Optimal flat-rate taxes on income and consumption in an endogenous growth model

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OPTIMAL TAX STRUCTURE IN A TWO-SECTOR MODEL OF ENDOGENOUS GROWTH*

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Abstract. The optimal tax structure is calculated in a two-sector endogenous growth model of the U.S. economy. The welfare gains of shifting to the optimal tax mix are much smaller than those obtained in the optimal taxation exercises when tax rates are time-varying and confiscatory levels of taxation are possible in the short run. The optimal tax rate on capital income is found to be extremely robust to parameter variations. However, the balance between wage and consumption taxation depends markedly on the tax treatment of goods invested in human capital accumulation and, to a lesser extent, the intertemporal elasticity of substitution.

JEL classification: H21; O41

Keywords: Tax structure; Endogenous growth; Welfare

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Abstract. The optimal tax structure is calculated in a two-sector endogenous growth model of the U.S. economy. The welfare gains of shifting to the optimal tax mix are much smaller than those obtained in the optimal taxation exercises when tax rates are time-varying and confiscatory levels of taxation are possible in the short run. The optimal tax rate on capital income is found to be extremely robust to parameter variations. However, the balance between wage and consumption taxation depends markedly on the tax treatment of goods invested in human capital accumulation and, to a lesser extent, the intertemporal elasticity of substitution.

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1. Introduction

In their seminal papers, Chamley (1986) and Judd (1985) have analyzed the optimal dynamic factor taxation that a social planner would choose to maximize the welfare of an infinite-lived representative agent in a neoclassical exogenous growth model. They showed that the optimal tax rate on capital income is zero in the long-run. Jones et al. (1993, 1997), Bull (1993) and Milesi-Ferretti and Roubini (1995) generalized this result to a wider class of endogenous growth models with human capital accumulation, and introduced in the analysis consumption taxes in addition to labor and capital income taxes. They found that the optimal long-run tax rates on both labor income and consumption are zero, too.

Implementing the optimal dynamic tax policy would require an initial phase of relatively high taxation followed by tax rates converging to zero. The government expenditure would be financed by the interest earned on the assets raised by accumulating budget surpluses. To some authors (e.g., Jones et al., 1993, Milesi-Ferretti and Roubini, 1995, and Coleman, 2000) this feature might cast some doubt on the practical relevance of the optimal dynamic tax policy. Perhaps as a consequence, much of the literature has focused on the long-run properties of such a tax policy. However, as pointed out by Coleman (2000), the tail end of the optimal dynamic tax policy need not be optimal by itself and welfare may even fall if initial confiscatory tax rates are not implemented. Thus, more restrictions need to be placed on the tax codes to obtain an optimal policy plan that seems reasonable.

Limiting the analysis to time-invariant tax rates has been relatively common (e.g., Lucas, 1990; Cooley and Hansen, 1992; Stokey and Rebelo, 1995; Mendoza and Tesar, 1998). As Coleman (2000) points out, such a tax policy would naturally avoid confiscatory levels of tax rates and would not attain a welfare gain by the promise of significant tax reductions far in the future. Due to its simplicity, this policy seems to be among the most realistic in practice.
Furthermore, Cooley and Hansen (1992) and Coleman (2000) provide evidence suggesting that the extra gains made with time-variant taxes relative to time-invariant taxes can be relatively small.

The computation of the optimal tax structure of the U.S. economy has recently attracted the attention of several authors. İmrohoroğlu (1998) determined the optimal tax mix of labor and capital income taxation in a general equilibrium model with overlapping generations. His analysis is restricted however to steady-state welfare, consumption taxation is not considered, and labor supply is exogenous. Coleman (2000) calculated the optimal time-invariant tax rates on capital income, labor income and consumption. However, he abstracts from human capital accumulation as the analysis relies on a neoclassical exogenous growth model. Grüner and Heer (2000) computed the optimal constant tax rate on capital income in a Lucas’ (1990) supply side model.

The analysis of Grüner and Heer (2000) is limited in certain key aspects. First, by assuming that human capital is produced using effective time as the only input, it fails to take account of some channels through which taxes can affect growth and welfare. For instance, Milesi-Ferretti and Roubini (1998a, 1998b) have analyzed the channels through which taxes affect growth, and their results clearly show that capital income taxation, wage taxation and consumption taxation have quite different effects when leisure as raw time is introduced in the model and market goods are employed in the production of human capital. Second, the specification of fiscal policy is restrictive; specially the fact that government has no access to consumption taxation. The debate concerning fundamental U.S. tax reform has led to a number of proposals that involve a shift toward a consumption-based system, as the one proposed by Hall and Rabusha (1995) (see also, e.g., the conference papers in Aaron and
The purpose of this paper is to derive the optimal tax structure and estimate the welfare gain that the U.S. could attain by switching from its current tax policy to an optimal flat-rate tax policy. Here, tax structure refers to the mix of flat-rate taxes on capital income, labor income and consumption that keep the present discounted value of government revenues equal to the present discounted value of government expenditures. Thus, we shall consider deficit-neutral tax reforms; i.e., sustainable tax reforms, meaning that the tax rates ensure the intertemporal viability of the government’s budget without any corrective action on its part (Bruce and Turnovsky, 1999). As noted by Jorgenson and Wilcoxen (1997), this is an effective device for separating the discussion of tax reform from the budget debate. The restriction that tax rates are constant over time will be placed on the tax code. We also compute the optimal tax structure when income is taxed at a uniform rate; i.e., the tax rates on capital income and labor income are equal. Furthermore, we assess the optimal structure of income taxation, keeping the consumption tax rate constant. These issues are important because the composition of the income tax as well as the use of income versus consumption taxes lie in the center of discussion over tax policy.

In addition, we compute the time it takes before the welfare gain sets in after a change to the optimal tax structure. This issue is interesting since, as Islam (1995) argues, if the government has a relatively short planning horizon, the gain in welfare that occurs in the distant future is not likely to be of major concern, and so it could be interested in instituting policies that result in a gain in welfare in a shorter time. We also analyze the effect that a shift to the optimal tax mix has on the welfare of different generations. This issue could also have influence on the government willingness to adopt the optimal policy. Finally, we perform a
thorough sensitivity analysis to investigate the robustness of the optimal tax structure to variations in crucial parameters. Related work has been made by Gómez (2003). However, he focused on the quantitative implications that the specification of the leisure activity has on the effects of alternative tax structures.

The analysis in this paper is based upon an endogenous growth model where human capital is a non-market good that is produced with effective labor and a flow of market goods and services. The model is calibrated to match U.S. data. In the baseline, we find that a movement to the optimal structure of income and consumption taxation would involve reducing the tax rate on capital income by approximately one quarter, practically halving the tax rate on labor income, and increasing by about six-fold the consumption tax rate. If income from capital and labor is taxed the same rate, switching to the optimal tax structure would entail a moderate shift from income to consumption taxation. A movement to the optimal structure of income taxes, keeping the consumption tax rate constant, would entail a slight shift from capital income to labor income taxation. The welfare gain sets in immediately after an immediate permanent change to the optimal structure of income taxation. However, it would take approximately 72 periods before instituting the optimal structure of capital income, labor income and consumption taxation results in a gain in welfare; about 55 periods if capital and labor income is taxed at the same rate. These optimal policies have different effects on welfare of different generations. Following a change to the optimal structure of capital income, labor income and consumption taxation, generational welfare rises for all generations, even if capital and labor income is taxed at the same rate. After switching to the optimal structure of income taxation, while the consumption tax is kept constant, individuals born in the initial 4 periods are made better off, but generational welfare decreases for the subsequent generations.
The bulk of the literature has assumed that goods inputs in human capital accumulation are not tax-deductible. However, goods used in job training are paid for by lower wages, so these inputs are effectively tax-deductible in actual tax systems. The tax treatment of goods invested in human capital accumulation, i.e., the fraction that is paid for by lower earnings, is an important determinant of the effects of taxation (see Trostel, 1993, for an estimation of effects on human capital, and Hendricks, 1999, for an analysis of growth effects). In this paper, we show that this is a key determinant of the welfare effects of taxation as well, since the balance between wage and consumption taxation in the optimal tax structure depends markedly on this issue. The sensitivity analysis also reveals the importance of having reliable estimates of the intertemporal elasticity of substitution. The value of the optimal tax rate on capital income is found to be extremely robust to parameter variations instead, and significantly different from zero, ranging from 29 percent to 32 percent in all simulations.

The paper is organized as follows. Section 2 describes the model. Section 3 calibrates the model to match U.S. data. Section 4 conducts the welfare analysis. Section 5 presents the sensitivity analysis. Section 6 concludes.

2. The model

Our analysis is based upon a two-sector endogenous growth closed-economy with human capital accumulation. The first sector produces goods and services that can be consumed, accumulated as physical capital or utilized in the production of human capital; the second produces human capital.

2.1. Households

The economy is inhabited by infinitely lived identical households. The representative household derives utility from the consumption of both a private consumption good, \( c \), and
from leisure, \( l \). The intertemporal utility derived by the representative household is represented by the intertemporal isoelastic utility function

\[
U(c, l) = \int_0^\infty e^{-\rho t} \left( \frac{e^{\sigma \rho}}{1 - \sigma} \right) dt.
\]  

(1)

The time argument has been suppressed in this and all subsequent equations. Here, \( \rho \) is the rate of time preference, \( \sigma \) is the inverse of the intertemporal elasticity of substitution (IES) for consumption, and \( \eta \) reflects preferences for leisure.

Time can be allocated to work, \( u \), learning, \( z \), or leisure, \( l \). If the endowment of time is normalized to one per period, the time constraint is

\[
1 = u + z + l.
\]  

(2)

Income is earned from physical capital, \( k \), and human capital, \( h \). The rate of return on physical capital is denoted \( r \). The wage rate, \( w \), is defined as the rate of return per unit of effective labor, \( uh \). Income is spent on consumption and investment in physical and human capital, \( i_k \) and \( e \), respectively. Following Trostel (1993), we assume that on-the-job training is paid for by lower wages. As these inputs are deducted from labor earnings, a constant fraction of goods invested in human capital, \( \kappa \), is bought with foregone earnings. Goods invested in human capital purchased directly by individuals are subsidized by the government at a rate \( s_h \). The government also imposes flat-rate taxes on capital income, labor income and consumption, \( \tau_k \), \( \tau_h \) and \( \tau_c \), respectively, and provides lump-sum transfers, \( s \). Thus the budget constraint is

\[
(1 - \tau_c)rk + \tau_h \delta_h k + (1 - \tau_h)(wh - \kappa e) + s = i_k + (1 + \tau_c)c + (1 - s_h)(1 - \kappa)e,
\]  

(3)

where \( \delta_h \) is the rate of depreciation of physical capital. The second term in the left side of (3) reflects the depreciation allowance built into the tax code.

The stocks of physical and human capital evolve according to the dynamic equations
\[ \dot{k} = i - \delta_h k, \]  
\[ \dot{h} = Be^\beta (zh)^{1-\beta} - \delta_h h, \]

where \( \delta_h \) is the rate of depreciation of human capital. This specification of human capital accumulation was suggested by Ben-Porath (1967), and was recently used by Jones et al. (1993, 1997), Trostel (1993) and Hendricks (1999), among others.

2.2. Firms

Output, \( y \), is a function of the stocks of physical and human capital, and the time individuals supply as labor, \( u \). The production function is Cobb-Douglas:

\[ y = A k^\alpha (uh)^{1-\alpha}. \]  

Competitive firms rent physical capital at the rate of interest \( r \) and hire effective labor at the wage rate \( w \). Profit maximization implies that labor and capital are used up to the point at which marginal product equates marginal costs:

\[ r = \alpha A(k/h)^{\alpha-1} u^{1-\alpha}, \]  
\[ w = (1-\alpha) A(k/h)^\alpha u^{-\alpha}. \]

Measured output in national accounts does not take into account on-the-job training investments in human capital that are paid for by lower wages. From now on, we will use the term GDP for measured output,

\[ \text{GDP} = y - \kappa e. \]

2.3. The government

The government levies flat-rate taxes on labor income, \( \tau_h \), capital income, \( \tau_k \), and consumption, \( \tau_c \), to finance lump-sum per capita transfers to households, \( s \), subsidies to education, and public consumption, \( g \). If capital and labor income is taxed at the same rate \( \tau_y \),

\[ \tau_h = \tau_k = \tau_y, \]  
\[ \text{GDP} = y - \kappa e. \]
then $\tau_c = \tau_k = \tau_h$. The tax policy must satisfy the requirement that the present discounted value of government expenditure equals the present discounted value of government revenue. We shall assume that government claims a fraction, $g_c$, of GDP, for expenditure on consumption:

$$g = g_c \text{GDP},$$

and a fraction, $g_s$, of GDP for lump-sum transfers to households representing welfare programs. Adjustment in lump-sum transfers to households balance the government budget constraint each period, so that

$$s = g_s \text{GDP} - b,$$

and the government’s budget can be expressed as

$$b + \tau_c (r - \delta_c) k + \tau_h (wuh - \kappa e) + \tau_e c = (g_s + g_c) \text{GDP} + s_h (1 - \kappa) e. \quad (10)$$

Thus, $b$ represents the amount of lump-sum taxes (or transfers) needed to balance the current budget and, therefore, expresses the primary budget deficit. For simplicity, we do not explicitly consider financing by debt issue since, because of Ricardian Equivalence, the lump-sum tax is equivalent to debt.

2.4. The equilibrium

A competitive equilibrium for this economy is defined as a set of market-clearing prices and quantities such that i) the household’s choice of $c, k, h, i, e, u, z$ and $l$ maximizes (1) subject to the constraints (2), (3), (4) and (5), taking as given the path of factor returns and fiscal policy variables; ii) firms set factor prices so as to maximize profits (eqs. (7) and (8)); and iii) the government obeys its budget constraint (10).
The constraint (2) will be used to express \( u \) as a function of \( z \) and \( l \), and thus to eliminate it from the problem. Eqs. (3), (6), (7), (8) and (10) yield the market-clearing condition in the goods market:

\[ y = i_k + c + g + e. \]  

(11)

Let \( J \) be the current value Hamiltonian and \( \lambda, \mu \) and \( \theta \) be the multipliers for the constraints (3), (4) and (5) in the household’s problem:

\[
J = \frac{(c_i^\rho)^{1-\sigma}}{1-\sigma} + \lambda[(1-\tau_k)r_k + \tau_k\delta_k k + (1-\tau_h)(w(1-z-l)h - \kappa e) + s - i_k - (1+\tau_c)c \nonumber \\
- (1-s_h)(1-\kappa)e] + \mu[i_k - \delta_k k] + \theta[B\epsilon^\beta(z)h^{1-\beta} - \delta_h h].
\]

Then, the first order conditions are

\[
\frac{\partial J}{\partial c} = c^{-\sigma} \rho^{1-\sigma} - (1+\tau_c)\lambda = 0, \tag{12a}
\]

\[
\frac{\partial J}{\partial l} = \eta c^{-\sigma} \rho^{(1-\sigma)-1} - \lambda(1-\tau_h)wh = 0, \tag{12b}
\]

\[
\frac{\partial J}{\partial i_k} = -\lambda + \mu = 0, \tag{12c}
\]

\[
\frac{\partial J}{\partial z} = \lambda(1-\tau_h)wh - \theta(1-\beta)Bz^{1-\beta}(e/h)^\beta h = 0, \tag{12d}
\]

\[
\frac{\partial J}{\partial e} = -\lambda((1-\tau_h)\kappa + (1-\kappa)(1-s_h)) + \theta Bz^{1-\beta}(e/h)^{\beta-1} = 0, \tag{12e}
\]

\[
\mu = (\rho + \delta_h)\mu - \lambda((1-\tau_k)r + \tau_k\delta_k), \tag{12f}
\]

\[
\dot{\theta} = [\rho + \delta_h - (1-\beta)Bz^{1-\beta}(e/h)^\beta]\theta - \lambda(1-\tau_h)wh, \tag{12g}
\]

where \( u = 1-z-l \). From (12a), (12b) and (8), we get

\[
\eta(1+\tau_c) c/h = (1-\tau_h)A(1-\alpha)lu^{1-\alpha}(k/h)^\alpha. \tag{13}
\]

Combining equations (12d) and (12e), recalling (8), we find

\[
(1-\tau_h)(1-\alpha)\beta Azu^{1-\alpha}(k/h)^\alpha = (1-\beta)((1-\tau_h)\kappa + (1-s_h)(1-\kappa))(e/h). \tag{14}
\]
Hereafter, let $\gamma_x = \dot{x}/x$ denote the growth rate of the variable $x$. Taking logarithms and differentiating (12a) with respect to time, and using (12c) and (12f), we obtain the law of motion of consumption as

$$\gamma_c = (1/\sigma)[(1-\tau_k)(r-\delta_k) - \rho + \eta(1-\sigma)\gamma_z]. \quad (15)$$

Denoting $\zeta = \mu/\theta$, (12c), (12d), (12f) and (12g) imply that

$$\gamma_\zeta = \gamma_\mu - \gamma_\theta = (1-\beta)Bz^{-\beta}(e/h)^\beta (1-l) - \delta_k - (1-\tau_k)(r-\delta_k). \quad (16)$$

From (12c) and (12e), we get

$$\zeta = \frac{\beta Bz^{1-\beta}(e/h)^{\beta-1}}{(1-\tau_k)\kappa + (1-\kappa)(1-s_h)},$$

which taking logarithms and differentiating, yields

$$\gamma_\zeta = (\beta - 1)(\gamma_{e/h} - \gamma_z). \quad (17)$$

The evolution of physical and human capital can be expressed as

$$(\gamma_k + \delta_k)(k/h) = Au^{1-\alpha}(k/h)^\alpha - g_c(Au^{1-\alpha}(k/h)^\alpha - \kappa e/h) - c/h - e/h, \quad (18)$$

$$\gamma_h = Bz^{1-\beta}(e/h)^\beta - \delta_h. \quad (19)$$

2.5. The balanced growth path

Along the balanced growth path, consumption, investment in physical and human capital, physical capital and human capital grow at the same constant rate $\gamma$, and time allocations remain constant. Imposing these conditions in (15), (16), (18) and (19), and recalling (7) and (8), provides

$$\gamma = (i - \rho)/\sigma, \quad (20a)$$

$$i = (1-\tau_k)(\alpha Au^{1-\alpha}(k/h)^{\alpha-1} - \delta_k), \quad (20b)$$

$$i = (1-\beta)Bz^{-\beta}(e/h)^\beta (1-l) - \delta_h, \quad (20c)$$
\[
\gamma = B(e/h)^\beta z^{1-\beta} - \delta_h, \tag{20d}
\]

\[
(\gamma + \delta_h)(k/h) = (1 - g_e)Au^{1-\alpha} (k/h)^\alpha + g_e \kappa e/h - c/h - e/h, \tag{20e}
\]

which, together with (2), (13) and (14), form the system that characterizes the balanced growth path in terms of the variables \(\gamma, i, k/h, u, z, l, c/h, \) and \(e/h.\)

Equation (13) reflects the equality between the marginal rate of substitution between consumption and leisure and the real return rate on human capital. Equation (14) follows from the equality of the returns of each factor in both sectors. Equation (20a) relates the long-run growth rate with the net return on physical capital. Equations (20b) and (20c) equal the real return rates of each factor, net of taxes and depreciation, with the interest rate. Equation (20d) establishes that in the long-run human capital grows at the same rate than consumption and physical capital. Equation (20e) is the resource constraint for the economy.

2.5. Transitional dynamics

Let \(\chi\) denote \(c/h, \psi\) denote \(k/h\) and \(\varphi\) denote \(e/h.\) Equations (2), (13) and (14) yield

\[
u(\chi, \psi, l) = \left[ \frac{A(1-\alpha)(1-\tau_h)l}{\eta(1+\tau_e)\chi} \right]^{1/\alpha} \psi, \tag{21}\]

\[
z(\chi, \psi, l) = 1 - l - u(\chi, \psi, l), \tag{22}\]

\[
\varphi(\chi, \psi, l) = \frac{A(1-\alpha)\beta (1-\tau_h)z(\chi, \psi, l)e^{\alpha}}{(1-\beta)((1-\tau_h)\kappa + (1-s_h)(1-\kappa))u(\chi, \psi, l)^\alpha}. \tag{23}\]

The laws of motion of \(\chi\) and \(\psi\) as functions of \(\chi, \psi\) and \(l\) are given by

\[
\gamma_{\chi}(\chi, \psi, l) = \gamma_c(\chi, \psi, l) - \gamma_h(\chi, \psi, l), \tag{24}\]

\[
\gamma_{\psi}(\chi, \psi, l) = \gamma_k(\chi, \psi, l) - \gamma_h(\chi, \psi, l). \tag{25}\]

The laws of motion of human and physical capital can be readily obtained from (18) and (19):
\[\gamma_h(\chi, \psi, l) = Bz(\chi, \psi, l)^{-\beta} \varphi(\chi, \psi, l)^\beta - \delta_h,\]

\[\gamma_k(\chi, \psi, l) = (1-g_c)Au(\chi, \psi, l)^{-a} \psi^{a-1} - (1-\kappa g_c)\varphi(\chi, \psi, l)\psi - \chi \psi - \delta_k.\]

Using (16), the law of motion of \(\zeta\) can be expressed as

\[\gamma_\zeta(\chi, \psi, l) = (1 - \beta)Bz(\chi, \psi, l)^{-\beta} \varphi(\chi, \psi, l)^\beta (1 - l) - \delta_h\]

\[- (1 - \tau_k)(\alpha Au(\chi, \psi, l)^{-a} \psi^{a-1} - \delta_k).\]

Differentiating the expressions of \(u\), \(z\) and \(\varphi\) as functions of \(\chi\), \(\psi\) and \(l\) in (21), (22) and (23) with respect to time and combining the results with (17), provide a system of four equations:

\[\gamma_u(\chi, \psi, l) = (1/\alpha)(\gamma_1(\chi, \psi, l) - \gamma_x(\chi, \psi, l)) + \gamma_y(\chi, \psi, l),\]

\[0 = \gamma_u(\chi, \psi, l)u(\chi, \psi, l) + \gamma_z(\chi, \psi, l)z(\chi, \psi, l) + \gamma_1(\chi, \psi, l)l,\]

\[\gamma_v(\chi, \psi, l) = \gamma_x(\chi, \psi, l) + \alpha(\gamma_y(\chi, \psi, l) - \gamma_u(\chi, \psi, l)),\]

\[\gamma_w(\chi, \psi, l) = (1 - \beta)(\gamma_z(\chi, \psi, l) - \gamma_v(\chi, \psi, l)).\]

Solving this system for \(\gamma_u\), \(\gamma_z\), \(\gamma_v\) and \(\gamma_l\) we obtain the following expression for \(\gamma_r\):

\[\gamma(\chi, \psi, l) = \gamma_x(\chi, \psi, l) + \gamma_z(\chi, \psi, l)/(1 - \beta),\]

which, using (24), is equivalent to

\[\gamma_1(\chi, \psi, l) = \gamma_x(\chi, \psi, l) - \gamma_h(\chi, \psi, l) + \gamma_z(\chi, \psi, l)/(1 - \beta).\]

Equation (15) can be expressed as

\[\gamma_x(\chi, \psi, l) = (1/\sigma)[(1 - \tau_k)(\alpha Au(\chi, \psi, l)^{-a} \psi^{a-1} - \delta_k) - \rho + \eta(1 - \sigma)\gamma_y(\chi, \psi, l)].\]

Solving the system (29)-(30) yields

\[\gamma_x(\chi, \psi, l) = \frac{1}{\sigma + \eta(\sigma - 1)} \{[1 - \tau_k)(\alpha Au(\chi, \psi, l)^{-a} \psi^{a-1} - \delta_k) - \rho + \eta(\sigma - 1)[\gamma_h(\chi, \psi, l) - \gamma_z(\chi, \psi, l)/(1 - \beta)](1 - \beta)]\}

\[+ \eta(\sigma - 1)[\gamma_h(\chi, \psi, l) - \gamma_z(\chi, \psi, l)/(1 - \beta)]\]
\[ \gamma_i(\chi, \psi, l) = \frac{\sigma}{\sigma + \eta(\sigma - 1)} \left\{ (1/\sigma) \left[ (1 - \tau_k)(\alpha A u(\chi, \psi, l)^{1-\alpha} \psi^{\alpha - 1} - \delta_k) - \rho \right] - \gamma_k(\chi, \psi, l) + \frac{\gamma_z(\chi, \psi, l)}{(1 - \beta)} \right\}. \] (32)

Equations (24), (25) and (32) form the system that characterizes the dynamics of the economy, and allow us to express \( \gamma_i, \gamma \psi \) and \( \gamma \) as functions of \( \chi, \psi \) and \( l \) through (26), (27), (28) and (31), recalling (21), (22) and (23).

3. Calibration of the model

To calibrate the model, we shall view the postwar U.S. economy as if it were a closed economy on a balanced growth path. Measuring aggregate tax rates is a complex and difficult task and there is little consensus on tax rate measures. In this paper, we use the estimates computed by Mendoza et al. (1994), as extended in Mendoza and Tesar (1998), who calculated effective tax rates for the U.S. by combining detailed tax revenue statistics with information from the aggregate balanced sheets of households, corporations and government from national income accounts. Recently, these authors (Mendoza and Tesar, 1998) analyzed the effects of tax reforms in a neoclassical growth model calibrated to the post-war U.S. economy, where estimates for the tax rates are computed following their methodology. In this paper, we follow the detailed calibration of Mendoza and Tesar (1998) with two minor modifications, imposed by the consideration of human capital accumulation in our model, which are described below.

First, since our model considers human capital accumulation, we need estimates for the time in market work, \( u \), and the time in school and training, \( z \), which are taken from Jones et al. (1993, fn 2) over the period 1960-1985. Second, using data for the 1968-1991 period, Mendoza and Tesar (1998) divide GDP in the fractions 0.65 to private consumption, 0.19 to government spending, and 0.16 to gross investment on physical capital, which includes net
exports. However, our model includes investment on human capital, so these figures must be modified accordingly. What should be considered investment in human capital accumulation is very troublesome (see Trostel, 1993, for a discussion). Here, we will assume that it comprises only educational and job-training expenditures. Educational expenditures will be the part of investment in human capital accumulation that is included in measured output, while job-training expenditures will be an unmeasured component of output. Data of expenditure on education are based on the *Digest of Education Statistics* 1996 (Table 30) and 1997 (Table 409) (U.S. Department of Education). The share of expenditure on education averaged 6.7 percent of GDP over the 1969-1991 period, and public expenditure on education averaged 76 percent of total expenditure on education over the 1988-1991 period. Since public expenditure on education is treated as a subsidy (as in Trostel, 1993), this is the subsidy rate to goods invested in human capital purchased directly by individuals, $s_h$. Private and public expenditure on education constitute investment on human capital in this model, so they must be subtracted from private consumption and government spending, respectively, in the national income accounts to obtain the 63.4 percent share of consumption in GDP and the 13.9 percent share of government spending in GDP reported in Table 1.

We follow Mendoza and Tesar (1998) to set the ratio of the stock of physical capital (fixed nonresidential) to GDP, $k/GDP$, the long-run growth rate, $\gamma$; the share of physical capital income in GDP, $\alpha_E$, and the IES, $1/\sigma$ (see that reference for details). The fraction of goods invested in human capital that are bought with forgone earnings, $\kappa$, is difficult to estimate. We use the value of 0.25 in the baseline, as suggested by Trostel (1993), but the sensitivity of the results to this choice will be studied in the following section. Predetermined parameters, data to be matched by the model, and calibration results are shown in Table 1.

[TABLE 1 AROUND HERE]
Now, we shall briefly explain how the calibration results reported in Table 1 are obtained. The capital income’s share of GDP, \( \alpha_E \), must not coincide with the capital income’s share of output, \( \alpha \), as there is an unmeasured component of output, \( \kappa e \). Using (9), the ratio of output to physical capital can be obtained as

\[
\frac{y}{k} = \frac{y}{\text{GDP}} \frac{\text{GDP}}{k} = \left( \frac{\kappa (1-\kappa) e}{1-\kappa} \right) \text{GDP} = 0.473,
\]

substituting the values of \( \kappa \), \( (1-\kappa)e/\text{GDP} \) and \( k/\text{GDP} \) reported in Table 1. Thus, the capital income’s share of output is calculated as \( \alpha = 0.352 \) from the condition \( \alpha y/k = \alpha_E \text{GDP}/k \).

Equation (4) (or (20e)), which can be expressed as \( \delta_k = (i_k/\text{GDP})(\text{GDP}/k) - \gamma \), yields the calibrated value of the rate of depreciation of physical capital. Equation (20b), \( i = (1-\tau_k)(\alpha y/k - \delta_k) \), allows us to compute the interest rate, \( i \), using (33). Then, the value of the rate of time preference \( \rho \) is obtained from the expression for the growth rate (20a). From \( y/k = Au^{1-\alpha}(k/h)^{\alpha - 1} \), recalling (33) and using the estimated value of the work time, \( u \), we can obtain the ratio of physical to human capital, \( k/h \). From \( e/h = (e/\text{GDP})(\text{GDP}/k)(k/h) \) and \( c/h = (c/\text{GDP})(\text{GDP}/k)(k/h) \), we can compute the stationary values of \( e/h \) and \( c/h \). The reported values of \( \eta \) and \( \beta \) and are then obtained from (13) and (14), respectively. Now, (20c) and (20d) yield the values of \( B \) and \( \delta_h \), after substituting the value of the learning time, \( z \). From the government budget constraint (10), we obtain the share of GDP claimed by the government for lump-sum transfers, \( g \). Since the economy is on its balanced growth path in its pre-tax-reform equilibrium, the requirement that the present discounted value of government revenues equal that of government expenses entails that the government budget is balanced in the pre-tax-reform equilibrium; i.e., \( b = 0 \).
4. Welfare analysis

In this section, the welfare-maximizing tax structure is determined. Here, tax structure refers to the mix of flat-rate taxes on capital income, labor income and consumption that keeps the present discounted value of government revenues equal to the present discounted value of government expenditures; i.e., that keeps the present discounted value of the lump-sum taxes (or transfers) necessary to balance the government’s budget over time,

$$
\Omega = \int_0^\infty b(t) e^{\int_0^t (\delta - r_s) ds} dt,
$$
equal to zero. The quantity $b$ is a measure of the current fiscal imbalance, whereas $\Omega$ is a measure of the intertemporal fiscal imbalance (see Bruce and Turnovsky, 1999), expressed as a deficit. Hence, in this paper we focus on deficit-neutral tax policies. The values of $g_c$, $g_s$ and $s_h$ remain fixed at their pre-tax-reform levels displayed in Table 1. The consumption tax rate is also kept fixed at its pre-reform level when the optimal structure of capital and labor income is computed. The government adjusts lump-sum taxes $b$ (or equivalently issues debt) as needed to make up for any shortfall or excess of tax revenue over expenses. The welfare gain of a reform is measured as the constant permanent percentage increase in consumption, $\epsilon$, keeping leisure constant, that leaves the household indifferent between remaining in the pre-tax-reform equilibrium or undertaking the tax reform. We explicitly solve for the non-linear transitional dynamics by using the time elimination method (Mulligan and Sala-i-Martin, 1993, Brunner and Strulik, 2002).

4.1. Optimal tax structure

Table 2 reports the optimal tax structures when the tax on consumption is optimally set or is kept constant. Switching to the welfare-maximizing structure of capital income, labor income, and consumption taxation involves roughly reducing the tax rate on capital income
by approximately one quarter to 30.58 percent, practically halving the tax rate on labor income to 14.48 percent, and increasing the tax on consumption by about six-fold to 25.21 percent. Therefore, welfare rises by 0.46 percent and growth rises by 0.18 percentage points. If capital and labor income is taxed at the same rate, switching to the optimal tax structure involves setting the tax rate on income, $\tau_y$, at 24.50 percent and increasing the consumption tax, $\tau_c$, to 16.77 percent. As a result, growth rises by 0.01 percentage points and welfare rises by 0.21 percent. If the consumption tax is kept constant, the movement to the optimal structure of income taxation involves lowering the tax on capital income, $\tau_k$, to 36.92 percent and increasing the wage tax, $\tau_h$, to 30.93 percent. Therefore, growth falls by 0.07 percentage points and a 0.08 percent welfare gain is achieved. For the sake of comparison, Grüner and Heer (2000) estimate an optimal tax on capital income of approximately 32 percent, drawing on the Lucas’ (1990) supply side model.

It should be noted that the ability to manage the consumption tax leads to welfare gains that are considerably larger than those attainable when the consumption tax is kept constant. Actually, when the consumption tax is kept constant, the welfare gain attainable is fairly small (0.08 percent), whereas the welfare gain attainable when consumption can be set in an optimal manner amounts to 0.46 percent (0.21 percent if capital and labor income is taxed at the same rate). This result is in accordance with those reported by Coleman (2000), who finds that the welfare gains attainable when consumption can be taxed are quite large but they are rather small if consumption cannot be taxed in a neoclassical model. In any case, the welfare gains of shifting to the optimal tax mix are much smaller than those obtained in the optimal taxation exercises considered in the literature, when tax rates are time-varying and
confiscatory levels of taxation are possible in the short run (see, e.g., Jones et al., 1993, and Coleman, 2000).

The results reported in Table 2 differ to a great extent from those obtained in the optimal taxation literature. Milesi-Ferretti and Roubini (1995, 1996, 1998b) found that the optimal long-run tax rates on capital income, labor income, and consumption are zero. The government expenditure would be financed by the interest earned on the assets raised by accumulating budget surpluses during an initial phase of relatively high taxation. When taxes are constrained to be constant, the optimal tax rate would be a compromise between these two opposite forces, which could result in imposing nonzero constant tax rates throughout. This would explain intuitively the result of nonzero tax rates in the optimal tax mix. Moreover, government expenditure is exogenously given in the aforementioned papers, whereas here it is a constant fraction of output, and the tax reforms considered are deficit neutral. The magnitude of the tax rate would then be given by the distortion that it induces along with the requirements of the government’s budget constraint.

Furthermore, the subsidy to educational expenditures included in the model introduces a distortion on human capital accumulation that can be reversed by using a tax on labor income as follows. The subsidy to education lowers the private cost of monetary investments in human capital. This naturally encourages investments in human capital, and changes the mix of these investments by stimulating monetary investments more than time investments. Even though the principal cost of investing in human capital is foregone earnings, which is effectively tax-deductible, the cost of most goods and services used in producing human capital is not reduced by wage taxation (see Trostel, 1993, for a discussion). Hence, a tax on labor income discourages explicit monetary investments more than investments in time,
therefore reversing the distortionary nature on human capital accumulation of subsidizing expenditure on education.

4.2. Transitional dynamics

Figures 1 and 2 illustrate the dynamic effects of an immediate permanent tax-reform that imposes the optimal tax structure ($\tau_k=30.58\%, \tau_h=14.48\%, \tau_c=25.21\%$) at date $t=0$. Figure 1 presents the transitional paths of the variables $l$, $c/h$, $c/GDP$, $e/GDP$, $u$ and $z$, from their pre-tax-reform equilibrium values to their after-tax-reform long-run equilibrium values, as it changes $\nu$, the percentage of deviation of $k/h$ with respect to its after-tax-reform long-run equilibrium value. The point OSS indicates the old steady state value of the variable, where $k/h$ is about 13 percent lower than its after tax-reform long-run equilibrium value.

[FIGURE 1 AROUND HERE]

The reduction of the capital income tax rate after the tax reform encourages investing in physical capital rather than in human capital. The reduction of the labor income tax rate incentives instead to invest in human capital. However, the principal cost of investing in human capital is foregone earnings, which, as noted above, is effectively tax-deductible. Only a fraction of the cost of goods and services used in producing human capital is not reduced by taxation. Hence, the effects of reducing the tax on physical capital income outweigh those of reducing the tax on labor income, and the net effect is to encourage investment in physical capital rather than in human capital. Therefore, $k/h$ rises from 0.54 at the outset to 0.62 at the new balanced growth path after the tax reform. Furthermore, at date 0, learning time, $z$, and the investment on human capital to GDP ratio, $e/GDP$, fall by about 9 percent (from 0.12 to 0.11, and from 8.9 percent to 8.1 percent, respectively), while work time, $u$, increases from 0.17 to 0.185. The increment in the consumption tax, along with the increase of savings caused by a lower taxation of interest income, causes the consumption to GDP ratio to fall by
more than 8.5 percent from 63.4 percent to 58.0 percent, while leisure time slightly decreases from 0.71 to 0.706 at date 0.

The increment in the ratio of physical to human capital over time leads to a rise in the wage rate, which positively affects human capital investment. It also entails a decrease in the interest rate over time, which gradually reverses the negative influence from its initial rise. Therefore, learning time, \( z \), and investment on human capital to GDP ratio, \( e/GDP \), grow over the transition to the new balanced growth path, while the labor supply declines towards its long-run equilibrium value. The fall in the interest rate over time also causes a reduction in savings and a subsequent increase in the consumption to GDP ratio.

**[FIGURE 2 AROUND HERE]**

Figure 2 presents the evolution, in logarithmic terms, of consumption, physical capital, human capital, and investment on human capital after the optimal tax-reform has been instituted. The evolution of each variable is compared with the trajectory that it would follow if the tax reform had not been undertaken, i.e., if each variable remains on the old balanced growth path. The effects of a higher long-run growth rate after the tax reform become apparent in the evolution of these variables. The stock of physical capital increases quickly over time relative to its pre-tax-reform equilibrium, and the economy recovers rapidly from the initial decrease in consumption, human capital, and investment on human capital.

4.3. Setting in of the welfare gain

Now, we calculate and compare the time it takes before the welfare gain sets in after a change to each of the optimal tax structures calculated above. Let \( V(T, \tau) \) denote the utility attained at time \( T \) following a change to the flat-rate policy plan \( \tau=(\alpha, \tau_l, \tau_c) \) at time \( t=0 \), i.e.,

\[
V(T, \tau) = \int_0^T e^{-\rho t} \left( \frac{c_l^{\eta}}{1-\sigma} \right)^{\frac{1-\sigma}{\eta}} dt .
\]
As a welfare measure, we will use the ratio of the utility attained at time $T$ under the new policy plan, $\tau$, and the utility obtained by remaining under the current tax structure, $\bar{\tau}=(41.5\%, 29.1\%, 4.4\%)$, displayed in Table 1:

$$\Phi(T, \tau) = -V(T, \tau)/V(T, \bar{\tau}).$$

Since $V(T, \tau) < 0$, a value in excess of $-1$ indicates that the utility attained at time $T$ increases as a consequence of undertaking the tax reform. Figure 3 illustrates the effects of a change to each of the optimal flat-rate tax policies displayed in Table 3: (i) an immediate permanent change to the optimal structure of capital income, labor income, and consumption taxation ($\tau_k=30.58\%, \tau_h=14.48\%, \tau_c=25.21\%$); (ii) an immediate permanent change to the optimal structure of income and consumption taxation, when income from capital and labor is taxed at the same rate ($\tau_k=\tau_y=24.50\%, \tau_h=\tau_y=24.50\%; \tau_c=16.77\%$), and (iii) an immediate permanent change to the optimal structure of capital and labor income taxation, keeping the tax rate on consumption constant ($\tau_k=36.92\%, \tau_h=30.93\%, \tau_c=4.4\%$).

The welfare gain sets in immediately under policy (iii) (dotted line). On the contrary, it would take about 72 periods before instituting policy (i) results in a gain in welfare (solid line), and about 55 periods before instituting policy (ii) results in a welfare gain (dashed line). Furthermore, switching to policy (iii) reports a higher welfare during approximately the first 80 periods than switching to policies (i) or (ii). The lower leisure and also lower consumption in the first periods under policies (i) and (ii) than under policy (iii), as a consequence of a lower tax rate on labor income and a higher tax rate on consumption, explains that in the short-run policies (i) and (ii) report less welfare than policy (iii). In the long-run, however, the effects of a higher growth, and therefore a higher consumption, under policies (i) and (ii) than under policy (iii) outweigh those short-run effects.
It takes a long time before instituting the optimal tax structure of income and consumption taxes results in a welfare gain, and other flat-rate policies, such as policy (iii), could report a higher welfare during a long period of time than the optimal tax policies (i) and (ii). As Islam (1995) argues, these facts could influence the government policy if the government has a relatively short planning horizon, so the gain in welfare that occurs in the distant future is not likely to be of major concern.

4.4. Generational welfare

Following Grüner and Heer (2000), the integral

\[
\int_{-\infty}^{\infty} e^{-\rho t} \frac{(e^{\eta t})^{1-\sigma}}{1-\sigma} dt
\]

can be interpreted as the utility of an individual born at time \( T \), after a once-and-for-all change at time \( t=0 \) to the policy plan \( \tau \). The ratio of welfare under the new policy plan, \( \tau \), and the current policy plan, \( \bar{\tau} \), is used as a welfare measure. Since \( W(T, \tau) < 0 \), a value in excess of \(-1\) indicates that the welfare of generation \( T \) increases as a consequence of undertaking the tax reform.

\[\Psi(T, \tau) = -W(T, \tau)/W(T, \bar{\tau}),\]

is used as a welfare measure. Since \( W(T, \tau) < 0 \), a value in excess of \(-1\) indicates that the welfare of generation \( T \) increases as a consequence of undertaking the tax reform.

[FIGURE 4 AROUND HERE]

Figure 4 depicts the generational welfare \( \Psi(T, \tau) \) undertaking, as in the preceding subsection, three different tax-reforms: (i) an immediate permanent change to the optimal structure of capital income, labor income, and consumption taxation; (ii) an immediate permanent change to the optimal structure of income and consumption taxation, when income from capital and labor is taxed at the same rate, and (iii) an immediate permanent change to the optimal structure of capital and labor income taxation, keeping the tax rate on consumption constant. As we can observe, policies (i), (ii) and (iii) have different impacts on
welfare of different generations. For policies (i) (solid line) and (ii) (dashed line), generational welfare rises for all generations, and the increase is relatively higher the later the generation under consideration. Following a change to policy (iii) (dotted line), individuals born in the initial 4 periods are made better off. However, for the subsequent generations, generational welfare decreases below the one attainable under the current tax structure. Furthermore, the welfare of every generation is higher under policy (i) than under policy (ii) which, in turn, is higher than under policy (iii).

5. Sensitivity analysis

In this section, we consider the sensitivity of the optimal tax structure to the choice of the fraction of goods invested in human capital accumulation paid for by lower wages, $\kappa$, and the inverse of the IES, $\sigma$. The remaining parameters are not considered in this sensitivity analysis since they were not chosen in advance but determined in the calibration of the model. For the inverse of the IES, we consider the value $\sigma=3$, to cover the range of 0.32-0.45 estimated by Ogaki and Reinhart (1998) for the IES. We also consider the value $\sigma=1$, because the logarithmic utility case has been frequently considered in the literature. For the fraction of goods invested in human capital accumulation paid for by lower wages, $\kappa$, we consider the values 0.1 and 0.4, following Trostel (1993). Only one predetermined parameter is changed at a time, while the others are kept constant. Engen et al. (1997) point out that when testing for the sensitivity of the results, the recalibrated model should reflect the original long-run data. Thus, instead of keeping the values of the remaining parameters (which were determined in the calibration in the baseline) constant, the model is recalibrated to match the U.S. data reported in Table 1. The changes in the optimal tax structures as each parameter changes are reported in Table 3.
When we consider an immediate permanent change to the optimal tax structure of capital income, labor income, and consumption taxation, two main results emerge from Table 3. First, the tax rate on capital income is extremely robust to parameter variations, and significantly different from zero, ranging in all simulations between 29 percent and 32 percent. Second, the balance between wage and consumption taxation varies noticeably, instead. We have already noted that both taxes distort the consumption-leisure margin of choice, which explains that the trade-off occurs between these two taxes.

Table 3 shows that the optimal tax structure depends markedly on the fraction of goods devoted to human capital accumulation that are bought with foregone earnings, $\kappa$. When all taxes are optimally set, as $\kappa$ rises from 10 to 40 percent, the tax rate on labor income falls from about 26 percent to zero, while the tax rate on consumption increases from almost 12 percent to about 41 percent. We have already noted that taxing labor income may reduce the distortionary nature of subsidizing education. The distortion introduced by the subsidy to education diminishes as the fraction of goods paid for by lower earnings augments, since the fraction of goods that are subsidized is reduced and, thus, the higher the value of $\kappa$ the lower the distortion introduced by the subsidy to education. Therefore, as $\kappa$ increases, the optimal tax rate on labor income falls, being progressively substituted by a higher tax rate on consumption to keep a balanced budget. The same reason explains that, when capital and labor income is taxed at the same rate, as $\kappa$ augments the optimal tax rate on income, $\tau_y$, falls from 29.18 to 20.47 percent, while the tax rate on consumption increases from 9.52 to 22.98 percent. When the tax rate on consumption is kept constant, the lower wage taxation is compensated by a higher taxation on physical capital income as $\kappa$ augments, although in this case the variation is fairly small.
Table 3 also shows that the optimal tax structure depends moderately on the IES, $1/\sigma$, when all taxes are optimally chosen. As the value of $\sigma$ rises from 1 to 3, the tax rate on labor income falls from 21.34 to 10.57 percent, and the tax rate on consumption increases from 19.01 to 29.43 percent. Thus, as the value of $\sigma$ increases, which makes agents less inclined to substitute intertemporally, the optimal tax rate on labor income falls, being replaced by a higher optimal consumption tax rate. When capital and labor income are taxed at the same rate, the optimal tax structure is, instead, relatively robust to changes in the value of $\sigma$. As $\sigma$ rises from 1 to 3, the optimal income tax rate increases slightly from 23.53 to 24.84 percent, whereas the consumption tax rate falls from 19.54 to 16.17 percent. The optimal structure of capital and labor income taxation, keeping the consumption tax rate constant, is also rather robust to changes in the value of $\sigma$.

6. Conclusions

In this paper, we derive the optimal tax structure and estimate the welfare gain that the U.S. could attain by switching from its current tax policy to an optimal constant flat-rate tax policy. The restrictions placed on the tax code are shown to lead to different results from those obtained in the optimal taxation literature, especially regarding the desirability of taxation on capital income. An immediate and permanent shift to the optimal tax structure would imply reducing the tax rate on capital income by approximately one quarter, practically halving the tax rate on labor income, and increasing by about six-fold the tax rate on consumption. The optimal tax rate on capital income is found to be extremely robust to parameter variations, and significantly different from zero, ranging from 29 percent to 32 percent in all simulations. On the contrary, the balance between wage and consumption taxation depends markedly on the fraction of goods devoted to human capital accumulation.
paid for by lower wages and, to a lesser extent, the intertemporal elasticity of substitution. Precise estimates of these key parameters are then needed to determine accurately the optimal tax structure.

The results suggest that the ability to manage the consumption tax can lead to welfare gains that are considerably larger than those attainable when the consumption tax is kept constant. Nonetheless, in any case, the welfare gains attainable from instituting the optimal tax structure are much smaller than those obtained in the optimal taxation exercises considered in the literature, when tax rates vary with time and confiscatory levels of taxation are possible in the short run. The ability to manage the consumption tax also leads to radically different implications of a shift to the optimal tax structure on the welfare of different generations. When the consumption tax can be set in an optimal manner, generational welfare rises for all generations after instituting the optimal structure of income and consumption taxation, and the increase is relatively higher the later the generation under consideration. However, if consumption cannot be taxed, individuals born in the initial periods are made better off but, for the subsequent generations, generational welfare decreases below the one attainable under the current tax structure.

If the government has a relatively short planning horizon, the gain in welfare that occurs in the distant future might not be of major concern, and so it could be interested in instituting policies that result in a gain in welfare in a shorter time. It would take a long period before the welfare gain sets in after instituting the optimal structure of income and consumption taxation, and switching to other flat-rate tax policies could report a higher welfare during a long period of time than switching to the optimal tax structure. This fact could influence the government willingness to adopt the optimal policy.
Appendix A

Variable and parameter definitions

\[ y \] output
\[ k \] physical capital
\[ h \] human capital
\[ c \] consumption
\[ i_k \] investment on physical capital
\[ e \] investment on human capital
\[ l \] leisure time
\[ u \] work time
\[ z \] learning time
\[ U(c,l) \] intertemporal utility function
\[ r \] rate of return on physical capital
\[ w \] wage rate
\[ s \] government lump-sum transfers
\[ g \] government consumption
\[ g_s \] share of GDP devoted to lump-sum transfers representing welfare programs
\[ g_c \] share of GDP devoted to government consumption
\[ A \] productivity constant in goods production
\[ \alpha \] capital income’s share of output
\[ \alpha_E \] capital income’s share of GDP
\[ \delta_k \] depreciation rate of physical capital
\[ \delta_h \] depreciation rate of human capital
$1/\sigma$ intertemporal elasticity of substitution

$\rho$ rate of time preference

$\eta$ parameter reflecting preference for leisure in the utility function

$B$ productivity constant in the production of human capital

$\beta$ elasticity of investment on human capital in the production of human capital

$\kappa$ fraction of investment on human capital paid for by lower earnings

$s_h$ subsidy rate to expenditure on education

$\tau_y$ flat income tax rate

$\tau_k$ flat capital income tax rate

$\tau_l$ flat labor income tax rate

$\tau_c$ flat consumption tax rate

$b$ current government’s fiscal imbalance

$\gamma_x$ growth rate of the variable $x$

$\gamma$ long-run growth rate

$i$ after-tax net interest rate

$\chi$ ratio of consumption to human capital, $c/h$

$\psi$ ratio of physical to human capital, $k/h$

$\varphi$ ratio of investment on human capital to human capital, $e/h$

$\Omega$ intertemporal government’s fiscal imbalance

$\varepsilon$ welfare gain of a tax reform

$V(T, \tau)$ utility attained at time $T$ after a change to the tax structure $\tau$

$W(T, \tau)$ utility of an individual born at time $T$ after a change to the tax structure $\tau$

$\nu$ percentage of deviation of $k/h$ with respect to its after-tax-reform long-run value
References


TABLE 1. Calibration of the model

<table>
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Data to match by the model

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<th>( i_k/\text{GDP} )</th>
<th>( (1-\kappa)e/\text{GDP} )</th>
<th>( k/\text{GDP} )</th>
<th>( \alpha_E )</th>
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Fiscal policy parameters

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Calibration results

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### TABLE 2. Optimal tax structure in the baseline

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<th>Optimal</th>
<th>$\tau_y$ (%)</th>
<th>$\tau_k$ (%)</th>
<th>$\tau_h$ (%)</th>
<th>$\tau_c$ (%)</th>
<th>$u^*$</th>
<th>$z^*$</th>
<th>$\gamma^*$ (%)</th>
<th>$\epsilon^*$ (%)</th>
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TABLE 3. Sensitivity analysis of the optimal tax structure to parameter variations

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<th>$\tau_x^*$ (%)</th>
<th>$\tau_h^*$ (%)</th>
<th>$\tau_c^*$ (%)</th>
<th>$u^*$</th>
<th>$z^*$</th>
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<td>$\tau_h, \tau_h$</td>
<td>---</td>
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<td>0.118</td>
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<td></td>
<td>$\tau_y, \tau_c$</td>
<td>29.18</td>
<td>---</td>
<td>---</td>
<td>9.52</td>
<td>0.168</td>
<td>0.118</td>
<td>1.53</td>
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<tr>
<td></td>
<td>$\tau_h, \tau_h, \tau_c$</td>
<td>---</td>
<td>32.01</td>
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<td>0.120</td>
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<td>38.50</td>
<td>30.34</td>
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<td>0.169</td>
<td>0.119</td>
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<td>41.33</td>
<td>0.181</td>
<td>0.129</td>
<td>1.94</td>
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CAPTIONS FOR THE FIGURES

FIGURE 1: Policy functions after instituting the optimal structure of capital income, labor income, and consumption taxation

Note: The abscissa, υ, is the percentage of deviation of k/h from its new steady state value.

OSS = Old Steady State; NSS = New Steady State.

FIGURE 2: Evolution of some variables after instituting the optimal structure of capital income, labor income, and consumption taxation

Note: The value of each variable in the old balanced growth path at t=0 is normalized to unity. OBGP = Old Balanced Growth Path.

FIGURE 3: Setting in of the welfare gain after instituting the optimal tax structure

FIGURE 4: Welfare of generation T after instituting the optimal tax structure
FIGURE 1: Policy functions after instituting the optimal structure of capital income, labor income, and consumption taxation

Note: The abscissa, $\nu$, is the percentage of deviation of $k/h$ from its new steady state value.

OSS = Old Steady State; NSS = New Steady State.
FIGURE 2: Evolution of some variables after instituting the optimal structure of capital income, labor income, and consumption taxation

Note: The value of each variable in the old balanced growth path at \( t=0 \) is normalized to unity. OBGP = Old Balanced Growth Path.
FIGURE 3: Setting in of the welfare gain after instituting the optimal tax structure

\[ \tau_k, \tau_h, \tau_c = \text{optimal}\tau_k, \tau_h, \tau_c \]

\[ \tau_y, \tau_c = \text{optimal}\tau_y, \tau_c \]

\[ \tau_k, \tau_h = \text{optimal}\tau_k, \tau_h \]
FIGURE 4: Welfare of generation $T$ after instituting the optimal tax structure

---

$T^*=\text{optimal } \tau_k, \tau_h, \tau_c = \text{optimal } \tau_y, \tau_c = \text{optimal } \tau_k, \tau_h$

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