounts for their claim that $\delta$ is mysterious and pointless. Naturally they are amazed that we should be interested in the probability of a cause ($A$ or $B$) given its effect ($H$). Of course we are not; all of this simply has nothing to do with our $\delta$.

Kaye and Aickin then proceed to revise the basic apportionment ratio. They first "reinterpret" $\delta$ in a way that makes sense, using their own notation. They refer to our equation (1) above as "$P(H | A \cap B)$." This is the probability of an effect $H$ (for example, an individual's death) given

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12 Rizzo & Arnold, supra note 1, at 1410.

13 Suppose we have two causes, $A$ and $B$, that can produce a harm. If $\alpha = .5$ and $\beta = .5$, then $\delta = .75$. If the actual joint probability ($\gamma$) is equal to .90, the causes are physically (causally) interactive. The degree of synergism is measured by $\gamma - \delta$, or .15 in this case.

14 See Kaye & Aickin, supra note 3, at 198.

15 Id. at 198, equation 4.

16 Id. at 200, equation 7"
causes A and B taken together. Making use of this relationship, they construct an apportionment ratio that supposedly "contain[s] the elements of the Rizzo and Arnold presentation" and then arrive at a "conclusion [that] is different [from ours] even in [a] simple situation."\textsuperscript{17} The reason for this difference seems to lie in their incorrect belief that δ has something to do with physical interaction between two causes. Thus their apportionment ratio allocates the last term of δ (αβ in equation 1) equally between the causes.

To see the error in this consider the following example. Suppose, first, that an individual dies after two gun shots, one of which (A) hits him in the heart, the other of which (B) hits his brain, and, second, that there is no physical interaction between the two wounds. Suppose further that A is considered more serious and B less serious (α > β) but, ex hypothesi, we cannot determine which actually killed him in this case. The victim might have died in any one of three indistinguishable scenarios: (1) through cause A, but not B; (2) through cause B, but not A; or (3) through both. (This is analogous to the coin toss example above.) Kaye and Aickin would apportion the probability of death through the third scenario (αβ) equally between A and B. This would make sense if δ and, hence, αβ represented a physical interaction rather than simply a notional accumulated probability. Quite the contrary, however, case three is a situation where A and B simultaneously but independently kill the victim. Accordingly, the PMP of A must encompass those situations where A caused harm but B did not, and where A caused harm and B did as well. The fact that B also caused harm should not detract from A's causal potency.

The probability α that A will result in H can be decomposed as α(1 − β) + αβ to compare it with Kaye and Aickin's interpretation. Similarly β, the PMP of B, can be decomposed as β(1 − α) + αβ. The first term in each of these expressions is the portion of A's or B's PMP calculated in the absence of the other cause. The second term in each is αβ, or the portion of A's and B's PMP when the other cause is operating. Note that both expressions can be multiplied through to yield α and β, respectively. This makes clear that, in our approach, the causal potency of A (B) was originally defined without reference to B's (A's) existence. Moreover, the final term in each of these expressions is not αβ/2, as Kaye and Aickin believe,\textsuperscript{18} but simply αβ. This is not double counting because αβ represents a state of the world in which both causes are efficacious. Our appor-

\textsuperscript{17} Id. at 200.

\textsuperscript{18} Id. at 200, equation 8.
tionment of liability between $A$ and $B$ is based on the PMP of each cause irrespective of the existence of the other. Thus we can write the ratio of $A$’s share to $B$’s share (using the expanded forms of the PMP’s of $A$ and $B$) as $[(\alpha(1 - \beta) + \alpha\beta)/(\beta(1 - \alpha) + \alpha\beta)]$. Note that the $\alpha\beta$ term is included in the PMP of both $A$ and $B$. It is not, as Kaye and Aickin propose, split in half between $A$ and $B$ (as $\alpha\beta/2$). In our formulation the numerator simplifies to $\alpha$ and the denominator to $\beta$. When the PMP’s are thus computed correctly, the Kaye and Aickin ratio of damage shares reduces to our $\alpha/\beta$. This is the appropriate ratio in the absence of physical synergism.

III. THE UNIQUENESS OF THE PMP

The central problem Kaye and Aickin have in interpreting our $\delta$ (discussed in the previous section) is also related to their inability to arrive at unique expressions for the probabilistic marginal product. Kaye and Aickin hypothesize two gunslinging tortfeasors, both of whom appear simultaneously to kill an individual. $^{19}$ The probability that the “first” wrongdoer killed the victim is $.8 \ (= \text{PMP}_a)$, and that the “second” killed him is $.4 \ (= \text{PMP}_b)$. If damages are apportioned according to our theory, A would pay twice as much as B. Kaye and Aickin recognize this in their first computation. But then they go on to a second allegedly equally valid analysis in which the PMP’s are construed differently. Instead of calculating each tortfeasor’s marginal contribution in comparison to a baseline state in which no wrongful conduct by the other party has occurred, they alter the baseline to include the other tortfeasor’s wrongful conduct. The first method (which is our approach) assumes that initially no other causes are operating and then adds one cause to determine its PMP. That is, the risk that each cause imposes is computed relative to a baseline in which the other cause has not occurred. $^{20}$ The second method (Kaye and Aickin’s) assumes that “the other cause already is acting” $^{21}$ and then adds a second cause to determine its PMP. These two methods do not give the same result. To illustrate, assume that there is no physical synergy or interaction between the causes, so that $\gamma = \delta = .9$ $^{22}$ Kaye and Aickin’s method assumes that if $B$ is already acting, then $A$ adds $.9 - .4$, or $.5$. If $A$ is already acting, $B$ adds $.9 - .8$, or $.1$. The PMP’s are thus dramatically different from those in the original analysis above.

$^{19}$ Id. at 202.

$^{20}$ Nevertheless, our calculations of the PMP’s of each cause, as discussed in the previous section, include cases in which both causes may simultaneously result in the harm.

$^{21}$ Kaye & Aickin, supra note 3.

$^{22}$ $\delta$ is exactly equal to .88 given the values of $\alpha$ and $\beta$ dictated by Kaye and Aickin’s example; .9 is used here for ease of exposition.
Despite a three-dimensional diagram designed to support the general point, the problem of ambiguous probabilistic marginal products is illusory. This can be seen by looking at the issue in general terms. The PMP’s computed under the second technique are, for $A$ and $B$, respectively, $\delta - \beta$ and $\delta - \alpha$. For convenience let us call them $\alpha'$ and $\beta'$. Recalling that $\delta = \alpha(1 - \beta) + \beta(1 - \alpha) + \alpha\beta$, and after collecting terms, we can write the alternate PMP’s as

$$\alpha' = \alpha(1 - \beta)$$  \hspace{1cm} (2)

$$\beta' = \beta(1 - \alpha).$$  \hspace{1cm} (3)

Comparing these with the expanded forms of $\alpha$ and $\beta$ in the previous section, it is easy to see that the Kaye and Aickin computation excludes the states of the world in which $A$ and $B$ simultaneously, but independently, cause harm. (This is apparently how Kaye and Aickin define $\alpha$ and $\beta$, as we saw above.)

The logical foundation of our apportionment scheme is at variance with these alternative PMP’s. The mere presence of $B$ does not vitiate or neutralize $A$’s causal potency, nor does the mere presence of $A$ neutralize $B$’s causal potency. Thus, as we have argued above, in computing the riskiness of each factor all of the circumstances in which it results in harm must be included. This, however, cannot be done if $\alpha$ (or $\beta$) is merely the residual after $\beta$ (or $\alpha$) has been subtracted from $\delta$. For in each case the $\alpha\beta$ term would be implicitly attributed to the other factor. But since the same computation would be consecutively applied to both factors, the term would then be excluded from both. Hence the Kaye and Aickin method of computing the PMP yields an implication that is inconsistent with our system’s underlying logic.

The analysis is essentially unchanged if physical synergy is present, that is, if $\gamma > \delta$. In this case, Kaye and Aickin apparently compute the PMP’s of $A$ and $B$ as $\gamma - \beta$ and $\gamma - \alpha$, respectively. Denoting these as $\alpha''$ and $\beta''$, substituting $\alpha(1 - \beta) + \beta(1 - \alpha) + \alpha\beta$ for $\delta$ as above, and collecting terms, these can be written as:

$$\alpha'' = \alpha(1 - \beta) + (\gamma - \delta)$$  \hspace{1cm} (4)

$$\beta'' = \beta(1 - \alpha) + (\gamma - \delta).$$  \hspace{1cm} (5)

However, their formulation is now inconsistent with our method for two reasons. In the case of physical synergy, Kaye and Aickin’s method still excludes $\alpha\beta$, as in the case of no synergy. In addition, Kaye and Aickin also include the entire amount of synergy $(\gamma - \delta)$ in each PMP, rather

23 Kaye & Aickin, supra note 3, at 203.
than dividing the synergistic effect equally between the two parties. These are errors in opposite directions, but only by chance will Kaye and Aickin’s method yield our apportionment of liability. Theirs is not an equally valid method and should be rejected.  

IV. A Case of Synergistic Chemical Interaction

Kaye and Aickin have difficulty with what they infer to be our position on an extreme, limiting case of chemical interaction. They postulate a situation in which A dumps an ordinarily harmless chemical X into a canal, and, a few minutes later, B dumps another ordinarily harmless chemical Y into the same canal. Each of these substances, taken alone, has a probability of causing harm equal to zero. Taken together, however, they interact and produce harm with a probability equal to unity. Kaye and Aickin believe that our apportionment scheme would unfairly allocate all of the liability to B because A’s emission is considered merely part of the environment in which B operates. This is incorrect. The case described by Kaye and Aickin is actually one of simultaneous causation. Although the discharges that give rise to the chemical interaction are successive, the causal relationship is simultaneous. Neither discharge, taken alone, has any probability of producing harm. Therefore the discharges themselves are causally inert. Only when the chemicals actually combine does this causally inert status change, making the chemical combination or interaction simultaneous. We would, therefore, treat this case as one of purely synergistic simultaneous causation. Accordingly, each tortfeasor would bear half of the total damages regardless of the time order of chemical discharge.

24 A numerical example may be helpful. Suppose $\alpha = .4$, $\beta = .2$, and $\gamma = .9$. Clearly there is positive physical synergy in this case because $\delta = .52$ and $\gamma = .9$. Kaye and Aickin’s apparent method of apportioning liability ignores the $\alpha\beta$ term and includes the full synergy (.38) for each party. Thus, they would calculate the ratio of A’s to B’s share of liability as 

$\left[\frac{.4}{.4 + .2} (.52) + .19\right]/\left[\frac{.2}{.4 + .2} (.52) + .19\right] = 1.40.$

Our apportionment formula, on the other hand, includes $\alpha\beta$ and only one-half of the synergistic effect in A’s and B’s shares. Hence, we calculate the shares of liability according to formula (7) in our original article (supra note 1, at 1411) as

$\left[\frac{.4}{.4 + .2} (.52) + .19\right]/\left[\frac{.2}{.4 + .2} (.52) + .19\right] \approx 1.48.$

Note that the combination of Kaye and Aickin’s mistaken exclusion of $\alpha\beta$ and their overinclusion of all of the physical synergy in each party’s share results in a ratio of liability that just happens to be close to the correct one in this case.

25 Kaye & Aickin, supra note 3, at 207.

26 See discussion in Rizzo & Arnold, supra note 1, at 1411.