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Manuel A. Gómez**

Department of Applied Economics II, University of A Coruña

Abstract.
This paper devises a consumption tax policy that allows to decentralizing the efficient equilibrium in an AK endogenous growth model with external habit formation. The equilibrium dynamics, including that of the optimal taxes, is characterized by means of phase-diagram analysis.

Keywords and Phrases: Endogenous growth; Habit formation; Optimal taxation.

JEL Classification Numbers: O41, H21.

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** Facultad de Ciencias Económicas y Empresariales, Campus de Elviña, 15071 A Coruña, Spain. Phone: +34-981-167000, Fax: +34-981-167070, E-mail: mago@udc.es.
1. Introduction

The role of habits in dynamic equilibrium models has recently attracted a great attention in the literature. For the most part, the objective has been to account for empirical facts that cannot be explained under more traditional specifications of preferences (e.g., Abel, 1990, Carroll et al., 2000, Fuhrer, 2000, Diaz et al., 2003). The formation of habits from some external benchmark taken as given by agents raises the question of whether the competitive equilibrium is efficient and, if it is not so, calls for the design of a tax policy capable of decentralizing the efficient path attainable by a central planner.

This paper analyzes the equilibrium efficiency in an AK one-sector endogenous growth model with external habit formation. In this model, habits arise from the average past consumption levels in the economy that are taken as given by the individuals and, therefore, generate an external effect that renders the competitive equilibrium inefficient. The equilibrium dynamics of this model has been analyzed by Carroll et al. (1997). However, they have not analyzed the issue of equilibrium efficiency.

The competitive equilibrium is inefficient because the agent does not take into account the indirect effect that present consumption has on future utility through its effect on the habit stock. The efficient equilibrium can be decentralized by means of a consumption tax at an increasing (decreasing) rate if the shadow cost of the habit stock grows at a greater (smaller) rate than the shadow price of the capital stock. The equilibrium dynamics is analyzed by means of phase diagrams. This allows to performing a qualitative analysis of the global dynamics of the economy, including the evolution of the optimal tax rates.

This paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the decentralized economy, and Section 4, the centralized economy. Section 5 devises the optimal tax policy. Section 6 concludes.
2. Setup of the model

Consider an economy populated by $N$ identical infinitely-lived representative agents that grows at the exogenous rate $\dot{N}/N=n$. The intertemporal utility derived by the agent is

$$\Omega = \frac{1}{1-\varepsilon} \int_0^{\infty} (C_i/H_i)^{1-\varepsilon} e^{-\beta t} dt \quad \beta > 0, \ \varepsilon > 1, \ 0 < \gamma < 1,$$

(1)

where $C_i$ and $H_i$ are agent’s $i$ consumption and reference consumption level (habits), respectively, $\gamma$ reflects the importance of habits in utility, $\beta$ is the rate of time preference, and $1/\varepsilon$ is the intertemporal elasticity of substitution in the time-separable case ($\gamma = 0$).

The habit stock is formed as an exponentially declining average of past consumption,

$$H_i(t) = \rho \int_{-\infty}^{t} e^{\rho(s-t)} \bar{C}(s) ds \quad \rho > 0,$$

(2)

where $\bar{C} = \sum_{i=1}^{N} C_i/N$ denotes the economy-wide average consumption of agents. Differentiating (2) with respect to time, the rate of adjustment of the reference stock is

$$\dot{H}_i = \rho (\bar{C} - H_i).$$

(3)

Individual output, $Y_i$, is determined by the AK technology

$$Y_i = AK_i \quad A > 0,$$

(4)

where $K_i$ is the individual’s capital stock. The government taxes consumption at a rate $\tau_c$, and the income raised is rebated as lump-sum transfers, $S_i$. The agent’s budget constraint is then

$$\dot{K}_i = AK_i - (1 + \tau_c)C_i - (n + \delta)K_i + S_i,$$

(5)

where $\delta$ is the rate of depreciation of capital, and the government’s budget constraint is

$$\tau_c C_i = S_i.$$

(6)

3. The market economy

The agent maximizes utility (1) subject to the budget constraint (5), taking as given $\bar{C}$. Let $J$ be the current value Hamiltonian of the agent’s problem:
\[ J = (C_i/H_i^\gamma)^{1-\varepsilon} \left/ (1-\varepsilon) + \lambda_i \left[ AK_i - (1+\tau_c)C_i - (n+\delta)K_i + \delta \right] \right. \]

The first-order conditions for an interior optimum are\(^1\)

\[
\frac{\partial J}{\partial C_i} = C_i^{-\varepsilon}H_i^{\gamma(1-\varepsilon)} - (1+\tau_c)\lambda_i = 0 , \quad (7a)
\]

\[
\dot{\lambda}_i = \beta\lambda_i - (A-n-\delta)\lambda_i , \quad (7b)
\]

plus the usual transversality condition. Henceforth, the equilibrium condition \( \bar{C} = C_i \) will be taken into account. Differentiating (7a) with respect to time, we get

\[
-\varepsilon C_i^{-\varepsilon-1}H_i^{-\gamma(1-\varepsilon)} \dot{C}_i \gamma(1-\varepsilon)C_i^{-\varepsilon}H_i^{\gamma(1-\varepsilon)-1} \dot{H}_i - \dot{\tau}_c \lambda_i - (1+\tau_c)\dot{\lambda}_i = 0 . \quad (8)
\]

Substituting for \( \dot{H}_i \) from (3), \( \dot{\lambda}_i \) from (7b), and \( \lambda_i \) from (7a), we can obtain

\[
\dot{C}_i = (C_i/\varepsilon)[A-n-\delta - \beta + \gamma(\varepsilon-1)\rho(C_i/H_i-1) - \dot{\tau}_c/(1+\tau_c)] . \quad (9)
\]

From (5) and (6), the overall resources’ constraint is

\[
\dot{K}_i = AK_i - C_i - (n+\delta)K_i . \quad (10)
\]

Defining \( c \equiv C_i/H_i \) and \( h \equiv H_i/K_i \), we can get the system that drives the dynamics of the market economy in terms of \( c \) and \( h \) as

\[
\dot{c} = (c/\varepsilon)[A-n-\delta + \rho - \dot{\tau}_c/(1+\tau_c) + (1-\gamma)(\varepsilon-1)\rho(1-c) - \rho c - \beta] , \quad (11a)
\]

\[
\dot{h} = -h(A-n-\delta + \rho - c(h + \rho)) , \quad (11b)
\]

Eq. (11a) is obtained from (9) and (3), and Eq. (11b) is obtained from (3) and (10).

4. The centrally planned economy

The central planner chooses all quantities directly taking all the relevant information into account. She maximizes (1) subject to the resources’ constraint (10) and

\[1\] In this case \( H_i \) is an externality, and the utility function (1) is concave in \( C_i \). Given that the constraints are concave functions, these conditions are sufficient for a maximum.
\[ \dot{H}_i = \rho(C_i - H_i). \]  

(12)

Let \( J \) be the current value Hamiltonian of the central planner’s optimization problem, and let \( \lambda_i \) and \( \mu_i \) be the multipliers for the constraints (10) and (12), respectively:

\[ J = (C_i / H_i^\gamma)^{\gamma - \epsilon} / (1 - \epsilon) + \lambda_i (AK_i - C_i - (n + \delta)K_i) + \mu_i \rho(C_i - H_i). \]

The first-order conditions for an optimum are\(^2\)

\[ \frac{\partial J}{\partial C_i} C_i^{-\epsilon} H_i^{-\gamma(1-\epsilon)} + \rho \mu_i - \lambda_i = 0, \]  

(13a)

\[ \dot{\mu}_i = (\beta + \rho) \mu_i + \gamma C_i^{-\epsilon} H_i^{-\gamma(1-\epsilon)-1}, \]  

(13b)

\[ \dot{\lambda}_i = \beta \lambda_i - (A - n - \delta) \lambda_i, \]  

(13c)

\[ \lim_{t \to \infty} e^{-\beta t} \lambda_i K_i = \lim_{t \to \infty} e^{-\beta t} \mu_i H_i = 0. \]  

(13d)

Differentiating (13a) with respect to time, we get

\[ -\epsilon C_i^{-\epsilon-1} H_i^{-\gamma(1-\epsilon)} \dot{C}_i - \gamma(1 - \epsilon) C_i^{-\epsilon} H_i^{-\gamma(1-\epsilon)-1} \dot{H}_i + \rho \dot{\mu}_i - \dot{\lambda}_i = 0. \]  

(14)

Defining \( q = -\mu_i / \lambda_i \), using (13a) we can get

\[ \dot{\lambda}_i = C_i^{-\epsilon} H_i^{-\gamma(1-\epsilon)}/(1 + \rho \lambda), \]  

(15a)

\[ \dot{\mu}_i = -C_i^{-\epsilon} H_i^{-\gamma(1-\epsilon)} q/(1 + \rho \lambda). \]  

(15b)

Substituting for \( \dot{H}_i \) from (12), \( \dot{\lambda}_i \) from (13c) and \( \dot{\mu}_i \) from (13b) into (14), and using (15a) and (15b), we can obtain

\[ \dot{C}_i = (C_i / \epsilon) [(A - n - \delta + \rho)/(1 + \rho \lambda) + \gamma \epsilon \rho (C_i / H_i - 1) - \rho(1 - \gamma) - \beta]. \]  

(16)

The system that drives the dynamics of the centralized economy is

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\(^2\) The utility function (1) is not concave in \( C_i \) and \( H_i \), and so, the first-order conditions may fail to characterize the maximum. In this case, Alonso-Carrera et al. (2005) argue that the interior solution will indeed be a maximum if \( \epsilon > 1 \), as empirical evidence suggests.
\[ \dot{c} = (c/\varepsilon)[(A-n-\delta + \rho)/(1+\rho q) + (1-\gamma)(\varepsilon - 1)\rho (1-c) - (1-\gamma)\rho c - \beta] , \]  
Eq. (17a) is obtained from (16) and (12); (17b) is obtained from (13b) and (13c), using (15a) and (15b), and Eq. (17c) is obtained from (12) and (10). It should be noted that, unlike the market economy, the dynamics of the shadow price \( q \) must also be taken into account.

The steady state of (17) is

\[ \dot{c} = \frac{A-n-\delta - \beta + (\gamma + \varepsilon(1-\gamma))\rho}{(\gamma + \varepsilon(1-\gamma))\rho} , \]  
\[ \dot{h} = \frac{((\varepsilon - 1)(1-\gamma)(A-n-\delta) + \beta)\rho}{A-n-\delta - \beta + (\gamma + \varepsilon(1-\gamma))\rho} , \]  
\[ \dot{q} = \frac{\gamma(A-n-\delta - \beta + (\gamma + \varepsilon(1-\gamma))\rho)}{\beta\gamma\rho + \rho\varepsilon(1-\gamma)(A-n-\delta) + (\gamma + \varepsilon(1-\gamma))\rho^2(1-\gamma)} , \]  

The corresponding long-run growth rate of capital, output and consumption per capita is

\[ \dot{g} = (A-n-\delta - \beta)/(\gamma + \varepsilon(1-\gamma)) . \]  
We shall assume that the economy grows at a positive endogenous growth rate, \( \dot{g} > 0 \):

\[ A > n + \delta + \beta . \]  

It can be easily shown that the steady state \((\dot{c}, \dot{h}, \dot{q})\) is feasible if (20) is satisfied. After simplification, the transversality condition (13d) is found to be equivalent to

\[ \beta + \dot{g}(\varepsilon - 1)(1-\gamma) > 0 , \]  
so that it is satisfied given the assumptions made that \( \varepsilon > 1 \) and \( \dot{g} > 0 \).

**[FIGURE 1 AROUND HERE]**

The system (17) is accessible to phase-diagram analysis. It can be decoupled since \( \dot{c} \) and \( \dot{q} \) depend only on \( c \) and \( q \) and, therefore, their dynamics are independent of \( h \). The top right panel of Figure 1 is a phase diagram in the \((q,c)\)-space. From (17a), the \( \dot{c} = 0 \)–locus is
\[
l_c(q) = \frac{1}{\rho \varepsilon (1-\gamma)} \left[ \frac{A - n - \delta + \rho}{1 + \rho q} + (\varepsilon - 1)(1-\gamma)\rho - \beta \right],
\]
which is decreasing, convex and stable, with \( l_c(0) > \dot{c} > \lim_{q \to \infty} l_c(q) \). From (17b), the \( \dot{q} = 0 \)–locus is
\[
l_q(q) = q(A - n - \delta + \rho)/(\gamma(1 + \rho q)).
\]
which is increasing, concave and unstable, with \( l_q(0) = 0 < \dot{\gamma} < \lim_{q \to \infty} l_q(q) \). Given the configuration of the two loci, there exists a unique and saddle-point steady state \((\dot{c}, \dot{q})\). The top left panel of Figure 1 depicts a phase diagram in the \((h,c)\)-space. Given that the economy is on its saddle path in the \((q,c)\)-space, \( c \) converges monotonically. Thus, the \( \dot{c} = 0 \)–locus is horizontal and stable in the \((h,c)\)-space. The \( \dot{h} = 0 \)–locus is
\[
l_h(h) = (A - n - \delta + \rho)/(h + \rho),
\]
which is decreasing, convex and unstable, with \( l_h(0) > \dot{h} > \lim_{h \to \infty} l_h(h) = 0 \). Given the configuration of the two loci, there exists a unique and saddle-point steady state \((\dot{h}, \dot{c})\). It remains open the question on how \( h_0 \), the initial value of the predetermined variable \( h \), determines the initial point on the stable saddle-path. The top left panel of Figure 1 shows that \( h_0 \) determines \( c_0 \), the initial value of \( c \), on the stable saddle-path \( c(h) \). Then, the top right panel shows that \( c_0 \) determines \( q_0 \), the initial value of \( q \), on the stable saddle-path \( c(q) \).

The local stability analysis confirms the saddle-point property of the steady state. Linearizing (17) around the steady state we get
\[
\begin{pmatrix}
\dot{c} \\
\dot{h} \\
\dot{q}
\end{pmatrix} =
\begin{pmatrix}
d_{11} & 0 & d_{13} \\
d_{21} & d_{22} & 0 \\
d_{31} & 0 & d_{33}
\end{pmatrix}
\begin{pmatrix}
c - \hat{c} \\
h - \hat{h} \\
q - \hat{q}
\end{pmatrix} = D
\begin{pmatrix}
c - \hat{c} \\
h - \hat{h} \\
q - \hat{q}
\end{pmatrix},
\]
\[
d_{11} = -\frac{\varepsilon(1-\gamma)\rho \hat{c}}{\varepsilon}, \quad d_{13} = -\frac{(\beta + \rho \hat{c} (1-\gamma) + \rho (\hat{c} - 1)(\varepsilon - 1)(1-\gamma)) \rho \hat{c}}{\varepsilon (1 + \rho \hat{q})},
\]
\[ d_{21} = \hat{h}(\hat{h} + \rho), \quad d_{22} = \hat{c}h, \]
\[ d_{31} = -\gamma (1 + \rho \hat{q}), \quad d_{33} = \gamma \hat{c} / \hat{q} = A - n - \delta + \rho (1 - \gamma) . \]

One eigenvalue is \( d_{22} = \hat{c}h > 0 \), and the other two eigenvalues are those of the submatrix
\[
D_{13} = \begin{pmatrix}
 d_{11} & d_{13} \\
 d_{31} & d_{33}
\end{pmatrix}.
\]

After simplification, we can get that
\[
\text{det}(D_{13}) = -\rho \hat{c}(\beta + \hat{\phi}(\varepsilon - 1)(1 - \gamma) + (1 - \gamma) \rho \hat{c})(\gamma + \varepsilon(1 - \gamma))/\varepsilon < 0 ,
\]
so that \( D_{13} \) has one stable and one unstable root. Definitively, the matrix \( D \) has one stable and two unstable roots. As (17) features one pre-determined variable, \( h \), and two jump variables, \( c \) and \( q \), the steady state \( (\hat{c}, \hat{h}, \hat{q}) \) is saddle-path stable.

5. Optimal tax policy

This section devises a fiscal policy capable of decentralizing the optimal equilibrium attainable by a central planner. First, note that (11b) and (17c), which describe the dynamics of \( h \) in the decentralized and centralized economies, coincide. Equating the right hand sides of (11a) and (17a), and using (17b), we find the following relationship:
\[
\frac{\dot{\tau}_c}{1 + \tau_c} = \frac{\rho \dot{q}}{1 + \rho q} . \tag{21}
\]

Eq. (21) shows that \( \tau_c \) tends to a constant whatever its initial value may be, since \( q \) tends to its steady state \( \hat{q} \). The bottom right panel of Figure 1 is a phase diagram in the \((q, \tau_c)\)-space. Given that the economy is on its saddle path in the \((q, \tau_c)\)-space, \( q \) converges monotonically. Thus, the \( \dot{q} = 0 \)–locus is vertical and stable in the \((q, \tau_c)\)-space. Eq. (21) shows that this is also the \( \dot{\tau}_c = 0 \)–locus. Since when \( q \) decreases (increases) \( \tau_c \) does so, the arrows point south (north) to the right (left) of the \( \dot{\tau}_c = 0 \)–locus. Therefore, irrespective of the initial
value of the consumption tax, \(q\) and \(\tau_c\) converge. But now there is a degree of indeterminacy on the behavior of the optimal \(\tau_c\) in that its initial value may be arbitrarily chosen. The bottom right panel of Figure 1 depicts two possible trajectories starting from \(\tau_0^1\) and \(\tau_0^2\) when the initial value of \(h\) is greater or smaller than its steady state, \(h_o^0 > \hat{h}\) and \(h_o^0 < \hat{h}\), respectively.

Some intuition for Eq. (21) may be given. Comparing (7a) with (13a), we see that the agent in the market economy does not take into account the indirect effect that consumption has in utility through its influence on the habit stock (the term \(\rho \mu_h\) in (13a)). Note that \(q\) is the ratio of the shadow cost of habits, \(-\mu_h\), to the shadow price of capital, \(\lambda_i\). If the shadow cost of habits increases at a greater rate than the shadow price of capital (i.e., \(q > 0\)), the agent overvalues the benefit of future consumption relative to the efficient solution because she does not take into account the (negative) indirect effect of the rising habit stock on future utility. Hence, agent’s willingness to shift present consumption to the future would be suboptimally high along the efficient solution. Equilibrium efficiency can achieved by taxing consumption with a tax rate increasing over time. This tax policy increases the relative price of future consumption and discourages individuals to shift present consumption to the future.

6. Conclusions

We devise a tax policy that allows to decentralizing the efficient path in an AK model with external habit formation. Habits are formed from average past consumption levels in the economy that are taken as given by the agent. Inefficiency arises because the agent does not take into account the indirect effect that consumption has in future utility through its influence on the habit stock. The efficient equilibrium can be decentralized by taxing consumption at an increasing (decreasing) rate if the shadow cost of habits grows at a greater (smaller) rate than the shadow price of capital. The dynamics of the economy, including that of the optimal consumption tax, is characterized by means of a phase-diagram analysis.
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Figure 1. Phase diagram