Equilibrium dynamics in the one-sector endogenous growth model with physical and human capital

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Abstract

This paper concerns the transitional dynamics of the one sector endogenous growth model with physical and human capital when gross investments are irreversible. It has been claimed that the transition path is on the stable saddle path of the system that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding. We show that this does not have to be the case. The dynamics can be determined by noting that the continuity of the shadow prices involves the continuity of the consumption path.

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1. Introduction

The one sector model with physical and human capital is among the simplest models of endogenous growth. If investments in physical and human capital are reversible, the model features no transitional dynamics: The ratio of physical to human capital jumps immediately to the optimal value, in which the net returns on each type of capital are equalized, and the economy is on a balanced growth path after that. Transitional dynamics arises, however, if investments are irreversible, i.e. must each be nonnegative. Now, there can be an initial phase when one of the nonnegativity constraints on investment is binding for one type of capital.

Barro and Sala-i-Martín (1995) claim that during the transition the economy will be on the stable saddle path corresponding to the system that describes the dynamics of the economy as long as the constraint on nonnegative investment in one of the factors is binding. We shall show that this does not have to be case. Instead, transitional dynamics can be determined by noting that the continuity of the shadow prices involves the continuity of the consumption path. Thus, the transition path has to be chosen such that when the balanced growth path is reached, consumption reaches its balanced growth value without jumping. This transition path does not have to be on the stable saddle path of the initial dynamical system.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 performs a phase diagram analysis. Section 4 shows that the transition path does not have to be on the stable saddle path of the initial dynamical system. Section 5 explains how the true transition path can be computed. Section 6 concludes.

2. The model

Consider a closed economy inhabited by an infinitely-lived representative household that maximizes the intertemporal utility function \( \int_0^\infty e^{-\rho t} \left( \frac{c^{1-\theta}}{1-\theta} - 1 \right) dt \), where \( c \) denotes
consumption, $\rho>0$ is the rate of time preference and $1/\theta > 0$ is the elasticity of intertemporal substitution. Output $y$ is produced with the Cobb-Douglas production function $y = A k^{\alpha} h^{1-\alpha}$, 0$<\alpha<1$, where $k$ is physical capital, and $h$ is human capital. The economy’s resource constraint is $y = i_k + i_h + c$, where $i_k$ and $i_h$ are gross investment in physical and human capital, respectively, which must each be nonnegative. The stocks of physical and human capital evolve according to the dynamic equations $\dot{k} = i_k - \delta k$ and $\dot{h} = i_h - \delta h$, respectively.

This model has been analyzed by Barro and Sala-i-Martín (1995, Ch. 5). The current value Lagrangian of the representative household’s maximization problem is

$$L = (c^{1-\theta} - 1)/(1-\theta) + \lambda(i_k - \delta k) + \mu(i_h - \delta h) + \psi(A k^{\alpha} h^{1-\alpha} - i_k - i_h - c) + \eta_i_k + \xi i_h,$$

where $\lambda$ and $\mu$ are the shadow prices of physical and human capital, $\psi$ is the multiplier associated with the economy’s resource constraint, and $\eta$ and $\xi$ are the multipliers associated with the nonnegativity constraints on gross investment. The necessary conditions are

$$\partial L / \partial c = c^{-\theta} - \psi = 0, \quad (1a)$$

$$\partial L / \partial i_k = \lambda - \psi + \eta = 0, \quad i_k \geq 0, \quad \eta \geq 0, \quad \eta i_k = 0, \quad (1b)$$

$$\partial L / \partial i_h = \mu - \psi + \xi = 0, \quad i_h \geq 0, \quad \xi \geq 0, \quad \xi i_h = 0, \quad (1c)$$

$$A k^{\alpha} h^{1-\alpha} = i_k + i_h + c, \quad (1d)$$

$$\dot{\lambda} = (\rho + \delta)\lambda - \psi\alpha A k^{\alpha-1} h^{1-\alpha}, \quad (1e)$$

$$\dot{\mu} = (\rho + \delta)\mu - \psi(1-\alpha) A k^{\alpha} h^{-\alpha}, \quad (1f)$$

$$\lim_{t \to \infty} e^{-\delta t} \lambda k = \lim_{t \to \infty} e^{-\delta t} \mu h = 0. \quad (1g)$$

These conditions are also sufficient since, imposing the condition $i_k = A k^{\alpha} h^{1-\alpha} - i_h - c$, the current value Hamiltonian, $H = (c^{1-\theta} - 1)/(1-\theta) + \lambda(A k^{\alpha} h^{1-\alpha} - i_h - c - \delta k) + \mu(i_h - \delta h)$, and the constraints are jointly concave in the states and the controls.

Hereafter, let $\chi \equiv c/k$, $\omega \equiv k/h$, and let $\gamma = \dot{x}/x$ denote the growth rate of any variable
x. The interior solution, when the nonnegativity constraints are non-binding \((i_k>0, \ i_h>0)\), can be easily obtained. Eqs. (1b) and (1c) yield \(\lambda=\mu=\psi\). Eqs. (1e) and (1f) then imply that the net returns on physical and human capital must be equal, \(\alpha A(k/h)^{\alpha-1}-\delta = (1-\alpha)A(k/h)^\alpha-\delta\). Thus, the ratio of physical to human capital is constant at any time, \(\omega=\omega^* = \alpha/(1-\alpha)\). Eqs. (1a) and (1e) involve that \(\gamma_c\) is constant and equal to \(\gamma^*=(A\alpha^\alpha(1-\alpha)^{-\alpha}-\delta-\rho)/\theta\). We shall assume that the parameters are so that \(\gamma^*>0\). It is readily shown that \(c, k, h, i_k, i_h\) and \(y\) must all grow at the same constant rate \(\gamma^*\). Gross investments in physical and human capital are positive since \((i_k/k)^*= (i_h/h)^* = \gamma^*+\delta > 0\). The economy’s resource constraint entails that

\[
\chi^* = A\omega^{\alpha^\alpha-1} - (\gamma^*+\delta)(1+\omega^*) = (\rho - (1-\theta)(A\alpha^\alpha(1-\alpha)^{\alpha-\alpha}-\delta))/(\alpha\theta).
\]  

The transversality conditions (1g) are satisfied if \(\rho > (1-\theta)(A\alpha^\alpha(1-\alpha)^{-\alpha}-\delta),\) which also entails that \(\chi^*>0\). Hence, if the initial ratio \(\omega(0)=\omega^*\), the model exhibits no transitional dynamics: At the initial time, the ratio of consumption to physical capital, \(\chi\), jumps to its steady state value, \(\chi^*\), and all variables grow at the common constant rate \(\gamma^*\) after that.

Suppose now that \(\omega(0)<\omega^*\), so that \(h\) is initially abundant relative to \(k\). Without nonnegativity constraints, Barro and Sala-i-Martin (1995) show that the adjustment entails increasing \(k\) and decreasing \(h\) by discrete amounts so that the ratio of physical to human capital jumps at the initial time to its steady state value \(\omega^*\), and all variables grow at the same constant rate \(\gamma^*\) after that. This solution requires negative gross investment in human capital at an infinite rate. Thus, when the nonnegativity constraints on gross investment are considered, the one corresponding to human capital would be violated. The desire to lower \(h\) entails that the inequality \(i_h\geq0\) will be binding in an interval \([0,T]\), whereas \(i_k>0\). Since \(\psi=\lambda\), Eqs. (1a) and (1e) imply that \(\gamma_c = (\alpha A\omega^{\alpha^\alpha-1}-\delta-\rho)/\theta\). As the growth rates of \(h\) and \(k\) are
given by \( \gamma_s = -\delta \) and \( \gamma_k = A\omega^{a-1} - \chi - \delta \), the dynamics of the economy can be described by

\[
\begin{align*}
\gamma_o &= A\omega^{a-1} - \chi, \quad (3a) \\
\gamma_x &= -A\omega^{a-1}(\theta - \alpha)/\theta + \chi - \delta(1 - \theta)/\theta - \rho/\theta. \quad (3b)
\end{align*}
\]

The system (3) has a unique positive steady state \( \hat{\omega} = (\delta(1 - \theta) + \rho)^{-\alpha/(1-\alpha)}(A\alpha)^{\alpha/(1-\alpha)} \) and \( \hat{x} = (\delta(1 - \theta) + \rho)/\alpha \) if and only if \( \delta(1 - \theta) + \rho > 0 \). The assumption \( \gamma^* > 0 \) involves that \( \omega^{a-1} < \alpha A/(\delta + \rho) < \alpha A/(\delta(1 - \theta) + \rho) = \omega^{1-\alpha} \) and, therefore, \( \hat{\omega} > \omega^* \). If \( \delta(1 - \theta) + \rho \leq 0 \), there is no positive steady state of system (3). Similar arguments to those used in Ladrón-de-Guevara et al. (1997) show that the economy converges to the interior balanced growth path with \( \gamma_s > \gamma^* \) and \( \gamma_h = -\delta < \gamma^* \). Hence, as the economy evolves the ratio of physical to human capital increases. At time \( T \), the net returns on each type of capital are equalized, \( \omega(T) = \omega^* \), and the constraint \( i_h \geq 0 \) becomes non-binding. From \( t = T \) on, the solution is given by \( \omega(t) = \omega^* \) and \( \chi(t) = \chi^* \), and all quantities grow at the constant rate \( \gamma^* \). Unlike in the model without nonnegativity constraints on gross investment, the initial ratio \( \chi(0) \) of consumption to physical capital is not known in advance and still needs to be determined. This will be done in the next section.

If \( \omega(0) > \omega^* \), the analysis is symmetrical to the former one. Now, \( k \) is initially abundant relative to \( h \), and the inequality \( i_h \geq 0 \) will be initially binding. Since \( \psi = \mu \), Eqs. (1a) and (1f) imply that \( \gamma_c = ((1 - \alpha)A\omega^\alpha - \delta - \rho)/\theta \), and the dynamics of the economy are described by

\[
\begin{align*}
\gamma_o &= -A\omega^\alpha + \chi \omega, \quad (4a) \\
\gamma_x &= (1/\theta)(A(1 - \alpha)\omega^\alpha - \delta(1 - \theta) - \rho). \quad (4b)
\end{align*}
\]

The system (4) has a unique positive steady state \( \tilde{\omega} = (A(1 - \alpha))^{-\alpha/(1-\alpha)}(\delta(1 - \theta) + \rho)^{\alpha/(1-\alpha)} \) and \( \tilde{x} = A^{1/\alpha}(1 - \alpha)^{1/(\alpha-1)}(\delta(1 - \theta) + \rho)^{1/\alpha} \) if \( \delta(1 - \theta) + \rho > 0 \). The assumption \( \gamma^* > 0 \) involves that \( \omega^{a-1} > (\delta + \rho)/(A(1 - \alpha)) > (\delta(1 - \theta) + \rho)/(A(1 - \alpha)) = \tilde{\omega}^\alpha \) and hence \( \omega^* > \tilde{\omega} \). If \( \delta(1 - \theta) + \rho \leq 0 \)
there is no positive steady state. As the economy evolves the ratio of physical to human capital decreases up to the point in which the returns on both factors are equalized. After that, the solution is given by \( \omega(t)=\omega^* \) and \( \chi(t)=\chi^* \), and all variables grow at the constant rate \( \gamma^* \).

3. Phase diagram analysis

Figure 1 is a phase diagram in the \((\omega, \chi)\)-space when \( \omega(0) < \omega^* \). From (3a) the \( \gamma_\omega=0 \)-locus, \( \chi = A\omega^{\alpha-1} \), is decreasing and stable. From (3b), there are three possible shapes for the \( \gamma_\chi=0 \)-locus, \( \chi = A\omega^{\alpha-1}(\theta - \alpha)/\theta + \delta(1-\theta)/\theta + \rho/\theta \). It is decreasing when \( \theta > \alpha \), increasing when \( \theta < \alpha \), and constant when \( \theta = \alpha \). In any case, given (3), the \( \gamma_\chi=0 \)-locus is unstable and its slope is greater than that of the \( \gamma_\omega=0 \)-locus. The assumption \( \gamma^* > 0 \) entails that \((\omega^*, \chi^*)\) is below the \( \gamma_\chi=0 \)-locus, as substituting \( \omega^* \) into the expression for the \( \gamma_\chi=0 \)-locus, and using Eq. (2), yields

\[
(A\omega^{\alpha-1}(\theta - \alpha)/\theta + \delta(1-\theta)/\theta + \rho/\theta) = A\omega^{\alpha-1} - (\gamma^* + \delta) > A\omega^{\alpha-1} - (\gamma^* + \delta)(1 + \omega^*) = \chi^*.
\]

The \( \gamma_\omega=0 \)-locus is decreasing with \( \chi \to +\infty \) as \( \omega \to 0 \) and \( \chi \to 0 \) as \( \omega \to +\infty \). The \( \gamma_\chi=0 \)-locus is in any case below the \( \gamma_\omega=0 \)-locus at least in a neighbourhood of \( \omega=0 \), and \( \chi \to \delta(1-\theta)+\rho \) as \( \omega \to +\infty \). Hence, if \( \delta(1-\theta) + \rho > 0 \), there is a unique positive state \((\hat{\omega}, \hat{\chi})\), which is a saddle point. If the \( \gamma_\chi=0 \)-locus is decreasing, it is flatter than the \( \gamma_\omega=0 \)-locus. This implies that the stable saddle path must be decreasing and flatter than the \( \gamma_\chi=0 \)-locus. Panels (a) and (b) in Figure 1 are phase diagrams in this case. If the \( \gamma_\chi=0 \)-locus is increasing (constant), the stable saddle path is increasing (constant). Panel (c) in Figure 1 depicts a phase diagram in this case.

The stable saddle paths are represented as dashed lines.

[Insert Figure 1 here]

If \( \delta(1-\theta) + \rho \leq 0 \), the \( \gamma_\chi=0 \)-locus lies entirely below the \( \gamma_\omega=0 \)-locus so the two loci do not cross. As \( \delta \) and \( \rho \) are positive, the former condition requires that \( \theta > 1 > \alpha \). Therefore, the \( \gamma_\chi=0 \)-locus is decreasing. Panel (d) in Figure 1 depicts a phase diagram in this case.
Figure 2 is a phase diagram in the \((\omega, \chi)\)-space when \(\omega(0) > \omega^\ast\). From (4a) the \(\gamma_{\omega}=0\)-locus, \(\chi = A\omega^{\alpha-1}\), is decreasing and unstable. From the assumption \(\gamma^\ast > 0\), it is clear that \((\omega^\ast, \chi^\ast)\) is below the \(\gamma_{\omega}=0\)-locus. From (4b), \(\gamma_{\chi}=0\) requires \(\omega = \tilde{\omega}\), which is positive if \(\delta(1-\theta) + \rho > 0\).

In this case, a value of \(\omega\) above (below) \(\tilde{\omega}\) corresponds to \(\gamma_{\chi} > 0\) (\(\gamma_{\chi} < 0\)). Since the \(\gamma_{\omega}=0\)-locus is decreasing with \(\chi \to \infty\) as \(\omega \to 0\) and \(\chi \to 0\) as \(\omega \to \infty\), there is a unique positive steady state \((\tilde{\omega}, \tilde{\chi})\), which is a saddle point. The stable saddle path is decreasing and steeper than the \(\gamma_{\omega}=0\)-locus. We have already noted that \(\gamma^\ast > 0\) involves that \(\tilde{\omega} < \omega^\ast\). This, in turn, implies that \(\tilde{\chi} > \chi^\ast\) because the \(\gamma_{\omega}=0\)-locus is decreasing and \((\omega^\ast, \chi^\ast)\) is below the \(\gamma_{\omega}=0\)-locus. Panel (a) in Figure 2 is a phase diagram if \(\tilde{\omega}\) is positive.

[Insert Figure 2 here]

If \(\delta(1-\theta) + \rho \leq 0\), then \(\gamma_{\chi} > 0\) for all \(\omega > 0\) and there is no positive steady state. Panel (b) in Figure 2 depicts a phase diagram in the \((\omega, \chi)\)-space in this case.

### 4. The transition path

In this section we shall show that the transition path does not have to be on the stable saddle path corresponding to the system that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding.

The main result is that consumption must be continuous on the optimal path.

**Proposition 1.** The path of consumption is continuous on the optimal solution.

**Proof.** We have shown that one of the inequality restrictions on gross investment in physical or human capital is never binding. Hence, Eqs. (1b) and (1c) shows that \(\psi(t) = \lambda(t)\) (if \(\omega(0) < \omega^\ast\)) or \(\psi(t) = \mu(t)\) (if \(\omega(0) > \omega^\ast\)) at any time. The continuity of the shadow prices \(\lambda(t)\) and \(\mu(t)\) entails that \(\psi(t)\) is continuous as well. The continuity of the consumption path \(c(t)\) follows then from Eq. (1a). **q.e.d.**
Since the state variables $k(t)$ and $h(t)$ are continuous, the ratios $\omega(t)$ and $\chi(t)$ must be continuous as well. Furthermore, the economy’s resource constraint implies that $i_k(t) + i_h(t)$ must be continuous although $i_k(t)$ and $i_h(t)$ can jump (as they do in effect). Proposition 1 entails that the transition path has to be chosen such that when the balanced growth path is reached, consumption reaches its balanced growth value without jumping. In other words, when the ratio of physical to human capital $\omega$ reaches its steady state value, $\omega^*$, the ratio of consumption to physical capital $\chi$ must also reach its steady state value, $\chi^*$, with no jump.

Barro and Sala-i-Martín (1995, pp. 202-3) claim that the economy evolves along the stable saddle path of the system (3) or (4), as the case may be, that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding, in the direction of the “hypothetical target,” $\hat{\omega}$ or $\tilde{\omega}$, respectively, up to the point where $\omega$ reaches its steady state value $\omega^*$. Accordingly, in their Figures 5.10 and 5.11 the adjustment path is depicted as the stable saddle path of the system (3) or (4) truncated between $\omega(0)$ and $\omega^*$. Proposition 2 below shows that this procedure is generally incorrect. In addition, it leaves open the question on how the transition path is computed when there does not exists such saddle path. In the next section we will present a method to compute the transition path that applies even if no such stable saddle path exists.

**Proposition 2.** Let $\delta(1-\theta) + \rho > 0$. If $\alpha(0) < \omega^*$ and i) $\alpha < \theta \leq 1$ or ii) $\alpha = \theta < 1$ or iii) $\theta < \alpha < 1$, the transition path cannot be on the stable saddle path of the system that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding.

**Proof.** i) If $\alpha < \theta \leq 1$, the $\gamma \chi = 0$-locus is decreasing and the stable saddle path is decreasing and flatter than the $\gamma \chi = 0$-locus. Therefore, it must be above $\hat{\chi}$ to the left of $\hat{\omega}$. Since $\chi^* - \hat{\chi} = (\theta - 1)(\gamma^* + \delta)/\alpha$, the assumptions $\gamma^* > 0$ and $\theta \leq 1$ entail that $\chi^* \leq \hat{\chi}$. As $\omega^* < \hat{\omega}$, this
implies that the steady state \((\omega^*, \chi^*)\) must be below the stable saddle path of system (3).

Panel (a) in Figure 1 illustrates this case.

ii) If \(\alpha = \theta < 1\) then \(\chi < \dot{\chi}\). Now the stable saddle path is constant, so it must be above \((\omega^*, \chi^*)\).

iii) If \(\theta < \alpha < 1\), the \(\gamma_{\dot{\chi}}=0\)-locus is increasing and is above \((\omega^*, \chi^*)\). The stable saddle path slopes upward and is above the \(\gamma_{\dot{\chi}}=0\)-locus. Hence, \((\omega^*, \chi^*)\) and the stable saddle path are in distinct regions of the phase diagram. Panel (c) in Figure 1 illustrates this case.

In any case, the result follows from the continuity of \(\dot{\chi}(t)\) on the optimal solution.

q.e.d.

Proposition 2 is not exhaustive in the sense that even if the conditions do not apply, the transition path may still be off the stable saddle path of the system that describes the dynamics of the economy as long as the constraint on nonnegative gross investment in one of the factors is binding when such saddle path does exist, i.e., when \(\delta(1-\theta) + \rho > 0\). If \(\omega(0)<\omega^*\) and \(\theta>1>\alpha\), or \(\omega(0)>\omega^*\), Panel (b) of Figure 1 and Panel (a) of Figure 2, respectively, depict numerical examples that show that the steady state \((\omega^*, \chi^*)\) does not have to be on the stable saddle path of the initial dynamical system in each of these cases for the parameter values quoted. The stable saddle paths have been computed by means of the time elimination method (Mulligan and Sala-i-Martín, 1993) and are represented as dashed lines.

5. Computation of the transition path

In this section, we shall indicate how the true transition path can be computed. Suppose that \(\omega(0)<\omega^*\). In the interval \([0, T]\) the dynamics of the economy are described by the system (3). At time \(T\) the economy reaches its steady state when, as the paths of \(\chi\) and \(\omega\) are continuous, \(\chi(T)=\chi^*\) and \(\omega(T)=\omega^*\). Note that at this stage the time \(T\) is unknown. Thus, the time paths of \(\chi\) and \(\omega\) in \([0, T]\) and the time \(T\) can be obtained by solving the dynamical system.
\[ x'(t) = \gamma_x(x(t), \omega(t)) x(t), \]
\[ \omega'(t) = \gamma_\omega(x(t), \omega(t)) \omega(t), \]

with initial condition \( \omega(0) = k(0)/h(0) = k_0/h_0 \), and terminal conditions \( x(T) = x^* \) and \( \omega(T) = \omega^* \).

From time \( t = T \) on, we have that \( x(t) = x^* \) and \( \omega(t) = \omega^* \). Although a shooting method can be used, the time elimination method (Mulligan and Sala-i-Martín, 1993) is a simpler and more efficient numerical technique. The policy function \( x(\omega) \) can be computed by solving the initial value problem:

\[ x'(\omega) = \frac{d}{d\omega} \left( \frac{\gamma_x(x(\omega), \omega) x(\omega)}{\gamma_\omega(x(\omega), \omega) \omega} \right), \quad x(\omega^*) = x^*. \]

The free final time boundary problem has been transformed in an initial value problem in which time does not appear. Note that the slope of the policy function is never undefined since \( \gamma_\omega > \gamma^* > 0 \) along the transition to the balanced growth path. Standard numerical methods can be used to solve this first order differential equation in \( \omega \). For instance, we have used the Mathematica v4.1 command NDSolve to compute the policy functions depicted in every panel of Figures 1 and 2. Once the policy function \( x(\omega) \) has been computed time must be reintroduced. The time path of \( \omega \) can be computed by solving the initial value problem:

\[ \omega'(t) = \gamma_\omega(x(\omega(t)), \omega(t)) \omega(t), \quad \omega(0) = k(0)/h(0) = k_0/h_0. \]

Then we compute the time \( T \) at which the net returns on physical and human capital are equalized, i.e., the time \( T \) at which \( \omega(T) = \omega^* \). From time \( t = T \) on, the optimal path is \( \omega(t) = \omega^* \).

The time path of \( x \) can be obtained by substituting the optimal time path \( \omega(t) \) into the policy function \( x(\omega) \) in \([0, T]\), \( x(t) = x(\omega(t)) \). From time \( t = T \) on, we have that \( x(t) = x^* \). The time paths determined in this manner are clearly continuous. Once the paths of \( \omega \) and \( x \) have been computed, the time paths of the remaining variables can be readily obtained (see Mulligan and Sala-i-Martín, 1993). The method described above allows to compute the transition path...
even if $\tilde{\omega}$ or $\dot{\omega}$ is undefined (see panels (d) in Figure 1 and (b) in Figure 2).

It is worth noting that other computational methods, as the backward integration method (Brunner and Strulik, 2002), can also be readily adapted to compute the transition path.

6. Conclusions

In this paper, we described how the transition path can be correctly determined in the one-sector endogenous growth model with physical and human capital when gross investments are irreversible. A genuine understanding of its transitional dynamics can be useful to analyse the dynamics of other more complex models when some constraint may switch from binding to nonbinding, leading to corners in the optimal path, as continuity arguments similar to that presented here could be applicable.
References


Figure 1. Phase diagram for the model when $\omega(0) < \omega^*$

Note. Parameter and steady state values:

1. $A=0.21$, $\alpha=0.33$, $\theta=0.5$, $\rho=0.08$, $\delta=0.02$. $(\omega^*, \chi^*) = (0.49, 0.21)$, $(\hat{\omega}, \hat{\chi}) = (0.68, 0.27)$.

2. $A=0.25$, $\alpha=0.5$, $\theta=1.25$, $\rho=0.09$, $\delta=0.02$. $(\omega^*, \chi^*) = (1, 0.19)$, $(\hat{\omega}, \hat{\chi}) = (2.16, 0.17)$.

3. $A=0.2$, $\alpha=0.4$, $\theta=0.3$, $\rho=0.07$, $\delta=0.02$. $(\omega^*, \chi^*) = (3/4, 0.10)$, $(\hat{\omega}, \hat{\chi}) = (0.92, 0.21)$.

4. $A=0.24$, $\alpha=0.4$, $\theta=1.25$, $\rho=0.02$, $\delta=0.08$. $(\omega^*, \chi^*) = (3/4, 0.06)$. 
Figure 2. Phase diagram for the model when $\omega(0) > \omega^*$

Note. Parameter and steady state values:

1. $A=0.21, \alpha=0.33, \theta=0.5, \rho=0.08, \delta=0.02$. $(\omega^*, \chi^*) = (0.49, 0.21), \ (\bar{\omega}, \bar{\chi}) = (0.26, 0.52)$.

2. $A=0.24, \alpha=0.4, \theta=1.25, \rho=0.02, \delta=0.08$. $(\omega^*, \chi^*) = (3/4, 0.06)$. 