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Developers, Wall Street, and the Taxmen: A Theory of Real Estate Development*

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Abstract

We propose a general equilibrium model of commercial real estate (CRE) development in a multi-jurisdictions economy with segmented commercial good and equity markets. The CRE assets' capital structures and cash flows, the prices of equity, debt, and commercial goods, and the jurisdictions' property taxes and land use policies are all endogenous. The capital structures of CRE assets in different jurisdictions are interdependent. Shocks to a developer's funding capacity, the production of a CRE asset, and a jurisdiction's property tax illustrate this result. Our model also captures the impact that global real estate markets have on local jurisdictions' fiscal policies.

JEL Classification: D52, D53, H73, R13, R33, R38

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1 Introduction

In the last decades, regional and local real estate markets have rapidly changed in part because of the dominant presence of Wall Street financial intermediaries in real estate development deals. The most common types of real estate investors are Real Estate Investment Trusts (REITs), pension funds, insurance companies, and, to some extent, foreign investors. REITs own more than \$3 trillion in gross assets, support an estimated 1.8 million full-time workers, and contribute more than \$56 billion in new construction and capital expenditures. REITs are corporate entities that can be privately or publicly held. Only in the United States (U.S.), there are more than 200 stock exchange-listed REITs with a total equity market capitalization of approximately \$1 trillion. The presence of Wall Street – rather than Main Street investors – leads to larger questions regarding the role of global investors in urban and metropolitan economic development.

The prevalence of REITs in the acquisition of commercial real estate goes back to 1991 and since then, there has been a spirited debate in the political arena of their impact on real estate finance, urban policy, and the local economy (Mills 1993 and Dinsmore 1998). Today, in the presence of a more globalized financial market, there are also ongoing discussions about the ability of American cities to attract international capital investments. Economic strains in other parts of the world, such as Brexit and the cooling of the Chinese economy, are now driving international capital into commercial real estate in the US in search of higher returns and stability. Not only traditional gateway U.S. cities, such as New York, Los Angeles and Miami, are the focus of investors' attention, but also other places such as Denver, Phoenix and Nashville. Investors search for diversification, not only across real estate sectors, but also across different regions. The population and job growth of a city, its property taxes, and its zoning and land use regulations are key determinants of the rise of a city. Developers in cities with worse fundamentals fall behind and struggle to get financing. Local and regional economic shocks can exacerbate these dynamics, with some cities rising and others falling, in turn bringing hope or desperation to their citizens.

Another important issue in the current discussion is whether highly acquisitive REITs are able to influence the local economy. REITs own and finance the development of real estate properties

in both urban and suburban areas, such as office buildings, luxury hotels, urban apartments, regional and strip malls, storage, and warehouse.¹ Depending on the magnitude of a REIT's portfolio, it may be a major stakeholder able to influence whether and how a local community deals with a particular issue. An alarming example is the case of Texas, where REITs own more than 29,600 properties. REITs have a clear self-interest in reducing some of the costs decided by the local municipality, such as local property taxes, zoning and land use controls, and utility rates. The problem becomes more intricate when Wall Street investors have an overwhelming impact on the fate of the local community.

These important practical questions deserve a careful economic analysis and a general equilibrium model happens to be the natural way to analyze these issues. The financing aspects of a real estate development project cannot be analyzed without taking into consideration the local fiscal and land use policies, nor can they be analyzed without a proper evaluation of the city's economic fundamentals. Local households are the final buyers of the goods produced and sold by the commercial real estate assets and their prices determine the cash flows used to evaluate the equity price of a commercial real estate asset. Neither we can ignore a developer's access to funding in the debt market because debt ceilings might determine the funding gap that a developer needs to fill with equity. Also, a developer's choice of capital structure has general equilibrium implications in an economy with segmented equity markets. A more leveraged development in a jurisdiction may be the result of equity investments flowing to other development projects located in more attractive cities. Moreover, because interests on debt are tax exempt but not the equity dividends, a local fiscal policy may determine not only the capital structure of a local development project, but also the capital structures of other projects outside the jurisdiction.

We propose a general equilibrium model of real estate development that captures these economic interactions. We focus on equity REIT investments and commercial (income-producing)

¹Commercial real estate development plays an integral role in the economy. It creates spaces for jobs and provides a permanent source of revenue to investors. In the U.S. alone, there is about \$5 trillion worth of commercial real estate. The construction of these assets contributes approximately 5% percent of the U.S. Gross Domestic Product (GDP), which in 2016 terms amounts to a contribution of \$861 billion to the U.S.'s economic output. In terms of employment, commercial real estate development supports 6.25 million American jobs.

real estate properties, such as office buildings, industrial space (e.g., heavy manufacturing, light assembly, and warehouses), retail space (e.g., strip centers and regional malls), multifamily buildings (e.g., student housing and mid-rise apartments), land (e.g., greenfield land), and hotels. We consider an economy with multiple jurisdictions and pay special attention to the financing aspects of the real estate development deals, as well as their interaction with the local governments' fiscal and land use policies. For each development project in each jurisdiction, the capital structure, composed by the developer's debt, the developer's (or entrepreneurial partner's) equity, and the investors' (or capital partners') equity, is endogenous in our model and is driven by factors such as the developer's funding capacity, the jurisdiction's property taxes, and the performance of the income-producing real estate asset. Other important elements such as real estate equity cash flows and development construction costs are also endogenous in our model.

For the sake of simplicity, we build our model in a two-period setting. Uncertainty enters into our model because we consider several states of nature in the second period. Our notion of equilibrium is in the tradition of general competitive equilibrium models with incomplete financial markets and restricted participation, where agents do not choose their location (jurisdiction), but are exogenously assigned to jurisdictions. There are multiple jurisdictions and each jurisdiction has both local households and local developers. Global investors can buy CRE equity in all jurisdictions.

Commercial real estate (CRE) development projects are capital intensive. Developers finance their CRE development projects by issuing debt in a global market and selling equity to investors. Developers use these funds to buy construction inputs, including land, in the first period. Given the jurisdiction's choice of the type of CRE asset that a developer is allowed to construct, these production inputs are transformed into a CRE asset using a Cobb-Douglas production technology. A CRE asset can produce multiple commercial goods in the second period, possibly in different amounts across states of nature, which are purchased by local households. Because households may have different preferences for the consumption of commercial goods, the price of similar commercial goods can differ across jurisdictions. This in turn implies that similar CRE assets in

different jurisdictions may generate different cash flows in equilibrium.

Our interpretation of commercial goods produced by CRE assets and sold to local households is in the spirit of Debreu (1959)'s classical book *Theory of Value*. For example, for the case of hotels, the consumption of a "hotel good" should be interpreted as the purchase of a certain number of nights in a hotel room with specific characteristics in a given location. Similar interpretations apply for the consumption of office space or student housing.

Given the fiscal and land use policies of the jurisdictions, we identify mild conditions under which a competitive equilibrium exists for our economy. This result is not trivial because of the additional economic structure that our model requires in order to accommodate the development (aka creation) of CRE assets and the segmentation of commercial good and CRE equity markets. One particular difficulty is to guarantee that a developer's budget set correspondence takes convex value. This is because the market value of a CRE asset depends on the developer's choice of materials and also because the equity return in the second period depends on the (endogenous) type of CRE asset chosen in the first period. Another subtlety of our existence proof is the property of lower semicontinuity of the budget correspondence (the standard approach of embedding all prices in the simplex does not guarantee the existence of an interior point with segmented real estate equity and commercial good markets). We circumvent this problem by finding an endogenous upper bound for real estate equity prices (previous results in the literature of market segmentation with a fixed point theory approach are not useful in our setting).

Having established this general result, we provide a simplified version of our general equilibrium model with only two jurisdictions and whose equilibrium happens to be unique. This simplified model allows us to understand the impact of several economic shocks and public policies on the feasibility of CRE development projects and capital structures.

Our first characterization result shows that global real estate equity investors increase their exposure to CRE equity in those CRE development projects whose developers become more debt constrained. In terms of financial ratios, this means that when a developer's funding capacity diminishes, the developer's debt-to-equity ratio – a proxy of leverage – decreases, while the

investor's equity-to-total-equity ratio increases. Because developers compete for the CRE equity investments of global real estate investment funds, the debt-to-equity ratio must increase in other development projects where developers have a better funding capacity in order to compensate for the reduction of investors' equity in those jurisdictions. Thus, our model captures a substitution effect not only within the capital structure of a real estate development project, but also across jurisdictions. Roughly speaking, our example conveys the idea that leverage in the real estate construction sector shifts from one jurisdiction to another when access to the debt market changes for developers in a jurisdiction. This result contributes to the recent theoretical literature on risk shifting and asset substitution (see the seminar paper of Stiglitz and Weiss 1981, and Martinez-Miera and Repullo 2010 for a recent contribution).²

Another result that we are able to generate is that if the a jurisdiction decides to increase its property tax rate to improve the provision of local public goods, the developer of that jurisdiction substitutes equity with tax free debt, so leverage increases for this developer. This in turn induces global investors to increase their equity holdings in the *high* tax jurisdiction because the equity price diminishes due to the developers' strategy of rebalancing toward debt. There is also an inter-jurisdictional equilibrium effect because developers in low property tax jurisdictions find debt relatively more expensive than equity given the additional price pressure on risk-free debt and choose to issue less debt and buy more CRE equity. The result is a less leveraged capital structure in the low property tax jurisdictions.³

We also extend our two-period model to incorporate a Nash game in a pre-stage of the economy where strategic jurisdiction authorities choose their property taxes and the types of CRE developments allowed in their jurisdictions. These decisions will then influence the type of competitive equilibrium (e.g., the composition of the different capital structures and the commercial good and equity prices). Because these two variables become endogenous, this extension of the model is appealing as public policy for municipalities. We provide an example to illustrate

²Sun, Titman, and Twite (2015) for evidence of the impact of the recent financial crisis on the limits to debt capacity for commercial real estate assets.

³Importantly, we obtain these findings without modifying the jurisdiction subsidies (out of jurisdiction's profits) to local households.

this. There we find that high property taxes are not always optimal for jurisdictions that seek to maximize profits. The underlying reason is that jurisdictions compete to attract global real estate equity investments. This result resembles the “race to the bottom” concept in literature of financial competition. Finally, another interesting aspect of our example is that it highlights the impact that global real estate markets have on local jurisdictions. Rather than having fiscal policy driving financial markets, our economy captures the dominant role of Wall Street on the fiscal policies of “small” municipalities.

Contribution to the literature:

The question of what determines the capital structure of a firm has been extensively researched in the last decades since the seminal work of Modigliani and Miller (1958, 1963). Significant contributions are Miller (1977) on the impact of taxes, Jensen and Meckling (1976) on agency costs, Kraus and Litzenberger (1973) on bankruptcy costs, and DeAngelo and Masulis (1980) on non-debt tax shields. Titman and Wessels (1988) analyze the explanatory power of these and other related theories of optimal capital structure. The literature has expanded and recently the focus has been on financial intermediation and the implications of certain regulations on the banks’ optimal choice of capital structure (Gale 2004, DeAngelo and Stulz 2015, Gornall and Strebulaev 2015, Allen, Carletti and Marquez 2015, Gale and Gottardi 2015, and Amaral, Corbae, and Quintin 2017). However, capital structure theory is not as well developed for real estate development and investments.

While there is some work that has highlighted the importance of the importance of capital constraints, taxes on equity dividends, and default risk on the capital structure of a real estate asset (Gau and Wang 1990, Giambona, Mello, and Riddiough 2016), most research is empirical and existing theoretical papers rely on a partial equilibrium reduced form analysis. We contribute to this literature by incorporating and endogenizing for the first time and in a consistent way several important financial variables of real estate development and investment decisions, such as the developer’s capital structure – composed by debt, common equity of the capital partners, and common equity of the developer partner –, the property taxes chosen by jurisdiction managers,

the cash flows of the real estate asset, and the types of development projects chosen by developers and jurisdiction managers. Our model captures and exploits the trade-off between equity taxes and leverage, but leaves aside other considerations such as agency costs and default risk.

Our equilibrium model is related to the literature of competitive market economies with incomplete markets, developed by Diamond (1967), Radner (1974), and Grossman and Hart (1979), among others. Our main departure from this literature is our focus on income-producing real estate assets. This particular type of assets generates cash flows which are dependent on the market price of the goods sold in local segmented markets. Thus, the real estate asset's cash flows are endogenous because commercial good prices are determined by market clearing given the demand of households at the jurisdiction level. Therefore, in our model similar real estate assets in different jurisdictions can generate different cash flows if the fundamentals of their respective local economies are different.

In our setting, developers are subject to short sale constraints and markets can be incomplete. In addition, we depart from the standard one good economy because we want to allow for multiple construction inputs in the development phase and also for multiple consumption goods that are sold in the jurisdiction where the commercial real estate asset was constructed. Our setting comes with a cost because with multiple goods and incomplete markets, we are not able to establish the efficiency property of a decentralized competitive economy (Geanakoplos and Polemarchakis 1986). In this sense, our work departs from Gale and Gottardi (2017), who follow a similar approach than Makowski (1983) and Hart (1979) by considering an alternative general equilibrium model of financial intermediation with one good and complete markets where the efficiency property holds.

Our work is also closely related to the the literature on financial innovation – also referred to as the security design literature – pioneered by Allen and Gale (1991) (see Allen and Gale 1994 and Duffie and Rahi 1995 for surveys). This literature has focused on the design of financial securities, such as bonds, stocks, mortgages, and mortgage-backed securities, and the relationship

with economic development.⁴ Up to our knowledge, we are the first paper to uncover the specific case of real estate development (creation) in a model with segmented equity and commercial good markets. In this respect, our paper is related to Rahi and Zigrand (2009), who study financial innovation in a setting with segmented markets.

Rahi and Zigrand (2009) propose an elegant two stage financial equilibrium model to analyze the equilibrium and welfare properties of financial innovation across segmented markets. These markets (jurisdictions in our terminology) are characterized by differential marginal valuations and can list the same assets. Their model, although elegant and powerful in predictions, does not allow investors to trade assets in multiple markets (jurisdictions). The ability to trade across markets is left for arbitrageurs, who turn out to be the issuers of assets in the first stage of their model. However, as Rahi and Zigrand recognize, this interpretation of issuance and implied listing is rather specific and not realistic in many cases.⁵ In our theory of real estate development and investments, we depart from these assumptions and allow global investors to buy real estate equity in multiple jurisdictions. In addition, besides the obvious difference in motivations, our model also departs from Rahi and Zigrand (2009) in that financial innovation is not driven by arbitrageurs, but by the sequential actions of local jurisdiction authorities and developers. The former choose the type of commercial real estate asset that developers can construct in their respective jurisdictions (e.g., hotel versus shopping mall). The latter choose, for a given type of real estate asset, the combination of construction inputs that determines the size of the development.

So far, and up to our knowledge, the literature has approached the modelling of real estate development with either a “real options” pricing partial equilibrium model (see e.g. Titman 1985, Capozza and Helsley 1990, Capozza and Sick 1990, Williams 1991, Childs, Riddiough, and Triantis 1996, and Grenadier 1995a,b) or in a general equilibrium setting with entrepreneurial developers and a housing/land market (see e.g. Henderson 1974, Helsley and Strange 1997, and

⁴See, for example, Amaral, Corbae, and Quintin (2017) for a recent study of the relationship between financial engineering (repackaging) and development.

⁵In reality, when a company lists its shares in an jurisdiction, arbitrage possibilities are not the main reason to go public.

Konishi 2013).⁶ Our paper proposes a different approach than these two literatures by considering a general equilibrium model that focuses on the financing aspects of commercial real estate development when jurisdictions compete to attract real estate equity investments. In addition to having developers choosing their optimal capital structure, our paper also departs from this literature in that it captures how global capital markets influence local fiscal policy.

Finally, our paper is also related to the regional science literature (see Berliant and ten Raa 1994 for a discussion of the different methodologies in the fields of regional science, regional economics, and urban economics). Our focus is not on where agents choose to live, but on the implications of allowing global real estate investors to do business in all jurisdictions. In this sense, our approach differs from those Tiebout models in which consumers “vote with their feet” where to live and work (see e.g. Tiebout 1956, Konishi 2008, and Luque 2013)

The remainder of this paper is structured as follows. In Section 2, we present the model, the equilibrium concept, and the result of equilibrium existence. Section 3 builds a simplified economy with two jurisdictions and provides results regarding the impact of the developers’ funding capacity and changes in the property tax on the feasibility and composition of the capital structure of a real estate development project. Section 4 proposes an extension to the model in which the jurisdiction’s fiscal and land use policies become endogenous variables. Section 5 concludes. The Appendix is devoted to the proofs.

2 The model

We start by providing a brief informal description of the main elements of our multi-jurisdictional economy. There are two periods and several states of nature in the second period. In the first period, there is one consumption good and several inputs for CRE development, such as land in

⁶Henderson (1974) studies the evolution of cities when intracity Marshallian externalities in production are present and discusses the role of entrepreneurship. Helsley and Strange (1997) consider the case of (city) developers that provide local public goods with limited power and an explicit geographical structure of the city. Konishi (2013) considers an economy with a large number of atomless land developers who can enter the market freely in an idealized version of Tiebout (1956).

a specific jurisdiction and construction materials. We refer to the consumption good as the *numeraire* and think of it as any good that facilitates trade. We refer to the elements used for the development of a CRE asset as *CRE construction inputs*, independently of whether they are land or construction materials. For the sake of simplicity, we assume that agents trade their endowments of the numeraire good and CRE inputs in a global (i.e., non-local) competitive market. Developers are the only agents that need CRE construction inputs to build CRE assets and thus they are the only ones on the demand side of this market.

The transformation of CRE construction inputs into CRE assets in the first period occurs as follows. There is a Cobb-Douglas production function that assigns positive weights to certain CRE construction inputs. Developers in each jurisdiction can only construct one type of CRE asset, which is determined by the Cobb-Douglas production exponential coefficients.⁷ If a developer in a jurisdiction buys positive amounts of non-zero weight CRE construction inputs, the CRE asset is created (instantaneously) in the first period. The developer can sell part or all of the CRE asset to global investors in the first period. Importantly, the cost to develop a CRE asset in the first period is endogenous because it depends on the (endogenous) prices of CRE construction inputs.

In the second period, there is one numeraire good that is transacted at the global level. In addition, in the second period, each CRE asset produces multiple consumption commodities, which we call CRE goods. The demand for these goods is *local* – only agents belonging to the CRE asset’s jurisdiction can consume and enjoy the consumption of these CRE goods. This assumption can be justified by different reasons, such as a prohibitive distance among jurisdictions or the agents’ consumption preferences for domestic goods. Interestingly, CRE cash flows are endogenously determined in our model because they depend on the prices and amounts of the CRE goods sold in the jurisdiction.

Local developers can finance the CRE development project by raising debt and selling equity.

⁷In the baseline model introduced in Section 2, we take these weights as given; later, in Section 4, we shall consider a game in which strategic jurisdiction managers choose their respective vectors of Cobb-Douglas exponential coefficients.

Debt is assumed risk free (for simplicity, we ignore default considerations in this model) and is subject to a no-short sale constraint. CRE equity cash flows are endogenous because they depend on the prices of CRE goods. In equilibrium, the developer's real estate capital structure is endogenous, as are the equity and debt prices.

2.1 Primitives

Consider a multi-jurisdictional economy with $\mathcal{K} > 0$ jurisdictions. In each jurisdiction k , there is a representative developer d_k and $\mathcal{H}_k \geq 1$ households. We denote the set of developers by $\mathbf{D} = \{d_1, \dots, d_k, \dots, d_{\mathcal{K}}\}$, and the set of households in jurisdiction k by $\mathbf{H}_k = \{1, \dots, h_k, \dots, \mathcal{H}_k\}$. For simplicity, we assume that \mathcal{H}_k is the same across jurisdictions, i.e., $\mathcal{H}_k = \mathcal{H}$, for all k . In addition, in this economy there are $\mathcal{I} \geq 1$ global investors that have access to all jurisdictions. The set of investors is $\mathbf{I} = \{1, \dots, i, \dots, \mathcal{I}\}$. We write $a \in \mathbf{A}$ to denote an agent independently of its type, where $\mathbf{A} = \mathbf{I} \cup \mathbf{D} \cup \{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$. Finally, we write $\mathbf{K} = \{1, \dots, \mathcal{K}\}$ to denote the set of all jurisdictions. We assume that the distribution of agents into jurisdictions is exogenously given and satisfies the following conditions:

Assumption 1: (i) *Households and developers each belong to one (and only one) jurisdiction;* (ii) *each jurisdiction has one developer and at least one household ($\mathcal{H}_k \geq 1$); and* (iii) *global investors can invest in all jurisdictions.*

Item (i) of Assumption 1 guarantees that there is at least one household that buy (local) CRE goods. In addition, this assumption makes a developer subject to the local public policy in the jurisdiction where it operates. Items (ii) and (iii) of Assumption 1 allow us to write the capital structure associated with a CRE asset as a function of both the developer and the investors' debt and equity positions.⁸ Assumption 1.iii is key to introduce interdependence of the capital

⁸Extending the model to allow for multiple developers in each jurisdiction is feasible; all that is needed is to rewrite the market clearing equations of debt and equity to accommodate the additional CRE assets of the same type constructed by the developers in a jurisdiction. However, this extension would complicate the analysis without adding much additional economic insight, as it would introduce sharing rules for investors' equity investments among the different CRE assets within the same jurisdiction.

structures of CRE assets across jurisdictions.⁹

In this economy, there are two periods, $t = 1, 2$, and S states of nature in the second period. \mathbf{S} denotes the set of states of the second period. Because we allow for the construction of CRE assets, the space of commodities changes between periods 1 and 2. There is a consumption good, indexed by $l = 0$ and called the “numeraire”. This good can be any good that facilitates trade, e.g., cash. We assume that the numeraire good is transacted in the global market in both periods. In addition, in period 1, there are also L_1 CRE construction inputs, which for simplicity we assume are traded in the global market.¹⁰ We denote the set of CRE construction inputs by $\mathbf{L}_1 = \{1, \dots, L_1\}$. We denote an agent a ’s endowments of the numeraire good and CRE construction inputs in the first period by $\omega_1^a \in \mathbb{R}_{++}^{1+L_1}$. For simplicity, we do not explicitly model the production of construction materials (e.g., the process of cutting trees and using timber pieces to construct the frames of large structures) and take their supply as given. CRE construction inputs can also take the form of land. For example, suppose, without loss of generality, that good $l \in \mathbf{L}_1$ is land in a given jurisdiction k . Then, the total amount of available land in jurisdiction k that is available for developers is $\sum_{a \in \mathbf{A}} \omega_{l1}^a$. Because $\omega_{l1}^a < \infty$ for all $a \in \mathbf{A}$ and all $l \in \mathbf{L}_1$, this upper bound limits the supply of land in each jurisdiction.

In the second period, each CRE asset produces L_2 commercial goods (also referred to as CRE goods), with corresponding set $\mathbf{L}_2 = \{1, 2, \dots, L_2\}$. Consumption goods and CRE construction inputs are perfectly divisible. We denote agent a ’s endowment of the numeraire good at state s of the second period by $\omega_0^a(s) \in \mathbb{R}_{++}$.

We refer to the numeraire, CRE construction inputs, and CRE produced goods as “commodities”. The vector of commodities purchased by an agent a is $x^a = (x_1^a, (x^a(s), s = 1, \dots, S)) \in \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{S(1+\mathcal{K}L_2)}$. Notice that the consumption possibilities in the second period at a state $s \in \mathbf{S}$ are $1 + \mathcal{K}L_2$ goods (the numeraire good and $\mathcal{K}L_2$ CRE goods). This is because there are \mathcal{K} juris-

⁹We could restrict item (iii) of Assumption 1 to the case where an investor only has access to two jurisdictions and the main results of the model would not change.

¹⁰Notice that in the first period, CRE produced goods do not exist yet and, therefore, the notion of a jurisdiction as a region where local households can buy CRE goods is not appealing. For this reason, we consider that the markets of the numeraire good and CRE construction inputs in the first period clear at the global level.

dictions in the economy and we assume that a jurisdiction's CRE asset produces L_2 CRE goods. We introduced, however, restrictions on the consumption of some of these goods. In particular, item (i) of Assumption 1 implies that, when an agent is a developer or a household in a jurisdiction k , the consumption of a CRE good $l \in \mathbf{L}_2(k')$ produced in another jurisdiction $k' \neq k$ is zero, i.e., $x_{lk'}^a(s) = 0$ if $a \in \{d_k\} \cup \mathbf{H}_k$. Investors belong to all jurisdictions, so they can consume all CRE goods produced in the economy.

We find convenient to denote the vector of all individual commodity purchases in the economy by $x^A = (x^a : a \in \mathbf{A})$. The vector of commodity prices is $p = (p_1, (p(s), s = 1, \dots, S)) \in \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{S(1+\mathcal{K}L_2)}$. We refer to the price of the numeraire good in the first period by p_{01} . For the second period, we denote the price of a CRE good $l \in \mathbf{L}_2$ sold to households of jurisdiction k at state s by $p_{lk}(s)$. We write the price of the numeraire good at state s as $p_0(s)$.

Agents derive utility from the consumption of the numeraire and the CRE goods available in the jurisdictions where they belong to. The consumption of CRE construction inputs does not provide agents any (direct) utility. We denote an agent a 's utility function by $u^a : \mathbb{R}_+^{(1+L_1)+S(1+\mathcal{K}L_2)} \rightarrow \mathbb{R}$. These hypotheses about utility functions are included in the following assumption.

Assumption 2: (i) For any agent $a \in \mathbf{A}$, the utility function u^a is continuous and strongly quasi-concave,¹¹ (ii) For all $k \in \mathbf{K}$ and for any agent $a \in \{d_k\} \cup \mathbf{H}_k$, $u^a(x_1, (x(s), s = 1, \dots, S)) = u^a(x_{01}, (x_k(s), s = 1, \dots, S))$, where $x_k(s) \in \mathbb{R}^{1+L_2}$ is the bundle of CRE goods available at jurisdiction k , and, for any investor $i \in \mathbf{I}$, $u^i(x_1, (x(s), s = 1, \dots, S)) = u^i(x_{01}, (x(s), s = 1, \dots, S))$; that is, developers, households, and investors do not assign any utility to the CRE construction inputs; moreover, developers and households do not assign any utility to CRE goods in jurisdiction where they do not belong to. (iii) For any agent $a \in \mathbf{A}$, u^a is strictly increasing in all commodities to which agent a assigns utility.

¹¹Given a convex set $X \subset \mathbb{R}^n$, a function $f : X \rightarrow \mathbb{R}$ is strongly quasi-concave if $f(\lambda x + (1 - \lambda)y) > \min\{f(x), f(y)\}$, for any $(x, y) \in X \times X$ such that $f(x) \neq f(y)$. This property is weaker than strict quasi-concavity, which requires $f(\lambda x + (1 - \lambda)y) > \min\{f(x), f(y)\}$, for any $(x, y) \in X \times X$ such that $x \neq y$.

2.2 CRE projects

In the first period, developer d_k can buy CRE construction inputs to construct one (and only one) CRE asset j_k . These CRE construction inputs are evaluated by a Cobb-Douglas production function, which transforms them into a specific CRE asset. In particular, we consider the following production function:

$$y_{j_k}^{d_k} = TFP_k \cdot \prod_{l \in \mathbf{L}_1} (x_{l1}^{d_k})^{\alpha_{lk}} \quad (1)$$

where

- TFP_k is a parameter that stands for the “total factor productivity” specific to jurisdiction k . TFP_k may differ across jurisdictions because their natural resources, their agglomeration economies, or the intensity of competition.¹² TFP_k may also capture the infrastructure and the amenities of a jurisdiction.¹³
- parameter $\alpha_{lk} \in [0, 1]$ denotes the weight assigned to CRE construction input $l \neq 1$ by jurisdiction manager k .
- $x_{l1}^{d_k}$ is the amount of CRE construction input $l \in \mathbf{L}_1$ that developer d_k buys in period 1. Materials, once employed for the construction of the CRE asset, cannot be further traded.
- the j_k subindex of y denotes the type of CRE asset that is produced in jurisdiction k . Type j_k is determined by the vector of production weights $\alpha_k = (\alpha_{lk})_{l \in \mathbf{L}_1}$, i.e., different vectors α_k leads to different types of CRE assets j_k .

To illustrate the last bullet item, let us consider a two-jurisdiction economy with only two CRE construction inputs. In this case, α_k is a two-dimensional vector. For example, we can associate the vector $\alpha_k = (\alpha_{1k}, \alpha_{2k}) = (1/2, 1/2)$ with a hotel, and the vector $\alpha_{k'} = (\alpha_{1k'}, \alpha_{2k'}) =$

¹²The fact that firms in large jurisdictions are more productive is a well-established fact in the empirical literature; see, e.g., Rosenthal and Strange (2004) and Duraton, Philippe-Combes, Gobillon, Puga, and Roux (2012).

¹³In addition, we could make TFP_k a function of the jurisdiction’s fixed and variable costs, which in turn determine its public investments and depend on parameter α_k (see Section 2.4). Here, for simplicity, we just take TFP_k as a parameter specific of a jurisdiction k .

$(1/3, 2/3)$ with a shopping mall. Thus, for a given vector α_k , different combinations of CRE construction inputs result in different sizes of the same CRE asset. Formally, let a developer d_k choose a vector of CRE construction inputs $x_1^{d_k} \in \mathbb{R}^{\mathbf{L}_1}$. If $(x_1^{d_k}, \alpha_k)$ is such that $y_{j_k}^{d_k} = \prod_{l \in \mathbf{L}_1} (x_{1,l}^{d_k})^{\alpha_{lk}} > 0$, the CRE asset j_k is produced with size equal to $y_{j_k}^{d_k} > 0$. For example, if the jurisdiction manager k chooses $\alpha_k = (\alpha_{1k}, \alpha_{2k}) = (1/2, 1/2)$ and if the representative developer d_k purchases 2 units of CRE construction input 1 and 3 units of CRE construction input 2 (i.e., $x_1^{d_k} = (2, 3)$), then CRE development of type j_k has a size equal to $y_{j_k}^{d_k} = 2.45$. Different combinations of components in $x_1^{d_k}$ result in different CRE asset sizes.

We write $y^A = (y^a : a \in \mathbf{A})$ to denote the production bundles of CRE projects of all agents in the economy. Since agents other than the developer d_k cannot construct the CRE assets y_k , we set $y_k^a = 0$ if $a \in \mathbf{I} \cup \{\mathbf{H}_k\}_{k=1}^{\mathbf{K}} \cup \mathbf{D} \setminus \{d_k\}$.

The production of CRE goods only depends on the type and size of the CRE asset. In particular, at state s of the second period, the supply of consumption goods by a CRE asset j_k is given by the following function:

$$f_{j_k}(y_{j_k}^{d_k})(s) : [0, +\infty) \rightarrow \mathbb{R}_+^{L_2}$$

with $f_{j_k}(0)(s) = 0$. Because TFP_k is a component of $y_{j_k}^{d_k}$ and it may capture the specific infrastructure and amenities of jurisdiction k , we have that the commercial goods produced and sold in that jurisdiction incorporate the TFP_k component through the type of jurisdiction (k) and the size of the CRE asset ($y_{j_k}^{d_k}$). For example, a local household that purchases a commercial good enjoys both the consumption of the good itself and a walk around the surrounding public and commercial spaces. In terms of Debreu's (1959) classical book *Theory of Value*, all these characteristics are embedded in the type of commercial good purchased by the household.

More generally, for CRE assets such as hotels, office space, and student housing, consumption of those CRE goods should be interpreted as the purchase of a particular type of space for a given period in a given location with specific surrounding amenities. For CRE assets that involve the production and sale of physical goods, we differentiate between industry and retail. The former produces some goods that are sold to retail owners. The latter buys those goods from the industry

and sells them to the individuals in the jurisdiction. For the sake of simplicity, our model does not differentiate between the two. Instead, we assume that households purchase commercial goods directly from the local CRE property. An extension of our model that differentiates between industry and retail real assets could be done by considering an additional period ($t = 3$), where the goods purchased in $t = 2$ by retail owners are sold to local households.

Let us go back to our interpretation of function f . When the size of CRE asset j_k is $y_{j_k}^{d_k}$, the amount of good $l \in \mathbf{L}_2$ supplied to local households at state s is $f_{lj_k}(y_{j_k}^{d_k})(s)$. This amount is sold at price $p_{lk}(s)$ by the CRE asset of jurisdiction k . Thus, a CRE asset with size $y_{j_k}^{d_k}$ generates an endogenous price-dependent cash flow equal to

$$c_{j_k}(y_{j_k}^{d_k}; p)(s) = \sum_{l \in \mathbf{L}_2} p_{lk}(s) f_{lj_k}(y_{j_k}^{d_k})(s) \text{ at } s \in \mathbf{S}.$$

Whenever $S > 1$, a CRE asset can pay different returns at different states of the second period. We impose the following assumptions on function f_{j_k} :

Assumption 3: (i) $f_{j_k}(\cdot)(s)$ is additively separable and homogeneous of degree 1; (ii) for every $k \in \mathbf{K}$ and $l \in \mathbf{L}_2$, f_{lj_k} is increasing and concave; (iii) and for all $k \in \mathbf{K}$ and $s \in \mathbf{S}$, $f_{j_k}(y)(s) \neq 0$ when $y \neq 0$.

Assumption 3.i implies that, if $y_{j_k}^{d_k} = \varkappa e_1 + e_2$ and $e_1, e_2 > 0$, then $f_{j_k}(y_{j_k}^{d_k})(s) = \varkappa f_{j_k}(e_1)(s) + f_{j_k}(e_2)(s)$. This assumption is useful because it allows us to split the CRE asset's cash flows among different property owners (possibly each with a different amount of equity ownership). We impose Assumption 3.ii to guarantee that the developer's budget constraints are convex in order to prove existence of equilibrium for the general model presented in this section.¹⁴ Assumption 3.iii is needed to prove that there is no excess of supply in the equity market.

¹⁴Later, in section 3, we shall provide an example of a simplified economy, where an equilibrium exists in a context where f_{lj_k} is a linear increasing function.

2.3 Debt and CRE equity

In this economy, the main difference between debt and CRE equity is that only equity is subject to taxes (this model of taxation is consistent with the so-called “pass-through taxation”, see discussion in Section 2.4). For simplicity, we ignore any issues regarding debt collateralization and accordingly we assume that debt is risk-free.¹⁵ In addition, we assume that debt is a nominal asset (we need this assumption for our proof of Theorem 1 below in order to guarantee the existence of an interior point in an agent’s budget constraint). An agent buying (selling) a face value of debt equal to D^a pays (receives) τD^a in the first period, where τ is the (endogenous) discount price of debt.¹⁶ As usual, short and long debt positions are denoted by $D^a < 0$ (borrower) and $D^a > 0$ (lender), respectively. The borrower (lender) then receives $r(s)D^a$ (negative if it is a borrower and positive if it is a lender) nominal units at state s of the second period. Since debt is risk-free, we take $r(s) = \bar{r}$. Also, to deal with the possibility of redundant assets in our setting with an endogenous financial structure, which in turn induces indeterminacy in the agents’ portfolios choices, we consider a short sale constraint on debt of the type $D^a \geq -\bar{D}^a$ with $\bar{D}^a > 0$ (notice that we allow this short sale constraint to be agent-type specific). This constraint allows us to bound agents’ debt choices. In economic terms, this short sale constraint is reminiscent of covenants imposed on unsecured debt contracts (see Giambona, Mello, and Riddiough 2016)

Next, we describe the equity market. Developers can sell all, part, or none of their CRE property to investors. Investors can buy equity in one or several CRE development projects and thus can be thought of as equity REITs (equity REITs are real estate companies that acquire commercial properties – such as office buildings, shopping centers and apartment buildings – and lease the space in the structures to tenants).¹⁷ We assume that households are not financially sophisticated in the sense that they do not have access to the equity market.¹⁸

¹⁵We leave for future research an extension of this model in which debt is risky and collateralized by the CRE asset. In such a model, developers could issue debt specific to the CRE asset, and investors could buy this debt. Such investors would be similar to mortgage REITs. Campello and Giambona (2013) and Cvijanović (2014) provide empirical insights on important aspects to consider in this modeling.

¹⁶Debt prices, which reflect the discounted cashflow of future debt payments, are endogenous in our model.

¹⁷See “Guide to REITs”: <https://www.reit.com/investing/reit-basics/guide-equity-reits>

¹⁸This modelling choice requires an impatience assumption on the utility function (see below)

With these considerations at hand, we proceed to introduce the following notation. Let $E_{j_k}^a$ denote an agent a 's equity positions on a CRE project j_k , where $a = d_k, i$. We write $E^A \equiv (E^a : a \in \mathbf{D} \cup \mathbf{I}) \in \mathbb{R}_+^{\mathcal{K} + \mathcal{I}\mathcal{K}}$ to denote the vector of equity positions corresponding to all developers and investors of the economy, respectively (recall that there are \mathcal{I} investors and \mathcal{K} developers – one per jurisdiction – and that we do not allow developers to buy CRE equity in jurisdictions other than their own). Since households are just consumers and do not have access to the CRE equity market, we set $E^{h_k} = 0$ for all h_k and all k .

An equity stake on a CRE asset is just a claim to the future payoffs generated by this asset. Since a developer initially owns the whole property, the sum of all equity stakes must be equal to $y_{j_k}^{d_k}$. Formally,

$$E_{j_k}^{d_k} + \sum_{i \in \mathbf{I}} E_{j_k}^i = y_{j_k}^{d_k}. \quad (2)$$

This equation is the market clearing condition for equity shares corresponding to a CRE asset j_k . Developers sell all their initial CRE equity and then decide how much to repurchase. When $E_{j_k}^{d_k} = 0$, the developer sells all the initial equity and does not keep any equity for himself. If instead $E_{j_k}^{d_k} \in (0, y_{j_k}^{d_k})$, the developer keeps part but not all of the ownership on the property. When $E_{j_k}^{d_k} = y_{j_k}^{d_k}$, the developer owns all equity of the project, so he is entitled to all of the property's cash flows.

Remark 1: *The CRE capital structure of a CRE asset j_k is endogenous and is composed by the developer's debt $D_{j_k}^{d_k}$, the common equity of the entrepreneurial (developer) partner $E_{j_k}^{d_k}$, and the common equity of the capital (investor) partners $(E_{j_k}^i)_{i \in \mathbf{I}}$.*

Let us denote by q_{j_k} the (endogenous) price of one unit of equity in CRE asset j_k in period 1. Then, an agent a that purchases an equity stake $E_{j_k}^a > 0$ on asset j_k pays $q_{j_k} E_{j_k}^a$ in the first period and receives $c_{j_k}(E_{j_k}^a; p)(s)$ units of the numeraire good at state s of the second period. Notice that market clearing equation (2) and Assumption 3 allow us to write

$$c_{j_k}(y_{j_k}^{d_k})(s) = c_{j_k}(E_{j_k}^{d_k})(s) + \sum_{i \in \mathbf{I}} c_{j_k}(E_{j_k}^i)(s),$$

where

$$c_{j_k}(E_{j_k}^a; p)(s) = \sum_{l \in \mathbf{L}_2} p_{lk}(s) f_{lj_k}(E_{j_k}^a)(s) \text{ at } s \in \mathbf{S}.$$

That is, the endogenous price-dependent cash flow that a CRE asset with size $y_{j_k}^{d_k}$ generates at state s equals the sum of all payments received by the equity owners of this asset.

2.4 The jurisdiction's profit function

Jurisdictions incur some costs when providing public goods. Examples of these costs are roads, sewerage, fire protection, police, legal advisors, supervision, and accounting. We split the costs for local authority k between fixed costs, $\lambda(\alpha_k) > 0$, and a variable cost $\varepsilon(\alpha_k) > 0$. Both cost functions are defined in terms of the numeraire good. We assume that $\varepsilon(0) = 0$ and $\lambda(0) = 0$, and also that $\varepsilon(\alpha)$ and $\lambda(\alpha)$ are, respectively, homogenous of degree 1 and 0 in α . The latter assumption guarantees that, without loss of generality, we can normalize the return vector of a CRE asset. In terms of economic interpretation, making fixed and variable costs a function of α_k suggests that the type of CRE asset dictates the associated cost of public infrastructure. In other words, the cost of public infrastructure is not independent of the type of CRE assets that the jurisdiction targets to be developed by private developers.

To finance these costs, the local authority imposes a CRE property tax that is proportional to the agent's equity holdings. In particular, an agent holding $E_{j_k}^a$ equity shares in asset j_k must pay $\gamma_k E_{j_k}^a$ units of the numeraire good to the jurisdiction, where $\gamma_k > 0$.¹⁹ Notice that property taxes indirectly depend on CRE cash flows. To see this, notice that property taxes are a function of CRE equity and equity is a function of the prices and quantities of commercial goods sold in a jurisdiction, which in turn determine the CRE cash flows.

We take tax rate γ_k as given in Sections 2 and 3 (later, in Section 4, we provide an extension

¹⁹This model of taxation is consistent with the so-called "pass-through taxation", in which the owners of the CRE property are personally responsible for paying taxes and expenses according to some pari-passu rule, which normally takes the form of a proportional sharing rule with respect to the owners' equity holdings. The "pass-through taxation" model includes Limited Liability Companies (LLC), which are one of the most prevalent business forms in the United States.

to the model that makes this variable endogenous).

Assumption 4: For all $k \in \mathbf{K}$, γ_k belongs to the compact set $\Gamma_k = \{\gamma_k \in \mathbb{R}_+ : \varepsilon(\alpha_k) \leq \gamma_k \leq \bar{\gamma}_k\}$.

Let us now define a jurisdiction by a triplet $(\mathbf{k}, \alpha_k, \gamma_k)$ that specifies the set of players in jurisdiction k , the type of CRE asset that developers can construct, and the property tax, respectively. Hereafter, \mathbf{K} represents the *jurisdiction structure*, i.e., $\mathbf{K} \equiv \{(\mathbf{k}, \alpha_k, \gamma_k), k = 1, \dots, \mathcal{K}\}$ satisfying Assumption 1. By defining the set of all jurisdictions in \mathbf{K} that contain agent a by $\mathbf{K}^a \equiv \{(\mathbf{k}, \alpha_k, \gamma_k) \in \mathbf{K} : a \in \mathbf{k}\}$, we can restrict agents' choices. For example, if household $h \in \mathbf{k}$, then it can only buy a positive amount of CRE consumption goods if they are produced and sold in its own jurisdiction $(\mathbf{k}, \alpha_k, \gamma_k)$, since $\mathbf{K}^h = \{(\mathbf{k}, \alpha_k, \gamma_k)\}$. Also, investors can buy positive amounts of CRE debt and CRE equity in those jurisdictions where they belongs to, i.e., in those $(\mathbf{k}, \alpha_k, \gamma_k)$, such that $(\mathbf{k}, \alpha_k, \gamma_k) \in \mathbf{K}^i$. Since the investor belongs to all jurisdictions – as indicated in Assumption 1.iii –, then $\mathbf{K}^i = \mathbf{K}$. By abuse of notation, k designates de jurisdiction and also stands for the triplet $(\mathbf{k}, \alpha_k, \gamma_k)$. Denote by N^a the number of jurisdictions to which an agent a belongs. If the agent is an investor in all jurisdictions, then $N^i = \mathcal{K}$. If the agent is a household h_k or a developer d_k , then $N^{h_k} = N^{d_k} = 1$.

Given E^A , the profits of a jurisdiction k in the first period are given by

$$\pi_k \equiv \sum_{a \in \mathbf{k}} \gamma_k E_{jk}^a - \left(\lambda(\alpha_k) + \varepsilon(\alpha_k) \sum_{a \in \mathbf{k}} E_{jk}^a \right)$$

CRE property taxes have a redistributive effect in our model because jurisdiction profits revert to the agents that live and do business in the jurisdiction according to some weights.²⁰ Consider an agent $a \in \mathbf{k}$ and let its share of jurisdiction k 's profit be $\delta_k^a \in [0, 1]$. By choosing a vector $(\delta_k^a)_{a \in \mathbf{k}}$, such that $\sum_{a \in \mathbf{k}} \delta_k^a = 1$, the jurisdiction manager is effectively redistributing resources among agents in the jurisdiction. A theory of political economy could be elaborated by endogenizing

²⁰This assumption is standard in competitive financial economies with intermediation costs. See e.g. Markeprand (2008) and Prechac (1996).

these shares (we leave this possibility for future research).

2.5 Optimization and equilibrium

We start by presenting the agents' optimization problems and then move to our notion of equilibrium.

Given equity prices $q \in \mathbb{R}_+^K$, commodity prices $p \in \mathbb{R}_+^{1+L'_1} \times \mathbb{R}_+^{S(1+L_2K)}$, and the price of debt $\tau \in \mathbb{R}_+$, an agent a 's *optimization problem* consists of choosing a vector

$$(x_1^a, x^a(1), \dots, x^a(S), D^a, E^a) \in \mathbb{R}_+^{1+L'_1} \times \mathbb{R}_+^{S(1+L_2K)} \times \mathbb{R} \times \mathbb{R}_+^K$$

that maximizes his utility function u^a , subject to his budget constraints of periods 1 and 2, his corresponding debt short sale constraint, and the constraints that impose a zero consumption and trade of commercial goods and equity outside its jurisdiction(s). The budget constraints of an agent $a \in \mathbf{A}$ in period 1 and state $s \in \mathbf{S}$ of period 2 are, respectively,

$$\begin{aligned} \sum_{l \in \{0\} \cup \mathbf{L}_1} p_{l1}(x_{l1}^a - \omega_{l1}^a) - p_{01} \sum_{k \in \mathbf{K}^a} \delta_k^a \pi_k + \tau D^a + \sum_{k \in \mathbf{K}^a} q_{jk}(E_{jk}^a - y_{jk}^a) + p_{01} \gamma_{jk} E_{jk}^a &\leq 0 \\ p_0(s)(x_0^a(s) - \omega_0^a(s)) + \sum_{k \in \mathbf{K}^a} \sum_{l \in \mathbf{L}_2} p_{lk}(s) x_{lk}^a(s) &\leq \bar{r} D^a + \sum_{k \in \mathbf{K}^a} c_{jk}(E_{jk}^a; p)(s) \end{aligned}$$

The formal definition of an equilibrium is as follows:

Definition 1: *Given a jurisdiction structure \mathbf{K} , a competitive equilibrium for this economy consists of a system $(x^A, D^A, E^A, p, q, \tau, (\pi_k)_{k \in \mathbf{K}})$, such that:*

- (i) *each agent a solves its optimization problem, given \mathbf{K}^a ;*
- (ii) *for each jurisdiction k , the profit function π_k is determined by the local developers and global investors' equity choices according to the following functional form:*

$$\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} E_{jk}^a - (\lambda(\alpha_k) + \varepsilon(\alpha_k)) \left(\sum_{a \in \{d_k\} \cup \mathbf{I}} E_{jk}^a \right)$$

(iii) the following market clearing conditions hold:

- global market clearing for the numeraire consumption good in period 1:

$$\sum_{a \in \mathbf{A}} (x_{01}^a - \omega_{01}^a + \sum_{k \in \mathbf{K}} (\lambda(\alpha_k) + \varepsilon(\alpha_k) y_{j_k}^{d_k})) = 0$$

- global market clearing for the CRE construction inputs in period 1:

$$\sum_{a \in \mathbf{A}} (x_{l1}^a - \omega_{l1}^a) = 0, \forall l \in \mathbf{L}_1$$

- global market clearing for the numeraire consumption good at state $s \in \mathbf{S}$ of period 2:

$$\sum_{a \in \mathbf{A}} (x_0^a(s) - \omega_0^a(s)) = 0,$$

- local market clearing for CRE goods:

$$\forall k \in \mathbf{K}, \forall l \in \mathbf{L}_2, \sum_{a \in \mathbf{K}} x_{lk}^a(s) - f_{lj_k}(y_{j_k}^{d_k})(s) = 0$$

- global market clearing for debt:

$$\sum_{a \in \mathbf{A}} D^a = 0$$

- local market clearing for equity:

$$\forall k \in \mathbf{K}, E_{j_k}^{d_k} + \sum_{i \in \mathbf{I}} E_{j_k}^i = y_{j_k}^{d_k}$$

Remark 2: Our Walrasian economy has segmented good and equity markets, and financial markets can be incomplete. This setting is similar to Section 3 of Rahi and Zigrand (2009) in the sense that we do not consider “arbitrageurs”. The main difference from our models is that we allow for global investors that can buy and sell CRE equity in multiple jurisdictions. An equilibrium exists for our economy and, therefore, we can rule out arbitrage opportunities even when global investors can operate in multiple jurisdictions.

A subtle condition in general equilibrium models is the lower semicontinuity property of the agent’s budget constraint in the first period. The usual approach to guarantee this property is assuming that all agents of the economy have positive endowments of all goods. In our model, we cannot impose this assumption for the CRE goods because these goods are endogenously

produced. To guarantee the existence of an interior point in the budget constraint set for each profile of prices, we consider the following assumptions on the endowments of CRE construction inputs ($l \in \mathbf{L}_1$) and the numeraire good (indexed by “0”):²¹

Assumption 5: For all agents $a \in \mathbf{A}$, $\omega_{01}^a > (\lambda(\alpha_k) + \varepsilon(\alpha_k)(1 + \mathcal{K}))\mathcal{K}$, $\omega_{l1}^a > 0, \forall l \in \mathbf{L}_1$, and $\omega_0^a(s) > 0$. Moreover, for all developers $d_k \in \mathbf{D}$, $\omega_{01}^{d_k} > \gamma_{j_k}(\tilde{y}_{j_k}^{d_k})$, where $\tilde{y}_{j_k}^{d_k} \equiv TPF_k \cdot \prod_{l \in \mathbf{L}_1} (\omega_{l1}^{d_k})^{\alpha_{lk}}$.

The last part of Assumption 5 allows to transfer the developer’s wealth from the first to the second periods in the proof of Theorem 1 below.

Another subtlety to guarantee the property of lower semicontinuity of the budget set correspondence has to do with the presence of portfolio constraints, which prevent the usual normalization of commodities and assets prices in the first period. Since we cannot impose additional restrictions on the agent’s budget constraints and portfolio sets – as these are written to capture our particular setting – we consider the following mild impatience assumption, proposed by Seghir and Torres-Martinez (2011) for an economy with restricted participation:

Assumption 6: For any agent $a \in \mathbf{A}$ and any $x \in X^a$, there exists a bundle $\varrho(\theta, x) \in \mathbb{R}_+^{1+\mathbf{L}_1}$, given $\theta \in (0, 1)$, such that $u^a(x_1 + \varrho(\theta, x), (\theta(x(s)))_{s=1, \dots, S}) > u^a(x_1, (x(s))_{s=1, \dots, S})$.

Assumption 6 says that we can always find a large consumption for an agent in period 1 such that this agent is better off with this extra consumption in period 1 but less consumption in every state of period 2.²² This assumption is satisfied by many different types of utility functions that are unbounded on the first period consumption, such as von-Neumann utility functions with quasi-linear, Cobb-Douglas, or Leontieff kernels, e.g., Cobb-Douglas, CES, and CARA. Also, notice that this type of utility function does not depend on the representation of individuals’ preferences and does not require further assumptions on the portfolio sets.

²¹We leave for future research the relaxation of some of the elements in Assumption 5, namely, getting rid of the interiority assumptions of agents’ endowments (see Rincon-Zapatero and Santos 2009 for similar issues). In Section 3, we provide a simplified version of our economy in which an equilibrium exists without requiring agents having positive endowments of all commodities in the first period.

²²This way to reframe the standard impatience assumption of an agent preferring to consume today rather than tomorrow is convenient to state in terms of the “primitives” of the model.

Theorem 1: *Let assumptions 1.i, 1.ii, 1.iii, 2.i-iii, 3.i, 3.ii, 4, 5, and 6 hold. Then, given a jurisdiction structure $\mathbf{K} \equiv \{(\mathbf{k}, \alpha_k, \gamma_k), k = 1, \dots, \mathcal{K}\}$, there exists a competitive equilibrium.*

The proof of Theorem 1 is left for the Appendix. Next, we discuss the technical subtleties of this proof in view of existing results in the literature of general equilibrium with segmented markets.

First of all, it is not trivial that a developer's budget set correspondence takes convex values. To see this, notice that the market value of a CRE asset depends on the developer's choice of materials and that the equity return in the second period depends on the CRE asset through functions $(f_{j_k})_{k \in \mathbf{K}}$, and so is endogenous. Then, to guarantee the convexity of a developer's budget set, we have to impose Assumption 3.ii, namely, we require that f_{j_k} is a concave and increasing function for every $k \in \mathbf{K}$.²³

The main difficulty has to do with the lower semicontinuity property of the agents' budget constraints. Since the CRE equity markets are segmented (CRE equity not available to all agents in the economy), we cannot take the usual approach where an auctioneer chooses both the commodity and security prices in the simplex. For if the auctioneer chooses the price of one type of CRE equity equal to 1, the remaining commodity, debt, and equity prices would be zero. But then, there would be jurisdictions with commodity and security prices equal to zero and the budget constraints of agents with single jurisdiction memberships would hold with equality. Lower semicontinuity of the budget correspondence would fail as a result since we could not guarantee the existence of an interior point. To circumvent this problem, we let the price auctioneer for the first period choose the prices for commodities in the simplex (a compact set). For asset prices, we have to find an endogenous upper bound (below we explain how we find it).

Another issue related to the lower semi-continuity of the budget set correspondence has to do with the existence of an interior point in the budget constraint. To see this, notice that, even if agents have strictly positive endowments of the numeraire good in the second period, the value of

²³The convexity of a developer's budget set is not compromised by the budget constraint in the first period because the production function for the CRE asset is Cobb-Douglas and this function is assumed to be concave.

endowments may not be strictly positive for some agents because we embed commodity prices into the simplex. To overcome this difficulty, we rely on two assumptions: 1) endowments of the numeraire and CRE construction inputs are strictly positive for all agents in the first period, and 2) the nominal return of risk-free debt is positive. With prices chosen in the simplex and strictly positive endowments of all commodities in the first period, the value of the endowment in the first period is strictly positive. The following bundle is an interior point of the budget constraints for all agents: all commodity purchases equal to zero, CRE equity positions also equal to zero, and a small position in risk-free debt.

As argued above, the segmentation of CRE equity markets precludes us from using standard techniques to bound equity prices. Here we argue that previous results in the literature of market segmentation with a fixed point theory approach are not useful for finding endogenous upper bounds for CRE equity prices.

In one strand of this literature, authors consider exogenous trading constraints, but impose financial survival assumptions or spanning conditions on the set of admissible portfolios.²⁴ These assumptions are not suitable for our particular setting. For instance, households do not satisfy the survival assumption because they do not trade equity. Also, the property tax on equity prevents us from considering a spanning condition on the set of admissible portfolios.

Another strand of the literature applies fixed point theory but in a setting with endogenous portfolio constraints. For instance, the paper by Cea-Echenique and Torres-Martinez (2016) imposes a super-replication condition that allows payments associated with segmented contracts to be super-replicated by durable goods and/or contracts that agents can short sell. This assumption does not fit into our framework for the following reasons: 1) we do not have durable goods, 2) there is a short sell restriction on risk-free debt, and 3) CRE equity returns are endogenous as they depend on the developers' choices. In a similar context, Faias and Torres-Martinez (2017) consider instead assumptions in the utility function – precisely, indifference curves through individuals' endowments do not intersect the consumption set boundary. However, the “essen-

²⁴See for instance Balasko, Cass, and Siconolfi (1990) for a seminal contribution, and, more recently, Angeloni and Cornet (2006), Aouani and Cornet (2009), and Cornet and Gopalan (2010).

tiality of commodities” assumption requires endowments to be strictly positive, which is not true in our setting for the case of CRE goods. Finally, the paper by Seghir and Torres Martinez (2011) considers an impatience assumption in the utility function, which requires that small reductions in the consumption of the second period can be compensated by an increase in the consumption of the first period. We adapt this assumption to our framework by imposing Assumption 6. A key difference from Seghir and Torres Martinez (2011) is that they assume that second period endowments are strictly positive, which is not true in our case. We overcome this difficulty by imposing Assumption 5, which says that developers have enough endowment of the numeraire in the first period to pay the property tax associated with the equity of the CRE asset that they could produce with their endowment of CRE construction inputs. With this trick, we make sure that developers can always transfer wealth from the first period to the second period.

3 Determinants of CRE capital structures

The seminar paper by Modigliani and Miller (1958) shows that the capital structure of a financial asset is irrelevant. However, this result ignores important institutional features, such as limits on debt issuance, different tax treatment between equity and debt, and default risk. See Admati and Hellwig (2013) for a recent paper that discusses these possibilities in the rather different context of the banking sector, Gau and Wang (1990) for a discussion of usual restrictions in real estate investments and their relationship with the capital structure of an income-producing property, and Sun, Titman, and Twite (2015) for evidence of the impact of the recent financial crisis on the limits to debt capacity for commercial real estate assets. In this section, we provide examples that illustrate the role of funding capacity and property taxes on the CRE capital structures in different jurisdictions. Because equity markets are segmented, we shall see that CRE capital structures in different jurisdictions are interdependent.

In order to obtain closed form equilibrium equations, we propose a simplified version of the model presented in Section 2. Here we state our main assumptions and main results, and leave

the interpretation of the assumptions and the formal proofs of our results for the Appendix.

We consider a simplified economy with two periods ($t = 1, 2$), two states in the second period, $s = s_1, s_2$, and two jurisdictions, k_1 and k_2 . In the first period, there is a numeraire good, denoted by 01, and two CRE construction inputs, denoted by 11 and 21. At state $s = s_1, s_2$ of the second period, there is also a numeraire good 0s that is traded in the global market. We normalize to 1 the prices of the numeraire good in period 1 and also at states s_1 and s_2 in period 2.

CRE asset development obeys the following functional forms: $y_{j_1}^{d_1} = TFP_1(x_{11}^{d_1})^{\alpha_1}$ and $y_{j_2}^{d_2} = TFP_2(x_{21}^{d_2})^{\alpha_2}$, i.e., developer d_1 (d_2) uses CRE construction input 11 (21, respectively) to produce CRE asset j_1 (j_2 , respectively) in jurisdiction k_1 (k_2 , respectively). Once CRE assets j_1 and j_2 are developed, they produce CRE goods $11k_1(s)$ and $12k_2(s)$, respectively, at state s . Function $f_{j_k}(y_{j_k}^{d_k})(s)$, which transforms the corresponding CRE construction input into a CRE good at state s , is assumed to be linear: $f_{j_k}(y_{j_k}^{d_k})(s) = Y_{1k}(s)y_{j_k}^{d_k}$, where $Y_{1k}(s) > 0$ for $s = s_1, s_2$ and $k = k_1, k_2$. Households purchase the CRE good produced in their respective jurisdictions and, therefore, the market clearing of the commercial good occurs at the jurisdictional (local) level.

Jurisdiction k_1 has two households and one developer, denoted by $h_1, H_1, d_1 \in \mathbf{k}_1$. We think of household h_1 as a young household who enjoys the consumption of the commercial good in the second period. Household H_1 is thought as an old household who enjoys the consumption of the numeraire good in the first period. Jurisdiction k_2 has one household and one developer, denoted by $h_2, d_2 \in \mathbf{k}_2$. Household h_2 enjoys the consumption of the commercial good in the second period. In addition, there is one investor, denoted by i , who belongs to both jurisdictions. Both developers and the investor enjoys the consumption of the numeraire good in the second period. Formally, we consider the following utility function for an agent a ²⁵

$$u^a(x^a) = \sum_{l=\{0,1,2\}} \theta_{l1}^a \ln x_{l1}^a + \sum_{s=s_1, s_2} (\theta_0^a(s) \ln x_0^a(s) + \theta_{11}^a(s) \ln x_{11}^a(s) + \theta_{21}^a(s) \ln x_{21}^a(s))$$

²⁵The θ -parameters represent the agent a 's weights corresponding to the different goods. The consumption of CRE goods is restricted to only those households that belong to the jurisdiction in question. For example, the consumption of the CRE good produced in the second jurisdiction, denoted by $x_{21}^a(s)$, can only be positive in this simplified economy if $a = \{h_2\}$.

where all θ -parameters are zero, except for $\theta_{01}^{H_1} > 0$, $\theta_{11}^{h_1}(s) > 0$, $\theta_{21}^{h_2}(s) > 0$, $\theta_0^{d_1}(s) > 0$, $\theta_0^{d_2}(s) > 0$, $\theta_0^i(s) > 0$, for $s = s_1, s_2$.

In the first period, young households h_1 and h_2 are endowed with the numeraire good 01. Households are excluded from the CRE equity. The old household H_1 is endowed with the CRE construction inputs 11 and 21 in the first period ($\omega_{11}^{H_1} > 0$ and $\omega_{21}^{H_1} > 0$), and also with the numeraire good in the second period. This last endowment is such that $\omega_0^{H_1}(1) = \omega_0^{H_1}(2) > 0$. For the sake of simplicity, we exclude the old household H_1 from the debt market, i.e., $D^{H_1} = 0$. Investor i is endowed with the numeraire good 01 in the first period. We think of this investor as an “equity REIT”, whose business only consists of buying CRE equity. Thus, we do not allow this investor to buy debt, i.e., $D^i = 0$. Developers have no commodity endowments whatsoever, so debt and equity are the only means to transfer wealth from the first to the second period. Developers, who prefer consumption of the numeraire good tomorrow, will become the debtors in this economy. They are subject, however, to the following lower bounds on short sales: $D^{d_1} \geq -\bar{D}^{d_1}$ and $D^{d_2} \geq -\bar{D}^{d_2}$, where $\bar{D}^{d_1} > 0$ and $\bar{D}^{d_2} > 0$.

Proposition 1: *For this simplified economy, we find an equilibrium with the following properties: (1) the aggregate costs of private development and local public good provision equal the total amount of the numeraire good available in the economy in the first period; (2) the price of a CRE good becomes more expensive at state s relative to state $s' \neq s$ if the amount of the CRE good produced and sold at s is smaller than at state s' ; (3) at each state of the second period, the sum of CRE cash flows and debt promises equals the total amount of the numeraire good in the economy; (4) the relative value of CRE goods produced in jurisdictions k_1 and k_2 is driven by the relative amounts of resources that households h_1 and h_2 have in the first period; (5) the CRE equity price accrued of the property tax in a jurisdiction is driven by the CRE asset’s cash flows (i.e., the value of the CRE good produced and sold in that jurisdiction); (6) when developers borrow at their maximum capacity, the total amount of debt in the economy equals the resources that households have in the first period; and (7) the difference in prices of CRE construction inputs in different jurisdictions is driven by the difference in marginal productivity of the CRE assets.*

We leave the closed form solutions of the different statements in Proposition 1 as well as the corresponding proofs for the Appendix. For the simplified economy presented above, we can find a unique equilibrium with interior allocations and endogenous and interdependent CRE capital structures. Next, we illustrate by means of numerical examples the role of a developer's funding capacity and a jurisdiction's property taxes on the equilibrium capital structures of the different CRE assets in this economy. For the sake of brevity, we leave all details about parameter and equilibrium values for the Appendix. Here, we report the main results.

Figure 1 illustrates the equilibrium capital structure of CRE assets j_1 and j_2 in an economy where the developer's debt, the developer's equity, and the investor's equity are the same in both jurisdictions. We shall refer to Figure 1 as our benchmark example.

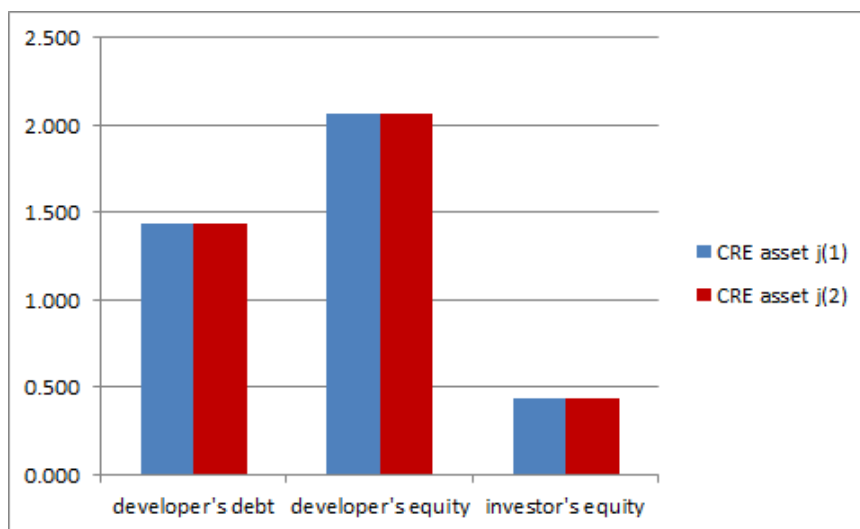


Figure 1: This figure illustrates the different capital structure components of CRE assets j_1 and j_2 , given the parameter values of Example 1 in the Appendix. Quantities are expressed in real terms, i.e., nominal amount times the corresponding price.

Funding capacity: Figure 2 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2 when the access to funding for the developer in the second jurisdiction (k_2) worsens with respect to developer in the first jurisdiction (k_1). There we see that, compared to our benchmark example in Figure 1, the developer with worse funding capacity (d_2)

decreases his absolute real exposure to CRE debt and equity, while the investor increases his equity exposure. The opposite happens in the CRE development project of the developer with better funding capacity. This suggests that when a developer has poorer access to debt financing, the equity investor finds optimal to fill the gap. Leverage, captured here by the debt-to-equity ratio, decreases (increases) in the jurisdiction with the developer with worse (better) funding capacity. Roughly speaking, leverage flies from a city with a shortage of debt to another city with better funding capacity for its developers.

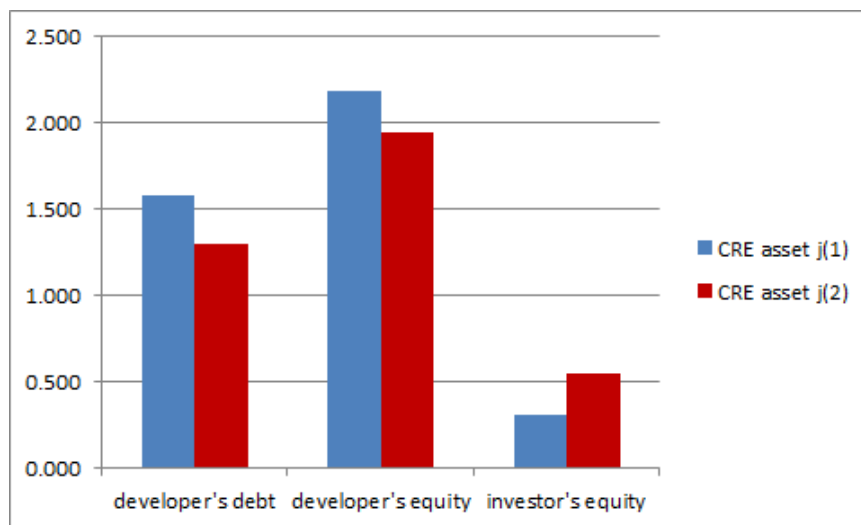


Figure 2: This figure illustrates the different capital structure components of CRE assets j_1 and j_2 , given the parameter values of Example 2 in the Appendix. Quantities are expressed in real terms.

Higher property taxes to finance an increase in local public spending: Figure 3 illustrates that, when jurisdiction k_2 finances an increase in public spending (higher fixed costs $\hat{\lambda}_2$) by increasing its tax rate γ_2 , the capital structure of CRE assets in both jurisdictions change. The increase in γ_2 decreases the developer d_2 's equity compared to the equilibrium value of our benchmark example in Figure 1. This change is due to the developer's substitution of equity for tax free debt. The developer d_2 's debt and the investor's equity contributions increase as a result. Roughly speaking, this example shows that cities with increasing government spending on infrastructure and higher taxes may end up with more levered developers, while cities with more conservative fiscal policies can effectively reduce developers' leverage.

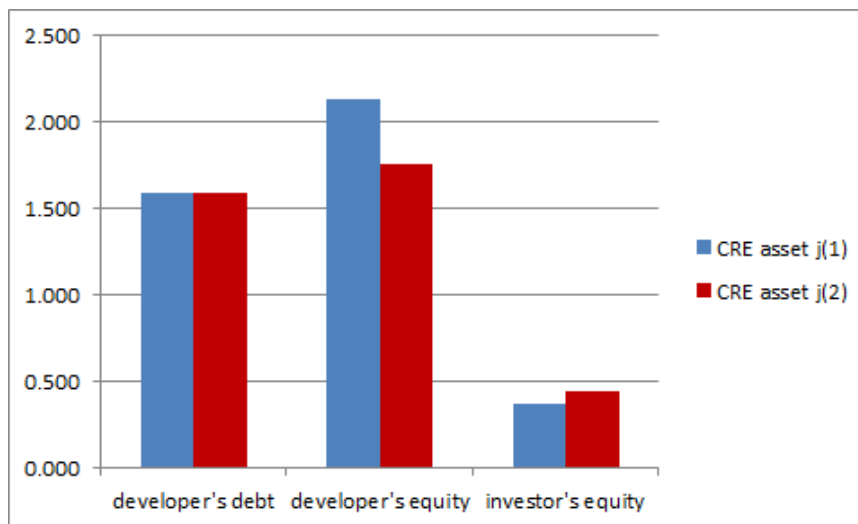


Figure 3: This figure illustrates the different capital structure components of CRE assets j_1 and j_2 , given the parameter values of Example 3 in the Appendix. Quantities are expressed in real terms.

4 Optimal property tax and selection of CRE assets

The presence of global investors makes public policy an important tool for local and national governments as they seek to attract capital for commercial real estate development. Recurrent policies used by local governments are fiscal and land use policies. The most important fiscal instrument for local governments is property taxes (e.g., property taxes contribute more than 60 percent of the City of Madison's revenues in Wisconsin). There is also a wide spectrum of land use policies, but one of the most important and effective ones for urban design is the restrictions that a jurisdiction imposes on the type of a real estate development, e.g., size, asset class, etc. Public policies of local governments are crucial in attracting CRE investments. If well executed, they create value for the jurisdiction, not only in terms of tax revenues, but also in terms of job creation and amenities for its citizens.

So far, we have taken the property tax γ_k and the Cobb-Douglas exponents α_k for all jurisdictions $k \in \mathbf{K}$ as parameters and proved that the equilibrium set is non-empty.²⁶ We denote by

²⁶Recall that developers in a jurisdiction can only construct one type of CRE asset, which is chosen by the jurisdiction manager. However, it is possible that developers in the same jurisdiction would choose a different combination of materials and thus develop the same CRE asset but with different sizes. Notice that allowing for the construction of multiple types of CRE assets would complicate our analysis because that would require having the jurisdiction

$\mathcal{E}(\alpha, \gamma)$ this set of competitive equilibria for a given profile of types of CRE assets and corresponding taxes, $(\alpha, \gamma) = ((\alpha_1, \gamma_1), \dots, (\alpha_K, \gamma_K))$. In this section, we propose an extension to our model of Section 2 in which strategic jurisdiction managers choose (α, γ) in a non-cooperative game. More precisely, we consider a strategic game in which jurisdiction authorities decide their respective types of CRE projects and property taxes. For this, each of these jurisdiction authorities evaluates a profile of strategies by replacing the equilibrium (or combination of equilibria) associated with that profile of policy variables into their corresponding profit function.

We consider that each jurisdiction authority has available a discrete set Λ_k of types of CRE development projects (that is, $\alpha_k \in \Lambda_k$ for all $k \in \mathbf{K}$) and a discrete set Γ_k of available policies for γ_k (that is, $\gamma_k \in \Gamma_k$ for all $k \in \mathbf{K}$).²⁷

Assumption 7: *The spaces Λ_k and Γ_k are discrete for all $k \in \mathbf{K}$.*

Examples of available CRE development projects in Λ_k are shopping centers, retail spaces, offices, and hotels. Examples of property taxes in Γ_k are a non-invasive fiscal policy with low property taxes and a redistributive fiscal policy with high property taxes. The jurisdiction manager chooses pairs $(\alpha_k, \gamma_k) \in \Lambda_k \times \Gamma_k$ that satisfy certain criteria. For example, we could consider only those CRE development projects that are compatible with a low tax scheme, i.e., those CRE development projects with low fixed and variable costs, i.e., low $\lambda(\alpha_k)$ and $\varepsilon(\alpha_k)$, respectively.

The aim of each jurisdiction manager is to maximize its profits, which consist of the difference between the property taxes the jurisdiction receives minus the costs it incurs to provide a CRE project. In particular, given a profile (α, γ) of CRE projects and taxes, let $(x^A, D^A, E^A, p, q, \tau, (\pi_k)_{k \in \mathbf{K}})$ be an equilibrium vector of the economy; then, for this equilibrium, the profit of a jurisdiction k is

$$\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_k E_{jk}^a - \left(\lambda(\alpha_k) + \varepsilon(\alpha_k) \sum_{a \in \{d_k\} \cup \mathbf{I}} E_{jk}^a \right).$$

As discussed by Allen and Gale (1989), it would be difficult to provide general conditions that guarantee uniqueness in this environment with endogenous security design. Here, we face the

manager choose multiple vectors of Cobb-Douglas production weights.

²⁷Recall that by Assumption 4.i, γ_k belongs to the compact set $\Gamma_k = \{\gamma_k \in \mathbb{R}_+ : \varepsilon(\alpha_k) \leq \gamma_k \leq \bar{\gamma}_k\}$.

same difficulty because, for each profile (α, γ) , the set of equilibria may not be single. We circumvent this problem by assuming that the jurisdiction manager of jurisdiction k evaluates its payoff using an index $\Psi^k(\mathcal{E}(\alpha, \gamma))$ that depends on the equilibrium set of profits that emerge given the profile of parameters (α, γ) .

Let $\Pi_k(\alpha, \gamma) = \{\pi_k : (x^A, D^A, E^A, p, q, \tau, (\pi_k)_{k \in \mathbf{K}}) \in \mathcal{E}(\alpha, \gamma)\}$, then, $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \Psi^k(\Pi_k(\alpha, \gamma))$. When the equilibrium set $\mathcal{E}(\alpha, \gamma)$ is unique, then for each profile $(\alpha, \gamma) \in \Lambda_k \times \Gamma_k$, $\Pi_k(\alpha, \gamma) = \{\pi_k\}$ is also unique and thus $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \pi_k$. When the equilibrium set for a profile (α, γ) is not single, we define this index using a measurable selector of the equilibrium correspondence $\mathcal{E}(\alpha, \gamma)$. In particular, let $(\tilde{x}^A, \tilde{D}^A, \tilde{E}^A, \tilde{p}, \tilde{q}, \tilde{\tau}, (\tilde{\pi}_k)_{k \in \mathbf{K}})(\alpha, \gamma)$ be the measurable selector of correspondence $\mathcal{E}(\alpha, \gamma)$. Then, $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \tilde{\pi}_k$. This index is well defined if there exists a measurable selector.²⁸

Lemma 7: *There exists a measurable selector $(\tilde{x}^A, \tilde{D}^A, \tilde{E}^A, \tilde{p}, \tilde{q}, \tilde{\tau}, (\tilde{\pi}_k)_{k \in \mathbf{K}})$ for the equilibrium correspondence \mathcal{E} .*

The proof of this lemma follows by applying the Kuratowski-Ryll-Nardzewski measurable selection theorem (see Aliprantis and Border 2006, p. 600). To see this, notice that in our model, the set of available CRE development projects and tax profiles is finite and, therefore, the equilibrium correspondence $\mathcal{E}(\alpha, \lambda)$ is trivially a weak measurable correspondence.

Considering equilibrium selections has implications for economic policy. To see this, notice that the index function $\Psi^k(\mathcal{E}(\alpha, \gamma))$ can be defined in many different ways. For example, if we assume that the jurisdiction manager is risk averse and has prudent behavior, then we can consider an index that corresponds to the minimal profit generated among the set of possible equilibria,

²⁸The notion of an equilibrium selector is well-known and has been used in different strands of the literature; see, for example, Miao (2006) in recursive macroeconomics, Berliant and Page (2001) in public economics, Simon and Zame (1990) in game theory, Faias, Moreno, and Pascoa (2002) and Luque and Faias (2017) in financial economics, and Stahn (1999) for a general equilibrium model with monopolistic behavior. In these models, in general, a profile of actions gives rise to a set of equilibrium outcomes. Then, to obtain an equilibrium existence result with well-defined payoff functions, which themselves depend on these profiles, authors use equilibrium selections. For example, this is the case of Cournot-Walras models with a continuous space of actions, where continuous random selections are used.

i.e.,

$$\Psi^k(\alpha, \gamma) = \min\{\pi_k : \pi_k \in \Pi_k(\alpha, \gamma)\}.$$

Because for each profile (α, γ) the set of equilibrium profits belongs to a compact set, the proof of existence of this minimum is trivial.

Local authorities play a strategic game $\mathbb{G} = \{(\Lambda_k \times \Upsilon_k, \Psi^k)_{k \in \mathbf{K}}\}$, where $\Lambda_k \times \Upsilon_k$ and Ψ^k are, respectively, the strategy set and the payoff function of local authority $k \in \mathbf{K}$. A Nash equilibrium in mixed strategies for this game consists of a probability measure over the set of property taxes and CRE projects.

The game \mathbb{G} pins down the local authorities' equilibrium strategies.

Definition 2: An equilibrium for the economy is a profile $(\alpha, \gamma) \in (\Lambda_1 \times \Upsilon_1) \times \cdots \times (\Lambda_{\mathcal{K}} \times \Upsilon_{\mathcal{K}})$ and a Walrasian equilibrium system $(x^A, D^A, E^A, p, q, \tau, (\pi_k)_{k \in \mathbf{K}}) \in E(\alpha, \gamma)$, such that

- (i) (α, γ) is a Nash equilibrium for the game $G = \{(\Lambda_k \times \Upsilon_k, \Psi^k(\alpha, \gamma))_{k \in \mathbf{K}}\}$, and
- (ii) for each $k \in K$, $\pi_k = \Psi^k(\varepsilon(\alpha, \gamma))$.

Theorem 2: *Let Assumption 7 hold. Then, there exists an equilibrium for the first stage of the economy.*

Theorem 2 guarantees the existence of an equilibrium, possibly in mixed strategies (due to the discreteness of sets Λ and Γ).

We finish this section with an example. For this, let us consider again the parameter values used to construct our benchmark example in Section 3, with the exception that now jurisdiction k 's set of strategies is $\Lambda_k \times \Gamma_k = \{(\alpha_k^{high}, \gamma_k^{high}), (\alpha_k^{low}, \gamma_k^{low})\} = \{(0.80, 0.53), (0.60, 0.50)\}$, for both $k = k_1, k_2$. The economic interpretation is that a high Cobb-Douglas exponential parameter α is associated with a high variable cost for the jurisdiction, which in turn requires a high property tax. Jurisdiction manager $k = 1, 2$ chooses (α_k, γ_k) in $\Lambda_k \times \Gamma_k$ in order to maximize the following

profit function:²⁹

$$\pi_k = \left(1 - \frac{\alpha_k}{6\gamma_k}\right) (\tau(\alpha, \gamma)\bar{D}^{d_k} + (q_k(\alpha, \gamma) + \gamma_1)E_k^i(\alpha, \gamma) - p_{1k}(\alpha, \gamma)\omega_{1k}^I) - \hat{\lambda}_k\alpha_k \quad (3)$$

where $(\alpha, \gamma) = (\alpha_1, \alpha_2, \gamma_1, \gamma_2)$. For simplicity, we assume that $\delta_k^{d_k} = 0$ and $\delta_k^i = 0$, for $k = 1, 2$. After computing the equilibrium associated with each pair of strategies and the corresponding profit functions for each jurisdiction manager, we obtain the following payoffs:

| | | Jurisdiction $k = 2$ | |
|----------------------|--------------------------------------|--------------------------------------|------------------------------------|
| | | $(\alpha_2^{high}, \gamma_2^{high})$ | $(\alpha_2^{low}, \gamma_2^{low})$ |
| Jurisdiction $k = 1$ | $(\alpha_1^{high}, \gamma_1^{high})$ | (0.13, 0.13) | (0.13, 0.20) |
| | $(\alpha_1^{low}, \gamma_1^{low})$ | (0.20, 0.13) | (0.20, 0.20) |

$(\alpha_k^{low}, \gamma_k^{low})$ is a dominant strategy for both $k = 1, 2$ in this game. Thus, the Nash equilibrium consists of the pair of strategies $((\alpha_1^{low}, \gamma_1^{low}), (\alpha_2^{low}, \gamma_2^{low}))$. This result illustrates that high property taxes are not always optimal for jurisdictions that seek to maximize profits. The underlying reason is that in our setting jurisdictions compete to attract global real estate equity investors.

5 Conclusions

In this paper we build a general equilibrium model of CRE development. The cash flows of a CRE asset depend on the amount of commercial goods sold to households in the jurisdiction where the asset is located. Global real estate investors compete to buy equity stakes on these assets. Because investors can buy CRE equity in different jurisdictions, the economy does not consist of isolated markets. Market interdependence means that the investors' decisions to buy more CRE equity in a jurisdiction affect the capital structure of CRE assets located in other jurisdictions.

²⁹We obtain function (3) by first writing the jurisdiction k 's profit function as $\pi_k = (\gamma_k - \alpha_k/6)TFP_k(x_{1k}^{d_k})^{\alpha_k} - \hat{\lambda}_1\alpha_k$, and then replacing $TFP_k(x_{1k}^{d_k})^{\alpha_k}$ with $(\tau(\alpha, \gamma)\bar{D}^{d_k} + (q_k(\alpha, \gamma) + \gamma_1)E_k^i(\alpha, \gamma) - p_{1k}(\alpha, \gamma)\omega_{1k}^I)/\gamma_k$ using (17a) and (17b), and taking $\delta_k^{d_k} = 0$.

We identify mild conditions that guarantee the existence of equilibrium for this economy. This result contributes to the literatures of optimal security design and market segmentation in general equilibrium (Allen and Gale's 1991, Allen and Gale 1994, Duffie and Rahi 1995, and Rahi and Zigrand 2009) and also to the literature pioneered by Modigliani and Miller (1958) and Bradley, Jarrel, and Kim (1994) on the existence of an optimal capital structure in equilibrium by considering the case of commercial real estate assets.

Our model endogenizes many important variables, such as the capital structure of a CRE asset, the CRE cash flows, the construction costs, the prices of commercial goods, the property taxes, and the type of CRE assets that a jurisdiction selects. Thus, our model seems appealing not only to finance, but also to other related disciplines such as macroeconomics, public policy, and urban economics. To motivate this aspect of our theory, we propose a simplified version of our general equilibrium model that illustrates the market mechanism of different shocks on the economy. In addition, we provide several numerical examples. For instance, we see that a negative shock to a developer's funding capacity increases the equity-to-total-equity ratio of the capital partner and decreases the debt-to-equity and the developer's equity-to-total-equity ratios. Because the capital structures of CRE assets in different jurisdictions are interconnected, this shock increases the leverage ratio in those CRE assets belonging to jurisdictions that are not hit by the shock. Moreover, developers in those jurisdictions must increase their equity contributions in order to offset the investors' decrease of CRE equity purchases in their jurisdictions. We also explore other shocks to the economy, such as a decrease in the production capacity of a CRE asset and an increase in the property tax in one jurisdiction. These examples also offer interesting insights regarding the inflation of CRE goods and the changes in the capital structures of CRE assets.

There are many other issues that can be explored in future research under the lens of our model. For example, our economy could be extended to accommodate the difference between industrial real estate and retail properties. This is briefly explored in Section 2.

Another interesting direction for future research would allow for default on both debt and

CRE equity investments. In that case, collateral, which is the CRE property itself, might reveal interesting economic dynamics. General equilibrium models with collateral constraints seem to be the correct approach for this extension (see e.g. Geanakoplos and Zame 2014, Gale and Gottardi 2015, and Fostel and Geanakoplos 2016). An interesting question that could be addressed following this approach would be to quantify the importance of collateral versus taxes for the capital structures of different CRE assets (see Li, Whited, and Wu 2016 for a similar question in the context of corporations). Another interesting question would be to characterize the relationship between the CRE asset collateral and the developer's funding capacity (see Campello and Giambona 2013 and Cvijanović 2014 for empirical work on this issue).

Another avenue for future research is to understand the role that transfers of property tax revenue to the agents that live and do business in a jurisdiction have in the economy. When the jurisdiction's profits revert to local households, taxes can be seen as a standard redistributive device from CRE developers and investors to households. When the jurisdiction profits revert to developers, transfers can be seen as Tax Incremental Financing. And if the jurisdiction transfers tax revenues to investors, the subsidies can be seen as tax credits (see Minnassian 2016). As in the classical theory of general equilibrium, all that we would require is that the profit sharing weights sum up to one across the agents of the jurisdiction.

Our model can also guide empirical evaluations of the role that CRE property taxes have on attracting REIT investments in a globalized economy in which jurisdictions compete for financial resources. Another relevant empirical issue is sizing up the impact of taxes on CRE property. Existing research on the impact of taxes on residential real estate transfers suggests that CRE property taxes might have a significant effect on the economy. For example, Duranton, Dachis, and Turner (2012) provide empirical evidence of the impact of real estate transfer taxes on the market for single family homes and find that taxes levied on the sale or purchase of real estate are pervasive. In particular, they show that Toronto's 1.1 percent tax caused a 15 percent decline in the number of sales and a decline in housing prices about equal to the tax. We are not aware of any empirical paper that documents the impact of taxes on CRE property.

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A Appendix

This Appendix is devoted to the proof of Theorem 1. This proof relies on fixed point theory and requires some assumptions that are not standard in the general equilibrium literature. Next, we present the formal proof. At the end of the Appendix, we provide further explanations on these assumptions and main difficulties that arise in the proof of Theorem 1.

A.1 Proof of Theorem 1

Our approach is to first construct a generalized game of the economy introduced in Section 2, then prove that the set of equilibria for our generalized game is non-empty, and then show that an equilibrium of the generalized game is in fact a competitive equilibrium that satisfies all conditions in Definition 1.

To this aim, let us first define the following thresholds:

- $W_{01} \equiv \sum_{a \in \mathbf{A}} \omega_{01}^a$ denotes the aggregate endowment of the numeraire good in the first period. We shall use this threshold to bound consumption of the numeraire in the first period.
- We shall use threshold $\bar{E} \equiv \max_{k=1, \dots, \mathcal{K}} (1/\gamma_{j_k})(W_{01})$ to find an upper bound on equity purchases. Notice that we chose threshold \bar{E} in such a way that the property tax payment, denominated in terms of the numeraire good, cannot exceed the total amount of the numeraire good in the economy.
- We shall use threshold

$$W \equiv \max\{W_{01}, \max_{l \in \mathbf{L}_1} \sum_{a \in \mathbf{A}} \omega_{l1}^a, \max_{l \in \mathbf{S}} \sum_{a \in \mathbf{A}} \omega_0^a(s), \max_{(s,l,k) \in \mathbf{S} \times \mathbf{L}_2 \times \mathbf{K}} f_{lj_k}(s)(\bar{Y})\}$$

to bound consumption in the second period, where $\bar{Y} = \max_{k=1, \dots, \mathcal{K}} Y_k$.

- $n \in \mathbb{N}_+$ is a parameter that we shall use for bounding consumption of commodities in the first period. The subtlety is that in this economy the prices of debt and CRE equity stakes can be very large and, if this happens, the consumption bundles of commodities in the first period can also be very large. However, we shall find an upper bound for these variables that depend only on the primitives of the economy and thus we shall conclude that this constant n will not be binding.

The set of players in this generalized game consists of the set of agents $\mathbf{A} = \mathbf{I} \cup \{d_k\}_{k=1}^{\mathcal{K}} \cup \{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$ together with the following auctioneers: an auctioneer that chooses period 1 prices, an auctioneer that chooses period 2 prices, and an auctioneer per jurisdiction that chooses profits.

Next, let us truncate the set of admissible consumption bundles and financial positions corresponding to households, developers, and investors.

- Each household $h \in \{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$ chooses a vector (x^h, D^h) in the compact set $\Omega^h(n) = [0, n]^{1+L_1} \times [0, 2W]^{(1+L_2)S} \times [-2D, 2(\#\mathbf{A})D] \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{(1+L_2)S} \times \mathbb{R}$.
- Each developer $d \in \mathbf{D} \equiv \{d_k\}_{k=1}^{\mathcal{K}}$ chooses a vector (x^d, D^d, E^d) in the compact set $\Omega^d(n) = [0, n]^{1+L_1} \times [0, 2W]^{(1+L_2)S} \times [-2D, 2(\#\mathbf{A})D] \times [0, 2\bar{E}] \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{(1+L_2)S} \times \mathbb{R} \times \mathbb{R}_+$, where $\#\mathbf{A}$ denotes the number of agents in the economy.
- Each investor $i \in \mathbf{I}$ chooses a plan (x^i, D^i, E^i) in the compact set $\Omega^i(n) = [0, n]^{1+L_1} \times [0, 2W]^{S(1+L_2\mathcal{K})} \times [-2D, 2(\#\mathbf{A})D] \times [0, 2\bar{E}]^{\mathcal{K}} \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{S(1+L_2\mathcal{K})} \times \mathbb{R} \times \mathbb{R}_+^{\mathcal{K}}$.

The goal of households, developers, and investors is to maximize their utility function by choosing a bundle in their respective compact sets

$$((x^h, D^h)_{h \in \mathbf{H}}, (x^d, D^d, E^d)_{d \in \mathbf{D}}, (x^i, D^i, E^i)_{i \in \mathbf{I}}) \in (\Omega^h(n))^{\mathcal{H}\mathcal{K}} \times (\Omega^d(n))^{\mathcal{K}} \times (\Omega^i(n))^{\mathcal{I}}$$

which satisfies their respective budget constraints. The consumption sets are $X^a = \mathbb{R}_+^{1+L_1+S(1+L_2)}$ if $a \in \mathbf{D} \cup \mathbf{H}$, and $X^i = \mathbb{R}_+^{1+L_1+S(1+\mathcal{K}L_2)}$ if $i \in \mathbf{I}$.

We now define the feasible sets of the auctioneers.

- The price-auctioneers in periods 1 and 2 choose prices in the following simplexes: $\Delta^{L_1} = \{p \in \mathbb{R}_+^{1+L_1} : \sum_{l=1}^{1+L_1} p_l = 1\}$ and $\Delta^{(1+L_2\mathcal{K})S-1} = \{p \in \mathbb{R}_+^{(1+L_2\mathcal{K})S} : \sum_{l=1}^{(1+L_2\mathcal{K})S} p_l = 1\}$, respectively. These sets are non-empty, convex, and compact.
- The profit-auctioneer chooses profits π_k in the compact set $[-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1 + \mathcal{I}), \sum_{a \in \mathbf{A}} \omega_{01}^a]$. The lower bound $-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1 + \mathcal{I})$ on the feasible set for profit π_k follows from the fact that parameter γ_k must be such that $\varepsilon(\alpha_k) \leq \gamma_k$, for all $k = 1, \dots, \mathcal{K}$. The upper bound on π_k , $\sum_{a \in \mathbf{A}} \omega_{01}^a$, is given by the aggregate of the numeraire good in the first period since profits are denominated in units of the numeraire. Notice that we fix an exogenous upper bound for the sake of simplicity; in fact, we could obtain the upper bound on profits endogenously.

Auctioneers solve the following optimization problems:

- The price auctioneer in period 1 chooses $(p_1, \tau, q) \in \Delta^{L_1} \times [0, m]^{1+\mathcal{K}}$, with $m \in \mathbb{N}_+$, in order to maximize the following function:

$$p_{01} \sum_{a \in \mathbf{A}} (x_{01}^a - \omega_{01}^a) - p_{01} \sum_{a \in \mathbf{A}} \sum_{k \in \mathbf{K}} \delta_k^a \pi_k + \sum_{l \in \mathbf{L}_1} p_{l1} \sum_{a \in \mathbf{A}} (x_{l1}^a - \omega_{l1}^a) + \tau \sum_{a \in \mathbf{A}} D^a + \\ + p_{01} \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} E_{jk}^a + \sum_{k \in \mathbf{K}} q_{jk} \left(\sum_{a \in \{d_k\} \cup \mathbf{I}} E_{jk}^a - y_{jk}^{d_k} \right)$$

- The price auctioneer in period 2 chooses $(p(s), s = 1, \dots, S) \in \Delta^{(1+L_2\mathcal{K})S-1}$ in order to maximize the following function:

$$\sum_{s \in \mathbf{S}} p_0(s) \sum_{a \in \mathbf{A}} (x_0^a(s) - \omega_0^a(s)) + \sum_{(s,l,k) \in \mathbf{S} \times \mathbf{L}_2 \times \mathbf{K}} p_{lk}(s) \left(\sum_{a \in \mathbf{H}_k \cup \mathbf{I} \cup \{d_k\}} x_{lk}^a(s) - f_{lk}(y_{jk}^{d_k})(s) \right).$$

- For all $k = 1, \dots, \mathcal{K}$, the profit-auctioneer in jurisdiction k chooses π_k in the closed set

$[-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1 + \mathcal{I}), \sum_{a \in \mathbf{A}} \omega_{01}^a]$ in order to minimize the following function:

$$\left(\pi_k - \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} E_{jk}^a + \lambda(\alpha_k) + \varepsilon(\alpha_k) \sum_{a \in \{d_k\} \cup \mathbf{I}} E_{jk}^a \right)^2$$

We refer to the above generalized game as $\mathcal{G}(n, m)$. Next, we verify that the player's best response correspondences for this game satisfy the conditions of Kakutani's fixed point theorem.

First, the objective functions of households, developers, and investors are continuous and strongly quasi-concave as stated in Assumptions 2.i-iii, and their choice sets are non-empty, convex, and compact.

Second, for each vector $(p_1, (p(s), s = 1, \dots, S), \tau, q, (\pi_k)_{k \in \mathbf{K}})$ of prices and profits, the choice set of each agent $a \in \mathbf{A}$ has an interior point. According to Assumptions 5 and 6, the endowments of the numeraire good and CRE construction inputs are strictly positive for every agent. Moreover, for every agent $a \in \mathbf{A}$, $p_1 \omega_1^a + p_{01} \sum_{k \in \mathbf{K}} \delta_k^a \pi_k > 0$ (this follows because $\omega_{01}^a > (\lambda(\alpha_k) + \varepsilon(\alpha_k)(1 + \mathcal{K}))\mathcal{K}$ and $p_1 \in \Delta^{1+L_1}$). Thus, $x^a = 0$ and $E^a = 0$, together with a D^a satisfying inequalities $\tau D^a < p_1 \cdot \omega_1^a + p_{01} \sum_{k=1}^{\mathcal{K}} \delta_k^a \pi_k$ and $\bar{r} D^a(s) > 0$ for all $s \in \mathbf{S}$, is an interior point of the budget correspondence. This guarantees the lower hemicontinuity property of the agent's budget set correspondence. Since upper hemicontinuity also holds in our setting, we can use Berge's Maximum Theorem to claim that, for these players, the best response correspondence is upper-hemicontinuous with non-empty and compact values. The best response correspondence also takes convex values – this follows from the convexity of the budget set correspondence and strongly quasi-convavity of the objective function. For developers, this is also true, but the convexity property is not immediate. That property follows from the fact that the production function, which transforms CRE construction inputs into a CRE asset, is concave, and also the fact that production functions $(f_{lj_k})_{l \in \mathbf{L}_1, k \in \mathbf{K}}$ that assign CRE assets into CRE goods are increasing and concave by Assumption 3.ii.

Third, the objective function of the profit-auctioneer in each jurisdiction is continuous and convex and its choice set is non-empty, convex, and compact. In addition, the price-auctioneers'

objective functions are linear and, therefore, continuous and strictly quasi-concave in their choice variable. Moreover, their choice sets are non-empty, convex, and compact. Thus, for each of these auctioneers, its best response correspondence is also upper-hemicontinuous and takes non-empty, compact and convex values. Kakutani's fixed point theorem guarantees that the generalized game $\mathcal{G}(n, m)$ has a Nash equilibrium, which is the fixed point of the product of the best response correspondences.

Lemma A.1: *Suppose that Assumptions 5 and 6 hold. Then, if $\bar{x}_{01}^a < W_{01}$ for all $a \in \mathbf{A}$, there exists a threshold $\bar{m} \in \mathbb{N}$ for an equilibrium $(\bar{x}^A, \bar{D}^A, \bar{E}^A, \bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$ of the generalized game $\mathcal{G}(n, m)$, such that $\max\{\bar{q}_{j_{k_1}}, \bar{q}_{j_{k_2}}, \dots, \bar{q}_{j_{k_K}}, \bar{\tau}\} < \bar{m}$.*

Proof of Lemma A.1: Let $\tilde{y}_{j_k}^{d_k} \equiv TPF_k \cdot \prod_{l \in L_1} (\omega_{l1}^{d_k})^{\alpha_{lk}}$ and $(\tilde{f}_{j_k}^{d_k}(s), s = 1, \dots, S) \equiv (f_{j_k}(\tilde{y}_{j_k}^{d_k})(s), s = 1, \dots, S)$ (here $\tilde{f}_{j_k}^{d_k}(s)$ is the bundle of CRE goods that can be produced in period 2 with the CRE assets developed by developer d_k using his endowment of CRE construction inputs). According to Assumption 5, a developer d_k has enough endowment $\omega_{01}^{d_k}$ of the numeraire good in the first period to buy equity of the CRE asset that he produces using his own endowment of CRE construction input goods. Thus, a developer can always transfer at least an amount of wealth $p(s)\tilde{f}_{j_k}^{d_k}(s)$ from period 1 to each state of period 2. We conclude that the bundle $(\omega_{01}^{d_k} - g_{j_k}(\tilde{y}_{j_k}^{d_k}), 0, (\omega_0(s), \tilde{f}_{j_k}^{d_k}(s)), s = 1, \dots, S)$ is always feasible for developer d_k .

According to Assumption 6, given $\theta \in (0, 1)$, there exists $\varrho \in \mathbb{R}_+^{1+L_1}$ such that

$$u^{d_k}(W_{01}, (2W(1, \dots, 1), s = 1, \dots, S)) < u^{d_k}(W_{01} + \varrho, (2\theta W(1, \dots, 1), s \in \mathbf{S})),$$

where $\varrho = \varrho(W_{01}, (2W(1, \dots, 1), s = 1, \dots, S))$. If we take $\theta \in (0, 1)$, such that $2W(1, \dots, 1)\theta < 0.7\tilde{f}_{j_k}^{d_k}(s)$ and $2W\theta < 0.7\omega_0^{d_k}(s)$, for all $s \in \mathbf{S}$, then

$$u^{d_k}(W_{01}, (2W(1, \dots, 1), s = 1, \dots, S)) < u^{d_k}(W_{01} + \varrho, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}^{d_k}(s)), s \in \mathbf{S}))$$

or

$$u^{d_k}(W_{01}, (2W(1, \dots, 1), s = 1, \dots, S)) < u^{d_k}(\omega_{01}^{d_k} - 0.7\gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \tilde{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}(s)), s \in \mathbf{S})),$$

where $\tilde{\varrho} = W_{01} - \omega_{01}^{d_k} + 0.7\gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \varrho$ (notice that parameter $\tilde{\varrho}$ only depends on the fundamentals of the economy). Since $\bar{x}_{01}^a < W_{01}$, for all $a \in \mathbf{A}$, by monotonicity it is also true that $u^{d_k}(\bar{x}_{01}^{d_k}, (\bar{x}^{d_k}(s), s \in \mathbf{S})) < u^{d_k}(W_{01}, (2W(1, \dots, 1), s \in \mathbf{S}))$, which by transitivity implies that

$$u^{d_k}(\bar{x}_{01}^{d_k}, (\bar{x}^{d_k}(s), s = 1, \dots, S)) < u^{d_k}(\omega_{01}^{d_k} - 0.7\gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \tilde{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}(s)), s \in \mathbf{S})).$$

This means that developer d_k cannot buy $\tilde{\varrho}$ units of numeraire in period 1 with the resources that he saves when buying only part of the CRE equity, which is $p_{0,1}g_{j_k}(0.3\tilde{y}_{j_k}^{d_k}) + q_{j_k}0.3\tilde{y}_{j_k}^{d_k}$. That is,

$$p_{0,1}0.3\gamma_{j_k} \tilde{y}_{j_k}^{d_k} + q_{j_k}0.3\tilde{y}_{j_k}^{d_k} < \bar{p}_{0,1}\tilde{\varrho} \Leftrightarrow q_{j_k} < \bar{m}_{j_k} = \frac{\bar{p}_{0,1}\tilde{\varrho} - 0.3p_{0,1}\gamma_{j_k} \tilde{y}_{j_k}^{d_k}}{0.3\tilde{y}_{j_k}^{d_k}}.$$

Finally, set $\bar{m}_E = \max_{k \in \mathbf{K}} \bar{m}_{j_k}$ (observe that, for every j_k and $k \in \mathbf{K}$, threshold \bar{m}_{j_k} depends only on the primitive parameters of the economy). Inequality $u^{d_k}(W_{01}, (2W(1, \dots, 1), s \in \mathbf{S})) < u^{d_k}(W_{01} + \varrho, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}(s)), s \in \mathbf{S}))$ also implies

$$u^{d_k}(W_{01}, (2W(1, \dots, 1), s = 1, \dots, S)) < u^{d_k}(\omega_{01}^{d_k} - \gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \hat{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}(s)), s \in \mathbf{S})),$$

where $\hat{\varrho} = W_{01} - \omega_{01}^{d_k} + \gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \varrho$ (notice that $\hat{\varrho}$ only depends on the fundamentals of the economy). Then, developer d_k cannot afford the bundle $(\omega_{01}^{d_k} - \gamma_{j_k} \tilde{y}_{j_k}^{d_k} + \hat{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_{j_k}(s)), s = 1, \dots, S)$; in particular, d_k cannot buy the bundle $\hat{\varrho} \in \mathbb{R}$ in period 1 using the debt payment that he would receive by selling the bundle $(0.3\omega_0^{d_k}(s), 0.3\tilde{f}_{j_k}(s)), s = 1, \dots, S$.

Finally, let D^{d_k} be such that $\bar{r}D^{d_k} < \min\{0.3\omega_0^{d_k}(s), 0.3\tilde{f}_{j_k,1}(s), \dots, 0.3\tilde{f}_{j_k,1}(s)\}$. Then, $-\tau D^h < \bar{p}_{01}\hat{\varrho}$, that is, $\tau < \bar{m}_D = \hat{\varrho}/(-D^{d_k})$. It just remains to set $\bar{m} \equiv \max\{\bar{m}_E, \bar{m}_D\}$ and $\bar{n} = 2W + \bar{m}$.

This concludes the proof of Lemma A.1. ■

Lemma A.2: *An equilibrium $(\bar{x}^A, \bar{D}^A, \bar{E}^A, \bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$ of the generalized game $\mathcal{G}(n, m)$*

for $(n, m) > (\bar{n}, \bar{m})$ is a competitive equilibrium as defined in Definition 1.

Proof of Lemma A.2: By adding the first period budget constraints of all agents in the economy, we obtain

$$p_{01} \sum_{a \in \mathbf{A}} (\bar{x}_{01}^a - \omega_{01}^a) + \sum_{l \in \mathbf{L}_1} p_{l1} \sum_{a \in \mathbf{A}} (\bar{x}_{l1}^a - \omega_{l1}^a) + \tau \sum_{a \in \mathbf{A}} \bar{D}^a +$$

$$p_{01} \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a + \sum_{k \in \mathbf{K}} q_{jk} \left(\sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{jk}^a - \bar{y}_{jk}^{dk} \right) \leq 0.$$

Then, taking into account the problem of the price-auctioneer in period 1, we conclude that there is no excess of demand for commodities and assets; that is,

1. $\sum_{a \in \mathbf{A}} (\bar{x}_{01}^a - \omega_{01}^a - \sum_{k \in \mathbf{K}} \delta_k^a \pi_k) + \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a \leq 0$
2. $\sum_{a \in \mathbf{A}} (\bar{x}_{l1}^a - \omega_{l1}^a) \leq 0$, for all $l \in \mathbf{L}_1$
3. $\sum_{a \in \mathbf{A}} \bar{D}^a \leq 0$
4. $\sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{jk}^a \leq \bar{y}_{jk}^{dk}$, for all $k \in \mathbf{K} \leq 0$

If $\sum_{a \in \mathbf{A}} (\bar{x}_{01}^a - \omega_{01}^a - \sum_{k \in \mathbf{K}} \delta_k^a \pi_k) + \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a > 0$, then the price auctioneer would choose $p_{01} = 1$ and a price equal to zero for the other commodities and assets. This allows this auctioneer to obtain a positive value for its objective function. However, this is in contradiction with the aggregation of the budget constraints. The same argument allows us to conclude that aggregate debt and equity holdings is less than or equal to zero. Finally, if $\sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{jk}^a - \bar{y}_{jk}^{dk} > 0$, for some asset jk , then the auctioneer would choose $\bar{q}_{jk} = \bar{m}$ (notice that $\bar{x}_{01}^a < W_{01}$), which would contradict Lemma A.1 for $n > \bar{n}$. The same argument applies for debt.

The inequality in the first numbered list item is equivalent to

$$\sum_{a \in \mathbf{A}} \bar{x}_{01}^a \leq \sum_{a \in \mathbf{A}} \omega_{01}^a + \sum_{k \in \mathbf{K}} \sum_{a \in \mathbf{A}} \delta_k^a \pi_k - \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a,$$

which in turn is equivalent to

$$\sum_{a \in \mathbf{A}} \bar{x}_{01}^a \leq \sum_{a \in \mathbf{A}} \omega_{01}^a + \sum_{k \in \mathbf{K}} \pi_k - \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a$$

because $\sum_{a \in \mathbf{A}} \delta_k^a = 1$. Given that $\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{jk} \bar{E}_{jk}^a - \lambda(\alpha_k) + \varepsilon(\alpha_k) (\sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{jk}^a)$, we get

$$\sum_{a \in \mathbf{A}} \bar{x}_{01}^a \leq \sum_{a \in \mathbf{A}} \omega_{01}^a - \sum_{k \in \mathbf{K}} (\lambda(\alpha_k) + \varepsilon(\alpha_k)) \sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{jk}^a.$$

By adding the budget constraints of all agents over all states of nature in period 2, we obtain

$$\begin{aligned} \sum_{s \in \mathbf{S}} \bar{p}_0(s) \sum_{a \in \mathbf{A}} (\bar{x}_0^a(s) - \omega_0^a(s)) + \sum_{(s,l,k) \in \mathbf{S} \times \mathbf{L}_2 \times \mathbf{K}} \bar{p}_{lk}(s) \left(\sum_{a \in \{d_k\} \cup \mathbf{H}_k \cup \mathbf{I}} \bar{x}_l^a(s) \right) &\leq \\ &\leq S\bar{r} \sum_{a \in \mathbf{A}} \bar{D}^a + \sum_{s \in \mathbf{S}} \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} c_{jk}(E_{jk}^a; \bar{p}(s)) \\ &\leq S\bar{r} \sum_{a \in \mathbf{A}} \bar{D}^a + \sum_{s \in \mathbf{S}} \sum_{k \in \mathbf{K}} c_{jk}(\bar{y}_{jk}^{d_k}) \end{aligned}$$

Using the definition of $c_{jk}(\bar{y}_{jk}^{d_k})$ and the fact that $S\bar{r} \sum_{a \in \mathbf{A}} \bar{D}^a \leq 0$, we can rewrite the above inequality as follows:

$$\sum_{s \in \mathbf{S}} \bar{p}_0(s) \sum_{a \in \mathbf{A}} (\bar{x}_0^a(s) - \omega_0^a(s)) + \sum_{(s,l,k) \in \mathbf{S} \times \mathbf{L}_2 \times \mathbf{K}} \bar{p}_{lk}(s) \left(\sum_{a \in \{d_k\} \cup \mathbf{H}_k \cup \mathbf{I}} \bar{x}_l^a(s) - f_{ljk}(\bar{y}_{jk}^{d_k})(s) \right) \leq 0.$$

Then, given the problem of the auctioneer of period 2, we conclude that there is no excess of demand for commodities at date 2, i.e.,

5. $\sum_{a \in \mathbf{A}} (\bar{x}_0^a(s) - \omega_0^a(s)) \leq 0$, for all $s \in \mathbf{S}$;
6. $\sum_{a \in \{d_k\} \cup \mathbf{H}_k \cup \mathbf{I}} \bar{x}^a(s) - f_{jk}(\bar{y}_{jk}^{d_k})(s) \leq 0$ for all $s \in \mathbf{S}$ and $k \in \mathbf{K}$.

In equilibrium, we also have that, for every jurisdiction $k \in \mathbf{K}$, profits are

$$\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_{j_k} \bar{E}_{j_k}^a - \left(\lambda(\alpha_k) + \varepsilon(\alpha_k) \sum_{a \in \{d_k\} \cup \mathbf{I}} \bar{E}_{j_k}^a \right).$$

Observe that this value for the profit function belongs to the profit-auctioneer's strategy set; moreover, this is the value that minimizes that auctioneer's objective function.

Next, we prove that there is no excess of supply of commodities other than CRE construction inputs. For if there were excess supply of one of the commodities in either period 1 or period 2, then the respective price-auctioneer would choose the price of that commodity equal to zero. However, this would be in contradiction with the existence of an optimal plan for the agents of this economy, since utility functions are increasing.

If there is excess supply of a CRE construction input, then the price-auctioneer would choose its price equal to zero and this would be in contradiction with the existence of an optimal plan for developers. To see this, notice that by increasing the purchased amount of CRE construction inputs, the developer could increase his income in period 1 to spend it on the consumption of the numeraire good 01 (recall that the utility function is increasing in the consumption of the numeraire good).

In addition, there is no excess supply of debt or equity. Again, we prove this by contradiction. If there were excess supply of debt or equity, the price-auctioneer would choose the price of that particular asset equal to zero and this would contradict the existence of an optimal plan because, by the monotonicity of the preferences, debt pays strictly positive returns in every state of nature and equity also pays strictly positive returns in all states of nature if $p_k(s) \gg 0$ and Assumption 3.iii hold. Observe that, in a Nash equilibrium of the generalized game, $p_k(s) \gg 0$ for all $s \in \mathbf{S}$, by monotonicity of preferences in period 2.

Finally, we establish the optimality of consumption plans. For this, first notice that, for each agent $a \in \mathbf{A}$, $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ satisfies the budget constraint given prices and profits $(\bar{p}, \bar{q}, \bar{r}, (\bar{\pi})_{k \in \mathbf{K}})$. Also, $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ belongs to the interior of $\Omega^a(n)$. Therefore, by the convexity of the budget sets

and the strongly quasi-concavity of the utility functions, we have that the bundle $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ is optimal in the budget set with prices and profits $(\bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$. ■

A.2 Proof of Proposition 1

Households h_1 and h_2 are excluded from the CRE equity market and, therefore, the only financial instrument that they can use to transfer wealth from the first to the second period is risk-free debt.³⁰ Because they prefer to consume tomorrow, they will sell their numeraire good endowment and purchase as much risk-free debt as possible (risk-free debt pays 1 unit of the numeraire good in both states of the second period). Thus, in equilibrium we expect³¹

$$D^{h_1} = \frac{1}{\tau} \left(\omega_{01}^{h_1} + \delta_1^{h_1} \left(\left((\gamma_1 - \varepsilon(\alpha_1)) TFP_1 (x_{11}^{d_1})^{\alpha_1} - \lambda(\alpha_1) \right) \right) \right) > 0 \quad (4)$$

$$D^{h_2} = \frac{1}{\tau} \left(\omega_{01}^{h_2} + \delta_2^{h_2} \left(\left((\gamma_2 - \varepsilon(\alpha_2)) TFP_2 (x_{21}^{d_2})^{\alpha_2} - \lambda(\alpha_2) \right) \right) \right) > 0 \quad (5)$$

Households purchase the CRE good produced in their respective jurisdictions and, therefore, the market clearing of the commercial good occurs at the jurisdictional (local) level. This implies that $x_{11}^{h_1}(s) = Y_{11}(s) TFP_1 (x_{11}^{d_1})^{\alpha_1}$ and $x_{21}^{h_2}(s) = Y_{21}(s) TFP_2 (x_{21}^{d_2})^{\alpha_2}$, for $s = s_1, s_2$.

For the sake of simplicity, we exclude the old household H_1 from the debt market, i.e., $D^{H_1} = 0$. Since the old household H_1 prefers to consume the numeraire good in the first period, in equilibrium we expect him to sell his CRE construction inputs to the developers and use the proceeds to purchase as much as he can of good 01.

Because investor i prefers consumption of the numeraire good in the second period, we expect him to sell his endowment of good 01 and buy CRE equity in one or both jurisdictions. The following condition follows from the investor's budget constraint in the first period:

$$(q_1 + \gamma_1) E_1^i + (q_2 + \gamma_2) E_2^i = \omega_{01}^i + \sum_{k=\{k_1, k_2\}} \delta_k^i \left(\left((\gamma_k - \varepsilon(\alpha_k)) TFP_k (x_{1k}^{d_k})^{\alpha_k} - \lambda(\alpha_k) \right) \right) \quad (6)$$

³⁰Household's debt could be seen as a deposit of this household in a bank account. For the sake of simplicity, we ignore banks as potential financial intermediaries in this economy.

³¹Conditions (4) and (5) are useful to obtain the following Lemmas 2 to 5 in the proof of Proposition 1.

Next, we prove Proposition 1 with a series of lemmas.

Lemma 1: *The aggregate costs of private development and local public good provision equal the total amount of numeraire good available in the economy in the first period, i.e.,*

$$\underbrace{p_{11}\omega_{11}^{H_1} + p_{21}\omega_{21}^{H_1}}_{\text{cost of CRE construction inputs}} + \underbrace{\sum_{k \in \mathbf{K}} (\lambda(\alpha_k) + \varepsilon(\alpha_k)TFP_1 (x_{11}^{d_k})^{\alpha_1})}_{\text{cost of local public good provision}} = \omega_{01}^{h_1} + \omega_{01}^{h_2} + \omega_{01}^i \quad (7)$$

Proof: Equation (7) follows from the old household H_1 's budget constraint in period 1 and the market clearing equation for the numeraire good 01. ■

Lemma 2: *Scarcity of a CRE good drives the price differential of this good between states of nature. In particular, the price of a CRE good becomes more expensive at state s relative to state $s' \neq s$ if the amount of the CRE good produced and sold at s is smaller than at state s' . In particular,*

$$p_{11}(1)/p_{11}(2) = Y_{11}(2)/Y_{11}(1) \quad (8)$$

$$p_{21}(1)/p_{21}(2) = Y_{21}(2)/Y_{21}(1) \quad (9)$$

Moreover, because the value of one unit of the CRE good produced in a jurisdiction is the same at both states, consumption of the numeraire good is the same at both states for the two developers and the investor, i.e., $x_0^{d1}(1) = x_0^{d1}(2)$, $x_0^{d2}(1) = x_0^{d2}(2)$, and $x_0^i(1) = x_0^i(2)$.

Proof: The budget constraint of households h_1 and h_2 at states s_1 and s_2 are such that

$$\bar{r}D^{h_1} = p_{11}(s)Y_{11}(s)TFP_1 (x_{11}^{d_1})^{\alpha_1}, \text{ for } s = s_1, s_2 \quad (10)$$

$$\bar{r}D^{h_2} = p_{21}(s)Y_{21}(s)TFP_2 (x_{21}^{d_2})^{\alpha_2}, \text{ for } s = s_1, s_2 \quad (11)$$

Conditions (8) and (9) follow from conditions (10) and (11), respectively (i.e., dividing the corresponding expression for state s_1 by the corresponding expression for state s_2). Moreover,

conditions (8) and (9), and the developers' budget constraints in the second period imply that $x_0^{d1}(1) = x_0^{d1}(2)$ and $x_0^{d2}(1) = x_0^{d2}(2)$. These equalities, together with the market clearing conditions of the numeraire good at states s_1 and s_2 , and assumption $\omega_0^{H1}(1) = \omega_0^{H1}(2)$, imply that $x_0^i(1) = x_0^i(2)$. ■

Lemma 3: *At each state of the second period, the sum of CRE cash flows and debt promises equals the total amount of the numeraire good in the economy, i.e.,*

$$\sum_{k \in \{k_1, k_2\}} p_{1k}(s) Y_{1k}(s) TFP_k \left(x_{1k}^{d_k} \right)^{\alpha_k} + r(D^{d1} + D^{d2}) = \omega_0^{H1}(s), \text{ for } s = s_1, s_2 \quad (12)$$

Proof: We obtain equation (12) using equalities $x_0^{d1}(1) = x_0^{d1}(2)$ and $x_0^{d2}(1) = x_0^{d2}(2)$, together with the investor and developers' budget constraints at states s_1 and s_2 , and the market clearing equation for the numeraire good in the second period. ■

Lemma 4: *The relative value of CRE goods produced in jurisdictions k_1 and k_2 is driven by the relative amounts of resources that households h_1 and h_2 have in the first period. In particular,*

$$\frac{p_{11}(s) Y_{11}(s) TFP_1 \left(x_{11}^{d1} \right)^{\alpha_1}}{p_{21}(s) Y_{21}(s) TFP_2 \left(x_{21}^{d2} \right)^{\alpha_2}} = \frac{\omega_{01}^{h_1} + \delta_1^{h_1} \left(\left(\gamma_1 - \varepsilon(\alpha_1) TFP_1 \left(x_{11}^{d1} \right)^{\alpha_1} - \lambda(\alpha_1) \right) \right)}{\omega_{01}^{h_2} + \delta_2^{h_2} \left(\left(\gamma_2 - \varepsilon(\alpha_2) TFP_2 \left(x_{21}^{d2} \right)^{\alpha_2} - \lambda(\alpha_2) \right) \right)} \quad (13)$$

Proof: Condition (13) follows by equating the price of debt (τ) that results from households h_1 and h_2 ' budget constraints of period 0 and state s_1 . ■

Roughly speaking, differences in wealth between households of different jurisdictions determine the difference in credit that these households extend to developers. Because households use their loan payments in the second period to purchase the CRE good of their respective jurisdiction, differences in loan amounts determine differences in the valuation of CRE goods and also differences in the (endogenous) cash flows generated by the different CRE assets.

Lemma 5: *The CRE equity price accrued of the property tax in a jurisdiction is driven by the*

value of the CRE good produced and sold in that jurisdiction. In particular,

$$r(q_1 + \gamma_1) = 2(p_{11}(1)Y_{11}(1)) \quad (14)$$

$$r(q_2 + \gamma_2) = 2(p_{21}(1)Y_{21}(1)) \quad (15)$$

Proof: Conditions

$$\lambda_1^i(q_1 + \gamma_1) = 2(\lambda^i(s_1) + \lambda^i(s_2))(p_{11}(1)Y_{11}(1))$$

$$\lambda_1^i(q_2 + \gamma_2) = 2(\lambda^i(s_1) + \lambda^i(s_2))(p_{21}(1)Y_{21}(1))$$

follow from investor i 's first order conditions with respect to CRE equity E_1^i and E_2^i , respectively, where λ_1^i and $\lambda^i(s)$ are the shadow values of investor's budget constraints in period 1 and state s of period 2, respectively. These equations take into account conditions (8) and (9).³² With risk-free debt, we have that $\lambda_1^i/(\lambda^i(s_1) + \lambda^i(s_2)) = r$ and, therefore, conditions (14) and (15) follow accordingly. ■

Lemma 6: *When developers borrow at their maximum capacity, the total amount of debt in the economy equals the resources that households have in the first period, i.e.,*

$$\tau(\bar{D}^{d_1} + \bar{D}^{d_2}) = \sum_{k=\{k_1, k_2\}} \left(\omega_{0k}^{h_k} + \delta_k^{h_k} \left((\gamma_k - \varepsilon(\alpha_k)) TFP_k (x_{1k}^{H_1})^{\alpha_k} - \lambda(\alpha_k) \right) \right) \quad (16)$$

Moreover, the equilibrium price of debt τ must satisfy the following equations:

$$\tau = \left(p_{11}\omega_{11}^{H_1} - (q_1 + \gamma_1)E_1^i + \gamma_1 TFP_1 (x_{11}^{d_1})^{\alpha_1} - \delta_1^{d_1} \left((\gamma_1 - \varepsilon(\alpha_1)) TFP_1 (x_{11}^{d_1})^{\alpha_1} - \lambda(\alpha_1) \right) \right) / \bar{D}^{d_1} \quad (17a)$$

$$\tau = \left(p_{21}\omega_{21}^{H_1} - (q_2 + \gamma_2)E_2^i + \gamma_2 TFP_2 (x_{21}^{d_2})^{\alpha_2} - \delta_2^{d_2} \left((\gamma_2 - \varepsilon(\alpha_2)) TFP_2 (x_{21}^{d_2})^{\alpha_2} - \lambda(\alpha_2) \right) \right) / \bar{D}^{d_2} \quad (17b)$$

³²Notice that (14) and (15) assume that the shadow values of sign constraints $E_{jk}^{d_k} \geq 0$ and $E_{jk}^{d_k} \leq y_{jk}^{d_k}$ are zero (below, we will verify that our equilibrium is such that these constraints are non-binding for both $k = k_1, k_2$).

Proof: Equations (16), (17a), and (17b) follow from conditions (4) and (5), together with the market clearing equation for debt, the market clearing condition for equity (at the jurisdiction level), and the developers' budget constraint in the first period. ■

Remark 3: *In the numerical examples of Section A.3, we have an equilibrium where developers borrow at their maximum capacity.*

Developers are the only agents in this economy who can create CRE assets. For that, they need to buy CRE construction inputs. The respective market clearing conditions imply that these purchases must be such that $x_{11}^{d_1} = \omega_{11}^{H_1}$ and $x_{21}^{d_2} = \omega_{21}^{H_1}$. In addition, the equilibrium must satisfy the following condition:

Lemma 7: *The difference in prices of CRE construction inputs in different jurisdictions is driven by the difference in marginal productivity of the CRE assets. In particular,*

$$p_{11} - p_{21} = \alpha_2 TFP_2 (\omega_{21}^{H_1})^{\alpha_2 - 1} (p_{21}(1)Y_{21}(1) - \delta_2^{h_2}(\gamma_2 - \varepsilon(\alpha_2))) - \alpha_1 TFP_1 (\omega_{11}^{H_1})^{\alpha_1 - 1} (p_{11}(1)Y_{11}(1) - \delta_1^{h_1}(\gamma_1 - \varepsilon(\alpha_1))) \quad (18)$$

Proof: (18) follows from the first order optimality conditions of developers d_1 and d_2 with respect to CRE construction inputs 11 and 21, respectively, and conditions (8) and (9). ■

A.3 Numerical examples

In this subsection, we provide all details regarding the computation of examples corresponding to Figures 1, 2, and 3 (Examples 1, 2, and 3, respectively). In addition, here we also discuss how poor CRE asset performance may affect price inflation of CRE goods in a jurisdiction (Example 4).

Example 1 (benchmark): Let us consider the following parameter values: $\omega_{11}^{H_1} = \omega_{21}^{H_1} = \omega_0^{H_1}(1) = \omega_0^{H_1}(1) = \omega_{01}^{h_1} = \omega_{01}^{h_2} = \omega_{01}^i = 1$, $\bar{r} = 1$, $\bar{D}^{d_1} = \bar{D}^{d_2} = 1$, $TFP_1 = TFP_2 = 1$,

$\alpha_1 = \alpha_2 = 0.6$, $Y_{11}(1) = Y_{21}(1) = 1$, $Y_{11}(2) = Y_{21}(2) = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\varepsilon(\alpha_1) = \alpha_1/6$, $\varepsilon(\alpha_2) = \alpha_2/6$, $\lambda(\alpha_1) = \hat{\lambda}_1\alpha_1$ with $\hat{\lambda}_1 = 0.333$, $\lambda(\alpha_2) = \hat{\lambda}_2\alpha_2$ with $\hat{\lambda}_2 = 0.333$, $\delta_1^{H_1} = 0$, $\delta_1^{h_1} = \delta_2^{h_2} = 0.7$, $\delta_1^{d_1} = \delta_2^{d_2} = 0.2$, and $\delta_1^i = \delta_2^i = 0.1$. All θ -parameters are equal to zero, except for $\theta_{01}^{H_1} = 3$, $\theta_{11}^{h_1}(1) = \theta_{21}^{h_2}(1) = 1$, $\theta_{11}^{h_1}(2) = \theta_{21}^{h_2}(2) = 0.5$, $\theta_0^{d_1}(s) = \theta_0^{d_2}(s) = 0.24$, and $\theta_0^i(s) = 0.52$. We obtain a unique equilibrium solution where $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_{11} = p_{21} = 1.500$, $p_{11}(1) = p_{21}(1) = 1.500$, $p_{11}(2) = p_{21}(2) = 3.000$, and $E_1^i = E_2^i = 0.173$.³³

Figure 1 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2 , namely, the developer's debt, the developer's equity, and the investor's equity. These quantities are expressed in real terms, i.e., nominal amount times the corresponding price.

When analyzing the capital structure of a CRE development project, analysts look at financial ratios, such as the debt-to-equity ratio or the ratio of an investor's equity-to-total-equity. A debt-to-equity ratio higher than 0.500 indicates that the CRE capital structure has a greater proportion of its capital funding from lenders rather than equity investors. An "investor's equity-to-total-equity" ratio higher than 0.500 indicates that the investor owns more than 50 percent of the equity of a CRE asset. Our theory obtains these ratios endogenously determined in equilibrium. For example, the ratios corresponding to our benchmark example in Figure 1 are 0.576 for the "developer's debt-to-total-equity" ratio, 0.827 for the "developer's equity-to-total-equity" ratio, and 0.173 for the "investor's equity-to-total-equity" ratio. The respective ratios are the same in both jurisdictions. ■

Example 2 (funding capacity):³⁴ Let us modify our benchmark example by increasing the debt limit for developer d_1 to $\bar{D}^{d_1} = 1.1$, while decreasing the debt limit for developer d_2 to $\bar{D}^{d_2} = 0.9$.³⁵ Roughly speaking, access to funding for developers in the first jurisdiction improves, while

³³We first obtain the value of equilibrium variables q_1 , q_2 , τ , p_{11} , p_{12} , $p_{11}(1)$, $p_{12}(1)$, $p_{11}(2)$, $p_{12}(2)$, E_1^i , and E_2^i by solving the following system of equations: (7), (8), (9), (12), (13), (6), (14), (15), (16), (17a), (17b), and (18). These equilibrium values, in turn, allow us to solve for the rest of the equilibrium variables by using the agents' budget constraints and market clearing equations.

³⁴See Sun, Titman, and Twite (2015) for evidence of the impact of the recent financial crisis on the limits to debt capacity for commercial real estate assets.

³⁵By offsetting the decreasing in \bar{D}^{d_2} with an increase in \bar{D}^{d_1} , we are able to keep the remaining components of

it worsens for those in the second jurisdiction. The new equilibrium is such that $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_{11} = p_{21} = 1.500$, $p_{11}(1) = p_{21}(1) = 1.500$, $p_{11}(2) = p_{21}(2) = 3.000$, $E_1^i = 0.125$, and $E_2^i = 0.221$. Figure 2 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2 for the new parameter values. As mentioned in Section 3, we see that, compared to our benchmark example, the developer with worse funding capacity (d_2) decreases his absolute real exposure to CRE debt and equity, while the investor increases his equity exposure. The opposite happens in the CRE development project of the developer with better funding capacity.

We can get further insights into the composition of the two capital structures by looking at and comparing financial ratios. For the parameter values of example 2, we find a “developer’s debt-to-total-equity” ratio equal to 0.634 in CRE asset j_1 and 0.518 in CRE asset j_2 ; a “developer’s equity-to-total-equity” ratio equal to 0.875 in CRE asset j_1 and 0.779 in CRE asset j_2 ; and an “investor’s equity-to-total-equity” ratio equal to 0.125 in CRE asset j_1 and 0.221 in CRE asset j_2 . We summarize the equilibrium values of the financial ratios under consideration for Examples 1 and 2 in table 1.

| | Example 1 (benchmark example) | | Example 2 (funding capacity) | |
|--|----------------------------------|-------|---------------------------------|-------|
| | k_1 | k_2 | k_1 | k_2 |
| developer’s debt to total CRE equity | 0.576 | 0.576 | 0.634 | 0.518 |
| developer’s equity to total CRE equity | 0.827 | 0.827 | 0.875 | 0.779 |
| investor’s equity to total CRE equity | 0.173 | 0.173 | 0.125 | 0.221 |

Table 1: This table reports the financial ratios of the equilibria corresponding to Examples 1 and 2.

Compared to our benchmark example, we conclude that both the developer’s debt-to-equity ratio and equity-to-total-equity ratio decrease (increase) for the CRE capital structure corresponding to the developer with worse (better) funding capacity, while the investor’s equity-to-total-equity ratio increases (decreases, respectively). ■

equations (12) and (16) with the same equilibrium value as in the benchmark example.

Example 3: Now suppose that jurisdiction 2 experiences an increase in its fixed costs $\hat{\lambda}_2$ from 0.2. to 0.5, and that the jurisdiction's manager responds by increasing its tax rate γ_2 from 0.5 to 0.8.³⁶ The other parameter values are as in Example 1. In this case, $q_1 = 2.500$, $q_2 = 2.200$, $\tau = 1.590$, $p_{11} = 1.572$, $p_{21} = 1.428$, $p_{11}(1) = p_{21}(1) = 1.500$, $p_{11}(2) = p_{21}(2) = 3.000$, $E_1^i = 0.147$, and $E_2^i = 0.199$.

Because the property tax γ_2 is paid by both the developer d_2 and the investor i , we expect that this tax increment has an impact on how these two agents allocate their resources. Figure 3 illustrates this. The increase in γ_2 decreases the developer d_2 's equity compared to the equilibrium value of our benchmark Example 1. As explained in Section 3, this change is due to the developer's substitution of equity for tax free debt. In particular, developer's debt increases because τ jumped from 1.440 (in Example 1) to 1.590 (in Example 3), while D^{d_2} remained equal to $\bar{D}^{d_2} = -1$. The developer d_2 's debt and the investor's equity contributions increase as a result.

In the other jurisdiction, the investor's equity contribution decreases. Interestingly, the increment in E_2^i and the decrease in E_1^i respond to the change in the equity price q_2 . Compared to Example 1, q_2 has decreased from 2.500 to 2.200, while q_1 has remained the same (2.500). Roughly speaking, equity in CRE asset j_2 has become relatively cheaper. In CRE asset j_1 , we also see that the developer's debt (τD^{d_1}) and equity contributions ($q_1 E_1^{d_1}$) increase.

| | Example 1 (benchmark) | | Example 3 (Tax policy γ_2) | |
|--|--------------------------|-------|---------------------------------------|-------|
| | k_1 | k_2 | k_1 | k_2 |
| developer's debt to total CRE equity | 0.576 | 0.576 | 0.636 | 0.723 |
| developer's equity to total CRE equity | 0.827 | 0.827 | 0.853 | 0.801 |
| investor's equity to total CRE equity | 0.173 | 0.173 | 0.147 | 0.199 |

Table 2: This table reports the financial ratios of the equilibria corresponding to Examples 1 and 4.

Table 2 reports the equilibrium values for the three financial ratios under consideration. It provides a complementary perspective on the comparison between the capital structures (in real

³⁶Notice that these changes are such that neither jurisdiction k_2 's profits nor the profit components of equilibrium equations described in the above simplified economy change.

absolute amounts) in the two CRE assets. There we see that the debt-to-equity ratio increases for j_2 , while the developer's equity-to-total-equity ratio $E_2^{d_2}/(E_2^i + E_2^{d_2})$ decreases. This responds to the developer d_2 's substitution effect between equity and tax free debt. The investor's equity-to-total-equity ratio $E_2^i/(E_2^i + E_2^{d_2})$ makes up the difference. ■

Example 4: Let us consider again the same parameter values as in the benchmark example, except that now we modify the amount of the CRE good 21 produced at state s_2 by CRE asset j_2 ; in particular, let $Y_{21}(2)$ decrease from 0.5 to 0.1. Possible reasons are a natural disaster that negatively impacts the CRE asset's production capacity, a reduction in the supply of intermediate inputs captured by $f_{j_2}(y_{j_2}^{d(2)})(2)$, or even political reasons, such as a policy of expropriation of resources. In this new equilibrium, only the the price of the CRE good 21 at state s_2 changes ($p_{21}(2) = 15.000$). The values of other equilibrium variables, including the financial ratios under consideration, do not change with respect to the benchmark example.³⁷ ■

³⁷In particular, $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_{11} = p_{12} = 1.500$, $p_{11}(1) = p_{12}(1) = 1.500$, $p_{11}(2) = 3.000$, $p_{12}(2) = 15.000$, and $E_1^i = E_2^i = 0.171$.