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Jaime Luque

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Assessing the role of TIF and LIHTC in an equilibrium model of affordable housing development*

Jaime Luque

University of Wisconsin - Madison

Abstract

The goal of this paper is to understand the impact of Tax Incremental Financing (TIF) and Low Income Housing Tax Credits (LIHTCs) in an equilibrium model of affordable housing development. Developers in different jurisdictions compete to sell their LIHTCs to a corporation, and LIHTC prices are determined in equilibrium. In this setting, when a jurisdiction decides to provide TIF to its developer, housing becomes more affordable, but both the construction costs and the leverage ratio of the project may increase with respect to a project without TIF money. In essence, TIF ameliorates the demand for LIHTC equity and makes equity less expensive.

JEL Classification : D52, D53, G12, G14, G15. G18.

Keywords: affordable housing development; Tax Incremental Financing (TIF), Low Income Housing Tax Credit (LIHTC); equity; debt

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1 Introduction

Los Angeles, New York, and other cities in America are struggling to cope with the problem of homelessness and the lack of affordable housing.[1] On a single night in January 2015, more than 560,000 people nationwide were homeless, meaning they slept outside, in an emergency shelter, or in a transitional housing program. Almost a quarter were children.[2] Meanwhile, homeownership is hovering at 20-year lows, while about half of renters struggle to pay their landlords. Housing affordability problems were rapidly transmitted into the vacancy rates, which fell sharply in almost all major metropolitan areas in the United States between 2010 and 2015 (see Gabriel and Painter 2017).[3] There is simply not enough housing available to provide affordable units to all of the community members who are struggling to pay their rents or are homeless (see Luque and Pope 2017 for a policy report on homelessness in Madison, Wisconsin). Policymakers are aware of these problems and are revising/expanding existing financial programs to increase the supply of affordable housing. Among these programs, the most important in the United States (U.S.) are the Tax Incremental Financing (TIF, hereafter) and the Low Income Housing Tax Credit (LIHTC, hereafter - often pronounced "lie-tech").

TIF enables municipalities to finance developments that, in theory, bring better infrastructure, more jobs, increased tax revenue, and other benefits to a city or district. TIF is also an important tool for affordable housing development. The mechanism is as follows. A developer applies for TIF and, if approved, the jurisdiction gives him a loan that is then paid back in subsequent periods using part (or all) of the property taxes that the new real estate asset generates. In a sense, TIF is seen as free money by the developer because existing property taxes must be paid no matter what. This cheap debt ameliorates the developer's need for additional equity and helps reduce the market rent charged to the low income tenants of the new building. The jurisdiction in turn is able to expand the supply of affordable housing in specific areas by allocating TIF to specific

[1]Other medium size US cities face similar problems. For example, in Madison, Wisconsin, the homeless population has dramatically increased in the last years, with a staggering 40% since 2010. Roughly one third of the homeless population is composed of children.

[2]For example, last fall, Los Angeles Mayor Eric Carcetti asked the City Council to declare "a state of emergency" on homelessness and committed US$100 million to solving the problem, suggesting that subsidies would play a role.

[3]Segregation and housing inequality is also an important issue in other developed countries. For example, Ben-Shahar and Warszawski (2017) find that in Israel, housing affordability inequality has considerably increased in the past decade.

districts, called Tax Increment Districts, or TIDs.

LIHTC is one of the most successful housing programs in U.S. history. More than 2.4 million affordable homes have been constructed or rehabilitated using LIHTC money since 1986. Under this program, each state receives an annual allocation of tax credits, and then distributes theses tax credits among developers according to a well-defined Qualified Allocation Plan (QAP). Developers raise income by selling their LIHTCs to national investors (e.g., Prudential Financial and AEGON USA Realty Advisors). Typical LIHTC transactions allocate a 99.99% share of the ownership entity to the investor that buys the tax credits, while the developers keeps the remaining 0.01%. The valuation of equity in this type of housing development projects is driven by the market prices of LIHTCs at the jurisdiction level. Hence, the phrase "LIHTC equity" is commonly used in the affordable housing development industry.

Our goal in this paper is to examine the role of TIF and LIHTC on affordable housing development. To this aim, we build an equilibrium model with two jurisdictions where local developers provide affordable housing to local households, and finance these investments with a mixture of TIF, LIHTC equity, and debt. We assume that each developer is endowed with a given amount of LIHTCs, which are then sold to a corporation at competitive prices. Because the development projects in different jurisdictions can differ in terms of productivity, size, availability of construction materials, or even in their capital structures, the competitive prices of LIHTCs can differ between jurisdictions. This feature of the model is consistent with the observed heterogeneity of LIHTC prices across different regions in the United States. In our model, LIHTC prices are endogenously determined in equilibrium given the demand for and supply of LIHTCs in different jurisdictions. The corporation's valuation of LIHTC equity in a development project is found by dividing the value of LIHTCs purchased by 0.9999.

Another key element of our model is the property taxes that real estate owners pay in their respective jurisdictions. The revenue from these taxes reverts to the agents of the jurisdiction according to some exogenous sharing rule. When the jurisdiction's profits revert to local households, taxes can be seen as a standard redistributive device from owners of real estate to households. When the jurisdiction profits revert to developers, transfers can be seen as TIF. Implementing a TIF policy makes all agents in the jurisdiction other than developers worse off in terms of transfers of the jurisdiction's profits because the jurisdiction profit weights must add up to 1 across all agents of the jurisdiction.

We use our model to study the impact of TIF on LIHTC equity pricing, housing rents, construction costs, and other equilibrium variables of our affordable housing development model. This question is particularly important in a setting with more than one jurisdiction and is relevant at the policy level in the U.S. because the strategies and policies of municipal governments often differ significantly, even between cities of similar size with similar demographics and similar industries supporting their existence. For instance, two cities can exist next to each other, in the same state, with roughly the same amount of people, with the majority of their workers employed in similar vocations, and yet they will be governed in entirely different manners. This individualized structure of local government in the U.S. makes standardization almost impossible, and very few policy tools exist that can be used across the country at the municipal level.

TIF is a rare animal in the world of local politics, and can credit much of its success to how easily it can be customized to each locality. However, because capital markets are global, it is unclear how a TIF policy implemented at the local level affects global investors' decisions of where to allocate their capital (in the form of LIHTCs for affordable housing investments). It is possible that a generous TIF policy in one jurisdiction reduces or even eliminates the necessity for equity in the capital structure of an affordable housing development project, but in turn may channel more equity into affordable housing development projects in other jurisdictions.

To illustrate this possibility, we provide an example of an equilibrium in which one of the two jurisdictions decides to provide TIF to its (representative) developer. TIF effectively makes housing more affordable (housing rents decrease in the second period). However, it also increases both the construction costs and the leverage ratio of the project compared with the other development project in the jurisdiction without a TIF policy.

In essence, TIF liberates resources that make the developer's budget constraint less binding, in turn ameliorating the demand pressure for LIHTC and with it making the valuation of equity less expensive. Also, the additional TIF resources that the developer receives end up increasing the price of construction materials in the jurisdiction that implemented the TIF policy, hence our result of higher construction costs relative to the jurisdiction without a TIF policy.

Importantly, we find that an equilibrium with affordable housing developed in both jurisdictions may not exist when there are additional restrictions on the equilibrium values, such as an upper bound on the amount of leverage (debt to total equity ratio) that the corporation tolerates when investing in affordable housing. We illustrate this with our leading example where TIF has the effect of increasing the leverage ratio for a given amount of a construction loan. The main insight from this exercise is that TIF may not always lead to a successful development project.

Another important question in municipal public policy is how to evaluate whether a proposed development project needs TIF to become feasible; in theory, a developer should only receive TIF if the project would not be feasible without it. There is an active and intense debate in the policy arena about this issue. Our model with two jurisdictions sheds light on this question by deriving equilibrium formulas (or rules of thumb) that express the TIF contribution both in terms of amounts and financial ratios. In terms of amounts, the equilibrium condition says that for affordable housing to be developed, the sum of TIF, income from the sale of LIHTCs to the corporation, and debt issued by the developer must cover the total construction costs and the developer's property taxes. In terms of financial ratios, we can express the TIF to total equity ratio as a function of the developer's property tax proceeds to total equity ratio, the construction costs to total equity ratio, the developer's debt to total equity ratio, and the corporation's LIHTCs to total equity ratio.

Relationship with the literature

The literature on affordable housing development is scarce. However, several academic[4] and government[5] initiatives suggest that this will rapidly change given the importance that low-income housing markets have had on people's welfare and public policy in recent years.

The theoretical literature has approached the economic modelling of real estate development with either a "real options" pricing partial equilibrium model (see e.g. Titman 1985 and Childs, Riddiough, and Triantis 1996) or in an equilibrium setting with entrepreneurial developers and a housing/land market (see, e.g., Helsley and Strange 1997). Recently, Faias and Luque (2017) have provided a third approach by considering an equilibrium model that focuses on the financing aspects of commercial real estate development when jurisdictions compete to attract commercial

[4]See for instance the 2017 AEI-BoI-BGFRS-TAU-UCLA Conference on Housing Affordability, a three-year academic partnership exploring policy approaches to the housing affordability challenge. The partnership consists of the American Enterprise Institute, Bank of Israel, Board of Governors of the Federal Reserve System, Tel Aviv University Alrov Institute for Real Estate Research, and UCLA Ziman Center for Real Estate.

[5]For example, we think of New York City mayor Bill de Blasio's unprecedented plan to create 200,000 units of affordable housing over ten years.

real estate equity investments.

Our equilibrium model builds on Faias and Luque (2017) to incorporate TIF and LIHTC in an equilibrium model of real estate development. However, there are important differences between the two papers. Faias and Luque's goal is to prove existence of an equilibrium in a model of commercial real estate development with segmented commercial equity markets. Our goal in the current paper is first to incorporate LIHTC and TIF into an equilibrium model of affordable housing development, and then to study the impact of TIF and LIHTC on the rents, construction costs, property taxes, and capital structure of two housing development projects in different jurisdictions. To our knowledge, our paper is the first to provide this type of analysis.

There are other papers in the literature, most of them empirical, that study the topic of affordable housing development, but which focus however on the impact of affordable housing on local neighborhoods; see, for instance, Baum-Snow and Marion (2009) for the impact of LIHTC financed developments in low income neighborhoods, Freedman and Owens (2011) for the impact of LIHTC developments on crime, and more recently, Diamond and McQuade (2017) for the impact of multi-family housing developments funded through the LIHTC program on surrounding property values; see also Green, Malpezzi, and Seah (2002) for an early study on this issue, and a review of the previous findings in the literature. We refer to Luque (2016) for a careful review of this literature and the different proposed methods for creating affordable cities that serve the interests of the disadvantaged.

The remainder of this paper is structured as follows. In Section 2, we introduce our economy with two jurisdictions, several types of agents (including local affordable housing developers and households), and property taxes at the jurisdiction level. Section 3 reviews the main financing tools available for developers to use to construct affordable housing (including LIHTC and TIF) and explains how we incorporate these tools into our model. Section 4 presents the definition of equilibrium and its characterization. Section 5 presents thought-provoking examples of equilibria with and without a TIF policy, and assesses the role of TIF on LIHTC equity pricing, housing rents, construction costs, and other equilibrium values in view of these examples. Section 6 provides a conclusion.

2 Model set-up

Let us consider an economy with two periods, denoted by $t = 1, 2$, and two states of nature in the second period, which we denote by s_1 and s_2. In the first period and at each state of the second period, there is a numeraire good that facilitates trade, e.g., cash. The numeraire good, whose price we normalize to 1, is traded in the global market at each node of the event tree. We label the numeraire good as "10" if we are in period 1 and "$s0$" if we are at state $s = s_1, s_2$ of the second period.

2.1 Agents

In our economy there are two jurisdictions, denoted by k_1 and k_2. Each jurisdiction $k = k_1, k_2$ has a representative local developer d_k, which finances the construction of affordable housing with its own income (developer's equity), the equity raised by selling ownership interests in the property to investors in exchange for LIHTCs, and TIF financing (developer's subsidized debt).

In jurisdiction $k = k_1, k_2$, there is also a representative low-income household h_k whose consumption only consists of affordable housing in the second period. A low-income household is defined as one led by a person with a precarious job whose income mostly depends on government aid, such as housing vouchers. For modeling purposes, we assume that the household's endowment (housing voucher) is denominated in units of the numeraire good. Also, for simplicity, we ignore the housing market segments for medium- and high-income households, and assume that affordable housing units are all filled with homogeneous low-income households. All construction in this model consists of affordable housing that relies on the LIHTC and TIF programs.

We also consider two types of companies: a corporation c that buys LIHTCs from affordable housing developers, and a company m that owns and sells materials to the developers for the construction of housing. We assume that these two companies belong to both jurisdictions k_1 and k_2. Thus, the sets of agents in jurisdictions k_1 and k_2 are $\mathbf{k}_1 = \{h_1, d_1, c, m\}$ and $\mathbf{k}_2 = \{h_2, d_2, c, m\}$, respectively. We denote by $\mathbf{K}^a$ the set of jurisdiction(s) to which agent a belongs. For example, if $a = h_1$, then $\mathbf{K}^{h_1} = \{k_1\}$. If instead $a = c$, then $\mathbf{K}^c = \{k_1, k_2\}$. Finally, let $\mathbf{A} = \{h_1, h_2, d_1, d_2, c, m\}$ denote the set of all agents in the economy.

2.2 Construction and consumption of affordable housing

For simplicity, we assume that company m owns two types of composite construction materials: one composite material that is needed for development in jurisdiction k_1 (e.g., adobe and wood) and another material that is needed for construction in jurisdiction k_2 (e.g., cement and wood). These differences may exist because of different land geographies, regulations, or simply households' tastes. For our purposes, the assumption of considering different construction materials for different jurisdictions is useful to simplify the equations that describe our equilibrium model. We label the composite good available for the construction of housing in jurisdiction $k = k_1, k_2$ as "$1k$". Company m's endowment of the composite good $1k$ and its market price are denoted by ω_{1k}^m and p_{1k}, respectively. We assume that the other agents of the economy have no endowments of construction materials. Thus, $\omega_{1k}^a = 0$, for all $a \neq m$ and $k = k_1, k_2$.

To construct affordable housing, developers purchase the composite material specific to their jurisdiction. We denote this choice variable for developer d_k by $x_{1k}^{d_k}$. The production function of affordable housing in jurisdiction k is Cobb-Douglas and has the following specific form:

$$y_k^{d_k} = TFP_k (x_{1k}^{d_k})^{\alpha_k},$$

where parameters $\alpha_k \leq 1$ and $TFP_k \geq 0$ stand for the exponent of the Cobb-Douglas function and the "Total Factor Productivity" of the housing construction project in jurisdiction k, respectively. We denote the housing development project in jurisdiction k by j_k.

Development of affordable housing occurs as follows. If developer d_k buys a positive amount of construction material $1k$, the construction of housing is initiated (instantaneously) in the first period. At the beginning of the second period, the construction is finalized, but the size of the project will depend on the state of nature. In particular, the final amount of affordable housing available in jurisdiction k at state s is given by the following function

$$f_k(y_k^{d_k})(s) = \varepsilon_k(s) y_k^{d_k},$$

with $\varepsilon_k(s) > 0$. We denote the (endogenous) rent per unit of affordable housing occupied in jurisdiction k at state s by $p_k(s)$. At state s, the low-income household h_k rents an amount $x_k^{h_k}(s)$

of affordable housing space.[6]

Uncertainty, captured in our model by the presence of two states of nature, reflects the possibility that development plans do not go as expected. For example, we can think of s_1 as the state when construction plans go as expected and take $\varepsilon_k(s_1) = 1$. We can also consider that at state s_2 there is a disruption in the construction process and that, as a consequence, the final project size of affordable housing is smaller than at state s_1. The latter would correspond to the case $\varepsilon_k(s_2) < 1$.

2.3 Preferences

Each agent $a = h_1, h_2, c, m$ assigns utility to the consumption bundle

$$x^a \equiv (x^a_{10}, x^a_{11}, x^a_{12}, (x^a_0(s), x^a_{k_1}(s), x^a_{k_2}(s))_{s=s_1,s_2})$$

composed of the agent's purchase of: (i) the numeraire good and construction materials in the first period, (ii) the numeraire good at states s_1 and s_2, and the housing goods in jurisdictions k_1 and k at states s_1 and s_2. We consider the following log linear functional form for an agent a's utility function:

$$u^a(x^a) = \sum_{l=0,1,2} \theta^a_{1l} \ln x^a_{1l} + \sum_{s=s_1,s_2} \left(\theta^a_0(s) \ln x^a_0(s) + \sum_{k=1,2} \theta^a_k(s) \ln x^a_k(s) \right)$$

where the θ-parameters represent the agent a's utility weights corresponding to the different consumption goods. Households are the only agents that enjoy the consumption of affordable housing in their respective jurisdictions (a household in jurisdiction k only enjoys the consumption of housing in its own jurisdiction k). Thus, $\theta^{h_1}_{k_1}(s) > 0$, $\theta^{h_2}_{k_2}(s) > 0$, $\theta^{h_1}_{k_2}(s) = 0$, and $\theta^{h_2}_{k_1}(s) = 0$. Roughly speaking, households h_1 and h_2 only enjoy the consumption of housing goods $k_1 s$ and $k_2 s$, respectively, at state $s = s_1, s_2$ of the second period, and thus prefer to postpone consumption until the second period.

The two local developers, d_1 and d_2, and the corporation c only get positive utility from the

[6]Because housing is only consumed in one period ($t = 2$), we use the words rents and prices for housing interchangeably.

consumption of the numeraire good in the second period. The only agent that prefers to consume in the first period is company m, which seeks to sell its construction materials to the representative local developer of each jurisdiction and use the proceeds to purchase as much as possible of the numeraire good 10. Thus, we assume that all remaining θ-parameters are zero, except for $\theta_{10}^{m} > 0$ and $\theta_0^{d_1}(s) > 0$, $\theta_0^{d_2}(s) > 0$, $\theta_0^{c}(s) > 0$, for $s = s_1, s_2$.

2.4 Jurisdiction property taxes and profits

We define a jurisdiction by a triplet $(\mathbf{k}, \alpha_k, \gamma_k)$ that specifies the set of players $\mathbf{k}$ in jurisdiction k, the technology α_k that developers use for the construction of affordable housing projects, and the property tax γ_k, respectively.

Each jurisdiction $k = k_1, k_2$ incurs some costs (e.g., public goods such as roads, sewerage, fire protection, police, etc.) in terms of the numeraire good. To finance these costs, jurisdictions charge property taxes to the owners of real estate properties. In our simple model, this amounts only to the owners of affordable housing. We consider the following linear functional form for property taxes:

$$g_k(E_k^a) = \gamma_k E_k^a$$

where $\gamma_k > 0$ is a parameter that we refer to as the property tax rate and E_k^a is the agent a's ownership interest in property j_k. Equity owners pay a property tax that is proportional to their affordable housing equity holdings.[7] For simplicity, we assume that this tax is charged in the first period and is a function of the housing development size (this tax can be seen as the taxes charged to equity holders in period 2 that are discounted with the corresponding equilibrium shadow price deflators).[8]

[7] We assume a "pass-through taxation" model, where the owners of the property are responsible for the property taxes and other expenses. This form of taxation includes Limited Liability Companies, one of most prevalent business forms in the United States.

[8] Optimality requires that equity taxes paid in the first period be equivalent to the discounted property taxes that the developer would pay in the second period using the developer's shadow prices of its budget constraints. We denote the shadow prices of budget constraint in the first period and at state s of the second period by $\beta_1^{d_2}$ and $\beta^{d_2}(s)$, respectively. Then, if $\gamma_k'(s)$ denotes the equity tax rate that a developer would pay at state s of the second period for holding one unit of equity in jurisdiction k_2, we would have

$$\gamma_k = \sum_{s=s_1,s_2} \frac{\beta^{d_k}(s)}{\beta_1^{d_k}} \gamma_k'(s)$$

We split the jurisdiction's costs to provide public goods between a fixed component and a variable component. Fixed costs are denoted by $\lambda_k > 0$. The variable cost for jurisdiction k is linear with functional form $\eta_k \sum_{a\in\mathbf{k}} E_k^a$, $\eta_k > 0$.

The profit of jurisdiction k in the first period is given by

$$\pi_k \equiv \sum_{a\in\mathbf{k}} \gamma_k E_k^a - \left(\lambda_k + \eta_k \sum_{a\in\mathbf{k}} E_k^a\right)$$

In our model, property taxes have a redistributive effect because jurisdiction profits revert to the agents that live and do business in the jurisdiction according to some weights. To see this, consider an agent $a \in \mathbf{k}$ and let its share of jurisdiction k's profit be $\delta_k^a \in [0,1]$. By choosing a vector $(\delta_k^a)_{a\in\mathbf{k}}$, such that $\sum_{a\in\mathbf{k}} \delta_k^a = 1$, the jurisdiction manager is effectively redistributing resources among agents in the jurisdiction.

3 Financing tools for affordable housing development

In this section we review the main financing tools available to developers in the construction of affordable housing and explain how we incorporate them into our model. The first financing tool for developers is uncollateralized debt. The modeling of debt issuance is standard, and thus for the sake of brevity we just introduce the notation and the short sale constraint that a developer is subject to. The other two financing tools, LIHTC and TIF, require more discussion because, to our knowledge, it is the first time they are being incorporated into an equilibrium model. Thus, for each of them, we first provide a brief description of how the corresponding program works in the United States, which is informative for our modeling purposes, and then explain how we incorporate them into our framework, which was introduced in the previous section.

3.1 Debt

The first financing tool for developers is uncollateralized debt issuance. For simplicity, we ignore any issues regarding debt collateralization.[9] An agent a buying (selling) a face value of debt equal

[9] We leave for future research an extension of this model in which debt is risky and collateralized by the housing asset. Also notice that assuming that debt is nominal is useful to guarantee the existence of an interior point in an

to D^a pays (receives) τD^a in the first period, where τ is the (endogenous) discount price of debt in the first period. We denote the short and long debt positions by $D^a < 0$ (borrower) and $D^a > 0$ (lender), respectively. Short sales are subject to the following constraint,

$$D^a \geq -\bar{D}^a, \tag{1}$$

where $\bar{D}^a > 0$ (notice that we allow this short sale constraint to be agent-type specific). At state s of the second period, the borrower (lender) pays (receives) $r(s)D^a$ nominal units.

3.2 Equity

An equity stake on an affordable housing project represents a claim to the future cash flows generated by this asset. The choice of how much equity the developer and the corporation agree to trade is endogenous in our model (see below). We denote these variables by $E_k^{d_k}$ and E_k^c, respectively. We assume that the low-income households and the company m that owns construction materials do not trade in the equity market of affordable housing development projects. Thus, we set $E_k^{h_k} = 0$ for $k = k_1, k_2$, and $E_k^m = 0$.

We denote by E_k the total amount of equity available in project j_k. If the corporation c buys $E_k^c < E_k$, then developer d_k holds the difference $E_k - E_k^c$. Formally, the market feasibility condition for equity shares corresponding to an affordable housing development project j_k is

$$E_k^{d_k} + E_k^c = E_k. \tag{2}$$

When $E_k^{d_k} = 0$, the developer sells the whole property to corporation c and does not keep any equity for itself. If instead $E_k^{d_k} \in (0, E_k)$, the developer keeps part but not all of the ownership on the property. When $E_k^{d_k} = E_k$, the developer owns all equity of the project.

In our model, the developer d_k and corporation c's equity stakes on property j_k are determined by the amount of LIHTCs traded between the two agents. We explain this in the next section.

agent's budget constraint. See Luque and Faias (2017).

3.3 LIHTC

With more than 2.4 million affordable homes constructed or rehabilitated since 1986, the Low-Income Housing Tax Credit is seen as one of the most successful housing programs in U.S. history.[10] These credits are also commonly called Section 42 credits in reference to the applicable section of the Internal Revenue Code. The creation of the LIHTC program goes back to 1986, when the administration implemented a federal income tax reform that provided a ten-year tax credit for investors in affordable housing.[11] This initiative was made permanent by Congress in 1993. The most innovative aspect of this program is that it makes federal subsidies a tax expenditure administered by the Internal Revenue Service (IRS) rather than a federal expense administered by the U.S. Department of Housing and Urban Development (HUD).

The National Association of Home Builders (NAHB) estimates that the one-year local impact of constructing 100 units for a typical family LIHTC development includes $7.9 million in local income, $827,000 in taxes and other revenue for local governments, and 122 local jobs. The annual recurring impact of those 100 family units includes $2.4 million in local income, $441,000 in taxes and other revenue for local governments, and 30 local jobs.[12]

Another important innovative aspect is that it allows for some local control because credit allocators usually are the state housing financial authorities. Each state receives an annual allocation of tax credits, and then distributes theses tax credits among local developers according to a well-defined Qualified Allocation Plan (QAP). The QAP is updated every year or two to reflect current priorities.

In their review of tax credit applications, the state agency in charge of allocating the tax credits uses a point scoring system to evaluate projects. These points are distributed among different categories. For example, the Wisconsin Housing and Economic Development Authority (WHEDA) 2017 QAP assigns a total of 284 points to the following categories (points in parentheses): lower-income areas (5), energy efficiency and sustainability (32), mixed-income incentive (12), serves large families (5), serves lowest-income residents (60), supportive housing (20), rehab/neighborhood stabilization (25), universal design (18), financial participation (25), eventual

[10]For a detailed description of the LIHTC program, see Hobart and Schwarz (1995). We also refer the interested reader to the U.S. Department of Housing and Urban Development's website for further details.

[11]The compliance period is 15 years, reflecting that investors are not out at year 10.

[12]See report by National Association of Home Builders (2010).

tenant ownership (3), development team (12), readiness to proceed (12), credit usage (30), and opportunity zones (25). Developers prepare their affordable housing proposals trying to score the maximum points for as many categories as possible, but sometimes financial and physical barriers prevent developers from getting high scores in some of these categories. The maximum annual credit allocation is usually either 4% or 9% of the eligible basis of a project[13], depending on factors such as the type of affordable housing development project (e.g., rehabilitation versus new construction) and the use of tax exempt bonds.[14]

The LIHTC program is a dollar-for-dollar tax credit, meaning that for each dollar of tax credits that an investor purchases, it can deduct a dollar from its federal income tax. Tax credits are sold at market prices and can vary across locations (depending on factors such as the size of the deals and the developers' expertise). The price per credit dollar is usually smaller than $1, but it is possible to see a price above $1 per credit dollar.[15]

Developers that obtain a state's tax credits raise equity by selling these credits to the investors. Typical transactions allocate a 99.99% share of the ownership entity to the investor that buys the tax credits, while the developers keep the remaining 1%. Thus, the equity raised by the developer to help finance the development is 0.9999 times the total equity value of the project. The following example illustrates how this transaction works.

Suppose a developer is allocated with $1 million in tax credits. This reflects a $100,000 annual allocation multiplied by 10 to reflect the 10-year period over which the credits can be taken. If the market price of tax credits is $0.90 for a dollar of credit, we have to multiply the $1 million by 0.90, indicating that the developer is willing to pay $900,000 for these credits. In actuality, the investor is buying a 99.9% share of the ownership entity, so the $900,00 would further be reduced to $899,910 when we multiply by 0.9999. So in this example, the equity raised by the developer to help finance the development is $899,910. Figure 1 illustrates this deal.

[13]The eligible basis of a project is the cost of acquiring an exisiting building if there is one (but not the cost of the land), plus construction and other construction-related costs to complete the project.

[14]The choice of the annual allocation rate comes down to whether the developer is using tax exempt bond financing or not. Tax exempt bonds are often combined with the 4% credits and are more often used for acquisition rehab projects. The bond financing can qualify the developer for 4% credits as long as thresholds are met, and is not as competitive as the 9% credit in many states. In a 4% deal, the amount of credit is so much less than the developer can generate with 9% credits that it is difficult to make new construction or adaptive re-use deals work without additional soft financing sources. Tax exempt bonds and 9% credits cannot be combined.

[15]In 2017, pricing on the cost has often risen above $1, and even in midwest states such as Wisconsin, pricing over $1 was not unheard of prior to the fall election of 2017.

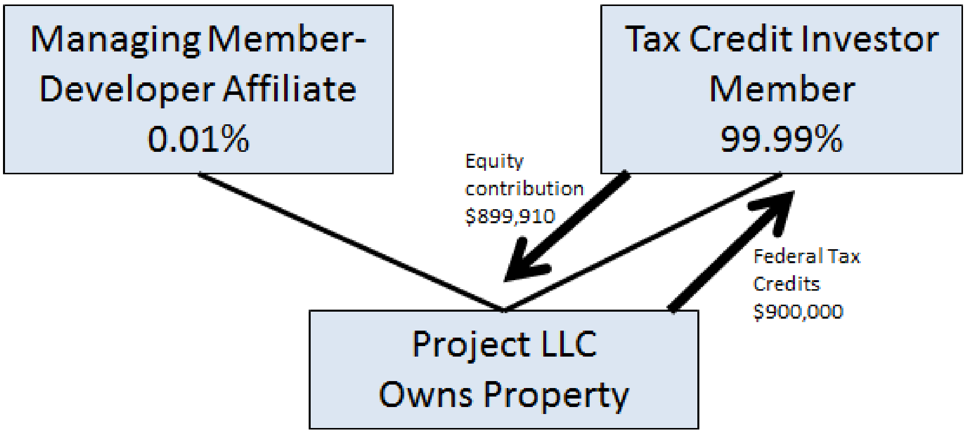

Figure 1: This figure illustrates a typical LIHTC deal, in which a developer is allocated $1 million in tax credits and the market price of these credits is $0.90.

3.3.1 Incorporating LIHTC into our model

Let us assume that company c sets aside an amount ω_{10}^{c} (in terms of the numeraire good) to buy LIHTCs (for simplicity, we ignore corporation c's decision of how much to allocate for purchasing LIHTCs). For the sake of exposition, we denote jurisdiction k's housing authority allocation of LIHTCs to agent a by $\hat{T}_k^a$, and assume that only the developer d_k receives a positive allocation of LIHTCs, i.e., $\hat{T}_k^{d_k} > 0$ and $\hat{T}_k^a = 0$ if $a \neq d_k$. This trick will help us simplify the budget constraints introduced below.

Corporation c chooses how many LIHTCs to buy. We denote this choice variable by $T_k^c \geq 0$. Because it is possible that the developer will keep some tax credits for itself ($T_k^{d_k} \geq 0$), market feasibility requires

$$T_k^c + T_k^{d_k} = \hat{T}_k^{d_k} \tag{3}$$

The following facts motivate our modelling of the relationship between the investor's tax credits purchased and its equity stakes on the property. First, recall that if a corporation buys all the tax credits ($\hat{T}_k$ in our terminology), then it becomes the owner of 99.99% of the property, and the developer keeps the remaining 1% of the ownership interest in the property. Second, although it is unusual, a corporation can buy fewer tax credits than the developer's total amount of LIHTCs (i.e., $T_k^c < \hat{T}_k^{d_k}$). In that case, the developer keeps the remaining tax credits ($T_k^{d_k} = \hat{T}_k^{d_k} - T_k^c$). Given these two facts, we can express the equity stakes that corporation c and developer d_k decide to hold in property j_k as

$$E_k^c = 0.9999T_k^c \tag{4}$$

and

$$E_k^{d_k} = (\hat{T}_k^{d_k} - T_k^c) + 0.0001T_k^c, \tag{5}$$

respectively.

We denote the price that clears the market of LIHTCs corresponding to the affordable housing development project j_k by q_k. In our equilibrium notion below, we let T_k^c and $T_k^{d_k}$ be choice variables and infer E_k^c and $E_k^{d_k}$ from variables T_k^c and $T_k^{d_k}$ and equations (3), (4), and (5). Notice that market clearing of LIHTCs in jurisdiction k (condition (3)) implies that the total amount of equity ($E_k = E_k^c + E_k^{d_k}$) must be such that

$$E_k = \hat{T}_k^{d_k} \tag{6}$$

In other words, the total amount of tax credits available to developer d_k is equal to the nominal amount of project j_k's total equity.

For our two period economy, when corporation c purchases an amount $T_k^c > 0$ of LIHTCs, it gives $q_k 0.9999T_k^c$ units of the numeraire good to developer d_k in the first period. At state s of the second period, corporation c is then entitled to an amount of tax credits T_k^c and the property cash flows $p_k(s)f_k(E_k^c)(s)$ corresponding to an amount of equity $E_k^c = 0.9999T_k^c$. The developer's revenue in the second period consists of the remaining tax credits ($T_k^{d_k} = \hat{T}_k^{d_k} - T_k^c$) and the cash flow $p_k(s)f_k(E_k^{d_k})(s)$ corresponding to an amount of equity $E_k^{d_k} = (\hat{T}_k^{d_k} - T_k^c) + 0.0001T_k^c$.[16] We can see these terms in the following budget constraints of an agent a:

$$(x_{10}^a - \tilde{\omega}_{10}^a) + \sum_{k=1,2} p_{1k}(x_{1k}^a - \omega_{1k}^a) + \tau D^a + \sum_{k \in \mathbf{K}^a} q_k 0.9999(T_k^a - \hat{T}_k^a) + \gamma_k E_k^a \leq 0 \tag{7}$$

$$(x_0^a(s) - \omega_0^a(s)) + \sum_{k \in \mathbf{K}^a} p_k(s)x_k^a(s) \leq \bar{r}D^a + T_k^a + \sum_{k \in \mathbf{K}^a} p_k(s)f_k(E_k^a)(s) \tag{8}$$

[16]Notice that from condition (6) and the linearity of function f, we have that $f_k(E_k^c)(s) + f_k(E_k^{d_k})(s) = f_k(\hat{T}_k^{d_k})(s)$.

Constraints (7) and (8) correspond to an agent a's budget constraints in period 1 and state s of period 2, respectively. The term $\tilde{\omega}_{10}^a$ in constraint (7) is defined as follows:

$$\tilde{\omega}_{10}^a \equiv \omega_{10}^a + \sum_{k \in \mathbf{K}^a} \delta_k^a \pi_k$$

That is, in addition to the endowment of the numeraire good in period 1, an agent a that belongs to jurisdiction k receives a transfer $\delta_k^a \pi_k$ from its jurisdiction, where δ_k^a is the redistribution weight of profits in jurisdiction k distributed to agent a, and is such that $\sum_{a \in \mathbf{k}} \delta_k^a = 1$. If an agent belongs to more than one jurisdiction – as it is in the case of the corporation c and the company m – this agent may receive transfers from the two jurisdictions. For the case of a developer d_k, the set of jurisdiction memberships is single – it only belongs to jurisdiction k – and therefore its after-transfer endowment is $\tilde{\omega}_{10}^{d_k} = \omega_{10}^{d_k} + \delta_k^{d_k} \pi_k$. Similarly, we can write $\tilde{\omega}_{10}^{h_k}$ for household h_k as $\tilde{\omega}_{10}^{h_k} = \omega_{10}^{h_k} + \delta_k^{h_k} \pi_k$. The interpretation of weight $\delta_k^{d_k}$ is key for our modeling of TIF, as we shall see below.

3.4 TIF

TIF starts with the creation of Tax Incremental Districts (or TID) in a city; these are catchments or neighborhoods that are designated with a specific development need, such as affordable housing. The first word of TIF is "Tax", and everything that occurs with a TIF project will always come back to taxation policy. This starts at the very beginning of a TIF project. Within a TID, all tax recipients have to agree before approving and offering TIF funds to a developer. These tax recipients include schools, fire stations, and many other essential programs to a local government: programs that often need significant financial support. TIF will limit the tax revenue received by these organizations, so it is very important that all of them understand the financial impact of offering TIF to a developer, and have an opportunity to veto a project that they think will not adequately improve the TID. These tax recipients, and occasionally local law makers as well, typically make up a TIF committee that approves or rejects projects requesting TIF assistance.

Once the TID is created, a developer can come forward with a proposal for their project. The developer will need the correct entitlements in place before asking for TIF, but not all TIDs place the same requirements on pre-development work before a TIF proposal is made. As we can see in Table 1 below, the developer's proposal needs to include the value of the project annually, typically for the next ten years (for simplicity, we restrict our model to two dates). The proposal should also include building renderings, budgets, and the project's current capital structure and shortfall.

There needs to be a budget shortfall for a project to be eligible for TIF; otherwise the developer can move forward without needing the city's financial assistance. While this may at first appear to be a market inefficiency, one should keep in mind that we are dealing with TIF projects that include an affordable housing component, and also that many of these developments occur in blighted areas, where soil contamination and other factors can complicate construction. There are complicating factors that surround these deals, thus placing them in the position where the market cannot fully support their weight. Also, keep in mind that taxes and regulations (such as soil requirements, building ordinances, and setbacks) create additional restrictions for developers that sometimes make their projects unfeasible. For instance, in downtown Madison, Wisconsin, there is significant demand for multi-family affordable housing, but because of the State Capitol Dome and beautiful lake views, there are strict height requirements which can prevent developers from being able to build to the needs of the downtown area, as they are only allowed to build so many stories. In a way, TIF remediates the burden of restrictions originally created by regulation through taxation policy.

The developer's submitted proposal is then reviewed by the city. Typically, there is a person or committee that analyzes deals and rejects undesirable proposals before the projects are reviewed and approved by the TIF committee. Ideally, this person or committee is intimately familiar with the ins and outs of the real estate industry. For the sake of presentation, the initial reviewer(s) (not the final TIF committee) will be referred to as the "analyst".

The first step for the city is valuing the project. The value might decrease initially if an existing, functioning structure needs to be demolished before construction of the project can begin, but should increase significantly upon completion of the improvements. The correct valuation of the proposed development is critical, because the new tax basis upon completion is the foundation for the TID to calculate available TIF monies, and typically the city will employ a consultant, either full-time or ad hoc, to independently appraise the project. The developer and the city may not come to the same conclusion about the project's completed value, but the TIF committee is not required to take the developer's valuation into account when valuing the project or at any point during the process of determining whether or not to support the development with TIF monies.

Once the analyst has established the value of the project, the difference in tax revenue from the project now versus the increased tax revenue from the new project must be determined. For instance, the city may currently receive $100,000 annually in property taxes from the existing structure, and the new development will increase tax revenue to $500,000 annually after construction is completed, a difference of $400,000 (as shown in Table 1, this adds up to a significant amount). This is the most critical part of a TIF project analysis, because it determines the amount of TIF funds available for the city to distribute to the developer. Typically, for reasons that will be explained shortly, these values are estimated for the next ten years. Simplistically, ten years in this case would mean there is a difference of $4,000,000. This difference is called the increment, hence the name Tax Increment Financing.

If the city approves the project after an intensive review process, then the city can offer a financing package for that amount to the developer, and take the increment as payment back on that financing. A municipality can often receive lower interest rates on mortgages or even use bonds to get the increment monies (in this case, $4,000,000) up front, and it hands those funds over to the developer. The developer then makes a payment back on these funds by simply paying its taxes. The tax recipients only take the original tax basis, the $100,000 annually in this example, and the remaining $400,000 is given back to the original lender, either the bond owners or the bank. Typically, a city would not lend out the entire $4,000,000 to the developer; a general benchmark is 50-60% of the increment, to mitigate the city's own risk. In this case, if the city gave the developer $2,000,000, the developer would pay back $400,000. However, the city is not using the full $400,000 for debt servicing; it is only using $200,000 for debt service. The remaining $200,000 can be distributed to the tax recipients, or it can be used for infrastructure development. For example, if a developer was putting up two hundred multifamily affordable units in a previously vacant lot, the city can anticipate more traffic, and plan to improve or widen the road. These additional TIF monies are used for almost any conceivable municipal improvement, from increasing sewer and storm water capacity to building parks and libraries.

Original Tax Basis	\$100,000
New Tax Basis	\$500,000
Increment	\$400,000
Ten Year Increment	\$4,000,000
Loan Amount	\$2,000,000
Debt Service (Annual)	\$200,000
Excess Funds (Annual)	\$200,000

Table 1: This table provides a numerical example of how to calculate the TIF, as well as the debt service and excess funds associated with the increment.

3.4.1 Incorporating TIF into the model

Now, let us go back to the term $\tilde{\omega}^a_{10} \equiv \omega^a_{10} + \sum_{k \in \mathbf{K}^a} \delta^a_k \pi_k$ in budget constraint (7). When agent a is a developer, say d_k, transfer $\delta^{d_k}_k \pi_k$ can be seen as TIF. Roughly speaking, the jurisdiction k transfers some resources to developer d_k in period 1, (partially) offsetting its property taxes, and thus making its budget constraint in the first period less binding. This in turn allows the developer to spend more resources on the development project if deemed necessary (optimality is a necessary condition for this argument to hold in equilibrium).

TIF can also benefit the equity partner (corporation c) if it is entitled to part of the jurisdiction's profits, i.e., if $\delta^c_k \pi_k > 0$. The intuition is similar to the case for developers presented above. The higher $\delta^c_k \pi_k$ is, the fewer "net property taxes" the corporation must pay to the jurisdiction.

Because in our model the developer d_k and the corporation c are the only agents that can own interest in a property, the interpretation of TIF only applies to these two agents. When $\delta^{h_k}_k > 0$, the interpretation is in terms of a lump-sum pure redistribution transfer to the household. Because households do not own equity, and thus do not pay property taxes, this lump-sum transfer is not offset by any other fiscal element in the household's budget constraint. The same argument applies to the case in which the agent is company m.

The jurisdiction can adopt different policies by choosing the value of parameters $\delta^{d_k}_k$, δ^c_k, δ^m_k, and $\delta^{h_k}_k$. For instance, the jurisdiction may choose to increase $\delta^{d_k}_k$. This in turn makes the other agents in the jurisdiction worse off relative to the developer after the new redistribution policy is implemented, since the jurisdiction profit weights must add to 1 across the agents of the jurisdiction.

Notice that in reality the jurisdiction may collect taxes during later periods (period 2 in our model), after the real estate asset is developed. For simplicity, we opted to model property taxes as being collected in the first period but, as discussed above, we could have incorporated this mechanism into our model by discounting future property taxes with the agent's Lagrangian multipliers of the budget constraints. Because in that case the jurisdiction does not collect taxes in the first period, the jurisdiction may need to issue a municipal bond to extend a TIF loan to the developer, and then in the second period collect the property taxes from the developer and make the corresponding payments to the municipal bond holders. Our approach of having the taxes paid in the first period greatly simplifies the exposition of our model and the computation of equilibrium.

4 Equilibrium

In this section, we first formalize our notion of equilibrium and then proceed to characterize the equilibrium in terms of intuitive economic conditions, which will be used later in Section 5 to construct numerical examples of an equilibrium.

4.1 Definition of equilibrium

Let $x^A = (x^{h_1}, x^{h_2}, x^{d_1}, x^{d_2}, x^m, x^c)$ denote the vector of all agents' consumption vectors. Similarly, we denote by $D^A = (D^{h_1}, D^{h_2}, D^{d_1}, D^{d_2}, D^m, D^c)$ and $T^A = (T^{h_1}, T^{h_2}, T^{d_1}, T^{d_2}, T^m, T^c)$ the vectors of all agents' debt and LIHTC choices, respectively.

Given commodity prices $p \in \mathbb{R}^3_+ \times \mathbb{R}^{2\cdot 3}_+$, LIHTC prices $q \in \mathbb{R}^2_+$, and the price of debt $\tau \in \mathbb{R}_+$, an agent a's *optimization problem* consists of choosing a vector $(x^a, D^a, T^a) \in \mathbb{R}^9_+ \times \mathbb{R} \times \mathbb{R}^2_+$ that maximizes its utility function $u^a(x^a)$, subject to budget constraints (7) and (8), the debt short sale constraint (1), and the corresponding constraints that impose a zero consumption and trade of the housing goods outside an agent's jurisdiction(s).

Definition 1: *A competitive equilibrium for this economy with two jurisdictions consists of a system* $(x^A, D^A, T^A, p, q, \tau)$, *such that:*

(i) each agent "a" solves its optimization problem.

(ii) the following market clearing conditions hold:

- *global market clearing of the numeraire consumption good in period 1:*

  $\sum_{a\in\mathbf{A}}(x^a_{10} - \omega^a_{10} + \sum_{k=k_1,k_2}(\lambda_k + \eta_k E_k)) = 0$

- *global market clearing of the construction input in period 1 in jurisdiction $k = k_1, k_2$:*

  $\sum_{a\in\mathbf{A}}(x^a_{1k} - \omega^a_{1k}) = 0$

- *global market clearing of the numeraire consumption good at state $s = s_1, s_2$ of period 2:*

  $\sum_{a\in\mathbf{A}}\left(x^a_0(s) - \omega^a_0(s)\right) = 0,$

- *local market clearing of affordable housing at state $s = s_1, s_2$ in jurisdiction $k = k_1, k_2$:*

  $\sum_{a=\{h_k,d_k,c,m\}} x^a_k(s) - f_k(y^{d_k}_k)(s) = 0$

- *global market clearing of uncollateralized debt:*

  $\sum_{a\in\mathbf{A}} D^a = 0$

- *local market clearing of LIHTCs corresponding to property j_k in $k = k_1, k_2$:*

  $T^{d_k}_k + T^c_k = \hat{T}^{d_k}_k$

The proof of existence of equilibrium for this economy follows Faias and Luque (2017) and is thus omitted. We move to the characterization of equilibrium for our specific economy with LIHTC and TIF.

4.2 Characterization of equilibrium

First, let us make a couple of remarks regarding the market clearing equations corresponding to construction materials and affordable housing. First, because market clearing for this good is jurisdiction-specific and because there is only one composite material per jurisdiction, we must have that developer d_k's purchase of the composite construction material ($x^{d_k}_{1k}$) equals the company m's jurisdiction-specific endowment ω^m_{1k}, i.e., $x^{d_k}_{1k} = \omega^m_{1k}$, for $k = k_1, k_2$. Second, because household h_k's utility is increasing in the amount of housing in period 2, we have that, by market clearing, the representative household h_k's housing consumption is $x^{h_k}_k(s) = \varepsilon_k(s)TFP_k(x^{d_k}_{1k})^{\alpha_k}$.

To obtain simple conditions that characterize the equilibrium of this economy, we find useful to impose the following set of assumptions into the model.

Assumptions on Equity: *Equity choices E_k^c and $E_k^{d_k}$ are implicitly determined by expressions (4) and (5), given T_k^c and $T_k^{d_k}$, and are assumed to be equal to zero for the remaining agents, i.e., $E_k^{h_k} = E_k^{h_{k'}} = E_k^m = E_k^{d_{k'}} = 0$.*

Assumptions on LIHTCs: *Because the LIHTC market of an affordable housing development project j_k is restricted to developer d_k and company c, we take $T_k^{h_k} = T_k^{h_{k'}} = T_k^m = T_k^{d_{k'}} = 0$ with $k' \neq k$.*

Assumptions on agents' shares of jurisdiction profits: *Because our goal is to understand the role of the jurisdiction's transfer of profits from households to developers, for simplicity we assume that company m does not receive any jurisdiction's transfer of profits, i.e., $\delta_1^m = 0$, $\delta_2^m = 0$.*

Assumptions on Debt: *We assume that debt is risk-free, in the sense that $r(s) = \bar{r}$ for $s = s_1, s_2$. We can think of this market as the Treasury bond market, where agents are allowed to short sell the bond up to an amount $-\bar{D}^a$. For simplicity, we assume that company m does not have access to the debt market. All it does is sell its endowments of construction materials to the developers in the first period. Thus, we impose $D^m = 0$. Also, we do not allow corporation c to buy LIHTCs with debt (for simplicity, we assume that the amount of cash allocated to buying LIHTCs is predetermined and thus we ignore the possibility of corporation c issuing debt to purchase LIHTCs); thus, $D^c = 0$.*

Assumptions on Endowments: *We assume that only households h_1 and h_2 and the corporation c are endowed with the numeraire good in the first period. Thus, $\omega_{10}^{h_1} > 0$, $\omega_{10}^{h_2} > 0$, $\omega_{10}^c > 0$, and $\omega_{10}^m = 0$. In the second period, only company m has positive endowments of the numeraire good: $\omega_0^m(s_1) = \omega_0^m(s_2) > 0$. Thus, $\omega_0^c(s_1) = \omega_0^c(s_2) = \omega_0^{d_k}(s_1) = \omega_0^{d_k}(s_2) = 0$. Developers, who prefer consumption of the numeraire good tomorrow, have no commodity endowments whatsoever (since $\omega_0^{d_k} = \omega_{11}^{d_k} = \omega_{12}^{d_k} = \omega_0^{d_k}(s_1) = \omega_0^{d_k}(s_2) = 0$), so issuing risk-free debt, raising LIHTC equity, and obtaining TIF are the only means for them to construct affordable housing and receive cash flows in the second period.*

Remark 1: *In the first period, young households h_1 and h_2 are endowed with the numeraire* *good* 10*. Households are excluded from the equity market and, therefore, the only financial instrument that they can use to transfer wealth from the first to the second period (i.e., saving) is the risk-free asset. Because they prefer to consume tomorrow, they will sell their numeraire good endowment and purchase as much of the risk-free asset as possible (risk-free debt pays 1 unit of the numeraire good in both states of the second period). Thus, in equilibrium we have that*

$$D^{h_1} = \frac{1}{\tau}\tilde{\omega}_{10}^{h_1} > 0 \tag{9}$$

$$D^{h_2} = \frac{1}{\tau}\tilde{\omega}_{10}^{h_2} > 0 \tag{10}$$

where $\tilde{\omega}_{10}^{h_k} \equiv \omega_{10}^{h_k} + \delta_k^{h_k}\pi_k$ *and* $\pi_k = \gamma_k - \eta_k TFP_k \left(x_{1k}^{d_k}\right)^{\alpha_k} - \lambda_k$*. In equilibrium, developers will be on the other side of the market of the risk-free asset (i.e.,* $D^{d_1} < 0$ *and* $D^{d_2} < 0$*).*

For this economy, we are able to find a number of closed form conditions that characterize the equilibrium of our economy. These conditions in turn allow us to construct our numerical examples below. Here we just highlight some interesting conditions regarding equilibrium price differentials and refer the reader to the Appendix for the additional closed form conditions that characterize the equilibrium of our economy.

Proposition 1: *In each jurisdiction, the relative scarcity of affordable housing drives the house price differential between states of nature in the second period, i.e.,*

$$p_1(1)/p_1(2) = \varepsilon_1(2)/\varepsilon_1(1) \tag{11}$$

$$p_2(1)/p_2(2) = \varepsilon_2(2)/\varepsilon_2(1) \tag{12}$$

Moreover, the difference in the prices of construction inputs between jurisdictions is driven by the difference in marginal productivity of the affordable housing assets. In particular,

$$\begin{aligned} p_{11} - p_{12} = {} & \alpha_2 TFP_2(\omega_{12}^m)^{\alpha_2 - 1}\left(p_2(1)\varepsilon_2(1) - \delta_2^{h_2}(\gamma_2 - \eta_2\right) \\ & - \alpha_1 TFP_1(\omega_{11}^m)^{\alpha_1 - 1}\left(p_1(1)\varepsilon_1(1) - \delta_1^{h_1}(\gamma_1 - \eta_1\right) \end{aligned} \tag{13}$$

We leave the proof of Proposition 1 for the Appendix.

Next, we provide simple formulas derived from our equilibrium conditions that express the amount of TIF as a function of other variables, such as LIHTCs, construction costs, property taxes and debt. After this, we shall provide expressions in terms of financial ratios.

Proposition 2: *For affordable housing to be developed in equilibrium, the sum of TIF, the income from the sale of LIHTCs to the corporation, and the debt issued by the developer must cover the total construction costs and the developer's property taxes, i.e.,*

$$TIF_k + LIHTC_k^c + Debt^{d_k} = ConstructionCost_k + PropertyTax_k^{d_k} \tag{14}$$

where $TIF_k \equiv \delta_1^{d_1}\left(\left(\gamma_1 - \eta_1 TFP_1\left(x_{11}^m\right)^{\alpha_1} - \lambda_1\right)\right)$, $ConstructionCost_k \equiv p_{1k}\omega_{1k}$, $LIHTC_k^c \equiv q_k 0.9999(\hat{T}_k^{d_k} - T_k^{d_k})$, *PropertyTax*$_k^{d_k} \equiv \gamma_k\left(\hat{T}_k^{d_k} - T_k^c + 0.0001 T_k^c\right)$, *and* $Debt^{d_k} = \tau D_k^{d_k}$.

When analyzing the capital structure of a development project, analysts look at financial ratios. For example, a debt-to-equity ratio higher than 0.5 indicates that the capital structure has a greater proportion of its capital funding from lenders rather than equity investors. Given this type of analysis in the real estate industry, we rewrite equation (14) in terms of the financial ratios by dividing each term by the total amount of equity $Equity_k \equiv q_k E_k$ corresponding to the affordable housing project j_k.

Corollary 1: *The TIF to total equity ratio is a function of the developer's equity tax proceeds to total equity ratio, the construction costs to total equity ratio, the developer's debt to total equity ratio, and the investor's LIHTC equity to total equity ratio, i.e.,*

$$\frac{TIF_k}{Equity_k} + \frac{LIHTC_k^c}{Equity_k} + \frac{Debt^{d_k}}{Equity_k} = \frac{ConstructionCost_k}{Equity_k} + \frac{PropertyTax_k^{d_k}}{Equity_k} \tag{15}$$

In equilibrium, equations (14) and (15) must hold with equality (otherwise market clearing conditions or the non-satiation property of agents' preferences condition may fail, a contradiction with equilibrium).

Notice also that it may happen that an equilibrium fails to exist when there are additional restrictions on the equilibrium variables. For example, if the corporation does not tolerate a Debt/Equity ratio above a given threshold in jurisdiction k, the amount of TIF and LIHTC granted by the local jurisdiction k may not be enough to cover the construction cost and property taxes. In that case, we say that the affordable development project envisioned for jurisdiction k is not feasible. A more generous TIF can make the difference in this case.

5 Assessing the role of TIF in the presence of LIHTCs

In this section, we provide numerical examples of an equilibrium for our economy. Our main goal in this section is to show the effect of TIF on equilibrium variables, as well as discuss the factors that might make an affordable housing development project unfeasible. We start with an example in which all jurisdiction profits go to the households. Thus, in this first example, neither jurisdiction k_1 nor jurisdiction k_2 adopt a TIF policy.

Example 1 (No TIF): Let us consider an economy that satisfies the assumptions considered in Section 4.2, and consider the following parameter values for materials and the numeraire good endowments: $\omega_{11}^m = \omega_{12}^m = \omega_0^m(1) = \omega_0^m(1) = \omega_{10}^{h_1} = \omega_{10}^{h_2} = 1$, $\omega_{10}^c = 3$. We set the risk-free rate and the lower bound on debt short sales equal to $\bar{r} = 1$ and $\bar{D}^{d_1} = \bar{D}^{d_2} = 1$, respectively. The Total Factor Productivity parameters are normalized to 1 (i.e., $TFP_1 = TFP_2 = 1$) and the α-parameters of the production function are $\alpha_1 = \alpha_2 = 0.5$. Uncertainty about the size of the affordable housing development project is captured by the following parameter values: $\varepsilon_1(1) = \varepsilon_2(1) = 1$, $\varepsilon_1(2) = \varepsilon_2(2) = 0.5$.

Property taxes are the same in both jurisdictions and equal to $\gamma_1 = \gamma_2 = 0.5$. Variable and fixed costs are $\eta_1 = \eta_2 = 0.1$ and $\lambda_1 = \lambda_2 = 0.01$, respectively. We assume that all jurisdiction profits are redistributed to the households and thus there is no TIF money in this benchmark example, i.e., $\delta_1^m = 0$, $\delta_2^m = 0$, $\delta_1^{h_1} = \delta_2^{h_2} = 1$, $\delta_1^{d_1} = \delta_2^{d_2} = 0$, $\delta_1^c = \delta_2^c = 0$. Finally, let us make the total amount of LIHTCs that the developer in each jurisdiction receives from its respective jurisdiction equal to 1, that is, $\hat{T}_1^{d_1} = \hat{T}_2^{d_2} = 1$.

For the above parameter values, we obtain a unique equilibrium solution where $q_1 = q_2 = 1.00$, $\tau = 1.39$, $p_{11} = p_{12} = 2.39$, $p_1(1) = p_2(1) = 0.50$, $p_1(2) = p_2(2) = 1.00$, $D^{d_1} = D^{d_2} = 1.00$, $T_1^c = T_2^c = 1.00$, and $T_1^{d_1} = T_2^{d_2} = 0.00$. Thus, $TIF_k = \delta_k^{d_k}\pi_k = 0$, $LIHTC_k^c = q_k T_k^c = 1.00$, $ConstructionCost_k = p_{1k}\omega_{1k}^m = 2.39$, $PropertyTax_k^{d_k} = \gamma_k E_k^{d_k} = \gamma_k(\hat{T}_k^{d_k} - T_k^c +$

$0.0001T^c_k) = 0.00025 = 0.00$, and $Debt^{d_k} = \tau D^{d_k} = 1.39$,for both $k = k_1, k_2$.[17]

In terms of financial ratios, our equilibrium is characterized as follows. For both $k = k_1, k_2$, $\frac{TIF_k}{Equity_k} = 0.00$, $\frac{LIHTC^c_k}{Equity_k} = 1.00$, $\frac{Debt^{d_k}}{Equity_k} = 1.39$, $\frac{ConstructionCost_k}{Equity_k} = 2.39$, $\frac{PropertyTax^{d_k}_k}{Equity_k} = 0.00$. Thus, financial ratios are the same in both jurisdictions. Figure 2 illustrates these financial ratios.

In the next example, we assume that jurisdiction k_2 adopts an active TIF policy.

Example 2 (The impact of TIF): Let us consider again the parameter values considered in our leading Example 1, but now suppose that jurisdiction k_2 introduces a TIF policy, which in terms of our parameters consists of choosing $\delta^{d_2}_2 = 0.5$ and $\delta^{h_2}_2 = 0.5$. That is, now developer d_2 receives half of the jurisdiction profits that before went to household h_2. For these new parameter values, we obtain a unique equilibrium solution.

We report the equilibrium solution and the corresponding financial ratios in Tables 2 and 3, respectively. In addition, for the sake of exposition, we illustrate the new financial ratios for each jurisdiction in Figure 2.

Compared to the equilibrium values of the benchmark model, we find that the new TIF policy for jurisdiction k_2 is now $TIF_2 = 0.20$ and has the following effects:

- TIF_2 increases by roughly 6 percent the construction cost in jurisdiction k_2 relative to jurisdiction k_1.

- TIF_2 decreases by roughly 8 percent the total equity value in jurisdiction k_2 relative to jurisdiction k_1.

- TIF_2 makes the capital structure in jurisdiction k_2 more levered than in jurisdiction k_1. In particular, $TIF_2 = 0.2$ increases by 8 percent the debt-to-equity ratio in jurisdiction k_2 relative to jurisdiction k_1.

- TIF_2 decreases the rent paid by household h_2 for consuming affordable housing by roughly 15 percent at both states s_1 and s_2.

In essence, TIF relaxes the developer d_2's budget constraint and as a result less LIHTC equity from outside investors (here, the corporation c) is needed to finance the development project.

[17] We write the equilibrium solution up to two decimals, and consider the rounding error as negligible.

Thus, for a given amount of debt issuance, the development project's debt-to-equity ratio increases relative to the project in the jurisdiction without a TIF policy. In terms of our numerical example, the affordable housing development project becomes 8 percent more levered in the jurisdiction with TIF relative to the project without TIF money. In addition, TIF liberates resources that end up being spent on the purchase of construction materials.[18] This in turn generates a 6 percent increase in construction costs relative to the no TIF jurisdiction.

	Example 1		**Example 2**	
	k_1	k_2	k_1	k_2
q_k	1.00	1.00	1.04	0.96
τ	1.39	1.39	1.29	1.29
p_{1k}	2.39	2.39	2.32	2.46
$p_k(1)$	0.50	0.50	0.54	0.46
$p_k(2)$	1.00	1.00	1.08	0.92
D^{d_k}	1.00	1.00	1.00	1.00
T_k^c	1.00	1.00	0.99	1.00
$T_k^{d_k}$	0.00	0.00	0.01	0.00
TIF_k	0.00	0.00	0.00	0.20
$LIHTC_k^c$	1.00	1.00	1.03	0.96
$Debt^{d_k}$	1.39	1.39	1.29	1.29
$ConstructionCost_k$	2.39	2.39	2.32	2.46
$PropertyTax_k^{d_k}$	0.00	0.00	0.01	0.00
$Equity_k$	1.00	1.00	1.04	0.96

Table 2: This table provides the values of the equilibrium variables for the parameters considered in Examples 1 and 2. To report the solutions, we consider two decimals, and thus assume that the rounding error is negligible.

[18]Notice that in a more sophisticated economy with additional investment opportunities, such as additional development projects, stocks, and savings, TIF money could be channeled to those assets instead, and that would not necessarily be in the interest of the jurisdiction.

	Example 1		Example 2	
	k_1	k_2	k_1	k_2
$TIF_k/Equity_k$	0.00	0.00	0.00	0.21
$LIHTC_k^c/Equity_k$	1.00	1.00	0.99	1.00
$Debt^{d_k}/Equity_k$	1.39	1.39	1.24	1.34
$ConstructionCost_k/Equity_k$	2.39	2.39	2.23	2.56
$PropertyTax_k^{d_k}/Equity_k$	0.00	0.00	0.01	0.00

Table 3: This table provides the values of the equilibrium variables for the parameters considered in Examples 1 and 2. To report the solutions, we consider two decimals, and thus assume that the rounding error is negligible.

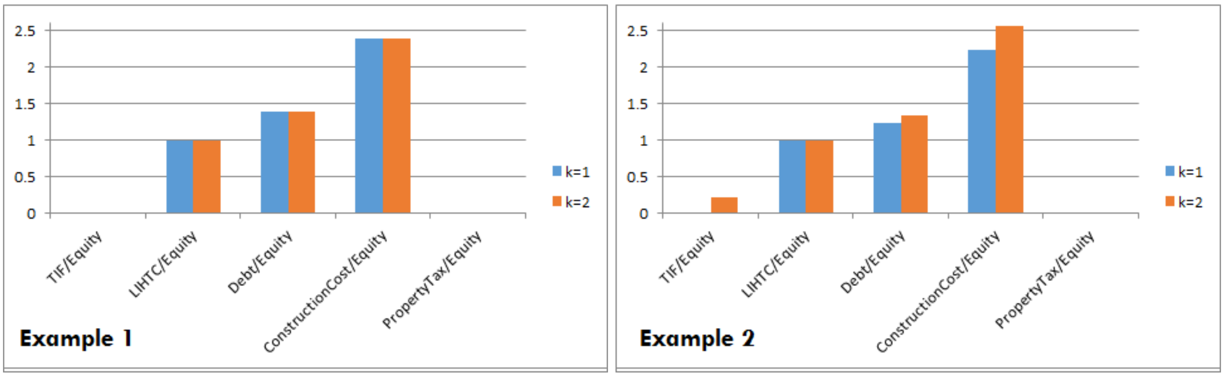

Figure 2: The figure on the left illustrates the financial ratios $TIF_k/Equity_k$, $LIHTC_k^c/Equity_k$, $Debt^{d_k}/Equity_k$, $ConstructionCost_k/Equity_k$, and $PropertyTax_k^{d_k}/Equity_k$ of the affordable housing development projects in jurisdictions $k = k_1, k_2$, given the parameter values of Example 1. The figure on the right does the same but using instead the equilibrium values of Example 2. We round solutions to two decimals, and consider the rounding error as negligible.

We finish this section with a remark on the existence of an equilibrium in the presence of additional constraints on the debt-to-equity ratio allowed by the corporation. This is a typical constraint that is present in affordable housing development projects, usually in the form of a maximum Debt Service Covered Ratio (DSCR).

Remark 2: *If the corporation has an upper bound on the amount of leverage that it tolerates in a project, then the project with TIF may fail.*

For example, suppose the corporation does not tolerate a debt-to-equity ratio above 2.40. In Example 1 (without the TIF policy), the projects in both jurisdictions are feasible and are developed in equilibrium. However, in Example 2, with the TIF policy in place for jurisdiction k_2, an equilibrium fails to exist with affordable housing developed in both jurisdictions, since equilibrium would require the debt-to-equity ratios in jurisdictions k_1 and k_2 to be 2.23 and 2.56, respectively. In this case, the equilibrium would turn into a corner solution, where only the project in jurisdiction k_1 (without TIF) would exist.

6 Conclusions and directions for future research

This paper provides a new model of affordable housing development to assess the role of the Tax Incremental Financing (TIF) and the Low Income Housing Tax Credit (LIHTC) programs. Our focus is not on where households choose to live, as in Tiebout (1956) and subsequent studies (e.g., Luque 2013); instead, we assume that households have already made the decision of where to live, and examine instead how developers finance their affordable housing development projects when they compete for LIHTCs and rely on TIF money provided by the jurisdiction. To this aim, we incorporate and endogenize in a consistent way several important financial variables, such as the rents that low income households pay for living in the newly constructed affordable housing units, as well as the developer's capital structure (composed of debt, TIF, LIHTC equity, and common equity of the developer partner). Our characterization of equilibrium and numerical examples illustrate thought-provoking ways in which TIF may affect the leverage ratio and the construction costs of an affordable housing development project.

The model proposed in this paper can be extended in many interesting directions. For example, a common concern in affordable housing development is financial efficiency. Often, developers are known to understate the equity available to them, as using more leverage increases their return. In addition, developers can often overstate the construction expenses and understate the final value of the project. These incorrect assumptions can be made in an effort to be cautious about setting expectations for the project, but they can also stem from intentionally trying to mislead the TIF board in order to receive more funds, as TIF is seen as "free" money for developers. The review board must carefully consider the project's current capital structure. Any budget shortfalls should be covered by finding additional debt or (more often) equity before TIF is utilized. Only when a budget shortfall cannot be addressed by private (non-government) sources of capital should TIF be considered. This is why the TIF reviewer must carefully analyze the project's estimated value upon completion or stabilization, all construction expenses, and all sources of capital.[19] These considerations suggest that extending our theory to allow for information asymmetries between the developer and the jurisdiction authorities would be a fruitful line of research with implications for public policy.

Other extensions of our model are the following. First, incorporate collateralized non-recourse mortgage debt, where in case of default the affordable housing asset would be seized by the lender. With this approach we would be able to understand the impact of default risk on affordable housing development. Second, allow for multiple housing projects and heterogeneous households within a jurisdiction. With this approach we would be able to understand the impact of TIF and LIHTC on house prices across the different residential real estate assets in a jurisdiction, connecting in this way with the empirical literature of housing externalities. Third, allow for the issuance of municipal bonds to finance the deficit of a jurisdiction with an ambitious housing program. In this setting, we would be able to examine the impact of liquidity in the municipal bond market on affordable housing development. These extensions may also provide us with a better understanding of the market mechanisms behind the different housing development programs.

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A Appendix

A.1 Proofs

Proof of Proposition 1: This proof follows Faias and Luque (2017). First notice that the budget constraints of households h_1 and h_2 at states s_1 and s_2 are such that

$$D^{h_1} = p_1(s)\varepsilon_1(s)TFP_1\left(x_{11}^{d_1}\right)^{\alpha_1}, \text{for } s = s_1, s_2 \tag{16}$$

$$D^{h_2} = p_2(s)\varepsilon_2(s)TFP_2\left(x_{12}^{d_2}\right)^{\alpha_2}, \text{for } s = s_1, s_2 \tag{17}$$

Conditions (11) and (12) follow from conditions (16) and (17). Equation (13) follows from the first order optimality conditions of developers d_1 and d_2 with respect to construction inputs 11 and 12, respectively, and conditions (11) and (12). ■

Proof of Proposition 2: Equation (14) follows from conditions (9) and (10), together with the market clearing for debt and the developer's budget constraint in the first period. Using the above definitions, we can write

$$\delta_1^{d_1}\left((\gamma_1 - \eta_1 TFP_1\left(x_{11}^{m}\right)^{\alpha_1} - \lambda_1)\right) = p_{1k}\omega_{1k} + q_k 0.9999(T_k^{d_k} - \hat{T}_k^{d_k}) + \gamma_k\left(\hat{T}_k^{d_k} - T_k^c + 0.0001T_k^c\right) - \tau\bar{D}_k^{d_k},$$

and obtain equation (14). If developer d_k borrows at maximum capacity, then we would write $D_k^{d_k} = \bar{D}_k^{d_k}$. ■

A.2 Numerical example

For the numerical examples, we first find the value of equilibrium variables q_1, q_2, τ, p_{11}, p_{12}, $p_{11}(1)$, $p_{12}(1)$, $p_{11}(2)$, $p_{12}(2)$, E_1^i, and E_2^i by solving the following system of equations: (11), (12), (13), LIHTC market clearing equations $T_1^{d_1} + T_1^c = \hat{T}_1^{d_1}$ and $T_2^{d_2} + T_2^c = \hat{T}_2^{d_2}$,

$$p_{11}\omega_{11}^m + p_{12}\omega_{12}^m = \omega_{10}^{h_1} + \omega_{10}^{h_2} + \omega_{10}^c - \sum_{k=k_1,k_2}\left(\lambda_k - \eta_k TFP_k\left(\omega_{1k}^m\right)^{\alpha_k}\right) \tag{18}$$

$$\sum_{k\in\{k_1,k_2\}} \left(p_k(s)\varepsilon_k(s)TFP_k\left(x_{1k}^{d_k}\right)^{\alpha_k} + \hat{T}_k^{d_k}\right) + r(D^{d_1}+D^{d_2}) = \omega_0^m(s)\,, \text{for } s=s_1,s_2 \tag{19}$$

$$\frac{p_1(s)\varepsilon_1(s)TFP_1\left(x_{11}^{d_1}\right)^{\alpha_1}}{p_2(s)\varepsilon_2(s)TFP_2\left(x_{12}^{d_2}\right)^{\alpha_2}} = \frac{\omega_{10}^{h_1}+\delta_1^{h_1}\left(\left((\gamma_1-\eta_1)\,TFP_1\left(x_{11}^{d_1}\right)^{\alpha_1}-\lambda_1\right)\right)}{\omega_{10}^{h_2}+\delta_2^{h_2}\left(\left((\gamma_2-\eta(\alpha_2))TFP_2\left(x_{12}^{d_2}\right)^{\alpha_2}-\lambda_2\right)\right)} \tag{20}$$

$$(q_1+\gamma_1)0.9999T_1^c + (q_2+\gamma_2)0.9999T_2^c = \omega_{10}^c + \sum_{k=\{k_1,k_2\}} \delta_k^c\left((\gamma_k-\eta_k)\,TFP_k\left(x_{1k}^{d_k}\right)^{\alpha_k}-\lambda_k\right) \tag{21}$$

$$0.9999\,(q_1+\gamma_1) = 1+0.9999p_1(1)\varepsilon_1(1) \tag{22}$$

$$0.9999\,(q_2+\gamma_2) = 1+0.9999p_2(1)\varepsilon_2(1) \tag{23}$$

$$\bar{D}^{d_1}+\bar{D}^{d_2} = \frac{1}{\tau}\sum_{k=\{k_1,k_2\}}\left(\omega_0^{h_k}+\delta_k^{h_k}\left((\gamma_k-\eta_k TFP_k\left(x_{1k}^m\right)^{\alpha_k}-\lambda_k)\right)\right) \tag{24}$$

$$\tau = \left(p_{11}\omega_{11}^m - q_1 0.9999(T_1^{d_1}-\hat{T}_1^{d_1}) + \gamma_1\left(\hat{T}_1^{d_1}-T_1^c+0.0001T_1^c\right) - \delta_1^{d_1}\left(\left((\gamma_1-\eta_1)y_1^{d_1}-\lambda_1\right)\right)\right)/\bar{D}^{d_1} \tag{25a}$$

$$\tau = \left(p_{12}\omega_{12}^m - q_2 0.9999(T_2^{d_2}-\hat{T}_2^{d_2}) + \gamma_2\left(\hat{T}_2^{d_1}-T_2^c+0.0001T_2^c\right) - \delta_2^{d_2}\left(\left((\gamma_2-\eta_2)y_2^{d_2}-\lambda_2\right)\right)\right)/\bar{D}^{d_2} \tag{25b}$$

Once the values of q_1, q_2, τ, p_{11}, p_{12}, $p_{11}(1)$, $p_{12}(1)$, $p_{11}(2)$, $p_{12}(2)$, $T_1^{d_1}$, $T_2^{d_2}$, T_1^c, and T_2^c are found, we solve for the rest of the equilibrium variables by using the agents' budget constraints and market clearing equations.

The economic interpretation of the above equilibrium conditions is the following. First, equation (18) says that the value of construction inputs equals the total amount of numeraire good available in the economy in the first period minus the resources that jurisdictions use for the provision of public goods. Second, equation (19) says that at each state of the second period, the sum of the total amount of LIHTCs, the cash flows generated by the affordable housing development projects, and the debt promises must equal the total amount of the numeraire good in the economy. Third, equation (20) says that the difference in cash flows generated by the affordable housing projects j_1 and j_2 at a given state of nature s in period 2 is driven by the relative amounts of resources that households h_1 and h_2 have in the first period. Fourth, equation (21) says that the value of corporation c's equity accrued of the taxes on its ownership interests in the affordable housing development projects must equal the value of endowments and transfers that corporation c has in the first period. Fifth, equations (22) and (23) are optimality conditions for corporation c to evaluate LIHTCs in jurisdictions k_1 and k_2, respectively. Sixth, condition (24) says that the maximum total amount of debt in the economy equals the resources that households have in the first period. Seventh, conditions (25a), and (25b) express the equilibrium price of debt τ in terms of other variables and parameters. Finally, equation (13) says that the difference in prices of construction inputs in different jurisdictions is driven by the difference in marginal productivities of the affordable housing assets.

Proposition 2: *Equations (18),(19), (20), (21), (22), (23), (24), (25a), (25b), and (13) follow from the economic structure imposed on our economy and the equilibrium conditions of Definition 1.*

Proof: First, notice that equation (18) follows from the company m's budget constraint in period 1 and the market clearing equation for the numeraire good 10, together with assumptions $\delta_1^m = 0$, $\delta_2^m = 0$, $\omega_{10}^m = 0$, and $\theta_{10}^{h_1} = \theta_{10}^{h_2} = \theta_{10}^c = 0$.

Second, to prove equation (19), we first notice that conditions (11) and (12), and the developers' budget constraint in the second period, imply that $x_0^{d1}(1) = x_0^{d1}(2)$ and $x_0^{d2}(1) = x_0^{d2}(2)$. These equalities, together with the market clearing conditions of the numeraire good at states s_1 and s_2, and assumption $\omega_0^{H1}(1) = \omega_0^{H1}(2)$, imply that $x_0^c(1) = x_0^c(2)$. Roughly speaking, the consumption of the numeraire good is the same in both states for the two developers and corporation c. Now, given these conditions, we can prove equation (19) using the following items: (i) equalities $x_0^{d1}(1) = x_0^{d1}(2)$ and $x_0^{d2}(1) = x_0^{d2}(2)$, (ii) the corporation c and the developer d_k's budget constraints at states s_1 and s_2, (iii) our assumptions $\theta_0^{h_1}(s) = \theta_0^{h_2}(s) = \theta_0^m(s) = 0$ and $\omega_0^{d_1}(s) = \omega_0^{d_2}(s) = \omega_0^c(s) = 0$ for $s = s_1, s_2$, (iv) our assumption $D^c = 0$, and (v) the market clearing equation for the numeraire good in the second period.

Third, condition (20) follows by equating the price of debt (τ) that results from households h_1 and h_2's budget constraints for period 0 and state s_1.[20]

[20]Because households use their loan payments in the second period to purchase affordable housing in their respective jurisdiction, differences in loan amounts determine differences in the valuation of affordable housing, and

Fourth, equation (21) follows from the corporation c's budget constraint in the first period and $x^c_{10} = x^c_{1k} = \omega^c_{10} = D^c = \hat{T}^c_k = 0$. Recall that if a corporation buys tax credits, it becomes the owner of 99.99% of the ownership interests in property (see expression (4)).

Fifth, condition (22) and (23) follow from corporation c's first order conditions with respect to equity T^c_1 and T^c_2, respectively. Notice that conditions (11) and (12) allow us to get rid of the shadow prices of budget constraints at state s_1 and s_2 because the term these shadow values would multiply is the same. For this, recall that the sum of shadow values across states of nature in the second period is equal to 1. Also recall that the shadow value at state s can be written as the shadow value of the budget constraint at s divided by the shadow value of the budget constraint in the first period. In addition, notice that (22) and (23) assume that the shadow values of sign constraints $T^{d_k}_k \geq 0$ and $T^{d_k}_k \leq \hat{T}^{d_k}_k$ are zero (in our numerical example, we verified that this holds in equilibrium for both $k = k_1, k_2$).

Finally, conditions (24), (25a), and (25b) follow from conditions (9) and (10), together with the market clearing for debt and the developer's budget constraint in the first period. Here, we assume that developers borrow at their maximum capacity (notice that in the numerical examples, we have an equilibrium in which developers borrow at their maximum capacity). ■

also differences in the (endogenous) cash flows generated by the different affordable housing properties.