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Heterogeneous Tiebout Communities with Private Production and Anonymous Crowding

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Heterogeneous Tiebout communities with private production and anonymous crowding ☆

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ABSTRACT

This paper provides a general equilibrium model where jurisdictions offer not only public goods, but also job opportunities. In a context of multiple types of consumers, labor complementarities, and anonymous crowding, heterogeneous populated communities form in equilibrium with an endogenous wage system that is labor-type and jurisdiction-type dependent. Equilibrium jurisdiction structures depend on the relative scarcity of labor types, unlike the situation in Berglas' (1976) partial equilibrium analysis. For a large economy, we prove that equilibrium exists and that the set of equilibria is equivalent to the core.

1. Introduction

Tiebout (1956) claimed that in a model with local public goods and many jurisdictions (among other conditions), equilibrium will feature sorting of consumers by type and that the resulting allocation will be efficient. Twenty years later, Berglas (1976) proposed a frictionless production Tiebout economy, in which private goods are produced within each community by means of its labor inputs, thereby dropping the unnecessary and unrealistic assumption of "no restrictions due to employment opportunities" from the Tiebout model. In an anonymous crowding scenario (the consumers care only about the level of congestion of the public goods and not about the identities of the other individuals making use of them), Berglas showed that if individuals differ in their productive skills (teachers, accountants, unskilled workers, etc.), if the distributions of tastes for public goods and labor skills are independent, and if labor skills in a community are complementary, then individuals may be better off forming mixed communities rather than sorting into homogeneous communities.

Berglas' purpose was to analyze the formation of mixed communities, its efficiency, and the associated tax structures. However, he did not demonstrate the existence of equilibrium and his existence conjecture was subject to Bewley's (1981) criticism. The issue of existence of equilibrium was also left aside in the subsequent literature that analyzes the formation of mixed communities (see, for example, McGuire, 1991). Berglas (1976) and other subsequent works suffer from several shortcomings that prevent the study of equilibrium existence. In particular, the approach through differential techniques is inappropriate when considering the population and locations as discrete sets (see Wooders, 1978).

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More concretely, Berglas (1976) considered a framework with two types of consumers with total populations M and N, but assumed the integer condition $n^*/m^* = N/M$, where n^* and m^* denote the optimal numbers of each type of consumer in a mixed community. Given the assumption $n^*/m^* = N/M$, it is unclear if Berglas' result holds in a general equilibrium setting, and in particular, whether Berglas' naive integer assumption prevents the existence of many different types of heterogeneous populated jurisdictions. Our general equilibrium approach to Berglas' (1976) problem highlights that depending on total population composition, there can be many possible jurisdiction types. For instance, doctors may prefer smaller jurisdictions, while teachers may like larger jurisdictions — all of them matter in determining the composition of existing jurisdiction types in equilibrium, their wages, and their utility levels. In Section 2 we provide three motivating examples that argue in favor of the necessity of a general equilibrium approach to Berglas (1976). In particular, Example 1 illustrates how in Berglas' (1976) set-up we cannot analyze the wage structures for various different jurisdiction structures, as population stratification is restricted by Berglas' naive integer assumption. Instead, our general equilibrium theory allows for the formation of multiple different mixed populated jurisdictions. Example 2 elaborates on the previous example and shows that indeed inefficient homogeneous jurisdictions can exist in equilibrium. Example 3 points out that jurisdiction composition affects the type of optimal mixed jurisdictions that will form in equilibrium through wage differentials and congestion. Unlike what we have in Berglas' (1976) partial equilibrium analysis, equilibrium jurisdiction structures depend on the relative scarcity of labor types.

Motivated by Berglas' (1976) work, we propose a general equilibrium model that incorporates Berglas' main assumptions: anonymous crowding, different labor skills among consumer types, and labor complementarities among consumer types in the private good production. Our equilibrium price system associates a wage with each type of consumer in a given jurisdiction type. These type dependent wages decentralize the optimum. Another feature of our economy is that jurisdictions are "small" in size. We model negligible jurisdictions following the works of Allouch et al. (2009) and Ellickson et al. (1999). In their interpretation, the world population splits into city districts, municipalities, villages, and counties, and not into countries of a "large" size.

The literature on community composition coming after Tiebout's tale has been extensive. It is well established (Scotchmer and Wooders, 1987) that communities should be taste-homogenous if crowding types are exogenous and crowding is anonymous. Allouch et al. (2009) and Ellickson et al. (1999, 2006) recover heterogeneity in the community composition by considering an economy with non-anonymous crowding (consumers have preferences for the other types of consumers with whom they wish to share a jurisdiction). We differ from these papers by showing that heterogeneous communities can exist and are optimal in a context of anonymous crowding and exogenous crowding types if a local collaborative production technology is added into the picture. To our knowledge, Conley and Wooders (2001) and Konishi (2010, 2011) are the only exceptions in the theoretical literature that obtain optimal taste-heterogeneous jurisdictions in a context of anonymous crowding. However, both models differ from the present one. On the one hand, Conley and Wooders (2001) obtain heterogeneity in community composition through different agents' genetic endowments and endogenous crowding types (through educational choices). Although these authors discuss the need to drop Tiebout's assumption of "no restriction due to labor opportunities,"[1] their model does not incorporate a private production sector associated with a jurisdiction. Konishi (2010, 2011) showed that mixed clubs are efficient when there are local consumption externalities and/or multiple facilities are subject to joint production, whereas in the present paper heterogeneous communities arise due to the labor complementarities in the production of a jurisdiction industry.

The rest of this paper is as follows. Section 2 provides three motivating examples. Section 3 reformulates Berglas (1976) into a general equilibrium model. Section 4 gives the equilibrium existence and the core decentralization results. Appendix is devoted to the proofs.

2. Motivating examples

Let us provide three motivating examples that illustrate how Berglas' (1976) model is unsatisfactory for the study of the formation of heterogeneous populated jurisdictions and their associated wage structures. Different problems are pointed out: Berglas' naive integer assumption (Example 1), the existence of inefficient homogeneous jurisdictions (Example 2), and how scarcity of labor types affects the wage structure in equilibrium (Example 3).

Example 1. Integer problem and mixed communities

Let us consider an economy with only one commodity and no crowding (no externalities from type composition). We consider two types of consumers, whose population measures are 7 for type 1 consumers and 3 for type 2 consumers. We consider the profiles of consumers $A = (3,2)$ and $B = (4,1)$ only. Each profile consists of a combination of type 1 and type 2 consumers (say, engineers and miners). Jurisdictions with profiles A and B can produce 100 units and 75 units of a single commodity, respectively. Wages will be 10 and 35 for consumer types 1 and 2, respectively, so production surplus is fully shared among consumers. It is clear that Berglas' (1976) naive integer assumption does not hold (for instance, jurisdictions with consumer profile A violate this condition since $3/2 \neq 7/3$). In our general equilibrium model this type of community exists if there is a measure 1 for each type of jurisdiction (population consistency). The bottom line of Example 1 is that in Berglas' (1976) set-up we cannot analyze the wage structures for various different jurisdiction structures, as population stratification is restricted in order to satisfy Berglas' naive integer assumption. Our general equilibrium theory will allow for the formation of multiple different mixed populated jurisdictions. ■

Example 2. Inefficient homogeneous jurisdictions

Let us consider a similar set-up as in Example 1, but now with the following consumer profiles: $C = (1,1)$ (heterogeneous populated jurisdiction), and $D = (2,0)$ and $E = (0,2)$ (homogeneous populated jurisdictions). The associated production output levels are 100, 80 and 80, respectively. Wages are 40 and 60 for types 1 and 2 consumers in C, respectively, and 40 for both consumer types in D and E. Population consistency holds by assuming a measure 3 of type C jurisdictions and a measure 2 of type D jurisdictions. Clearly, jurisdictions with profile C are efficient (in Berglas' sense), but in equilibrium we find that inefficient homogeneous jurisdictions (with profile D) can also exist. ■

Example 3. Wage differentials and scarce labor types

In this example the composition of consumer types in a jurisdiction matters. Let us consider two types of jurisdictions, ω_1 and ω_2. For simplicity, we assume that both jurisdictions offer the same public good, which costs 1. Both types of jurisdictions are populated by physicians (P) and miners (M), but in different amounts: $(n^P_{\omega_1}, n^M_{\omega_1}) = (1,\ 2)$ and $(n^P_{\omega_2}, n^M_{\omega2}) = (1,\ 20)$.[2] Each consumer (miner

[1] Konishi (2008) actually shows that "no restrictions due to employment opportunities" is not needed, but in a very different context. He assumes that there are different regions with different exogenous labor productivities, and in equilibrium he obtains homogeneous popuplated communities, due to a zoning constraint.

[2] Notation should be clear, e.g., $n^P_{\omega_1}$ denotes the number of physicians in jurisdiction ω_1.

or physician) is endowed with 1 unit of the commodity. The industry has the following production function: for $\omega=\omega_1,\omega_2$, $y_\omega(n^P_\omega, n^M_\omega)=n^P_\omega n^M_\omega$—we can think that physicians increase miners' productivity by improving their health conditions. Wages are determined as follows: $\alpha^P_\omega=(n^M_\omega/n_\omega)y_\omega(n^P_\omega, n^M_\omega)$ and $\alpha^M_\omega=(1-n^M_\omega/n_\omega)$ $y_\omega(n^P_\omega, n^M_\omega)$, for both jurisdictions, $\omega=\omega_1,\omega_2$. Given the above consumer profiles, wages are $\alpha^P_{\omega_1}=4/3$, $\alpha_{\omega_1}{}^M=1/3$, $\alpha^P_{\omega_2}=400/21$, and $\alpha^M_{\omega_1}=1/21$. Physicians earn 4 times the wage of miners in jurisdiction ω_1, and 400 times the wage of miners in jurisdiction ω_2. Congestion may decrease physicians' utility. For instance, let us consider that utility is always equal to the wage, except for physicians in ω_2, whose utility is 8/10 times his wage. Then, both physicians and miners will prefer to sort into jurisdictions with consumers' profile (1,2). In this example, the scarcity of labor types determines the equilibrium type of mixed jurisdictions through wage differentials and congestion. ■

3. Berglas (1976) reformulated

3.1. Consumers and jurisdictions

Let us consider a simple framework with one commodity and a continuum of consumers. Consumers sort into jurisdictions. Each jurisdiction provides a public project[3] and an industry to work with. The set of consumers' types is finite (e.g., physicians, engineers, and miners) and denoted by $\mathbf{\Theta}=\{1, \ldots, \Theta\}$. Each consumer h has an associated type $\theta\in\mathbf{\Theta}$. We refer to a representative consumer of type $\theta\in\mathbf{\Theta}$ by h_θ. A *consumer's type* is a complete description of consumer's endowments, membership characteristics, and preferences (described below). All consumers of the same type are endowed with the same positive amount of private good, i.e., $e(h_\theta)=e^\theta>0$, for all h_θ. *Membership characteristics* indicate the role of a consumer type in a jurisdiction. In particular, the consumer's membership characteristic describes the consumer's skills to carry out a certain type of work in the jurisdiction industry. Consumers of the same type have the same skills. Skills are observable and contractible.[4] In jurisdiction ω consumer h_θ's preferences are represented by a utility function $\tilde{u}^{h_\theta}(x, g_\omega, n_\omega)$ that increases with the level of private good consumption (x), depends on the type of public good g_ω provided in the jurisdiction, and decreases with the level of congestion n_ω of the jurisdiction. We assume that consumers care only about the level of congestion of the public goods and not about the identities of the other individuals making use of the public goods, i.e., *anonymous crowding* .

A *jurisdiction type* ω is described by the membership characteristics of its community and its organizational characteristics, in Ellickson et al. (2006) terminology. One of the organizational characteristics of a jurisdiction is the profile of consumers $\mathcal{C}_\omega\equiv(n^\theta_\omega)_{\theta\in\mathbf{\Theta}}$, where $n^\theta_\omega\in\mathbb{N}$ denotes the number of type θ consumers in a jurisdiction type ω.[5] Then, $n_\omega=\sum_{\theta\in\mathbf{\Theta}}n^\theta_\omega$ denotes the level of congestion of jurisdiction ω. In addition, we assume that the organization of a jurisdiction is also characterized by a public project g_ω and a single-output production technology y_ω (described below). A jurisdiction type is then characterized by a policy package $\omega\equiv(g_\omega, y_\omega, \mathcal{C}_\omega)$. Without loss of generality, we refer to a type ω jurisdiction by ω. The set of possible jurisdiction types is finite and is denoted by $\mathbf{\Omega}=\{1, \ldots, \omega, \ldots \Omega\}$. Thus, the sets of public projects and production technologies, denoted by $Y=\{y_\omega\}_{\omega\in\mathbf{\Omega}}$ and $G=\{g_\omega\}_{\omega\in\mathbf{\Omega}}$, respectively, are also finite.

For each public project $g_\omega\in G$, there is a pre-determined associated cost $inp(g_\omega)\geq0$ in terms of the private good (see also Ellickson et al., 2006; Konishi, 2008). A single-output *production technology* y_ω maps labor inputs (a body of consumers, possibly of different types, engaged in the production activity) in output (private commodity).[6] We assume that each consumer in a jurisdiction ω supplies his unit of labor inelastically to the jurisdiction industry, so the relevant vector of labor inputs consists of the jurisdiction profile of consumers $\mathcal{C}_\omega\equiv(n^\theta_\omega)_{\theta\in\Theta}$.[7] Each jurisdiction produces at its maximum capacity. We denote jurisdiction ω's production surplus by $y_\omega(\mathcal{C}_\omega)$.[8] The profile of consumers $\mathcal{C}_\omega$ may exhibit labor complementarities.[9] The jurisdiction surplus $y_\omega(\mathcal{C}_\omega)$ is shared among the consumers of the jurisdiction according to a rule s_ω that assigns the fraction $s^\theta_\omega\in(0,1)$ to consumers of type θ in jurisdiction ω, such that $\sum_\theta s^\theta_\omega=1$. Wages are both consumer-type and jurisdiction-type specific. The wage of a type θ consumer in jurisdiction ω is $\alpha^\theta_\omega=s^\theta_\omega y_\omega(\mathcal{C}_\omega)/n^\theta_\omega$.[10] Finally, let $\tilde{\alpha}^\theta_\omega$ denote the consumer θ's income associated with jurisdiction ω, i.e., the wage net of the visa price paid to access jurisdiction ω. For our purpose, we look only at the price variable $\tilde{\alpha}^\theta_\omega$ and will see how heterogeneous communities may form in equilibrium with a wage system $\tilde{\alpha}$. The consumer h_θ's budget constraint in jurisdiction ω is

$$x^{h_\theta}\leq\tilde{\alpha}^\theta_\omega+e^\theta \quad \left(BC_{h_\theta}(\omega)\right)$$

3.2. General equilibrium

In this section we enrich the simple set-up presented above. Let the set of consumers be represented by a nonatomic finite measure space $(\mathbf{H}, \mathscr{H}, \lambda)$, where $\mathscr{H}$ is a σ-algebra of subsets of the set of consumers $\mathbf{H}$, and λ is the associated Lebesgue measure. Considering a *large economy* is useful in order to avoid non-convexities associated with the consumer's choice problem. We guarantee that each jurisdiction is negligible with respect to the whole economy by assuming that *each jurisdiction has a finite number of consumers*. The set of type θ consumers is denoted $H(\theta)\in\mathscr{H}$ and has a finite measure $\lambda(H(\theta))>0$. For simplicity, we consider only one private good, with price normalized to 1. The *endowment* mapping e: $h_\theta\mapsto e(h_\theta)$ is an integrable function. Endowments are observable. We assume that endowments are uniformly bounded above and that the aggregate endowment is strictly positive, i.e., $\sum_\theta\int_\mathbf{H}e(h_\theta)d\lambda>E$, with $E>0$.

By acquiring a *visa permit* (or *membership*), a consumer gains access to the jurisdiction, so that he can consume its public goods and work in the jurisdiction industry. The visa is consumer-type (θ) and jurisdiction-type (ω) specific. We denote it by $m=(\theta,\omega)$. The set of visas is denoted by $\mathbf{M}$. A list is a function ι: $\mathbf{M}\to\{0,1,\ldots\}$, where $\iota(\theta, \omega)$ represents the number of visas of type (θ, ω). We write $\mathbf{Lists}=\{\iota:\iota$ is a list$\}$. A consumer h_θ also has an associated consumption set. We define the consumer h_θ's consumption set $\mathbf{X}^{h_\theta}\subset\mathbb{R}_+\times\mathbf{Lists}$ as the set of feasible bundles of private good consumption x^{h_θ} and visa permit μ^{h_θ} that consumer h_θ can choose. The consumption set correspondence $h_\theta\to\mathbf{X}^{h_\theta}$ is assumed to be a measurable correspondence. Because consumers choose non-negative numbers of jurisdiction memberships,

[3] A public project consists of a discrete set of public goods, such as a school or a park, in the sense of Mas-Colell (1980).

[4] By *contractible* we mean an agreement between two parties, the jurisdiction industry and the consumer with specific skills, that is enforceable by law.

[5] As in all previous Tiebout literature, we assume that every jurisdiction has the capacity to expel "illegal consumers" (exclusion).

[6] Following Berglas (1976), the production of the private good requires only labor. A model where production is a function of both labor inputs and physical capita would give us no further insights on the group composition problem, and is thus omited.

[7] In this paper we do not model externalities within a firm, such as poor working conditions and uncongenial co-workers (Ellickson et al., 2006) or contractual problems (Zame (2007)).

[8] This treatment of production differs from Wooders (1978), who makes production dependent only on the size of the jurisdiction, and also from Benabou (1993), who models production as a citywide activity (same industry for several jurisdictions).

[9] Labor complementarities in the sense that the production decreases when we change the consumers' profile by making the number of at least one type of consumer in ω equal to 0.

[10] The rule s_ω can simply be an equal division of the jurisdiction's production surplus among the consumers of the jurisdiction: $\alpha^\theta_\omega=y_\omega(\mathcal{C}_\omega)/n_\omega$ for all θ, where $s^\theta_\omega=n^\theta_\omega/n_\omega$, for all θ in ω.

we define the choice function $\mu : \mathbf{H} \rightarrow \mathbf{Lists}$. For our economy, we impose that if $(x^{h_\theta}, \mu^{h_\theta}) \in \mathbf{X}^{h_\theta}$ and $\hat{x}^{h_\theta} \geq x^{h_\theta}$, then $(\hat{x}^{h_\theta}, \mu^{h_\theta}) \in \mathbf{X}^{h_\theta}$. Also, we assume that each consumer chooses at most one jurisdiction membership (the place to live and work), i.e., $\sum_{m \in \mathbf{M}} \mu^{h_\theta}(m) \leq 1$. The set $\mathbf{Lists}(h_\theta) = \left\{ \mu^{h_\theta} \in \mathbf{Lists} : \sum_{m \in \mathbf{M}} \mu^{h_\theta}(m) \leq 1, \exists x^{h_\theta} \text{ s.t.} (x^{h_\theta}, \mu^{h_\theta}) \in \mathbf{X}^{h_\theta} \right\}$ can be seen as the consumer h_θ's restricted consumption set of visa permits compatible with his private consumption. The aggregate of type (θ,ω) memberships is $\hat{\mu}(\theta,\omega) \equiv \int_{H(\theta)} \mu^\theta(\theta,\omega) d\lambda$. We require consistent matching of consumers in terms of the aggregate of choices (see also Kaneko and Wooders, 1986; Ellickson et al., 1999). We say that the aggregate membership vector $\hat{\mu} \in \mathbb{R}^{|\mathbf{M}|}$ is *consistent* if for every jurisdiction type $\omega \in \mathbf{\Omega}$ there is a real number $\gamma(\omega)$ such that $\hat{\mu}(\theta,\omega) = \gamma(\omega) n_\omega^\theta$, $\forall \theta \in \mathbf{\Theta}$ Here, $\gamma(\omega)$ is read as the "number" of type ω jurisdictions. The choice function $\mu : \mathbf{H} \rightarrow \mathbf{Lists}$ is consistent for $H \subseteq \mathbf{H}$ if the corresponding vector is consistent. We write $\mathbf{Cons} \equiv \{ \hat{\mu} \in \mathbb{R}^{|\mathbf{M}|} : \hat{\mu} \text{ is consistent} \}$.

Utility function: Utility $\tilde{u}^{h_\theta}(\cdot, g_\omega, n_\omega)$ is continuous, strictly monotonic, and quasiconcave. Congestion is irrelevant when no public good is offered in the jurisdiction.[11] As mentioned above, we assume *anonymous crowding*. [12] As in Ellickson et al. (1999, 2006) we assume that *endowment is desirable*, which means that each consumer would prefer to remain single and consume his endowment rather than belong to any feasible jurisdiction and consume no private good.[13] Notice that for this one private good economy we do not need to worry about the "minimum expenditure situation" that may occur in a local public good economy with more than one private good, and therefore we do not need to impose the "jurisdiction irreducibility" assumption (see Ellickson et al., 1999). As in Berglas (1973), we assume that consumers' skills are linked in a one-to-one relationship with the consumers' tastes for public goods.[14] This assumption is essential in order to prove the core decentralization (Theorem 2 below), since skills (or endowments) act as an efficient fiscal discriminatory device among consumers. A visa permit $m = (\theta, \omega)$ gives access to the consumption of the public project g_ω with associated level of congestion n_ω. Thus, we can write consumer h_θ's utility function as a function of the private good consumption and jurisdiction membership, i.e., $u^{h_\theta} : X^{h_\theta} \rightarrow \mathbb{R}$, where $u^{h_\theta}(x, \mu^{h_\theta}(m)) \equiv \tilde{u}^{h_\theta}(x, g_\omega, n_\omega)$, for $m = (\theta, g_\omega, y_\omega, \mathcal{C}_\omega)$. The utility mapping $(h_\theta, x, \mu) \rightarrow u^{h_\theta}(x, \mu)$ is a jointly measurable function of all its arguments.

4. Equilibrium and core decentralization

We consider the notion of a price-taking equilibrium as an efficient summary of the equilibrium corresponding to a competitive theory of jurisdictions formation.

Definition 1. An equilibrium for this Berglas' economy is a vector $(x, \mu, \hat{\alpha})$ such that:

(E.1) Consumers choose optimally: if there exists $\left(\tilde{x}_{\tilde{\omega}}^{h_\theta}, \tilde{\mu}^{h_\theta}(\theta, \tilde{\omega})\right) \in \mathbf{X}^{h_\theta}$ such that $u^{h_\theta}\left(\tilde{x}_{\tilde{\omega}}^{h_\theta}, \tilde{\mu}^{h_\theta}(\theta, \tilde{\omega})\right) > u^{h_\theta}\left(x_\omega^{h_\theta}, \mu^{h_\theta}(\theta, \omega)\right)$, then the membership $\tilde{\mu}^{h_\theta}(\theta, \tilde{\omega})$ is such that $\tilde{x}_{\tilde{\omega}}^{h_\theta} - e^\theta - \hat{\alpha}_{\tilde{\omega}}^\theta > 0$.

(E.2) Exhausted jurisdiction surplus:

$$y_\omega(\mathcal{C}_\omega) - inp(g_\omega) = \sum\nolimits_\theta n_\omega^\theta \hat{\alpha}_\omega^\theta, \forall \omega \in \mathbf{\Omega}.$$

(E.3) Market clearing for private goods:

$$\sum\nolimits_\theta \int_{H(\theta)} \left(x^{h_\theta} - e^\theta\right) d\lambda + \sum\nolimits_\omega \gamma(\omega)(inp(g_\omega) - y_\omega(\mathcal{C}_\omega)) = 0.$$

(E.4) Consistency: $\hat{\mu}$ is consistent for $\mathbf{H}$.

Next, we show that there is an equilibrium outcome (existence) and that it passes the standard test of perfect competition (coincidence of the core with the set of equilibrium allocations).

Theorem 1. Existence

There exists an equilibrium for this economy.

The proof, in Appendix, follows a simultaneous optimization approach. For this, we construct a generalized game, prove that this game has equilibrium in pure strategies, and show that equilibrium is, in fact, a price taking equilibrium for our Berglas' economy. This proof constitutes by itself a contribution to the clubs/local public goods literature. Our approach is different than the core decentralization approach (Conley and Wooders, 1997; Allouch et al., 2009) and the non-excess demand approach (Ellickson et al., 1999). We now focus on the core decentralization.

The pair (x, μ) is said to be a *feasible state* for a measurable set $H \subseteq \mathbf{H}$ of positive measure if 1) $(x, \mu) \in \mathbf{X}^h$ satisfies the budget constraints for each $h \in \mathbf{H}$, 2) the aggregate membership vector $\hat{\mu}(\theta,\omega)$ is consistent for H, and 3) the private good market clears for H. We say that (x, μ) is in the *core* if there is no subset $H \subset \mathbf{H}$ with $\lambda(H) > 0$ and a feasible state $(\tilde{x}, \tilde{\mu})$ for H such that $u^{h_\theta}(\tilde{x}^{h_\theta}, \tilde{\mu}^{h_\theta}) \geq \mu^{h_\theta}(x^{h_\theta}, \mu^{h_\theta})$ for all $h_\theta \in H$ and all $\theta \in \mathbf{\Theta}$, and $u^{h'_\theta}\left(\tilde{x}^{h'_\theta}, \tilde{\mu}^{h'_\theta}\right) > u^{h'_\theta}\left(x^{h'_\theta}, \mu^{h'_\theta}\right)$ for all $h'_\theta \in H' \subset H$ with $\lambda(H') > 0$ and all $\theta \in \mathbf{\Theta}$.

Theorem 2. Core equivalence

For this economy, the core coincides with the set of equilibria.

The proof of Theorem 2 follows Ellickson et al. (1999, Theorem 5.1). In Appendix, we indicate how to accommodate Ellickson et al.'s proof to our jurisdictions production economy.

Appendix

To prove equilibrium existence (proof of Theorem 1 below) and core decentralization (Theorem 2 below) it will be useful to decompose $\hat{\alpha}_\omega^\theta$ as follows: $\hat{\alpha}_\omega^\theta \equiv \alpha_\omega^\theta - t_\omega^\theta - \tau_\omega$, where $\tau_\omega = inp(g_\omega)/n_\omega$ is a poll fee, common to all consumers in the jurisdiction, and $t_\omega^\theta \in \mathbb{R}^\Theta$ is the transfer paid by a type θ consumer in jurisdiction ω, which can be positive, negative, or zero (as transfers internalize the externalities among the consumers in the jurisdiction, given their tastes for the public goods, wealth, and share of jurisdiction's production surplus).[15] Consumers' types are observable and thus all consumers of type θ in jurisdiction type ω pay the same transfer t_ω^θ. Transfers must be such that $\sum_{\theta \in \mathbf{\Theta}} t_\omega^\theta = 0$. We say that $t_\omega \in \mathbb{R}^\Theta$ is a pure transfer system if $t_\omega \in \mathbf{Trans}$, where $\mathbf{Trans} = \{ t_\omega \in \mathbb{R}^\Theta : \sum_\theta t_\omega^\theta \hat{\mu}(\theta,\omega) = 0, \forall \hat{\mu} \in \mathbf{Cons} \}$ The poll fees cover the cost $inp(g_\omega)$.

Proof of Theorem 1. We investigate the problem of existence of a price taking equilibrium by transforming it into a problem of

[11] That is, $\forall (n_\omega, n'_\omega)$ with $n_\omega \neq n'_\omega$, $\tilde{u}^{h_\theta}(x, \emptyset, n_\omega) = \tilde{u}^{h_\theta}(x, \emptyset, n'_\omega)$, where $\emptyset$ means that there is no public project.

[12] Formally, we assume that for every consumer $h_\theta \in \mathbf{H}$ and any pair of jurisdictions $\omega = (g_\omega, y_\omega, \mathcal{C}_\omega)$ and $\tilde{\omega} = (g_{\tilde{\omega}}, y_{\tilde{\omega}}, \mathcal{C}_{\tilde{\omega}})$ with $g_\omega = g_{\tilde{\omega}}$ and $n_\omega = n_{\tilde{\omega}}$, but $\mathcal{C}_\omega \neq \mathcal{C}_{\tilde{\omega}}$, we have $\tilde{u}^{h_\theta}(x, g_\omega, n_\omega) = \tilde{u}^{h_\theta}(x, g_{\tilde{\omega}}, n_{\tilde{\omega}})$

[13] Formally, we assume that for every consumer $h_\theta \in \mathbf{H}$ and every list $\iota \in \mathbf{Lists}(h_\theta)$, we have $u^{h_\theta}(x^{h_\theta}, \iota) > u^{h_\theta}(e^{h_\theta}, 0)$. This assumption is needed in order to assure that the weak core and the strong core coincide (see Ellickson et al., 1999, for the distinction between both concepts).

[14] Individuals with different working conditions (miners and engineers) may demand different public health services and are thus taxed differently.

[15] Observe that, even if wage α_ω^θ is high, the consumer may find a jurisdiction membership prohibitive if the taxes are very high.

existence of a social system equilibrium. Our approach is by simultaneous optimization. There, a player's payoff function and constraint set are parameterized by the other players' actions. This second dependence does not occur in games. The extension is a mathematical object referred to as a generalized game by Debreu (1952). We carry out this analysis in the continuum of agents framework. Most of our extensions follow by application of Hildenbrand's (1974) results. This approach has never been applied to a local public goods non-atomic economy.[16]

The generalized game

In the generalized game a player k chooses his strategy $s(k)$ parameterized by the other agents' strategies $\bar{s}_{-k}$. For this economy the game is played by the consumers and three additional auctioneers. We divide the consumers' optimization problem in two stages.

Stage 1: Consumer h_θ chooses his most preferred consumption for a given jurisdiction membership $m=(\theta,\omega)$ with $\mu^{h_\theta}(\theta,\omega)=1$. That is,

$$\max_{\{x_\omega\}} u^{h_\theta}\left(\cdot,\bar{\mu}^{h_\theta}(\theta,\omega)\right)\ s.t.\ x_\omega - e^\theta + \bar{t}^\theta_\omega + \bar{\tau}_\omega - \bar{\alpha}^\theta_\omega \le 0$$

Observe that the terms in $BC_{h_\theta}(\omega)$ are all multiplied by $\bar{\mu}^{h_\theta}(\theta,\omega)$, but we omit it as we know that $\bar{\mu}^{h_\theta}(\theta,\omega)=1$ (the consumer is evaluating his utility at this specific jurisdiction type). Let us denote consumer h_θ's consumption demand for the private good at jurisdiction ω by $\psi\left(h_\theta,\omega\right)\equiv\left\{\arg\max u^{h_\theta}\left(\cdot,\bar{\mu}^{h_\theta}\quad(\theta,\omega)\right):\mathrm{BC}_\omega(h_\theta)\text{ holds}\right\}$. Note that, $h_\theta\to\psi(h_\theta,\omega)$ has a measurable graph (see Hildenbrand, 1974, p. 59, Proposition 1.b). $\psi(h_\theta,\omega)$ has nonempty convex compact values and is continuous. Compactness follows because $\sum_\theta\int_{H(\theta)}e^\theta d\lambda$; ;∞. By quasiconcavity of u^{h_θ} and convexity of the values of the budget consumption set $B^{h_\theta}\left(\bar{t}^\theta_\omega,\bar{\tau}_\omega,\bar{\alpha}^\theta_\omega,e^\theta,\omega\right)\equiv\left\{x^{h_\theta}_\omega\in\mathbb{R}_+ : BC(h_\theta)\text{holds}\right\}$, it follows that $\psi(h_\theta,\omega)$ has convex values. $B\left(h_\theta,\bar{t}^\theta_\omega,\bar{\tau}_\omega,\bar{\alpha}^\theta_\omega,e^\theta,\omega\right)$ and $u^{h_\theta}(x_\omega,\mu^{h_\theta}(\theta,\omega))$ are continuous in x. By Berge's maximum theorem and endowment desirability, $\psi(h_\theta,\omega)$ is upper semi-continuous. Lower semi-continuity of $\psi(h_\theta,\omega)$ follows by the interiority of endowment assumption. Actually, the consumer's budget constraint holds with strict inequality by choosing a sufficiently small consumption. Thus, $\psi(h_\theta,\omega)$ is continuous. Let $\int_{H(\theta)}\psi(h_\theta,\omega)d\lambda$ represent the measurable demand from the continuum of type ω consumers at jurisdiction ω. Because $\int_{H(\theta)}\psi(h_\theta,\omega)d\lambda$ is the integral of upper semicontinuous demands with respect to a nonatomic measure, $\int_{H(\theta)}\psi(h_\theta,\omega)d\lambda$ is upper semicontinuous. Moreover, $\int_{H(\theta)}\psi(h_\theta,\omega)d\lambda$ is compact, convex, and has nonempty values. The compact-valued function $h_\theta\to\psi(h_\theta,\omega)$ is bounded below by 0 and above by the integrable function $h_\theta\to\sum_\theta\lambda(H(\theta))e^\theta+\sum_{\omega,\theta}\int_{H(\theta)}\alpha^\theta_\omega d\lambda$. According to Hildenbrand (1974, p. 62, Theorem 2), $\int_{H(\theta)}\psi(h_\theta,\omega;p)d\lambda\neq\emptyset$. And according to Hildenbrand (1974, p. 73, Proposition 7) this set, which is bounded below by 0, is also compact. Therefore, $\sum_{\theta\in\mathbf{\Theta}}\int_{H(\theta)}\psi(h_\theta,\omega)d\lambda$ is nonempty and compact. The aggregate consumption demand being convex-valued is a consequence of Lyapounov's convexity theorem of an atomless finite dimensional vector measure (see Hildenbrand, 1974, p. 62, Theorem 3).

Stage 2: Given their optimal consumption, consumers choose their most preferred jurisdiction type (notice that the consumer being alone in a jurisdiction is a possibility). Let $U^{h_\theta}(\omega,\bar{t}^\theta_\omega,\bar{\tau}_\omega,\bar{\alpha}^\theta_\omega)\equiv u^{h_\theta}(\psi(h_\theta,\omega),\mu^{h_\theta}(\theta,\omega))$. Then, $\mu^{h_\theta}(\theta,\omega)$ for $\omega\in\arg\max U^{h_\theta}(\omega,\bar{t}^\theta_\omega,\bar{\tau}_\omega,\bar{\alpha}^\theta_\omega)$. We represent the pure strategy of consumer h_θ by a basis vector of dimension Ω. The vector $\mu^{h_\theta}(\theta,\omega)$ is the vector in $\mathbb{R}^\Omega$ with 1 as ω^{th} coordinate and zero otherwise. By a parallel argument as above, there is a measurable selection $h_\theta\to\mu^{h_\theta}(\theta,\omega)$ with an associated aggregate demand vector $\sum_\theta\int_{H(\theta)}\mu^{h_\theta}(\theta,\omega)d\lambda$, which is the integral of upper semicontinuous demands with respect to a non-atomic measure. Thus, $\sum_\theta\int_{H(\theta)}\mu^{h_\theta}(\theta,\omega)d\lambda$ is upper semicontinuous, with compact (by the requirement $\sum_m\mu^{h_\theta}(m)=1$, for a.e. h_θ), convex (by Lyapounov's convexity theorem) and nonempty values.

Auctioneer 1 chooses $\alpha=(\alpha^\theta_\omega)_{\theta\in\mathbf{\Theta},\,\omega\in\mathbf{\Omega}}$ to minimize $\sum_{\omega,\theta}\left((\alpha^\theta_\omega n^\theta_\omega - s^\theta_\omega y_\omega(\mathcal{C}_\omega))\cdot\int_{H(\theta)}\bar{\mu}^{h_\theta}(\theta,\omega)d\lambda\right)^2$.

Auctioneer 2 chooses τ_ω to minimize $\sum_\omega\left(\left(\tau_\omega-\frac{\mathrm{inp}(g_\omega)}{n_\omega}\right)\sum_\theta\int_{H(\theta)}\bar{\mu}^{h_\theta}(\theta,\omega)d\lambda\right)^2$.

Auctioneer 3 chooses $\{t^\theta_\omega\}_{\theta\in\mathbf{\Theta},\,\omega\in\mathbf{\Omega}}$, with $t_\omega\in\mathbf{Trans}$, for all $\omega\in\mathbf{\Omega}$, to minimize $\sum_\omega\left(\sum_\theta t^\theta_\omega\cdot\int_{H(\theta)}\bar{\mu}^{h_\theta}(\theta,\omega)d\lambda\right)^2$.

It is easy to see that Auctioneers 1, 2, and 3's strategy sets are nonempty, convex, and compact. For Auctioneer 3, the argument to prove compactness is well known: if some consumers are paying large negative lump-sum transfers, then others must be paying large positive lump-sum transfers, which implies that some transfers are canceled with some others (for $t_\omega\in\mathbf{Trans}$).

An equilibrium for the constructed generalized game consists of a vector $(\bar{x},\bar{\mu},\bar{t},\bar{\tau},\bar{\alpha})$ such that each player k chooses a strategy $s(k)$ to solve his respective optimization problem parameterized in the other players' actions $\bar{s}_{-k}$.

Proposition 1. *There exists an equilibrium in mixed strategies for the constructed generalized game.*

Proof. Note that a consumer's strategy of choosing his most preferred jurisdiction type in stage 2 has a finite and discrete space domain $\mathbf{\Omega}$. In order to circumvent this problem, we extend our generalized game to allow for consumers' mixed strategies in the set of jurisdiction types. Let us denote $\Pi(\mathbf{\Omega})=\{\pi=(\pi(\omega))_{\omega\in\mathbf{\Omega}}:\pi(\omega)\ge0,\sum_{\omega\in\mathbf{\Omega}}\pi(\omega)=1\}$. Then, $\Pi(\mathbf{\Omega})$ stands for the convex hull of $(1,\ldots,\omega,\ldots,\Omega)$ which is the set of mixed strategies for each consumer. A profile of strategies $\rho:\mathbf{H}\to\Pi(\mathbf{\Omega})$ brings the continuum of consumers into strategies (pure or mixed). Consumer h_θ's stage 2 optimization problem extended to mixed strategies is such that this consumer randomizes over the possible consumptions in the different jurisdiction types. We write $U^{h_\theta}(\pi,\bar{t}^\theta_\omega,\bar{\tau}_\omega,\bar{\alpha}^\theta_\omega)\equiv u^{h_\theta}(\sum_\omega\pi(\omega)\psi(h_\theta,\omega),\pi)$. That is, *consumer randomizes in* $\mathbf{\Omega}=\{1,\ldots,\Omega\}$, *but not directly in consumption*. Then, consumer h_θ's stage 2 maximization problem is $\max_{\pi\in\Pi(\mathbf{\Omega})}U^{h_\theta}(\pi,\bar{t}^\theta,\bar{\tau},\bar{\alpha}^\theta)$. The utility $u^{h_\theta}(\sum_\omega\pi(\omega)\psi(h_\theta,\omega))$ is a continuous bounded real valued function on $\sum_\omega\pi(\omega)\psi(h_\theta,\omega)$, and the mixed strategy π belongs to the convex compact set $\Pi(\mathbf{\Omega})$. $R(h_\theta)=\left\{\pi\in\Pi(\mathbf{\Omega}):\pi\in\arg\max U^{h_\theta}(\pi,\bar{t}^\theta,\bar{\tau},\bar{\alpha}^\theta)\right\}$ denotes the set of mixed strategies that solve consumer h_θ's second stage maximization problem.

We must extent the fictitious auctioneers' problems to allow for consumers' mixed strategies. Given a mixed strategy profile $\rho:\mathbf{H}\to\Pi(\mathbf{\Omega})$, we can rewrite Auctioneer 1, 2, and 3's

[16] Other approaches might also serve the objective of proving equilibrium existence.

objective functions extended to mixed strategies as follows: $\alpha \rightarrow \sum_{\omega\in\mathbf{\Omega}}\left(\sum_{\theta\in\mathbf{\Theta}}\left(\alpha_{\omega}^{\theta}n_{\omega}^{\theta}-s_{\omega}^{\theta}y_{\omega}(\mathcal{C}_{\omega})\int_{H(\theta)}\rho(h_{\theta})(\omega)d\lambda\right)^{2}\right)$ for Auctioneer 1, $\tau\rightarrow\sum_{\omega\in\mathbf{\Omega}}\left(\tau_{\omega}-\frac{inp(g_{\omega})}{n_{\omega}}\right)\sum_{\theta\in\mathbf{\Theta}}\int_{H(\theta)}\rho(h_{\theta})(\omega)d\lambda)^{2}$ for Auctioneer 2, and $t\rightarrow\sum_{\omega\in\mathbf{\Omega}}\left(\sum_{\theta\in\mathbf{\Theta}}t_{\omega}^{\theta}\int_{H(\theta)}\rho(h_{\theta})(\omega)d\lambda\right)^{2}$ for Auctioneer 3.

All the conditions of Debreu's (1952) theorem hold. Thus, we can assert that the extended generalized game has an equilibrium, possibly in mixed strategies. At this point it remains to observe that Auctioneers 1, 2, and 3's new objective functions depend only on the average of the consumers' profile, which satisfies Schmeidler (1973) hypotheses, and therefore a degenerate equilibrium of the extended generalized game is, in fact, an equilibrium of the original game. ■

Proposition 2. *An equilibrium for the generalized game (in pure strategies) is an equilibrium of our economy.*

Proof. Let us consider the generalized game introduced above. Let $(\psi, \mu, t, \tau, \alpha)$ be an equilibrium in pure strategies of the generalized game. We start by showing that all consumers are optimizing. We say that $(\psi(h_\theta, \omega), \mu^{h_\theta}(\theta, \omega))$ is an optimum of the consumer's problem if $\left(\tilde{x}_{\tilde{\omega}}^{h_\theta}, \tilde{\mu}^{h_\theta}(\theta, \tilde{\omega})\right)\in\mathbf{X}^{h_\theta}$ with $\tilde{x}_{\tilde{\omega}}^{h_\theta}\in B^{h_\theta}(p, t_{\tilde{\omega}}^{\theta}, \tau_{\tilde{\omega}}, \alpha_{\tilde{\omega}}^{\theta}, e^{\theta}, \tilde{\omega})$ and $u^{h_\theta}\left(\tilde{x}_{\tilde{\omega}}^{h_\theta}, \tilde{\mu}^{h_\theta}(\theta, \tilde{\omega})\right) > u^{h_\theta}(\psi(h_\theta, \omega), \mu^{h_\theta}(\theta, \omega))$, then $\tilde{x}_{\tilde{\omega}}^{h_\theta} + t_{\tilde{\omega}}^{\theta} + \tau_{\tilde{\omega}} - \alpha_{\tilde{\omega}}^{\theta} - e^{\theta} > 0$. Let us show, by contradiction, that there cannot be a nonnull set of consumers who are not optimizing.[17] Then, for each such consumer h_θ, there is an optimum $(\tilde{x}^{h_\theta}, \tilde{\mu}^{h_\theta})$ such that $u^{h_\theta}(\tilde{x}^{h_\theta}, \tilde{\mu}^{h_\theta}) > u^{h_\theta}(x^{h_\theta}, \mu^{h_\theta})$ and $\tilde{x}^{h_\theta} - e^{h_\theta} + t_{\tilde{\omega}}^{\theta} + \tau_{\tilde{\omega}} - \alpha_{\tilde{\omega}}^{\theta} \leq 0$. By continuity, we can choose $\bar{x}^{h_\theta}\in[0, \tilde{x}^{h_\theta})$ with $(\bar{x}^{h_\theta}, \tilde{\mu}^{h_\theta})\in\mathbf{X}^{h_\theta}$ and still have $u^{h_\theta}(\bar{x}^{h_\theta}, \tilde{\mu}^{h_\theta}) > u^{h_\theta}(x^{h_\theta}, \mu^{h_\theta})$ but $\bar{x}^{h_\theta} - e^{\theta} + t_{\tilde{\omega}}^{\theta} + \tau_{\tilde{\omega}} - \alpha_{\tilde{\omega}}^{\theta} < 0$, so $(\bar{x}^{h_\theta}, \tilde{\mu}^{h_\theta})$ costs strictly less than and is strictly preferred to $(\tilde{x}^{h_\theta}, \tilde{\mu}^{h_\theta})$, a contradiction.

From Auctioneers 1, 2, and 3's optimization problems in the generalized game, we have that, for all θ and ω, $\alpha_{\omega}^{\theta}n_{\omega}^{\theta} = s_{\omega}^{\theta}y_{\omega}(\mathcal{C}_{\omega}), \tau_{\omega} = inp(g_{\omega})/n_{\omega}$, and $\sum_{\theta}t_{\omega}^{\theta}\int_{H(\theta)}\bar{\mu}^{h_\theta}(\theta, \omega)d\lambda = 0$ (so $t_\omega \in$ **Trans**). Then, aggregating over consumer types on both sides of $\hat{\alpha}_{\omega}^{\theta} = \alpha_{\omega}^{\theta} - t_{\omega}^{\theta} - \tau_{\omega}$, we get $\sum_{\theta}n_{\omega}^{\theta}\hat{\alpha}_{\omega}^{\theta} = \sum_{\theta}n_{\omega}^{\theta}\alpha_{\omega}^{\theta} - \sum_{\theta}n_{\omega}^{\theta}t_{\omega}^{\theta} - n_{\omega}\tau_{\omega}$, and using the solutions of Actioneers 1, 2 and 3's optimization problems, we get the equilibrium condition (E.2) "exhausted jurisdiction surplus": $y_{\omega}(\mathcal{C}_{\omega}) - inp(g_{\omega}) = \sum_{\theta\in\mathbf{\Theta}}n_{\omega}^{\theta}\hat{\alpha}_{\omega}^{\theta}, \forall\omega\in\mathbf{\Omega}$. Also, using the solutions of Actioneers 1, 2 and 3's optimization problems, we can aggregate budget constraints as follows:

$$\zeta \equiv \sum_{\omega,\theta}\int_{H(\theta)}\left(\bar{x}_{\omega}^{h_\theta} - e^{\theta}\right)d\lambda + \sum_{\omega,\theta}\int_{H(\theta)}\left(\frac{inp(g_\omega)}{n_\omega} - s_{\omega}^{\theta}y_{\omega}(\mathcal{C}_{\omega})\right)\bar{\mu}^{h_\theta}(\theta, \omega)d\lambda \leq 0.$$

It is now easy to see that there is no private good excess consumption demand in equilibrium ($\zeta \leq 0$). Otherwise, we would contradict the above aggregation of budget constraints. In fact, the previous inequality holds with equality (i.e., the private good market clears). Suppose, by contradiction, that $\zeta < 0$. Then, there is a nonnull set of consumers with non-binding budget constraints, a contradiction with optimization. Thus, $\zeta = 0$. Finally, let us show that $\hat{\mu}$ is consistent. If consistency fails, then $\gamma(\omega)$ is such that $\sum_{\theta}(t_{\omega}^{\theta} + \tau_{\omega} - \alpha_{\omega}^{\theta})\gamma(\omega)n_{\omega}^{\theta} \neq \sum_{\theta}(t_{\omega}^{\theta} + \tau_{\omega} - \alpha_{\omega}^{\theta})\hat{\mu}(\theta, \omega)$ for some θ. But then, this inequality enters into contradiction with $\zeta = 0$. ■

Theorem 1 follows from Propositions 1 and 2. ■

Proof of Theorem 2. We first prove that for our economy any equilibrium (x, μ) belongs to the core. Suppose not. Then, there exists a blocking coalition $\tilde{H}\subseteq\mathbf{H}$ and a feasible allocation $((\tilde{x}^h, \tilde{\mu}^h)_{h\in\tilde{H}})$ with $u^h(\tilde{x}^h, \tilde{\mu}^h) \geq u^h(x^h, \mu^h)$ for all $h\in\tilde{H}\subset\mathbf{H}$ and $u^{h'}\left(\tilde{x}^{h'}, \tilde{\mu}^{h'}\right) > u^{h'}\left(x^{h'}, \mu^{h'}\right)$ for those consumers h' in $H'\subset\tilde{H}$ with $\lambda(H') > 0$. Feasibility of $(\tilde{x}, \tilde{\mu})$ implies consistency and market clearing for $\tilde{H}$, and budget balance for all consumers in $\tilde{H}$. Such a feasible and preferred state contradicts the equilibrium, where consumers choose optimally in their budget sets. Therefore, (x, μ) is in the core of this economy.

The proof that any core state (x, μ) can be supported as an equilibrium is similar to Ellickson et al. (1999, Theorem 5.1). Here we indicate how to adapt Ellickson et al.'s proof to our specific economy with jurisdictions production. First, to construct the net preferred trade correspondence, we should replace the consumer h_θ's good endowment e^θ by $e^\theta + \alpha_{\iota}^{\theta}$. The wage α_{ι}^{θ} is defined according to the production surplus sharing rule $\alpha_{\iota}^{\theta} = s_{\iota}^{\theta}y_{\iota}(\mathcal{C}_{\iota})/n_{\iota}^{\theta}$, in the same way as the poll tax $\tau(\iota)$ follows the equal division rule $\tau(\iota) = inp(g_\iota)/n_\iota$. Denote the net preferred trade correspondence by $\sigma(h_\theta) = \{(\tilde{x}, \iota)\in\mathbb{R}\times\mathbb{R}^{|\mathbf{M}|} : (\tilde{x} + e^\theta + \alpha_{\iota}^{\theta} - \tau_{\iota}, \iota)\in\Phi^{h_\theta}\}$, where $\Phi^{h_\theta} = \left\{(\tilde{x}, \iota)\in\mathbf{X}^{h_\theta} : u^{h_\theta}(\tilde{x}, \iota) > u^{h_\theta}(x^{h_\theta}, \mu^{h_\theta})\right\}$. The aggregate net trade correspondence, denoted by $Z = \int_{\mathbf{H}}\Sigma(h_\theta)d\lambda$ where $\Sigma(h_\theta) = \sigma(h_\theta)\cup\{0\}$, is a nonempty convex subset of $\mathbb{R}\times\mathbb{R}^{|\mathbf{M}|}$ (by Lyapounov convexity theorem).

To separate Z from a "fat" enough cone $C^* = \{(\bar{x}, \bar{\mu})\in\mathbb{R}\times\mathbb{R}^{|\mathbf{M}|} : \bar{x} < -\frac{W}{D}dist(\bar{\mu}, \mathbf{Cons})\}$ we proceed similarly as Ellickson et al. (1999, Theorem 5.1, Step 3), where it is shown, by contradiction, that there is no state $(\hat{x}, \hat{\mu})$ preferred to (x, μ) that is feasible for an "exactly" consistent coalition whose consumers choose in their preferred set.[18] The only difference is that the upper bound W now incorporates the private good production: let $W = \sum_{\theta}\lambda(\theta)e^{\theta} + \sum_{\omega}\gamma(\omega)y_{\omega}(\mathcal{C}_{\omega})$ — recall that if a state (x, μ) is in the core, then it is consistent by definition, and therefore, the measure $\gamma(\omega)$ exists. Also, notice that Lemma 7.1 of Ellickson et al. (1999), which asserts that such exactly consistent coalition can be chosen, does not depend on the bound W. As in Ellickson et al., we can use the separation theorem to find prices $(\tilde{p}, \tilde{t})\in\mathbb{R}\times\mathbb{R}^{|\mathbf{M}|}$, $(\tilde{p}, \tilde{t}) \neq (0, 0)$, such that $(\tilde{p}, \tilde{t})(\bar{x}, \bar{\mu}) \leq 0$ for each $(\bar{x}, \bar{\mu})\in C^*$, and $(\tilde{p}, \tilde{t})z \geq 0$, for each $z\in Z$. By an argument similar to Ellickson et al. (1999, Theorem 5.1, Step 4), we have $\tilde{p} > 0$ and $\tilde{t}\in$**Trans**. With one good we can normalize the private good price to 1, and redefine transfers by letting $t_{\omega}^{\theta} = \tilde{t}_{\omega}^{\theta}/p$, for all (θ, ω), so that consumers' budget constraints do not change and also have $t\in$ **Trans**. To show that almost all consumers choose in their budget sets, denote the wage, transfer, and poll tax associated with jurisdiction ω when membership is $m = (\theta, \omega)$ by $\alpha_{\omega}^{\theta}(\mu^{h_\theta}(m))$, $t_{\omega}^{\theta}\mu^{h_\theta}(m)$ and $\tau_{\omega}(\mu^{h_\theta}(m))$, respectively. Let $E_1 = \{h\in\mathbf{H}: x_{\omega}^{h_\theta} - e^{\theta} - \alpha_{\omega}^{\theta}(\mu^{h_\theta}(m) + \tau_{\omega}(\mu^{h_\theta}(m))) + t_{\omega}^{\theta}\mu^{h_\theta}(m) > 0, \forall\theta\}$ and $E_2 = \{h\in\mathbf{H}: x_{\omega}^{h_\theta} - e^{\theta} - \alpha_{\omega}^{\theta}(\mu^{h_\theta}(m) + \tau_{\omega}(\mu^{h_\theta}(m))) + t_{\omega}^{\theta}\mu^{h_\theta}(m) < 0, \forall\theta\}$. Feasibility of (x, μ) implies that if $\lambda(E_1) > 0$, then $\lambda(E_2) > 0$. By a procedure similar to Ellickson et al. (1999, Theorem 5.1, Step 4) we can show that $\lambda(E_1) = 0$, and also show that almost all consumers are quasi-optimizing. Monotonicity of preferences and positive good endowments suffice to show that a consumer's quasi-optimization implies consumer's optimization.[19] ■

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[17] Notice that with only one good we do not need to prove quasi-optimization in order to show consumer's optimization.

[18] D is constructed as EGSZ: $D \equiv \inf\{dist(conv(\mathcal{L}), \mathbf{Cons}) : \mathcal{L}\in\mathcal{D}\}$ where $conv(\mathcal{L})$ denotes the convex hull of the set $\mathcal{L}$ and $\mathcal{D} = \{\mathcal{L}\subset\mathbf{Lists} : conv(\mathcal{L})\cap\mathbf{Cons} = \emptyset\}$

[19] With more than one private good Ellickson et al. (1999), Proposition 3.3) need to use the "club linked" assumption to guarantee that private good prices are strictly positive, so that consumer's quasi-optimization does imply consumer's optimization.

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