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The Repo Channel of Cross-Border Lending in the European Sovereign Debt Crisis

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ABSTRACT

I build an equilibrium model of cross-border lending to understand how shocks to repo funding and leverage induce banks to reallocate credit by region and asset type during the European sovereign debt crisis of 2010–2013. A higher collateral risk on lower-rated sovereigns induces banks to fly to liquid collateral. Banks in the periphery sovereign crisis region with worse funding conditions rebalance their loan portfolios toward risky illiquid sovereigns. The European Central Bank's Long Term Refinancing Operations program stands as a mechanism whereby optimistic banks carried out the successful policy of the ECB to tamp down euro redenomination risk.

1. Introduction

The European Union project started in 1952 and since then, it engaged in a sequential widening and deepening process that resulted in a political, economic, and monetary union of 28 member states.[1] European Union (EU) capital markets were also part of the integration process. By 2002, the EU Commission enacted the Collateral Directive to provide a unified legal framework for the cross-border use of collateral and effectively institutionalized pure market-based governance in this area. These changes fueled the expansion of the European repo market, which between 2001 and 2008 saw its outstanding stocks tripling in value (Gabor and Ban, 2016).[2] Of this, large European banks generated around 80% of the repo volumes. Also, the share of cross-border repos increased from 36% in 2001 to 48% in 2008, while the share in "home" collateral declined from 63% to 31%. The European repo market became fully integrated in 2008 and, in contrast to the United States (U.S.), the EU offered a broad range of sovereign bonds that were used as repo collateral by European traders. Sovereign collateral accounted for more than 75% of repo transactions in 2011 (Hordahl and King, 2008; Gabor and Ban, 2016).[3]

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[1] See Luque et al. (2014) for a theory that rationalizes this sequential deepening process, and Spolaore (2013) for a discussion of the political economic aspects of European integration, including Monnet's chain mechanism of progressive integration.

[2] Repo is a very important element of European financial markets, with repo transactions reaching €25 trillion annually (ECB, 2015). This constitutes almost twice the volume in the unsecured inter-bank money market segment.

[3] GICIPS countries supplied around 25% of this sovereign European collateral (see Gabor and Ban, 2016). Of this, Italy accounted for around 11%. Also interesting is the fact that 99% of repo transactions in the BTEC electronic platform for the Euro Repo are in government bonds.

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The European sovereign debt crisis that started in the first quarter of 2010 and only remitted by the end of 2013 reversed investors' perceptions of the quality of sovereign debt issued by GICIPS countries (Greece, Ireland, Cyprus, Italy, Portugal, and Spain) and contrasted with the unprecedented low yields of some sovereign bonds, such as the German "bund." GICIPS sovereigns became the new toxic asset.[4] Tensions rapidly transmitted to the repo market. Repo exchanges, such as LCH Clearnet, raised haircuts on lower-rated sovereign bonds, and banks loaded with GICIPS sovereign collateral found it difficult to obtain private repo funding as fire sales of sovereign collateral intensified, a pattern also well documented in the literature of financial crises (Brunnermeier and Pedersen, 2009).[5] The increase in repo haircuts for GICIPS sovereigns was due to the investors' expectations of a government debt restructuring scenario and also the implicit "redenomination risk" underlying these bonds ("redenomination risk" refers to the bondholders' concern over which euro-denominated sovereign bonds – specifically, those of the GICIPS countries – may be at risk from a potential switch back into former national currencies).

The repo market is the main source for banks for funding their assets. This was true also during the sovereign crisis. For example, in 2011, large European banks were still funding 66% of their assets in wholesale funding markets, which was twice the level of U.S. or Asian banks (Hordahl and King, 2008; FSB, 2012). Motivated by these facts and the devastating sovereign debt crisis in Europe, I provide a two-period equilibrium model to understand how repo markets propagate shocks that induce banks to reallocate their cross-border loan exposures by geographic region and asset type. Other considerations such as bank deposits are left aside.

The model has two regions (region A and region B) and a continuum of banks in each region. Banks can invest in sovereign bonds issued in regions A and B and also in risky corporate loans. In addition, banks can lend to each other in the repo market using sovereign bonds as collateral. I assume that sovereign bonds issued in region A (henceforth, A-bonds) are safe. I then consider different scenarios for sovereign bonds issued in region B (henceforth, B-bonds). In one case, B-bonds are safe and trade with the same repo haircut as A-bonds. In another case, B-bonds lose collateral acceptance in repo and become risky.

The repo haircut, which is the difference between the market value of the collateral and the repo loan, plays an important role in my framework because banks fund their loan purchases by borrowing against sovereign bonds in the repo market. Repos allow banks to achieve a large long position on corporate loans by first constructing a leveraged short position on domestic bonds and then using the sale proceeds to buy corporate loans. Banks can also achieve a large long position on sovereign bonds by engaging in a series of bond purchases and repo short positions (i.e., pledging the collateral against a repo loan and then using the loan to buy more of the bond).

I show that when GICIPS sovereigns become risky and lose acceptability in the repo market, which I refer to as collateral risk –, banks choose to rebalance their loan portfolios toward safe and liquid sovereign collateral (e.g., German bonds). I refer to this lending strategy as flight-to-liquidity (FTL). Intuitively, when GICIPS lose collateral status, banks obtain less repo funding with GICIPS sovereigns, so banks assign a lower (shadow) value to possessing this type of bond. Safe sovereigns from core European countries become relatively more attractive, given their capacity to provide repo funding. This FTL increases the spread between core and GICIPS bond yields, and causes a credit squeeze in the markets of GICIPS bonds and corporate loans.

My model is also able to rationalize the fact that worse access to the funding market for banks in GICIPS countries (GICIPS banks henceforth) induces these banks to rebalance their loan portfolios toward risky and illiquid sovereigns. I refer to this lending strategy as risky-lending (RL) in sovereigns. The market mechanism behind this strategy is the following: as GICIPS banks obtain less repo funding with either bond type, the marginal value associated with holding one additional unit of a sovereign bond (from either region) decreases. This in turn induces these banks to reallocate their loan portfolios toward other assets that do not serve (or are not accepted) as collateral in repo. The lower pressure on safe and liquid sovereigns in turn induces banks with better funding conditions (core banks) to reallocate their loan portfolios toward these safe and liquid sovereign bonds. In other words, the "RL in sovereigns" strategy of GICIPS banks induces core banks' FTL.

I also extend the model to analyze the impact of the European Central Bank's (ECB's) Long Term Refinancing Operations (LTRO) program on the overall market capitalization of GICIPS bonds and the bond spread between safe and liquid sovereign bonds and GICIPS bonds. LTROs allow banks to take "bad" euro-denominated collateral to the ECB's long-term lending facilities at a lower repo haircut than in private repos.[6] An LTRO operation is similar to a repo transaction, where the ECB chooses the type of collateral, the repo rate, the haircut rate, and the rehypothecation rate (the fraction of the amount of a security that can be sold or lent after being borrowed). I show that by offering a better repo haircut on GICIPS sovereigns than the market, the ECB can effectively decrease the bond spread. This result is consistent with the ECB and other independent studies that argue that the LTRO program helped stabilize repo and collateral markets (ECB, 2010; Drudi et al., 2012; Bank for International Settlements, 2011) and was an important tool toward "normalizing" liquidity in "sovereign crisis hit" collateral markets (Gabor, 2014).

The market mechanism behind my result is the following. First, by accepting lower-rated sovereigns as repo collateral against an LTRO loan, the ECB is allowing optimistic banks to leverage their long position on GICIPS government bonds (demand effect). Second, by accepting GICIPS sovereign collateral from optimistic banks and not repoing it to pessimistic banks, the ECB constraints short sellers from leveraging their short positions (supply effect). The result is a significant decrease in the bond spread.

[4] To build on the analogy with the Great Recession, recall that the toxic assets during that time were U.S. subprime mortgage-backed securities. Banks with high exposures to subprime mortgages found it difficult, if not impossible, to access funding liquidity.

[5] This became evident during the summer of 2011, when the share prices of several large European banks plummeted. For example, between July and August of that year, the share price of Intesa Sanpaolo, the first banking group in Italy by market capitalization, went down by 29%. At the same time, the share price of Societe Generale, one of the three largest French banks, went down by 39%.

[6] Collateral policies encouraged private finance to treat AAA-rated Germany and A-rated Greece as sovereign issuers of similar creditworthiness, an implicit "subsidy" to lower-rated Member States (Buiter and Sibert, 2005).

From a policy point of view, I argue that the "normalization" of the "sovereign crisis hit" bond markets may not have only been due to moral suasion exerted by GICIPS governments on domestic banks as shown by Acharya and Steffen (2015),[7] but rather the result of the unprecedented ECB's LTRO program that accepted GICIPS bonds as repo collateral with the argument that these sovereigns were euro-denominated bonds (from the ECB's perspective, the euro was not in question). Thus, I identify an alternative channel whereby optimistic banks carried out the ECB's successful policy of tamping down euro redenomination risk.

I also discuss the potential costs that the ECB faces when running its repo facility. By potential cost I mean the exposure at default that the central bank may have if banks that are short in repo do not repurchase the collateral. I build on the repo collateral multiplier of Bottazzi et al. (2012) to construct the exposure at default as a function of the amount of leverage that the facility permits.

When banks are only allowed to pledge collateral at the ECB's facility, the ECB only enters in losses when the GICIPS sovereign bond's market price falls below the "haircutted" original transaction price. In this case, the ECB can always choose the haircut large enough to protect itself against future repo losses.

When banks can pledge collateral multiple times at the ECB's facility, the ECB may incur potential losses even if the ECB is entitled to the haircut benefit and the GICIPS bond price does not decrease below the initial transaction price. The difference between the value of collateral upon default and the levered loan, which is itself a function of the repo haircut, will determine the extent of the ECB's cost of running its repo lending facility. The potential loss increases the lower is the ECB's repo haircut.

This paper lies at the intersection of two literatures: the European sovereign debt crisis literature and the literature on repo markets. In particular, my paper provides the first equilibrium model that allows examination of the repo channel of cross-border lending in the European sovereign debt crisis. I do such analysis using a tractable marginal buyer model with a continuum of banks as in Geanakoplos (2003) instead of following Bottazzi et al. (2012) equilibrium analysis of repo markets and rehypothecation with a finite number of agents and interior allocations.

My model departs from Bottazzi et al. (2012) by studying how repo funding markets induce banks to reallocate their loan exposures by geographical region and asset type. This novel approach allows me to understand the market mechanism behind the FTL and RL strategies. In addition, this paper contributes to the literature in that I characterize the equilibrium outcomes of shocks to repo collateral and access to the repo funding market. I also examine for the first time the repo channel of central bank interventions on banks' cross-border lending strategies.

My work also contributes to the literature on repo by focusing on the European sovereign debt crisis. So far, most attention has focused on the run of U.S. repo markets following Lehman Brothers' collapse (e.g., Gorton and Metrick, 2012; Krishnamurthy et al., 2014; Martin et al., 2014). Uhlig (2013) examined the effect of repo on bank lending strategies in the European sovereign debt crisis context. However, he focuses on the role that regulation has on banks' incentives to hold home risky bonds and its effect on the ECB's balance sheet. In my model, I allow for collateral pledgability and rehypothecation, which make repo haircuts a trigger for banks to rebalance their cross-border loan portfolios. This mechanism is absent in Uhlig's model, in which repo haircuts do not matter for equilibrium.

The remaining few studies that deal with European repo markets do not engage with the impact on collateral markets (e.g., Boissel et al., 2014; Mancini et al., 2016). I contribute to this literature by providing an equilibrium approach to cross-border banking where repo sovereign collateral becomes "segmented," similar to what occurred in the European sovereign debt crisis.

The rest of this paper is as follows. In Section 2, I provide a simple baseline model to illustrate the role of bond collateral "desirability" on cross-border lending. I also formalize the different leverage trading strategies for banks and define an equilibrium for my economy using a marginal buyer approach. In Section 3, I examine the impact of collateral risk and a worse access to the funding market on banks' portfolio rebalancing strategies, corporate credit squeeze, and the bond spread. In Section 4, I focus on public policy, in particular on the role of the central bank's repo facility to reduce the bond spread between safe sovereign bonds and GICIPS sovereign bonds. I also analyze the potential cost that a central bank may incur with the repo facility. I conclude in Section 5. The proofs and extensions of the model are in the Appendix.

2. Baseline model

The baseline model departs from Bottazzi et al.'s (2012) equilibrium analysis of repo markets in that I consider a more tractable marginal buyer approach with a continuum of banks, as in Geanakoplos (2003), instead of a setting with a finite number of traders and interior allocations as in Bottazzi et al. (2012). The notation, modeling of repo and rehypothecation, description of individual constraints, and the concept of equilibrium presented in this section will be useful for my analysis in Sections 3 and 4.

2.1. Asset payoffs

My economy has two dates, $t = 0, 1$, a numeraire good that facilitates trade (which price I normalize to 1), and a continuum of banks that can invest in two possible assets: domestic sovereign bonds (denoted by j) and loans to domestic firms (or corporate loans, denoted by f). Because I only consider a two-date model, repo contracts signed at date 0 must be understood in terms of

[7] The premise behind the moral suasion hypothesis is that peripheral governments force domestic banks to absorb more of their own sovereign debt because overall demand is weak, in order to reduce sovereign bond yields (see Acharya and Steffen, 2015).

"repo-to-maturity" - see Bottazzi et al. (2016) for a multi-period infinite horizon model of repo markets, issuance of securities, and security bubbles.[8]

For the sake of brevity, I ignore the demand side of the corporate loan sector (firms) and consider an exogenous demand for corporate loans equal to D (in the Appendix, I show how to incorporate the demand side of the corporate loan market into the baseline model).

I model uncertainty by considering two states of nature at date 1: $s = s_A, s_B$. Sovereign bonds and corporate loans pay r_{js} and r_{fs} units of the numeraire good at state $s = \{s_A, s_B\}$, respectively. Asset payoffs have the following structure: $(r_{js_A}, r_{js_B}) = (1, \alpha_S)$ and $(r_{fs_A}, r_{fs_B}) = (1, \alpha_F)$. Sovereign bonds are riskless and corporate loans are risky; thus, $\alpha_S = 1$ and $\alpha_F < 1$.

2.2. Banks

The continuum of banks is uniformly distributed in the interval $\mathbf{B} \equiv (0, 1)$ described by the Lebesgue measure λ. Bank $b \in \mathbf{B}$ assigns a probability $\beta_s(b)$ to the occurrence of state $s = s_A, s_B$ at date 1. Probability $\beta_{s_A}(b)$ is strictly monotonically increasing and continuous in b (a higher b means that bank b assigns a higher probability to state s_A).[9]

Each bank b is endowed with $e > 0$ units of domestic sovereign bonds at date 0 and also with a vector $\omega = (\omega_{s_A}, \omega_{s_B})$ of numeraire good endowments at date 1. I find it convenient to normalize the price of the numeraire good at state $s = s_A, s_B$ to 1. For simplicity, I assume that banks do not consume the numeraire good at date 0 and also that their good endowments are null at the initial date.

Banks obtain funding for asset purchases by pledging their sovereign bond endowments as collateral in repurchase agreements (or repos). Financial transactions are subject to the following budget constraint at date 0:

$$(\lambda_0^b) : \Psi_0^b \equiv -hqz^b - qy^b - \gamma a^b \geq 0, \tag{1}$$

where λ_0^b is the shadow value of constraint ((1)); q and γ denote the discount prices of sovereign bonds and corporate loans, respectively[10]; y^b and a^b denote the face value of sovereign bonds and corporate loans, respectively; and z^b denotes the amount of bonds transacted in repo. The interpretation of signs for variables y^b and a^b is standard: the bank buys the sovereign bond if $y^b > 0$ and sells it if $y^b < 0$. Similarly, choice variable a^b is positive when bank b lends to firms and negative if the bank borrows from firms. Because I consider a model where banks are the providers of credit to firms, I rule out short positions on corporate loans for banks, i.e.,

$$(\eta^b) : a^b \geq 0. \tag{2}$$

(Because banks will end up lending to corporations ($a^b > 0$), the shadow value η^b will be 0 in equilibrium).

The interpretation of z^b is more subtle and requires some definitions. A repo is a collateralized transaction in which the borrower of cash (henceforth, the "repo short") lends a security j (e.g., a bond) to the lender of cash (henceforth, the "repo long") at date 0,[11] and then, at date 1, the repo short repurchases the security from the repo long at a repo rate ρ.[12] This collateralized transaction is subject to a repo haircut $1 - h > 0$ with $h \in (0, 1)$. The haircut may depend on the type of security used as repo collateral ("collateral risk") and also on the financial soundness of the repo short.

To illustrate how repo works, I present the following example, in which the repo long borrows sovereign bonds worth €100 million at a repo haircut of 2% and a repo rate of 1%. Then, the repo short only receives €98 million in cash at date 0, and at date 1 the repo short repurchases those bonds for €98.98 million. In terms of my notation, I denote the loan amount given at date 0 by hqz^b (€98 million) and the amount paid to repurchase the collateral at date 1 by $(1 + \rho)hqz^b$ (€98.98 million). When $z^b < 0$, bank b pledges bonds as collateral against a repo loan (b is repo short), whereas if $z^b > 0$, bank b extends a loan against sovereign collateral (b is repo long).

To understand how possession of sovereign bonds translates into a bank's valuation of collateral, I follow Bottazzi et al. (2012) by imposing the "box constraint":

$$(\mu^b) : e + y^b + z^b \geq 0. \tag{3}$$

Constraint (3) requires that the number of bond titles that a bank has in its "box" at date 0 cannot be negative. That is, the bank's bond endowments (e), plus what it purchases ($y^b > 0$) and borrows ($z^b > 0$) of the bond, minus what it sells ($y^b < 0$) and lends ($z^b < 0$) of the bond, cannot be negative. Box constraint (3) allows to model both bond short sales and the financing of bond purchases. Formally:

[8] Formally, a repo-to-maturity is a repurchase agreement that terminates on a bond's date of maturity. Considering a different maturity structure between the bond and the repurchase of collateral would complicate the model without additional insights on the main driving forces of the repo market mechanism.

[9] My modeling choice of heterogeneity in beliefs is supported by several empirical studies of the sovereign debt crisis (see, e.g., European Commission (2012), IMF, 2012, Corsetti et al., 2013, and De Grauwe and Ji (2013) for evidence on potential mispricing/over-pricing of debt in the course of the Eurozone sovereign debt crisis, and Grosse Steffen (2015) for direct evidence of disagreement between professional forecasters for different eurozone countries).

[10] Notice that the bond discount price q is endogenously determined in our model given bond payoffs r_{js_A} and r_{js_B}. This discount price is inversely related to the bond yield (or return), so when we say that a sovereign bond trades at a higher price, we are also saying that the bond is trading at a lower yield.

[11] Terminology is standard in the literature (see Duffie, 1996) and becomes natural provided one focuses on the effect of trades on the title balance of a given security, called the "amount in the box" in market parlance.

[12] We ignore any maturity mismatch considerations between corporate loan obligations and repo obligations. Such extension would require adding an additional date ($t = 2$) and letting bond payoffs and bond repurchases occurring at date 2 and date 1, respectively.

- A bank short sells sovereign bonds if $y^b < 0$ is such that $e + y^b < 0$ with $e \geq 0$. For this sale to be feasible, the bank needs to borrow through repo the additional bonds that it does not have (so $z_j^b > 0$).[13]
- To finance the purchase of additional sovereign bonds ($y^b > 0$), the bank can pledge its bonds in a repo transaction (at cost hqz^b with $z^b < 0$) and then use the cash proceeds to buy additional bonds.

I denote by μ^b the bank b's Lagrange multipliers associated with box constraint (3). This shadow price captures the value that bank b assigns to the possession of this type of bond at date 0. In terms of a short sale, μ^b measures the gain that a bank could get at date 0 if it is able to short sell one additional unit of the bond without having to borrow it first. In terms of the financing of bond purchases, μ^b captures the gain that a bank could get at date 0 if it is able to buy one additional unit of the bond without having to pledge it as collateral in repo. I refer to shadow price μ^b as the bank's desire to possess sovereign bonds (or bond desirability for short).

Bank b's consumption of the numeraire good at state $s = s_A, s_B$ (date 1) is

$$(\lambda_s^b) : \Psi_s^b \equiv \omega_s + (1+\rho)hqz^b + r_{js}(y^b + e) + r_{fs}a^b \geq 0 \tag{4}$$

In the bank's optimality condition below, each bank must respect its budget constraints in a setting where asset payoffs differ between states of nature in the second period. Thus, I do not consider bankruptcy in the model.

Maximization problem: *Each bank chooses a vector* $(y^b, z^b, a^b) \in \mathbb{R}^3$ *of secondary market sovereign bond trades, sovereign repo trades, and corporate loans in order to maximize the following*

$$\Psi_1^b \equiv d\left(\beta_{s_A}(b)\Psi_{s_A}^b + \beta_{s_B}(b)\Psi_{s_B}^b\right),$$

subject to constraints (1), (3), *and* (4). $d \leq 1$ *is the time discount factor between dates 0 and 1.*

2.3. Sovereign bond desirability

Uhlig (2013) considers an equilibrium model in which banks can use sovereign bonds for repurchase agreements with a common central bank, and claims (in Proposition 1) that the equilibrium does not depend on the repo haircut. Although our models are different, it is worth noting that his result does not hold true in my setting. It is precisely the box constraint that makes repo haircuts and bond desirability matter for equilibrium.

Lemma 1. *A higher repo haircut* $1 - h$ *makes the bond less desirable as collateral. In particular, when* $\alpha_S = 1$,

$$\mu^b = \frac{h}{1-h} \cdot d \cdot (1 - (1+\rho)q) \tag{5}$$

The proof of Lemma 1 (in the Appendix) relies on the bank's optimality conditions.[14]

Because of the strict monotonicity and continuity of $\beta_{s_A}(b)$ in b, the linear bank's objective function, as well as the connectedness of the set of banks $\mathbf{B} = (0, 1)$, there is a unique marginal bank $\hat{b}$ that is indifferent between holding sovereign bonds and corporate bonds. The following characterization of the marginal bank follows from optimality conditions (see the Appendix) and is specific to the case when $\alpha_S = 1$ (the left-hand side of (6) is independent of the bank type).

Definition 1. The marginal bank $\hat{b}$ is such that

$$\frac{d + \mu^b}{q} = \frac{d(\beta_{s_A}(\hat{b}) + (1 - \beta_{s_A}(\hat{b}))\alpha_F)}{\gamma}. \tag{6}$$

Condition (6) equates the return of holding sovereign bonds with the return of holding corporate loans. The pledgability property of sovereign bonds is captured by shadow price μ^b, which turns out to be independent of the bank type for the specific case $\alpha_S = 1$. Roughly speaking, the return on sovereign bonds increases for the marginal bank when it assigns a higher value for possessing sovereign bonds as repo collateral. Because the left-hand side of (6) is independent of the bank type, the bank's decision to go long or short in the sovereign bond is determined by its perception of the profitability of investing into corporate bonds. Finally, in Eq. (6), I take the shadow value η^b of the no-short sales restriction (2) for corporate loans equal to 0 because banks do invest in corporate loans in equilibrium (as shown below).

[13] Note that with the box constraint, I do not need to impose an exogenous bound on bond trades as in previous studies on general equilibrium with incomplete markets. The box constraint is enough to eliminate the well-known Hart (1975) counter-example (see Bottazzi et al., 2012).

[14] Lemma 1 takes the simple scenario where the repo rate does not decrease. But notice that even if the repo rate were allowed to decrease, the result of Lemma 1 would hold, as long as the negative impact of an increase in the repo haircut is offset by the positive impact of a decrease in the repo rate.

2.4. Leverage strategies

In the marginal buyer approach to equilibrium, all banks, except the marginal bank, choose to hold only one asset (corner solution). I consider a setting in which banks trade in the repo market to attain a large leveraged position on that asset. The "repo collateral multiplier" is a market mechanism proposed by Bottazzi et al. (2012), p 482–483) that shows how a bank can follow a sequence of security transactions and repo trades to attain a large leveraged position.

I evaluate the equilibrium using this multiplier process, which allows banks to achieve a repo position equal to $z^b = e/(1-h)$ (positive if the bank is repo long, and negative if the bank is repo short)[15] and the following positions on corporate loans and sovereign bonds:

Leveraged trading strategy for buying corporate loans: The bank sells its endowments of domestic sovereign bonds, e, at price q and uses the sale proceeds to borrow new titles of the bond through repo; the bank then short sells these new bond titles, with a difference in proceeds equal to the repo haircut $1-h$. By repeating a series of similar repo long transactions (borrowing more of the bond through repo) and short sales (selling the borrowed sovereign collateral), this bank can attain a large short position on the sovereign bond at date 0 equal to:

$$e + y^b = -\frac{h}{1-h}e. \tag{7}$$

At the same time, to attain this short sale, the bank must be long in repo with a position equal to $z = e/(1-h)$. The netting of these joint short sale and repo long transactions amounts to e units of the sovereign bond "in the box" of the short seller. Thus, the bank has qe available for investing in corporate loans at price γ, i.e.,

$$a^b = qe/\gamma. \tag{8}$$

Leveraged trading strategy for buying sovereign bonds: The bank pledges e units of the sovereign bond in repo against a cash loan, and uses the proceeds to buy more units of the bond. The bank then repeats this process until it achieves a leveraged long bond position equal to

$$e + y^b = \frac{1}{1-h}e. \tag{9}$$

2.5. Equilibrium

I incorporate the above trading strategies into the market clearing conditions of the economy. For this, first notice Lemma 2.

Lemma 2. *Banks $b < \hat{b}$ find it optimal to hold sovereign bonds.*

Given a fixed net asset supply of bonds in the market, the representative portfolio is isomorphic to the marginal bank type $\hat{b}$. Thus, in this setting a higher $\hat{b}$ means that the representative portfolio shifts away from corporate loans.

The economic intuition is the following. Consider, for instance, bank $b = 0$, which believes that state s_B will occur with probability 1. This bank prefers to hold sovereign bonds because it values the riskless asset more than the market does. As b increases toward $\hat{b}$, the diversification benefit that sovereign bonds provide ameliorates. When $b = \hat{b}$, the bank is indifferent between holding either of the two assets. Banks in the interval $(\hat{b}, 1]$ prefer to hold corporate loans instead. For example, consider bank $b = 1$, which believes that state s_A will occur with probability 1. This bank prefers to buy the cheaper risky asset (corporate loan), which pays 1 at s_A. By continuity, I can extend this argument to all banks in the interval $(\hat{b}, 1]$.

Using the aforementioned leveraged strategy for banks that prefer to hold corporate loans, I can write the market clearing equation for corporate loans as follows:

$$\int_{\hat{b}}^{1} \hat{q}e/\hat{\gamma}\ db = D. \tag{10}$$

To write the market clearing condition for sovereign bond positions, I first denote the bank b's sovereign bond position by $\varphi^b \equiv e + y^b$. Then, since market clearing of bond trades requires $\int_0^1 y^b = 0$, the market clearing of sovereign bond positions must be such that $\int_0^{\hat{b}} \varphi^b + \int_{\hat{b}}^1 \varphi^b = \int_0^1 e$. Using the repo collateral multiplier, I can rewrite this market clearing condition as follows:

$$-\int_{\hat{b}}^{1} \frac{h}{1-h}e\ db + \int_0^{\hat{b}} \frac{1}{1-h}e\ db = \int_0^1 e\ db. \tag{11}$$

Definition 2. I define an equilibrium as a triplet $(\hat{b}, \hat{q}, \tilde{\gamma})$, such that: (i) the marginal bank $\hat{b}$'s indifference condition (6) holds, and (ii) market clearing equations (10) and (11) hold.

[15] For example, when the haircut $1-h$ is 2%, the repo collateral multiplier of a long strategy is 50 to 1.

Equilibrium existence requires that both repo and bond positions are bounded, which holds in the economy.[16] I move to the characterization of equilibrium using a marginal buyer setting.

Because each bank chooses to hold only one asset in equilibrium (except for the marginal bank), the pair $(1-\hat{b}, \hat{b})$ can be seen as the weights on corporate loans and sovereign bonds corresponding to the representative loan portfolio of the economy. The market clearing equation (11) implies that $(1-\hat{b}, \hat{b}) = \left(\frac{h}{1-h}, \frac{1-2h}{1-h}\right)$ in equilibrium. Thus, the representative portfolio is driven by the repo haircut of sovereign bonds. I next provide a numerical example.

Benchmark example*:* Next consider the following parameter values: $\alpha_S = 1$, $\alpha_F = 0.9$, $\rho = 0.02$, $1-h = 0.1$, $d = 0.9$, $D = e = 1$, and $\omega_{s_A} = \omega_{s_B} = 0$.[17] Under these parameter values, I find an equilibrium where the representative portfolio composition consists of 47.37% corporate loans $(1-\hat{b})$ and 52.63% sovereign bonds $(\hat{b})$. The equilibrium prices of corporate loans and sovereign bonds are $\hat{\gamma} = 0.4064$ and $\hat{q} = 0.8580$, respectively. The box constraint for the sovereign bond is binding, with a shadow value equal to $\hat{\mu} = 1.0111$. ■

I conclude this section with a remark on capital requirements.

Remark 1. The banking industry has recently been subject to stricter capital requirements, in part due to the Great Financial Crisis. In Europe, however, the Capital Requirements Directive (CRD) allowed government debt issued in domestic currency (euros) at 0% "risk weighting." In the Appendix, I show how to incorporate capital requirements into the baseline model and show that stricter capital requirements on risky corporate loans induce banks to rebalance their loan portfolios toward sovereign bonds. This is in accordance with Popov and Van Horen (2014), who show that regulatory requirements provided banks with strong incentives to hold large amounts of sovereign debt on their balance sheets.

3. The sovereign debt crisis under the lens of the model

A larger repo market creates more liquidity, which in the absence of a crisis, stimulates economic growth through expansionary credit. In a monetary union, an integrated European repo market also means that investors are no longer concerned about exchange rate risk. However, a large integrated European repo market also has inconveniences. As pointed out by Galati and Tsatsaronis (2003), economic and monetary union (EMU) governments found themselves competing with each other as they lost the privilege of issuing risk-free in their currency. In a similar vein, the ECB highlighted in several reports that such competition for liquidity brought investors into liquid bond markets of large Member States (see ECB, 2000). Motivated by the integration process of capital markets in the EU, I extend the baseline model to a setting with two regions.

3.1. Baseline model extended to two regions

Next consider an economy with two regions, say A and B. i refer to the sovereigns issued in these regions as A-bonds and B-bonds, respectively. These payoffs are $(1, \alpha_S^A)$ and $(1, \alpha_S^B)$, respectively, and the corresponding repo haircuts are $1-h_A$ and $1-h_B$, respectively. For simplicity, I ignore differences in the repo rates between bonds. As before, loans to corporations are risky and pay $(1, \alpha_F)$.

By "liquidity" I mean the extent to which the asset is accepted as collateral in private repos. For convenience, I assume that a liquid asset has a repo haircut that is smaller than 1, while an illiquid asset has a repo haircut equal to 1.

For the sake of simplicity, I reduce the economy with three assets (corporate loans, B-bonds, and A-bonds) to an economy with two distinct assets: a safe and liquid asset, and a risky and illiquid asset. The trick is the following. I always assume that A-bonds are safe ($\alpha_S^A = 1$) and liquid, and that corporate loans are risky ($\alpha_F < 1$) and illiquid. I then consider two possibilities for B-bonds: (i) B-bonds are safe and liquid, i.e., $\alpha_S^B = 1$ and $1-h_B = 1-h_A < 1$, or (ii) B-bonds are risky and illiquid, i.e., $\alpha_S^B = \alpha_F < 1$ and $1-h_B = 1$. The former possibility makes B-bonds identical to A-bonds, while the latter makes B-bonds identical to corporate loans. In the equilibrium computations below, I take these considerations into account when calculating the demand and supply of an asset.[18]

The current setting requires additional notation. The set of banks in region $R = A, B$ is denoted by $\mathbf{B}_R$, and its representative bank by b_R. Bank b_R's endowments of sovereign bonds are now $e^{b_R} = (e_{j_A}^{b_R}, e_{j_B}^{b_R})$, where j_A and j_B denote the types of sovereign bonds issued in regions A and B, respectively. For simplicity, I assume that banks are only endowed with domestic bonds, i.e., $e_{j_R}^{b_R} = e$ and $e_{j_{R'}}^{b_R} = 0$ if $R \neq R'$.[19]

[16] Repo positions are bounded by $e/(1-h)$ (repo long positions are bounded by $(1/(1-h)) \int_{b' \neq b} e^b \, db = (1/(1-h))e$). The bound on bond positions follows from the box constraint and the bound on repo positions.

[17] Setting $\omega_{s_A} = \omega_{s_B} = 0$ allows me to focus on bond endowments, the main driver of long-term funding costs in the interbank lending market. Importantly, my results in the examples of this paper are qualitatively robust to the choice of parameter values.

[18] Reducing the economy to two distinct assets greatly simplifies our analysis and exposition of the extended model. Below we will extend again our model to deal with public policy issues, and then we will need to accommodate the more complex case of three assets with different haircuts and payoffs.

[19] This assumption on bond endowments is motivated by the high exposure that GICIPS banks had to domestic bonds prior to the European sovereign debt crisis. For example, the European Banking Authority's stress tests reveal that by the end of 2009 (a moment in time just before the onset of the European sovereign debt crisis), the percentage of home-country long term sovereign debt owned by home-banks was 94% for Greece, 65% for Ireland, 79% for Italy, 72% for Portugal, and 83% for Spain. Banks in GICIPS countries were also far more exposed in absolute terms to GICIPS sovereign debt than their peers in safe core European countries (see Luque, 2020).

I denote a bank b_R's position on bond $j = j_A, j_B$ by $\varphi_j^{b_R} = e_j^{b_R} + y_j^{b_R}$, and the corresponding repo transaction with this bond as collateral by $z_j^{b_R}$. The prices of sovereign bonds j_A and j_B are q_A and q_B, respectively. I refer to the difference $q_A - q_B$ as the bond spread.

For the case when B-bonds are safe and liquid, I denote the price of sovereign bonds (irrespectively of their type) by $\hat{q}$ and write the representative portfolio in region R as $(1-\hat{b}_R, \hat{b}_R/2, \hat{b}_R/2)$, where the first, second, and third elements correspond to the portfolio weights on corporate loans, B-bonds, and A-bonds, respectively (by assumption, the demand of the sovereign bond from banks in region R is split evenly between B-bonds and A-bonds; thus, I write $\hat{b}_R/2$ for the portfolio weight on both A-bonds and B-bonds). The corresponding market clearing equations of corporate loans and sovereigns for this case are, respectively,

$$\int_{\hat{b}_A}^{1} \hat{q}e/\hat{\gamma}\ db + \int_{\hat{b}_B}^{1} \hat{q}e/\hat{\gamma}\ db = D \tag{12}$$

$$-\int_{\hat{b}_A}^{1} \frac{h_A}{1-h_A} e\ db - \int_{\hat{b}_B}^{1} \frac{h_A}{1-h_A} e\ db + \int_{0}^{\hat{b}_A} \frac{1}{1-h_A} e\ db + \int_{0}^{\hat{b}_B} \frac{1}{1-h_A} e\ db = 2\int_{0}^{1} e\ db. \tag{13}$$

Notice that because the sovereign bonds in the two regions are identical, the supply of sovereigns in (13) is $2\int_0^1 e\ db$.

Case I: Next I consider the extended setting with two regions, where sovereign bonds in regions A and B are subject to the same repo haircut ($1-h_A = 1-h_B = 0.1$) and same payoffs ($\alpha_S^A = \alpha_S^B = 1$). As in the benchmark example, $\alpha_F = 0.9$, $\rho = 0.02$, $d = 0.9$, $D = e = 1$, and $\omega_{s_A} = \omega_{s_B} = 0$. Then, the representative portfolio in region $R = A, B$ is composed of 47.38% corporate loans ($1-\hat{b}_R$), 26.31% B-bonds ($\hat{b}_R/2$), and 26.31% A-bonds ($\hat{b}_R/2$). The equilibrium prices of corporate loans and sovereigns are $\hat{\gamma} = 0.9282$ and $\hat{q} = 0.9797$, respectively. Since A-bonds and B-bonds are identical, the bond spread is equal to zero. The desirability of repo sovereign collateral for marginal banks in region R, captured by the shadow price of the box constraint, equals $\mu_j^{\hat{b}_R} = 0.0056$, for both $R = A, B$. ■

3.2. Equilibrium when B-bonds lose acceptance as repo collateral

In 2010, LCH Clearnet, the second largest clearer of bonds and repos in the world, providing services across 13 government debt markets, increased haircuts on Irish sovereign collateral from 0% to 45% in a short period of time. Portugal went through the same process in early 2011, when LCH increased haircuts to 80% (Bank of England, 2011; Gabor and Ban, 2016). By December 2011, LCH no longer accepted Irish and Portuguese government bonds as collateral. The repo industry association claimed that the lack of acceptability of these bonds as repo collateral, in part due to redenomination risk, propagated stress in the GICIPS sovereign bond markets in 2011 and 2012.[20] In this section, I explore how this scenario may induce banks to rebalance their cross-border loan portfolios.

Next consider the case of B-bonds becoming risky[21] and losing their collateral pledgability property, i.e., $\alpha_S^B = \alpha_F < 1$ and $1-h_B = 1$.[22] A-bonds remain safe and liquid as repo collateral; thus, $\alpha_S^A = 1$ and $1-h_A < 1$. B-bonds and corporate loans can be regarded as the same asset with market price $\hat{\gamma}$.

I assume that $e = D$ in order to evenly split the holdings of the risky asset between B-bonds and corporate loans. For this case, I write the representative portfolio in region R as $x_R \equiv ((1-\hat{b}_R)/2, (1-\hat{b}_R)/2, \hat{b}_R)$, where the first, second, and third elements correspond to the portfolio weights on corporate loans, B-bonds, and A-bonds, respectively.

The market clearing equations for the risky asset and the safe liquid asset (A-bond) are, respectively,

$$\int_{\hat{b}_A}^{1} \frac{\hat{q}}{\hat{\gamma}} e\ db + \int_{\hat{b}_B}^{1} e\ db = D + e \tag{14}$$

$$-\int_{\hat{b}_A}^{1} \frac{h_A}{1-h_A} e\ db - \int_{\hat{b}_B}^{1} \frac{h_A}{1-h_A} \frac{\hat{\gamma}}{\hat{q}} e\ db + \int_{0}^{\hat{b}_A} \frac{1}{1-h_A} e\ db + \int_{0}^{\hat{b}_B} \frac{1}{1-h_A} \frac{\hat{\gamma}}{\hat{q}} e\ db = \int_{0}^{1} e\ db. \tag{15}$$

The supply of A-bonds in market clearing equation (15) is now $\int_0^1 e$ (this follows from assumption $e_{j_A}^{b_B} = 0$). In that equation, also observe that $\hat{\gamma}e/\hat{q}$ stands for the amount of A-bonds that a B-bank can purchase.[23] The supply of the risky asset in market clearing equation (14) is now $D + e$, where D and e account for the supply of corporate loans and B-bonds, respectively.[24]

[20] See http://www.icmagroup.org/Regulatory-Policy-and-Market-Practice/short-term-markets/Repo-Markets/frequently-asked-questions-on-repo/27-what-does-a-ccp-do-what-are-the-pros-and-cons/ and http://www.iflr.com/Article/3097680/Search/Results/Euro-exit-and-redomination-risk-a-UK-perspective.html.

[21] The default of B-bonds could be understood in terms of government debt restructuring, where investors receive a par bond with no haircut in the principal amount but with a substantially lower coupon (I ignore other possibilities such as haircuts on the principal and par bonds with longer maturity). Historically, sovereign bonds' realized losses have ranged between 13% (2003 Uruguay) and 73% (2005 Argentina). See Sturzenegger and Zettelmeyer (2008) for estimates of bond-by-bond realized investor losses in recent debt restructurings in Russia, Ukraine, Pakistan, Ecuador, Argentina, and Uruguay.

[22] The intermediate case $0 < 1-h_B < 1-h_A$ would complicate the analysis with additional restrictions on bond short sales, but without additional economic insights than the case with $1-h_B = 1$. To see this, notice that when $0 < 1-h_B < 1-h_A$, a bank could take a long bond position larger than the one that the repo collateral multiplier allows for given that this same bank could go short on the other bond. In that case, equilibrium would require additional assumptions, as in Bottazzi et al. (2012).

[23] B-banks sell their domestic endowments at price $\hat{\gamma}$ ($= \hat{q}_B$) and then use the sale proceeds to buy A-bonds at price $\hat{q}$. Because A-banks are endowed with domestic bonds, they can simply leverage their domestic bond endowment $e_{j_A}^{b_A}$ using the repo collateral multiplier process described above.

[24] In market clearing equation (14), all optimistic A-banks sell their A-bond endowments at price $\hat{q}$ to buy the risky illiquid asset at price $\hat{\gamma}$. Optimistic B-banks ($b > \hat{b}_B$) choose to keep their domestic bond endowments (risky assets that cannot be pledged in repo) and, therefore, each of these banks hold e units of the risky asset.

I focus on two portfolio rebalancing strategies for banks: flight-to-liquidity (FTL) and risky lending (RL). The following definition considers the representative portfolio of a region $R = A, B$ in two regimes, k and k', i.e., $((1-\hat{b}_R)/2, (1-\hat{b}_R)/2, \hat{b}_R)_k$ and $((1-\hat{b}_R)/2, (1-\hat{b}_R)/2, \hat{b}_R)_{k'}$.

Definition 3. For the case when $\alpha_S^B = \alpha_F < 1$ and $1 - h_B = 1$, I say that region R's representative portfolio exhibits the following:

- *FTL if* $(\hat{b}_R)_{k'} > (\hat{b}_R)_k$ *(if this condition is satisfied,* $FTL_R = 1$*).*
- *RL in B-bonds (or in corporate loans) if* $((1-\hat{b}_R)/2)_{k'} > ((1-\hat{b}_R)/2)_k$ *(if this condition is satisfied,* $RL_R = 1$*).*

Note that because I only consider one country per region, FTL could be understood in terms of flight-home (FH) if the representative portfolio corresponds to region A. Similarly, RL could be understood in terms of FH if the representative portfolio corresponds to region B.

Case II: Consider a similar situation to Case I, but now let $1 - h_B = 1$ and $\alpha_S^B = \alpha_F$. As before, let $1 - h_A = 0.1$, $\alpha_S^A = 1$ and $D = e$. The new equilibrium has the representative portfolio of each region composed of 24.33% corporate loans ($(1-\hat{b}_R)/2$), 24.33% B-bonds ($(1-\hat{b}_R)/2$), and 51.34% A-bonds ($\hat{b}_R$). Equilibrium asset prices are now $\hat{q}_A = 0.7432$, $\hat{q}_B = \hat{\gamma} = 0.2389$. Thus, the sovereign debt crisis in the B-region induces banks from both regions to rebalance toward A-bonds ($FTL_R = 1$, $R = A, B$) and away from risky and illiquid B-bonds and corporate loans. The price of B-bonds decreases with respect to Case I, and the bond spread now becomes positive ($\hat{q}_A - \hat{q}_B = 0.5043$). The desirability of A-sovereign bonds as repo collateral increases with respect to Case I, with the shadow price of the box constraint for A-bonds now equal to $\mu_{j_A}^{b_R} = 1.9598$ for marginal banks in both regions, $R = A, B$. Credit exposures to the risky asset (corporate loans and B-bonds), captured by $A\hat{\gamma}$ (see market clearing equation ((14))), decreases from 0.9282 in Case I to 0.2389 in Case II. ■

I summarize these findings in Lemma 3.

Lemma 3. *When B-bonds become risky and lose their collateral pledgability property, the representative portfolios of both regions rebalance toward liquid and safe A-bonds, and away from illiquid and risky assets. This FTL increases the bond spread and causes a credit squeeze in the markets of risky sovereigns and corporate loans.*

The novelty in Lemma 3 is the combination of segmented sovereign bond markets[25] and the possibility of rehypothecating sovereign collateral through repo in the context of the European sovereign debt crisis. On the one hand, going short in the B-bond requires borrowing it first before shorting it. Rehypothecation allows banks to repeat this strategy multiple times and as a result they can achieve large short sale positions on the B-bond that would be impossible using their own B-bond endowments. On the other hand, repo allows optimistic banks to finance the purchase of the A-bond. Rehypothecation allows banks to repeat this strategy multiple times and achieve a larger long position in the A-bond than they would get otherwise by selling their own endowments of the B-bond and then using the proceeds to buy the A-bond. Underlying in the bond pricing mechanism is the shadow value associated to the constraint that keeps track of the amount of the bond in possession, which must be positive (possession and position are different when repo is introduced).

The result of Lemma 3 regarding the negative impact of the GICIPS sovereign debt crisis on banks' lending to firms has been identified in the literature (e.g., Acharya and Steffen, 2015; Bocola, 2016).[26] My analysis complements this result by showing how collateral risk affects both banks' portfolio rebalancing strategies and lending to corporations through the repo channel.

3.3. Access to the funding market

Now consider the case where B-banks have worse access to the funding market than A-banks. Formally, I assume that $1-h_A^{A\text{-}short} < 1-h_A^{B\text{-}short}$, where the upper index indicates the bank's region and its repo position, and the lower index indicates the type of collateral accepted in repo (A-bonds). For example, $1-h_A^{B\text{-}short}$ denotes the haircut that applies to a B-bank when borrowing cash in repo with the collateral being a sovereign bond issued in region A. In other words, for each euro worth of A-bond collateral that a B-bank pledges in repo, it only receives $h_A^{B\text{-}short}$ euros in cash. If it is an A-bank pledging the same amount of collateral, it receives a larger loan amount ($h_A^{A\text{-}short}$ euros).

To economize on notation, let $1 - h_A$ (without upper index) denote the haircut that applies to banks that are lenders of cash in repo (longs in repo or funding providers) when the collateral is a bond issued in region A. For example, for each euro worth of collateral that a B-bank receives in repo, it lends h_A euros to its repo counterparty. If the repo counterparty of this B-bank (repo long) is another B-bank (repo short), the central clearing counterparty (CCP) keeps the difference in haircuts ($h_A^{B\text{-}short} - h_A$).[27]

[25] Modeling segmentation in the sovereign bond market is possible by differentiating between haircuts. This is different from other models with repo where equilibrium does not depend on haircuts (Uhlig, 2013).

[26] Bocola (2016) estimates a quantitative model of bank intermediation using Italian data and shows that news that the government may default in the future explains up to 45% of the impact of the sovereign debt crisis on firms' borrowing costs.

[27] For example, consider a repo transaction with A-sovereign collateral and let $1 - h_A = 0.02$ and $1 - h_A^{B\text{-}short} = 0.025$. In this case, for each euro worth of A-sovereign collateral that the repo short lends to the repo long, the repo short receives €0.975, the repo long lends €0.980, and the CCP keeps the difference (€0.005).

Table 1
This table reports the loan portfolio composition of representative banks in regions A and B, for Cases I, II, and III of our leading example. Here, x^R_{corps}, $x^R_{B-bonds}$, and $x^R_{A-bonds}$ denote the portfolio weight on corporate loans, B-bonds, and A-bonds, respectively, by the representative portfolio in region $R = A, B$.

Loan portfolios	x^A_{corps}	$x^A_{B-bonds}$	$x^A_{A-bonds}$	x^B_{corps}	$x^B_{B-bonds}$	$x^B_{A-bonds}$
Case I	47.36	26.32	26.32	47.36	26.32	26.32
Case II	24.33	24.33	51.34	24.33	24.33	51.34
Case III	20.14	20.14	59.72	37.07	37.07	25.86

Table 2
This table reports the equilibrium asset prices and bond spread for Cases I, II, and III.

Equilibrium prices	$\hat{\gamma}$	$\hat{q}_B$	$\hat{q}_A$	$\hat{q}_A - \hat{q}_B$
Case I	0.9282	0.9797	0.9797	0.0000
Case II	0.2389	0.2389	0.7432	0.5043
Case III	0.2363	0.2363	0.7385	0.5022

I modify the market clearing equation (15) by including the upper index B-short on the repo haircut of those B-banks that are short in repo (i.e., those banks that attempt to build a leveraged long position on the A-bond):

$$-\int_{\hat{b}_A}^{1} \frac{h_A}{1-h_A} e\, db - \int_{\hat{b}_B}^{1} \frac{h_A}{1-h_A} \frac{\hat{\gamma}}{\hat{q}} e\, db + \int_0^{\hat{b}_A} \frac{1}{1-h_A} e\, db + \int_0^{\hat{b}_B} \frac{1}{1-h_A^{B-short}} \frac{\hat{\gamma}}{\hat{q}} e\, db = 2\int_0^1 e\, db. \tag{16}$$

In addition, I modify $1 - h_A$ by $1 - h_A^{B-short}$ in expression (5) for the marginal bank $\hat{b}_B$'s shadow price $\mu_{j_A}^{\hat{b}_B}$.

In an equilibrium model with more than one asset and more than one type of bank, relative asset prices determine the demand for different assets. The following modification of the leading example illustrates that, when B-banks pay higher haircuts for funding their sovereign bond purchases, their demand for A-bonds decreases because A-sovereign collateral is less desirable for them; the lower pressure on the price of A-bonds induces optimistic A-banks to rebalance toward A-bonds. Observe this market mechanism at work in the following example.

Case III (*Access to the Funding Market*): Modify Case II by increasing by 0.5 basis points the repo haircut $1 - h_A^{B\text{-}\ short}$. That is, now $1 - h_A = 0.100$ and $1 - h_A^{B\text{-}short} = 0.105$. The representative portfolios are now different between the regions. In region A, the representative portfolio is composed of 20.14% corporate loans, 20.14% B-bonds, and 59.72% A-bonds. In region B, the representative portfolio is composed of 37.07% corporate loans, 37.07% B-bonds, and 25.86% A-bonds. Therefore, the representative portfolio in the A-region rebalances toward A-bonds and away from B-bonds and corporate loans, while the representative portfolio in the B-region does the opposite. In terms of Definition 3, I find that a worse access to the funding market for B-banks induces A-banks and B-banks to do FTL and RL, respectively. The new equilibrium asset prices are $\hat{q}_A = 0.7385$, $\hat{q}_B = \hat{\gamma} = 0.2363$. Thus, the bond spread slightly decreases from 0.5043 (Case II) to 0.5022 (Case III), given the opposite rebalancing forces between representative portfolios of the two regions — roughly speaking, the price impact on B-bonds of B-banks' higher demand for these bonds is more than offset by the lower demand from A-banks. Credit exposures to corporations, captured by $\hat{\gamma}$, decrease compared to Case II from 0.70 to 0.65. ■

In Lemma 4, I summarize the economic forces behind Case III.

Lemma 4. *A worse access to the funding market for B-banks induces A-banks and B-banks to do FTL and RL, respectively.*

3.4. Comparative statics

In Tables 1 and 2 I summarize the equilibrium loan portfolios and asset prices for Cases I, II, and III.[28]

4. Central bank intervention

In this section, I analyze the role of the ECB's repo facility in reducing the spread between safe and liquid sovereign bonds and illiquid GICIPS sovereign bonds. I approach this question by expanding the model set-up, now with three assets (and two marginal banks) per region. In the previous section, I avoided this additional complexity to gain clarity on the exposition.

[28] The results obtained in Cases I, II, and III are robust to different values for parameters α_S, α_F, d, and e.

4.1. ECB's LTRO program

The ECB conducts monetary policy by imposing reserve requirements on banks and supplying liquidity through refinancing operations. It also makes more unconventional policy decisions by choosing the collateral accepted by its repo operations and targeting a given level of excess demand for different currencies, among other policy decisions. Here, I examine the impact of the ECB's LTRO program on banks' cross-border lending strategies (see the Appendix for a discussion of other policies, such as the Outright Monetary Transactions program and the USD liquidity-providing program).

The LTRO program was established by the ECB to provide liquidity to EMU governments through the banking system. Banks were allowed to participate in this program as long as they provided eligible collateral as a pledge against an LTRO loan. I model an LTRO as a repo transaction, where the ECB chooses the type of collateral (j_B), the repo rate (ρ_B^{ECB}), and the haircut rate ($1-h_B^{ECB}$).

The bank's box constraint on the B-bond j_B is now as follows:

$$(\mu_{j_B}^b) : e_{j_B}^b + y_{j_B}^b + H_{j_B} z_{j_B}^{b\text{-}long} - z_{j_B}^{b\text{-}short} \geq 0 \tag{17}$$

where $z_{j_B}^{b\text{-}long} = \max\{0, z_{j_B}^b\}$, $z_{j_B}^{b\text{-}short} = -\min\{0, z_{j_B}^b\}$, $z_{j_B}^b \equiv z_{j_B}^{b\text{-}long} - z_{j_B}^{b\text{-}short}$, and H_{j_B} is the rehypothecation rate. By "rehypothecation rate," I mean the fraction of a borrowed sovereign bond that can be sold or lent. That is, I now explicitly differentiate between repo long and repo short positions. I assume that $H_{j_B} < h_{j_B}$. Without this restriction, equilibrium may not exist because with two or more bonds, the repo long is cash generating, e.g., the repo long borrows one unit of the bond against $h_j q_j$ euros and then sells that unit of the bond for q_j (see Bottazziet al., 2012).[29]

The ECB is on the long side in an LTRO repo ($z_{j_B}^{ECB\text{-}long} > 0$) because its objective is to provide liquidity to optimistic banks to leverage on their long B-bond positions. Thus, optimistic banks can pledge B-bonds to the ECB against an LTRO loan ($z_{j_B}^{b\text{-}short} < 0$).

By setting $z_{j_B}^{ECB\text{-}short} = 0$, the ECB precludes banks from using the ECB's LTRO loan lending facility to borrow B-bonds with the purpose of short selling them afterwards. Market clearing in the repo market then implies $z_{j_B}^{b\text{-}long} = 0$ (recall that there is no private repo market for B-bonds since $1 - h_B = 1$).

In this new setting, banks can choose among three assets:

- Safe A-bonds with payoffs $(1, \alpha_S^A)$, with $\alpha_S^A = 1$, and repo haircut $1 - h_A$.
- Risky corporate loans with payoffs $(1, \alpha_F)$ (not accepted in repo).
- Risky B-bonds with payoffs $(1, \alpha_S^B)$ and a repo haircut in the ECB's lending facility equal to $1 - h_B^{ECB}$.

As before, I assume that the payoffs of corporate loans and B-bonds are identical, thus $\alpha_F = \alpha_S^B < 1$. In addition, I make the following assumptions concerning haircuts in the private repo market and the ECB's lending facility, respectively: (i) $0 < 1 - h_A < 1 - h_B = 1$ and (ii) $0 < 1 - h_B^{ECB} < 1$. Shadow price $\mu_{j_A}^b$ corresponds to box constraint (3) for bond j_A with haircut $1 - h_A$ and no restrictions on z_{j_A}, and thus can be expressed as in Eq. (5). $\mu_{j_B}^b$ is the shadow price of the box constraint (17) for bond j_B. The first-order conditions with respect to $z_{j_B}^{b\text{-}short}$ (with $z_{j_B}^{b\text{-}short} < 0$) and $\varphi_{j_B}^b$ allows this shadow value to be written as follows:

$$\mu_{j_B}^b = \frac{h_B^{ECB}}{1 - h_B^{ECB}} d \sum_{s=s_A,s_B} (\beta_s(b) r_{j_B s} - (1 + \rho_B^{ECB}) q_B). \tag{18}$$

In a setting with three assets, there are two marginal banks in each region $R = A, B$, namely, $\hat{b}_1^R$ and $\hat{b}_2^R$, which satisfy the following respective indifference conditions:

- Marginal bank $\hat{b}_1^R$ in region R is indifferent between the leveraged return on the risky asset (i.e., buying B-bonds with leverage) and the unleveraged return on the risky asset (i.e., buying corporate loans with no leverage), i.e.,

$$\frac{d(\beta_{s_A}(\hat{b}_1^R) + (1 - \beta_{s_A}(\hat{b}_1^R))\alpha_S^B) + \mu_{j_B}^{\hat{b}_1^R}}{\hat{q}_B} = \frac{d(\beta_{s_A}(\hat{b}_1^R) + (1 - \beta_{s_A}(\hat{b}_1^R))\alpha_F)}{\hat{\gamma}}. \tag{19}$$

- Marginal bank $\hat{b}_2^R$ in region R is indifferent between the unleveraged return on the risky asset (i.e., buying corporate loans with no leverage) and the leveraged return on the safe asset (i.e., buying A-bonds with leverage), i.e.,

$$\frac{d(\beta_{s_A}(\hat{b}_2^R) + (1 - \beta_{s_A}(\hat{b}_2^R))\alpha_F)}{\hat{\gamma}} = \frac{d(\beta_{s_A}(\hat{b}_2^R) + (1 - \beta_{s_A}(\hat{b}_2^R))\alpha_S^A) + \mu_{j_A}^{\hat{b}_2^R}}{\hat{q}_A}. \tag{20}$$

Optimistic banks in $(\hat{b}_2^R, \hat{b}_1^R) \cup (\hat{b}_1^R, 1)$ are long in their respective preferred asset and short on the safe asset. Pessimistic banks in $[0, \hat{b}_2^R)$ are long in the safe asset. The leverage mechanism is analogous to Section 2.4. I consider the following trading strategies for the different banks in regions A and B, in a context with three different assets: an active private repo market for A-bonds, and the ECB's lending facility for B-bonds.

[29] A rehypothecation rate H_{j_B} equal to 0 would be equivalent to a "no short sell" constraint (i.e., the bank is not allowed to sell the bond that has previously borrowed in repo). I do not impose such a constraint to derive our results in this section.

Banks in $R = A$	$\varphi^{b_A}_{j_B}$	$\varphi^{b_A}_{j_A}$	a^{b_A}
$b_A > \hat{b}^A_1$	$\frac{1}{(1-h^{ECB}_{j_B})}\frac{\hat{q}_A e_A}{\hat{q}_B}$	$\frac{-h_A}{(1-h_A)}e_A$	0
$b_A \in (\hat{b}^A_2, \hat{b}^A_1)$	0	$\frac{-h_A}{(1-h_A)}e_A$	$\frac{\hat{q}_A e_A}{\hat{\gamma}}$
$b_A < \hat{b}^A_2$	0	$\frac{1}{(1-h_A)}e_A$	0

Banks in $R = B$	$\varphi^{b_B}_{j_B}$	$\varphi^{b_B}_{j_A}$	a^{b_B}
$b_B > \hat{b}^B_1$	$\frac{1}{(1-h^{ECB}_{j_B})}e_B$	$\frac{-h_A}{(1-h_A)}\frac{\hat{q}_B e_B}{\hat{q}_A}$	0
$b_B \in (\hat{b}^B_2, \hat{b}^B_1)$	0	$\frac{-h_A}{(1-h_A)}\frac{\hat{q}_B e_B}{\hat{q}_A}$	$\frac{\hat{q}_B e_B}{\hat{\gamma}}$
$b_B < \hat{b}^B_2$	0	$\frac{1}{(1-h_A)}\frac{\hat{q}_B e_B}{\hat{q}_A}$	0

Using the above trading strategies, I can rewrite the market clearing condition for A-bonds as follows:

$$-\int_{\hat{b}^A_2}^{1}\frac{h_A}{1-h_A}e_A\,db-\int_{\hat{b}^B_2}^{1}\frac{h_A}{1-h_A}\frac{\hat{q}_B e_B}{\hat{q}_A}\,db+\int_0^{\hat{b}^A_2}\frac{1}{1-h_A}e_A\,db+\int_0^{\hat{b}^B_2}\frac{1}{1-h_A}\frac{\hat{q}_B e_B}{\hat{q}_A}\,db=\int_0^1 e_A\,db. \tag{21}$$

The market clearing equations for corporate loans and B-bonds are, respectively,

$$\int_{\hat{b}^A_2}^{\hat{b}^A_1}\frac{\hat{q}_A e_A}{\hat{\gamma}}\,db+\int_{\hat{b}^B_2}^{\hat{b}^B_1}\frac{\hat{q}_B e_B}{\hat{\gamma}}\,db=D \tag{22}$$

$$\int_{\hat{b}^A_1}^{1}\frac{1}{(1-h^{ECB}_{j_B})}\frac{\hat{q}_A e_A}{\hat{q}_B}\,db+\int_{\hat{b}^B_1}^{1}\frac{1}{(1-h^{ECB}_{j_B})}e_B\,db=\int_0^1 e_B\,db. \tag{23}$$

Definition 4. An equilibrium for this economy consists of a vector $(\hat{b}^A_1, \hat{b}^A_2, \hat{b}^B_1, \hat{b}^B_2, \hat{q}_A, \hat{q}_B, \tilde{\gamma})$, such that: (i) indifference conditions (19) and (20) hold, and (ii) market clearing Eqs. (21), (22), and (23) hold.

Case IV: I modify Case II by considering the above setting with three distinct assets, two marginal banks, and the ECB accepting B-bonds as repo collateral. As before, $\alpha^A_S = 1$, $\alpha^B_S = \alpha_F = 0.9$, $1 - h_A = 0.1$, $\rho_{j_A} = 0.02$, and $D = e_A = e_B = 1$. The ECB's haircut and repo rate are $1 - h^{ECB}_B = 0.2$ and $\rho^{ECB}_{j_B} = 0.03$, respectively. For these parameter values, I find an equilibrium with marginal banks $\hat{b}^1_A = \hat{b}^1_B = 0.9025$ and $\hat{b}^2_A = \hat{b}^2_B = 0.5006$, and asset prices $\hat{\gamma} = 0.7505$, $\hat{q}_B = 0.9102$, and $\hat{q}_A = 0.9573$. Thus, the bond spread is 0.0471.[30] ■

In Case II of the leading example, I assumed that repo haircuts for A-bonds (core safe sovereigns) and B-bonds (GICIPS sovereigns) were 10% and 100%, respectively. In that case, the total market capitalization for illiquid GICIPS sovereigns, defined as $\hat{q}_B(((1-\hat{b}_A)/2)+((1-\hat{b}_B)/2))$, was 0.1162. Now, with the ECB accepting GICIPS collateral (B-bonds) against a repo loan at a substantially lower repo haircut (20%) than the private repo market, the GICIPS' market capitalization increases by 53% to a market value equal to 0.1774 (computed as $\hat{q}_B(((1-\hat{b}_A)/2)+((1-\hat{b}_B)/2))$). The policy is effective in significantly decreasing the bond spread from 0.5043 to 0.0471.

This result seems consistent with an ECB study and other independent studies that argue that the LTRO program helped stabilize collateral markets (ECB, 2010; (Drudi et al., 2012); Bank for International Settlements, 2011) and was an important tool toward "normalizing" liquidity in "sovereign crisis hit" sovereign bond markets (Gabor, 2014).[31] The result is also in accordance with Buiter and Sibert (2005), who argue that policies that diminish the difference in creditworthiness between sovereign issuers of different quality are an implicit "subsidy" to lower-rated Member States (e.g., AAA-rated Germany vs. A-rated Greece).

The market mechanism that leads to a decrease in the bond spread is the following. The ECB allows optimistic banks to leverage their long position on GICIPS government bonds by accepting lower-rated sovereigns as repo collateral against an LTRO loan (demand effect). Second, by accepting GICIPS sovereign collateral from optimistic banks and not repo'ing it to pessimistic banks, the ECB constrains short sellers from leveraging their short positions (supply effect). The result is a decrease in the bond spread.

Lemma 5. *The ECB's repo facility for lower-rated sovereign bonds increases the market capitalization of these bonds and reduces the bond spread between safe and liquidity sovereigns and lower-rated sovereigns.*

[30] Also notice that at both states s_A and s_B, the consumption of the numeraire good of both marginal banks in both regions is positive (thus, $\lambda_s^{\hat{b}^A_1} = \lambda_s^{\hat{b}^A_2} = \lambda_s^{\hat{b}^B_1} = \lambda_s^{\hat{b}^B_2} = 0$, for $s = s_A, s_B$). This result also holds for Cases IV.b and V.

[31] The ECB's LTRO program changed collateral standards in a pro-cyclical fashion by engaging in a sequence of mark-to-market and margin calls similar to what repo market participants do during periods of market stress (Domanski and Neumann, 2001; Gorton and Metrick, 2012). See Gabor and Ban (2016) for a discussion of the market turbulence that resulted following the ECB's decision in August 2010 that sovereign collateral rated BBB+ and lower would incur higher haircuts.

4.2. On the potential cost of the ECB's repo facility

I next discuss the potential costs that the ECB may incur when running the repo facility discussed above. By potential cost, I mean the exposure at default that the central bank would have if banks that are short in repo (i.e., borrow against collateral) do not repurchase the collateral ("exposure at default"). This exposure depends on the amount of leverage that the facility permits. For the sake of exposition, I consider two cases: (1) no repo collateral multiplier and (2) a repo collateral multiplier. The latter allows banks to pledge collateral in the repo facility an infinite number of times, whereas the former allows banks to pledge collateral only once.

In the "no repo collateral multiplier" case, the bank pledges the collateral into the ECB's repo facility only once[32] and receives a loan to finance the purchase of the bond pledged as collateral in repo. In this case, I find that the ECB will enter in losses if the new GICIPS sovereign bond market price q'_B falls below the "haircutted" original price $h_B^{ECB} q_B$, i.e., if:

$$q'_B < h_B^{ECB} q_B \tag{24}$$

where $h_B^{ECB} < 1$. Notice that the ECB can always choose h_B^{ECB} small enough (i.e., haircut $1 - h_B^{ECB}$ large enough) to protect itself against future repo losses.

In the "repo collateral multiplier" case, the ECB allows banks to rehypothecate multiple times the bond purchased through its repo facility. The maximum leverage that a bank can be obtained by iterating repo short and bond purchases positions an infinite number of times is:

$$\frac{h_B^{ECB}}{1 - h_B^{ECB}} q_B e_B$$

(see Bottazzi et al., 2012, p 482–483). If such leverage is attained, the ECB may incur potential losses even if the ECB is entitled to the haircut benefit and the GICIPS bond price does not decrease below the initial transaction price. To see this, notice that the ECB's repo long exposure exponentially increases the lower is the haircut $1 - h_B^{ECB}$, while the amount of collateral "in the box" does not change with the haircut. The difference between the new collateral value ($q'_B e_B$) and the levered loan amount originated ($-(h_B^{ECB}/(1 - h_B^{ECB}))q_B e_B$) will determine the extent of the ECB's cost of running its repo lending facility.

Lemma 6. *The ECB's potential loss of running its repo facility is limited to* $(h_B^{ECB} q_B - q'_B)e_B$ *if banks that are short in repo cannot further rehypothecate the pledgable GICIPS bond. If rehypothecation is allowed multiple times, the ECB's potential loss is:*

$$\left(q'_B - \frac{h_B^{ECB}}{1 - h_B^{ECB}} q_B \right) e_B.$$

This loss increases as the repo haircut $1 - h_B^{ECB}$ *decreases.*

5. Conclusion

Banks have always been an intensely political issue. Since Philip II, King of Spain from 1556 to 1598, sovereign debt crises have been recurrent in Europe, and bankers have played a key role in allocating debt across countries. Ruling over one of the largest and most powerful empires in history, King Philip accumulated towering debts and defaulted four times (Drelichmanm and Voth, 2011). German banking houses went into chaos while Genoese bankers became the de facto credit providers to the cumbersome Habsburg system. More than five centuries later, in 2010, the public finances of Spain and other peripheral European countries seriously deteriorated and banks were once again at the center of the crisis. The result was a credit squeeze of more than €11 trillion[33] and an intense reallocation of banks' credit across geographical regions and loan types.[34] History repeats itself, but now with a common supranational monetary authority and a more integrated and sophisticated financial market with sovereign bonds serving as collateral in repurchase agreements.

In this paper, I explore the role that segmented sovereign bond markets had in the transmission of the credit crisis through the banking system and the role of the European Central Bank's LTRO program in ameliorating the knock-on effects on bond spreads.

In particular, I provide a tractable equilibrium model to understand the role of repo funding on banks' incentives to fly-to-liquidity and engage in risky lending during the European sovereign debt crisis. Because banks rely on repo for their wholesale asset purchases, shocks to the collateral status and access to the funding market may result in banks changing their valuation of sovereign bonds. This change in relative asset prices, in turn, may induce banks to rebalance their loan portfolios toward other regions and loan types.

[32] The ECB is long in repo and the amount of money lent in repo to banks is smaller than the value of collateral received.

[33] Of this, only €38 billion corresponds to sovereign bonds, and the rest to productive investments (€3.2 trillion in residential mortgages, €1tr in commercial mortgages, €2 trillion in institutions, and €4.6 trillion in corporations). See Luque (2020).

[34] For example, between end-2009 and end-2013, Luque (2020) finds that German banks, on average, reduced their portfolio exposure to corporations from 30% to 14% and increased their exposure to sovereign bonds from 19% to 69%. There may be a number of reasons for this, including the liquidity associated with sovereign bonds, the Capital Requirements Directive, creditors' discrimination in favor of domestic countries, ad hoc domestic regulations, and risk-shifting motives (Uhlig, 2013; Broner et al., 2014; Acharya and Steffen, 2015; Popov and Van Horen, 2014, for seminal contributions on this issue).

I show that collateral risk on lower-rated sovereigns induces banks to fly to liquid and safe sovereign collateral and causes a credit squeeze in the markets of risky sovereigns and corporate loans. In addition, the model predicts that worse access to the funding market for banks in the GICIPS region induces these banks to engage in a risky-lending strategy, rebalancing their loan portfolios toward risky illiquid sovereigns, whereas core banks engage in a flight-to-liquidity strategy.

My paper also contributes to the policy arena by studying the market mechanisms by which "sovereign crisis hit" bond markets were "normalized." I show that the LTRO program can effectively reduce the bond spread by decreasing the repo haircut on lower-rated bonds below the corresponding haircut in the private repo market for that sovereign bond. In this sense, LTROs were an important tool toward "normalizing" liquidity in "sovereign crisis hit" collateral markets. The market mechanism behind my model is interesting because it deals with both leverage by optimistic banks and absence of rehypothecation by the ECB.

Specifically, the central bank's lending facility allows optimistic banks to leverage their long positions on GICIPS bonds and thus the demand for these bonds increases. At the same time, the ECB's lending facility also limits the availability of GICIPS sovereigns in the secondary bond market by not lending or short selling the collateral borrowed through repo. Rather than pointing to moral suasion by GICIPS governments on domestic banks as the reason demand for lower-rated sovereigns increased (Acharya and Steffen, 2015), I identify the ECB's LTRO program as a mechanism whereby optimistic banks carried out the successful policy of the ECB to tamp down euro redenomination risk.

Finally, I highlight new risks that emerge for the central monetary authority when banks are allowed to attain high levels of leverage by rehypothecating sovereign collateral through the LTRO facility. In particular, I show that the ECB's potential losses of running this program increase the lower is the repo haircut and the higher is the level of collateral rehypothecation.

Appendix

In this Appendix, I incorporate firms and capital requirements into the baseline model. Also, I derive the optimality conditions used to obtain the equilibrium indifference condition (6), and provide the proofs of Lemmas 1 and 2. In addition, I briefly discuss other relevant ECB interventions during the sovereign debt crisis.

A.1. Firms

Here, I show how to incorporate the demand side of the corporate loan market into the baseline model. Let $\mathbf{F}$ denote the set of firms with mass 1, i.e., $\lambda(\mathbf{F}) = 1$, where λ denotes the associated Lebesgue measure. Each firm f in $\mathbf{F}$ is endowed with a vector $(\omega_0^f, (\omega_s^f)_{s=s_A,s_B})$ of units of the numeraire good at dates 0 and 1 (state s), respectively. The probability that firm f assigns to states s_A and s_B are $\beta_{s_A}(f)$ and $\beta_{s_B}(f)$, respectively.

Each firm f can buy an amount $I^f \in \mathbb{R}_+$ of productive investment at discount price p and receive $(r_{is_A}, r_{is_B}) = (1, \alpha_I)$ per unit of investment at date 1. The firm can use its own capital (ω_0^f) and debt $(\gamma a^f < 0)$ for this investment. There is an exogenous supply $\bar{I}$ of inputs available for production and, for simplicity, I assume $\bar{I} = e$.

The optimization problem of a firm f is as follows. Firm f chooses an amount I^f of productive investment and a credit line a^f in order to maximize the following linear profit function:

$$\Psi^f \equiv d \sum_{s=s_A,s_B} \beta_s(f)\Psi_s^f, \tag{A.1}$$

with $\Psi_s^f \equiv \omega_s^f + r_{fs}a^f + r_{is}I^f$, subject to the sign constraints

$$(\zeta^f) \ : \ I^f \geq 0, \tag{A.2}$$

$$(\lambda_s^f) \ : \ \Psi_s^f \geq 0, \ s = s_A, s_B, \tag{A.3}$$

the budget constraint

$$(\lambda_0^f) : \omega_0^f - \gamma a^f - pI^f \geq 0, \tag{A.4}$$

and a no-short sales constraint

$$(\eta^f) : a^f \geq -L. \tag{A.5}$$

Here $L > 0$ is an exogenous lower bound on short sales, and ζ^f, λ_s^f, λ_0^f, and η^f are the shadow values of constraints (A.2), (A.3), (A.4), and (A.5), respectively. When $a^f > 0$, firm f has a long position (lender), and when a^f is negative, firm f has a short position (borrower). The equilibrium configuration is such that firms have short positions and use corporate loans to buy inputs for productive investments. Thus, I find it convenient to assume $L = 0$.

Optimality implies that discount prices of corporate loans and productive investment are, respectively,

$$\lambda_0^f \cdot \gamma = ((d\beta_{s_A}(f) + \lambda_{s_A}^f) + (d\beta_{s_B}(f) + \lambda_{s_B}^f)\alpha_F) + \eta^f \tag{A.6}$$

$$\lambda_0^f \cdot p = (d\beta_{s_A}(f) + \lambda_{s_A}^f) + (d(\beta_{s_B}(f) + \lambda_{s_B}^f)\alpha_I) + \zeta^f. \tag{A.7}$$

If $\alpha_F = \alpha_I$, $I^f > 0$, $-L < a^f$, for all f in $\mathbf{F}$, prices p and γ are the same. In the examples I take $D = e$. This follows from $\omega_0^f = 0$, constraint (A.4), and assumption $\bar{I} = e$, i.e., $D \equiv \int_{\mathbf{F}} a^f = e$ whenever $\omega_0^f = 0$.

A.2. Proofs

Proof of Lemma 1. Let $\varphi \equiv e + y$. Then, constraint (1) implies that $a = (1/\gamma)(-q(\varphi - e) - hqz)$. Replacing this expression in (4), I get $\Psi_s^b \equiv \omega_s^b + (1+\rho)hqz^b + r_{js}(y^b + e^b) + \hat{\imath}_{fs}(-q(\varphi - e) - hqz)$, where $\hat{\imath}_{fs} \equiv r_{fs}/\gamma$. Bank b's first-order conditions with respect to z and φ are, respectively,

$$\mu^b + (1+\rho)hq(d + \sum_s \lambda_s^b) = \lambda_0^b h + \sum_s (d\beta_s(b) + \lambda_s^b)(\hat{\imath}_{fs} qh)$$
$$\mu^b - \lambda_0^b + \sum_s (d\beta_s(b) + \lambda_s) r_{js} = \sum_s (d\beta_s(b) + \lambda_s^b)\hat{\imath}_{fs} q.$$

These two conditions imply that

$$\mu^b = \frac{h}{1-h} d \sum_{s=s_A,s_B} (\beta_s(b) r_{js} - (1+\rho)q)$$
$$\lambda_0^b = \frac{1}{1-h} d \sum_{s=s_A,s_B} \beta_s(b) r_{js} - dq(1 + \rho + \sum_s \beta_s(b)\hat{\imath}_{fs}).$$

Characterization of marginal bank: Optimality implies that the pricing of sovereign bonds (with respect to repo trades and bond trades) and corporate loans obeys the following equations, respectively:

$$(\text{wrt } z^b) \ : \ \mu^b = qh(\lambda_0^b - (1+\rho)(d + \lambda_{s_A}^b + \lambda_{s_B}^b)) \tag{A.8}$$
$$(\text{wrt } y^b) \ : \ q\lambda_0^b = \sum_{s=s_A,s_B} (r_{js}(d\beta_s(b) + \lambda_s^b)) + \mu^b \tag{A.9}$$
$$(\text{wrt } a^b) \ : \ \gamma\lambda_0^b = \sum_{s=s_A,s_B} (r_{fs}(d\beta_s(b) + \lambda_s^b)) + \eta^b \tag{A.10}$$

Shadow prices $\lambda_{s_A}^{\hat{b}}$ and $\lambda_{s_B}^{\hat{b}}$ are zero because the marginal bank is indifferent between consuming at states s_A and s_B. Also, because of the indifference between sovereigns and corporate loans, $\eta^{\hat{b}} = 0$. Then, after some algebra using (A.8), (A.9), and (A.10), I find that marginal bank $\hat{b}$ must satisfy the following indifference condition:

$$\frac{d(\beta_{s_A}(\hat{b}) + (1 - \beta_{s_A}(\hat{b}))\alpha_S) + \mu^b}{q} = \frac{d(\beta_{s_A}(\hat{b}) + (1 - \beta_{s_A}(\hat{b}))\alpha_F)}{\gamma}.$$

Condition (6) follows by making $\alpha_S = 1$.

Lemma 2. *Optimality conditions* (A.8) and (A.9) *for marginal bank $\hat{b}$ (with $\lambda_{s_A}^{\hat{b}} = \lambda_{s_B}^{\hat{b}} = 0$) imply that $d(1+\rho)qh = dh\sum_{s=s_A,s_B} r_{js}\beta_s(\hat{b}) - \mu^b(1-h) < 1$. Expression* (4) *for s_B, $r_{js_B} = 1$, and $d(1+\rho)qh < 1$, then, implies that $\Psi_{s_B}^b > \omega_{s_B}$ for $b < \hat{b}$ holding sovereign bonds, since $d\Psi_{s_B}^{\hat{b}} = d\omega_{s_B} + d\frac{e}{1-h}(1 - (1+\rho)hq) > d\omega_{s_B}$.*

A.3. Capital requirements

The banking industry has recently been subject to stricter capital requirements, in part due to the Great Financial Crisis of 2007–09. In Europe, however, the Capital Requirements Directive (CRD) allowed government debt issued in domestic currency (euros) at 0% "risk weighting." Here, I show how to incorporate capital requirements into the baseline model and discuss how stricter capital requirements on risky corporate loans may induce banks to rebalance their loan portfolios toward sovereign bonds. This is in accordance with Popov and Van Horen (2014), who show that regulatory requirements provided banks with strong incentives to hold large amounts of sovereign debt on their balances sheet.

I consider a well-known core measure of a bank's financial strength from a regulator's point of view: the core Tier 1 capital ratio. This measure is defined as the ratio of a bank's core equity capital to its total risk-weighted assets (RWA). The regulator puts this requirement into place to ensure that financial institutions do not take on excess leverage and become insolvent. RWAs are the total assets held by the bank, weighted by credit risk according to a formula determined by the regulator.

Here, I incorporate an stylized version of the core Tier 1 ratio into the baseline model. For this, I denote by ϕ_a, ϕ_y, and ϕ_z the credit risk weights on corporate loan exposures, sovereign bond exposures, and repo long positions with sovereign collateral. Credit risk weights are chosen by the financial regulator. In terms of my notation, I can express the bank b's RWA as follows:

$$RWA^b \equiv \phi_a a^b + \phi_y \varphi^b + \phi_z z^b.$$

Because the Capital Requirements Directive (CRD) allowed government debt issued in domestic currency (euros) a 0% "risk weighting", I assume $\phi_y = 0$. In addition, because repos are collateralized with sovereigns, I set $\phi_{z_R} = 0$.

I incorporate the stylized version of the core Tier 1 capital requirement into the model by adding the following constraint to the bank's optimization problem:

$$(v^b) \ : \ E^b \geq c \cdot RWA^b, \tag{A.11}$$

where $E^b \equiv \omega_0^b + qe^b$ and v^b denote the bank b's initial core equity and shadow value of constraint (A.11), respectively, and c is the core Tier 1 capital ratio chosen by the regulator. As before, I take $\omega_0^b = 0$. Indifference condition (6) becomes:

$$\frac{d(\beta_{s_A}(\hat{b}) + (1 - \beta_{s_A}(\hat{b}))\alpha_S) + \mu^{\hat{b}}}{q} = \frac{d(\beta_{s_A}(\hat{b}) + (1 - \beta_{s_A}(\hat{b}))\alpha_F) - v^{\hat{b}} c \phi_a}{\gamma} \tag{A.12}$$

where shadow value $\mu^{\hat{b}}$ is the same as in expression (5) because the extra terms related to capital constraint (A.11) cancel out from the first order conditions on z and φ.

In Eq. (A.12), I can see that, whenever the capital constraint is binding for the marginal bank, the return on corporate loans decreases if the credit risk weight on corporate loans (ϕ_a) increases. If that happens, the representative portfolio rebalances toward sovereign bonds to reestablish indifference condition (A.12).

A.4. ECB interventions during the sovereign crisis

The ECB has been lending to the banking system through the LTRO program with loans of several maturities. Long-Term Refinancing Operations (LTROs) became popular during the European financial crisis that began in 2008. Before the crisis hit, the ECB's longest tender offered was just three months. These LTROs amounted to just €45 billion. Probably the most important ECB decision during the sovereign crisis period only occurred at the very end of 2011, when investors became concerned about the quality of Italian sovereign collateral. The ECB responded by injecting more than €1 trillion into the banking sector in two consecutive LTROs, one in December 2011 and one in February 2012, with 3-year maturity loans at a rate fixed to the ECB's refinancing rate of 1%.

In addition, the ECB committed to preserving liquidity in government bond markets through direct market interventions (Buiter and Rahbari, 2012; Grauwe and Ji, 2012). The ECB defines the Outright Monetary Transactions (OMT) as a program regarding the Eurosystem's outright transactions in secondary sovereign bond markets that aims at safeguarding an appropriate monetary policy transmission and the singleness of the monetary policy. Under this program, the ECB makes purchases ("outright transactions") in secondary sovereign bond markets of bonds issued by eurozone Member-States with 1–3 year maturity. For the ECB to carry out this program, bond-issuing countries must agree to certain domestic economic measures. Note that the OMTs are not the same as quantitative easing (QE) operations, since the principle of "full sterilization" applies for the former, whereby the bank must reabsorb the money pumped into the system.

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