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The impact of trade costs on the European Regional Trade Network: An empirical and theoretical analysis

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Abstract

Using intra-European interregional trade data, we analyze the topology of the E.U. regional trade network. A triad census analysis confirms the intuition that the interregional trade network (and, thus, the European economic integration) is far from being complete. The majority of the E.U. interregional trade patterns are characterized by simple, at best bilateral, configurations. Moreover, we analyze the effect of trade costs in shaping the topological structure of the network. It emerges that the relative presence of simple trade configurations increases with distance, while the relative presence of more complex trade configurations decreases with distance. Finally, we discuss the theoretical underpinnings of these empirical facts through a simple new economic geography model with three regions. In this model, we analyze how trade costs shape the pattern of the trade network. On the whole we find a correspondence between theoretic and empirical results. However, details differ and they suggest directions for further research.

1 | INTRODUCTION

Intra-European trade is usually analyzed at the country level. Lack of intranational and international regional trade data has prevented, until 2015, a clear description of regional trade flows in Europe. In this paper, exploiting the availability of new European regional trade data at bilateral level (Thissen, Lankhuizen, & Jonkeren, 2015), and integrating network analysis and new economic geography modeling, we give evidence of regional trade flows in Europe in 2010, and describe their network structure. In this paper we extend the analysis started in Basile, Commendatore, De Benedictis, and Kubin (2016) by focusing on the role played by trade costs in shaping the topological structure of the network.

Trade costs are notoriously difficult to measure (Anderson & Van Wincoop, 2004), even more so if the quantification is undertaken at the regional level. The strategy we adopted in our analysis is to associate the theoretical bilateral trade cost between region r and region s, T_{rs}, to a latent variable that is empirically observed in its realization, through two components: one is the geographical distance between r and s, and the other is the presence of a border effect that generates a distinction between intranational and international regional trade.

The empirical investigation of the interregional trade network in Europe focuses on the results of a triad census analysis. First, it emerges that the interregional trade network (and, thus, the European economic integration) is far from being complete. The majority of the E.U. interregional trade patterns are characterized by simple, at best bilateral, configurations; and, when two regions establish a mutual trade relationship, this fosters them to export to, more than to import from, a third region. Then, by decomposing the overall interregional trade network graph in different subgraphs according to the quantile distribution of bilateral distances, we also analyze the effect of trade costs in shaping the topological structure of the network. From this analysis it emerges that the relative presence of simple trade configurations increases with distance, while the relative presence of more complex trade configurations decreases with distance. Finally, we consider a further dimension of trade costs by applying triad census analysis to regions within the same country (domestic trade) and regions of different countries (international trade). The triad census analysis applied to the corresponding subgraphs provides evidence of the occurrence of a border effect: bilateral trade flows are relatively more frequent between regions of the same country rather than between regions of different countries. In contrast, hierarchical structures, such as mutual stars, are relatively more frequent when only regions of different countries are selected, confirming the important role of hubs or export platforms played by core regions such as Île-de-France, Inner London, and Madrid.

Then, we discuss the theoretical underpinnings of these empirical facts through a three-region new economic geography (NEG) model that extends previous analyses (see Ottaviano, Tabuchi, & Thisse, 2002; Behrens, 2011; Ago, Isono, & Tabuchi, 2006; Basile et al., 2016). In Basile et al. (2016) we put forward a three-region NEG model that is more general than those previously existing in the literature, where the distance between the regions is not necessarily the same. Allowing for the possible presence or absence of trade links between the regions gives rise to 64 possible network structures. Without further specifications, these structures can be grouped into 16 isomorphic classes. In that contribution, we mostly limited our analysis to the conditions, expressed in terms of different combinations of trade costs and spatial factor allocation, corresponding to each of those classes. In the present article, we explicitly number the regions and specify their distances losing the isomorphisms between the network structures. We derive analytic conditions for the presence of trade links between regions and study how changes in trade costs affect the occurrence of the 64 possible trade network structures. Based on Proposition 1, we argue that some of these configurations (corresponding to the 16 different types of the triad census analysis) can be discarded a priori. Moreover, for each type of network structure or triad we establish its likelihood, given trade distances and the distribution of the economic activity (via conjecture 1 and simulations analysis). It turns out that more connected structures are a direct consequence of lower trade costs. We verify that, owing to competition, close regions engage more easily in mutual trade, and are less open to trade with more distant regions.

Finally, we use the theoretical analysis to shed some light on the empirical results looking at the impact of trade costs on the frequency of trade network structures. On the whole, we find a correspondence between theoretic and empirical results. However, details differ and they suggest directions for further research.

The paper is structured as follows: Section 2 presents the results of the empirical analysis, Section 3 develops the theoretical model, and Section 4 provides a discussion of the results and conclusions.

2 | EMPIRICAL ANALYSIS

2.1 | E.U. regional trade data

The data produced by the PBL Netherlands Environmental Assessment Agency (Thissen et al., 2015) is one of the first attempts to give account of E.U. regional trade. The data, obtained by breaking down international trade flows and national supply and use tables to the regional level,[1] is used by the E.U. Commission for regional policy simulations. Unlike gravity model estimations, the methodology used to calculate regional trade flows at the bilateral level stays as close as possible to observed data, without imposing any trade pattern constraint. Nevertheless, as pointed out by Thissen, Diodato, & Van Oort (2013), one has to keep in mind that the constructed interregional trade data are inferred from other data sources and are not measured as a flow from one region to another. Given the compatibility constraints with macro variables, some bias in the trade flows between regions inside a country or between regions of different countries might result from the weighting procedure used in the construction of the data. In order to reduce this bias, the original Project Based Learning (PBL) database has been adjusted to make the trade flows consistent with country totals from the national social accounting matrices. This is done by a generalized RAS method, which keeps the structure of bilateral trade flows as much unchanged as possible. The resulting data can therefore be used as such to describe and obtain information on the topology of the E.U. regional trade network. Moreover, as it will be clarified in Subsection 2.2, we only exploit the information on the values of the flows to identify a proper threshold value, according to which we transform the import–export matrix in a binary network matrix; and the last one will be the basic information used in the triad census analysis.

The original data used in our empirical analysis contains information, only for the year 2010, on bilateral trade flows (priced free on board and measured in millions of euros) between 268 regions belonging to 28 European countries.[2] Export and import flows refer to an aggregate category of sectors, which includes "Manufacturing" and two more sectors: "Mining and quarrying," and "Electricity, gas and water supply." From the original data we excluded the domestic use of domestic production, also known as the regional trade flow from region s to region s, imputing a zero-trade flow on the main diagonal of the trade matrix ($268 \times 268 = 71{,}824$ dyadic observations).

A descriptive analysis of the trade network and its visualization are reported in Appendix A; further information is also reported in Basile et al. (2016) for a slightly different version of the data.[3] In what follows, we focus on the topological properties of the network. Specifically, we start from a triad census analysis applied to the overall interregional trade network (Subsection 2.2); then we explore the role of physical distance and border effects in shaping the topological structure of the network (Subsection 2.3).

2.2 | Triad census

The structure of the European regional trade network can be studied through many different network analysis tools (Newman, 2003; Jackson, 2008; Hanneman & Riddle, 2005). We are interested, in particular, in the topological properties of the network concerning the relation between three nodes. Three is the minimum number to study interdependence. If the decision to trade from r to s is taken considering opportunity costs, the option of trading with an alternative region must be considered.

More specifically, we follow Basile et al. (2016) in studying the structure of the regional trade network focusing on the triad as the unit of analysis. The "Triad Census" (the count of the various types of triads in the network) is a classical network analysis tool (Wasserman and Faust, 1994), usually applied to a single, directed and binary network, and we follow the tradition in this respect. Taking the three nodes $\mathscr{V}_s$, $\mathscr{V}_r$, and $\mathscr{V}_k$, where $s \neq r \neq k$, we can call them a "triple." If we also consider the presence or absence of links between these different nodes, we have a triad ($\mathscr{T}_{srk}$). If the network is

composed of n nodes, there are $\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$ triads. In the European regional trade network there are, therefore, 3,172,316 triads.

As far as the possible realizations of triads, there are 16 isomorphic classes for the 64 different triad states.[4] These classes, represented graphically in the third column of Table 1, range from the empty subgraph to complete subgraph, in which all three dyads formed by the vertices in the triad have mutual directed links. The different classes are labeled with as many as four characters, according to the M–A–N labeling scheme of Holland and Leinhardt (1970), where the first character gives the number of mutual dyads in the triad, the second the number of asymmetric dyads, the third the number of null dyads, and lastly, the fourth one, if present, is used to distinguish further among the types (e.g., the two 030 triads—lines 9 and 10 in Table 1—can be distinguished by the transitivity of the dyad 9 and the cyclical links of dyad 10). The four letters in the fourth character are "U" (for up), "D" (for down), "T" (for transitive) and "C" (for cyclic).

In order to perform a triad census analysis, we need to consider the binary version of the trade matrix, where all real positive numbers (trade flows) are transformed into 1. However, there are no zero-trade flows in the matrix. Indeed, by construction, the E.U. regional trade network is fully connected, that is every vertex r is reachable from every s by a direct walk. In other words, the density of the trade matrix (also known as the ratio between the existing trade links and the possible trade links) is equal to 71,556/(268 × 267) = 1. Nevertheless, many dyadic observations are characterized by a negligible amount of exports. Thus, in order to make the E.U. regional data in line with the evidence coming from international trade data analysis (Helpman, Melitz, & Rubinstein, 2008; Baldwin & Harrigan, 2011; De Benedictis & Tajoli, 2011), that generally indicates the number of zero-trade flows around 30 percent of the data points in the trade matrix, at the aggregate level, and with the very rare data at the regional level (for the United States and Canada) used in the literature on the border effect (McCallum, 1995; Anderson & Van Wincoop, 2003), we operate a sparsification of the matrix. Specifically, we inflate the number of zeros, transforming the edges corresponding to small amount of trade value in a null one, and at the same time we do not reduce too much the total value of the E.U. regional flows included in the analysis. Excluding all dyadic observations lower than €25 million preserves 90 percent of the total value of intra-European regional trade.[5]

Adopting a threshold $w = 25$ million, the number of edges is substantially reduced, from 71,556 to 24,196 and the density drops from 1 to 0.34. A look at the distribution of the degrees of the truncated network indicates that the regional trade network is neither regular nor complete. The average degree is 90.28 and the skewness is 0.51. In other terms, if the heterogeneity in the strength of links is used to select their presence, this heterogeneity is reflected in the connectivity of the network.[6] To summarize, we use the information associated with the intensity of the links (the value of the trade flow) to define an appropriate threshold for the selection of links, and then we just exploit the binary information of the resulting truncated network.

In Table 1 we report the triad census for the European regional trade network, that is the absolute and relative frequencies of the 16 M–A–N classes. The column "Full network" in Table 1 shows that 29 percent of triads in the E.U. regional trade is represented by empty-graph structures, followed by mutual edges (21 percent), by single edges (14 percent), and then mutual stars (7 percent). Thus, about 65 percent of the E.U. interregional trade patterns are characterized by simple, at best bilateral, configurations. Moreover, it is worth noticing the prevalence of mutual edge + (double) Out structures over mutual edge + (double) In structures. It seems that when two regions establish a mutual trade relationship, this fosters them to export to, more that to import from, a third region. This aspect of the E.U. regional trade network will be discussed and theoretically motivated in the subsequent sections. Finally, only 12 percent of the triads are complete or almost complete graphs, suggesting that the interregional trade network (and, thus, the European economic integration) is far from being complete.

TABLE 1 Triadic census conditional on distance

	MAN code	Figure	Class	$\mathcal{N}$ Full network	$\mathcal{N}_1$ 1st decile <384km	$\mathcal{N}_2$ 2nd decile	$\mathcal{N}_5$ 5th decile	$\mathcal{N}_9$ 9th decile	$\mathcal{N}_{10}$ 10th decile >2,207km
1	003		Empty graph	905,607	2,255,196	2,323,572	2,215,740	1,335,148	353,637
				(0.29)	(0.75)	(0.79)	(0.87)	(0.93)	(0.92)
				[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
2	012		Single edge	437,075	96,686	140,101	145,396	63,308	21,253
				(0.14)	(0.03)	(0.05)	(0.06)	(0.05)	(0.06)
				[0.19]	[0.13]	[0.23]	[0.45]	[0.63]	[0.72]
3	102		Mutual edge	678,296	582,813	397,173	156,971	28,501	4,712
				(0.21)	(0.19)	(0.14)	(0.06)	(0.02)	(0.01)
				[0.30]	[0.79]	[0.66]	[0.48]	[0.28]	[0.16]
4	021D		Out-star	48,034	565	2,216	1,788	474	233
				(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.2]	[0.00]	[0.00]	[0.01]	[0.00]	[0.01]
5	021U		In-star	49,186	465	2,015	2,344	4,042	2,279
				(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
				[0.02]	[0.00]	[0.00]	[0.01]	[0.04]	[0.08]
6	021C		Line	20,078	478	836	733	408	157
				(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.02]
7	111D		Mutual edge + In	130,227	4,897	8,968	5,033	1,989	567
				(0.04)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.06]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
8	111U		Mutual edge + Out	178,248	5,851	10,467	4,267	921	270
				(0.06)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.08]	[0.01]	[0.02]	[0.01]	[0.01]	[0.01]
9	030T		Transitive	19,609	87	48	15	3	19
				(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
10	030C		Cycle	193	1	0	0	0	0
				(0.0)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.0]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
11	201		Mutual-star	209,472	26,455	40,819	9,513	1,016	166
				(0.07)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)

(Continues)

TABLE 1 (Continued)

	MAN code	Figure	Class	$\mathcal{N}$ Full network	$\mathcal{N}_1$ 1st decile <384km	$\mathcal{N}_2$ 2nd decile	$\mathcal{N}_5$ 5th decile	$\mathcal{N}_9$ 9th decile	$\mathcal{N}_{10}$ 10th decile >2,207km
				[0.09]	[0.04]	[0.07]	[0.03]	[0.01]	[0.01]
12	120D		Mutual edge + double In	33,641	472	109	22	2	4
				(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
13	120U		Mutual edge + double Out	63,328	1,105	205	31	5	8
				(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.03]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
14	120C		Mutual edge + Cycle	15,013	194	75	13	0	0
				(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
15	210		Almost complete graph	168,628	4,238	866	146	3	1
				(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.07]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
16	300		Complete graph	215,681	17,908	1,820	112	0	0
				(0.07)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
				[0.10]	[0.02]	[0.00]	[0.00]	[0.00]	[0.00]
			Total	3,172,316	2,997,411	2,929,290	2,542,124	1,435,820	38330
			Number of nodes	268	263	261	249	206	133
			Number of edges	24,196	5,898	4,568	2,173	701	300
			Number of edges before trimming	71,556	7,156	7,154	7,156	7,154	7,156
			Number of zero-trade flows	47,360	1,258	2,586	4,983	6,453	6,856
			Edge—example (top)		SI01→ SI02	ITC4→ ITI4	ITC4→ ITG1	ES21→ ES70	ES51→ ES70

Note. Our elaboration on E.U. bilateral regional trade flows. The numbers in parenthesis are percentages, the ones in square brackets are percentages excluding the number of empty graphs, e.g. $\frac{2,255,196\times 6}{263\times 262\times 261}=0.75$, in the first row and second column of the table; $\frac{96,686\times 6}{263\times 262\times 261-2,255,196}=0.13$ in the second row and second column of the table.

2.3 | Role of distance and border effects

Short- and long-distance trade relations are fundamentally different in terms of actors involved, the kind of goods traded and the quality of the same goods (Hillberry & Hummels, 2008). Following Abbate, De Benedictis, Fagiolo, and Tajoli (2017), we evaluate the structure of the network at different distance intervals, by splitting the distribution of bilateral distances between European regions in 10 deciles (Figure 1).[7] For every decile, we construct a different network containing only the edges between regions whose distance is included in the relevant interval (e.g., the network corresponding to the first decile contains only very short-distance relations; the one of the 10th decile only contains very

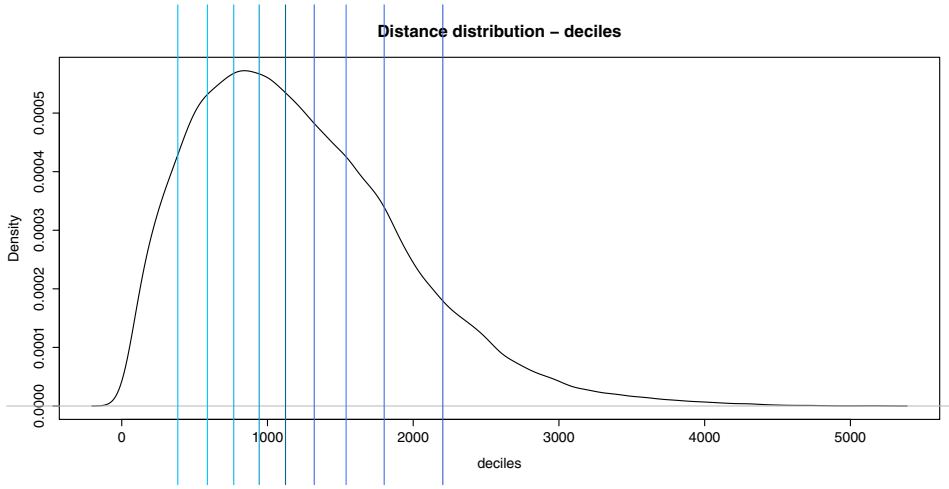

FIGURE 1 Kernel density of bilateral distance between E.U. regions

Note. Our elaborations on bilateral distance between E.U. regions. Distance is calculated using the "Haversine" great circle distance method, with a default value of 6,378,137 m for the radius of the earth. The deciles of the distribution are depicted by the vertical lines

long-distance relations). We end up having 10 networks, each one corresponding to different distance-related trade flows.

More formally, the original trade network $\mathscr{N} = (\mathscr{V},\mathscr{Q},\mathscr{P},\mathscr{W})$, consisting of the directed graph $\mathscr{G} = (\mathscr{V},\mathscr{Q})$, with $\mathscr{V}$ = 1, 2,. . ., 71,556 being the set of nodes (regions) and $\mathscr{Q}$ the set of edges between pairs of vertices (e.g., the binary categorical variable indicating trade partnership), plus $\mathscr{P}$, the additional information on the vertices (e.g., the nomenclature of territorial units for statistics (NUTS-2) code labels), and $\mathscr{W}$ the additional information on the links of the graph (e.g., the value of trade flow between *r* and *s*, in millions of euros), is the union of the 10 subgraphs corresponding to the 10 deciles of the distribution of bilateral distances: $\mathscr{N} = \mathscr{N}_1 \cup \mathscr{N}_2 \cup \ldots \cup \mathscr{N}_{10}$. The dimension of each subgraph is variable, being composed by a similar number of edges (between 7,154 and 7,156), but of a very different set of nodes (between 263 in the case of $\mathscr{N}_1$ and 133 for $\mathscr{N}_{10}$). The reason is quite natural: almost all regions (with the exception of distant islands and very remote regions) can trade with neighbors at short distance; instead, a much more limited number of regions trade with very far away regional trade partners.

For every one of the networks corresponding to the 10 deciles of the distribution of bilateral distances it is possible to calculate the frequencies of the 16 classes. In Table 1 we report the triad census for the European regional trade network for the first, second, fifth, ninth and 10th decile. Every triadic census is calculated excluding isolated regions and zero valued edges (the ones having a trade value below €25 million); for example, in the last column of Table 1 the triad census is calculated for the 133 regions and 300 edges of the trade network $\mathscr{N}_{10}$. In general, the trimming procedure that generates the zero-trade flows operates asymmetrically in the 10 networks. The number of zeros increases systematically from $\mathscr{N}_1$ to $\mathscr{N}_{10}$ in an inverse relation with the number of edges and the number of nodes.

With respect to the full network ($\mathscr{N}$), the number of empty graphs in the subnetworks $\mathscr{N}_1$ to $\mathscr{N}_{10}$ is inflated, ranging from 75 percent (in $\mathscr{N}_1$; bilateral trade flows between regions of no more than 384 km distance) to 93 percent (in $\mathscr{N}_9$) (see the relative frequencies in parenthesis). Excluding the empty graphs from the computation, around 90 percent of the remaining patterns are represented by single and mutual edges (see relative frequencies in square brackets). However, while the relative frequency of single edges increases with distance (being fairly high in $\mathscr{N}_9$ and $\mathscr{N}_{10}$ and quite small for $\mathscr{N}_1$ and $\mathscr{N}_2$), the relative frequency of mutual edges decreases with distance. Moreover, it is worth noticing that the relative frequency of more complex structures other than mutual edges (*in primis*

mutual stars, but also complete graphs and almost complete graphs) rapidly decreases with distance (cyclical structures are almost absent), while the relative frequency of simpler structure other than single edges (*in primis* In-stars, but also transitive and line) increases with distance.

Finally, we recognize that economic distance is not just a matter of geographical distance, but it depends on many different aspects (e.g., language, culture, institutions) that are country specific. To account for this general aspect of trade costs, we distinguish between trade that takes place between regions of the same country and the one that occurs between regions of different countries. When we separate intranational regional trade from international regional trade new information emerges, as reported in Tables A1 and A2 in the Appendix. Excluding empty graphs from the computation of relative frequencies, we may observe that mutual edges turn to be relatively more frequent when regions of the same country are involved (focusing on the first two subnetworks $\mathcal{N}_1$ and $\mathcal{N}_2$ the percentages of mutual edges are 88 percent and 83 percent, respectively, in Table A1), than in the case of regions belonging to different countries (70 percent and 62 percent, respectively, in Table A2). Thus, a sort of "border effect" seems to reduce the conditional probability of observing bilateral trade patterns. In contrast, mutual stars appear relatively more frequent when only regions of different countries are selected (international trade; see Table A2). This evidence is in line with the picture provided in Figure A1 (in the Appendix) obtained keeping only the two most important export ties, in terms of trade value, for every region, and discarding the rest (The reader can refer to the online version for the coloured version of Picture 1). In that picture most of the core regions in the network, such as Île-de-France, Inner London (UKI1), and Madrid (ES30), work as hubs or export platforms, bridging intranational and international regional trade flows.

3 | THEORETICAL MODEL

3.1 | General framework

In order to represent in the simplest way a triadic structure, we consider three regions—labeled 1, 2 and 3—in isolation. This three-region economy is characterized as follows: there are two sectors (agriculture A and manufacturing M) and two types of agents (workers and entrepreneurs) each one endowed with a different factor of production (unskilled labor L and human capital E). Labor is immobile across regions but mobile across sectors; human capital, instead, is spatially mobile and specific to manufacturing. The regions are identical with respect to the labor endowment, $L_1 = L_2 = L_3 = \frac{L}{3}$, technology, and consumer preferences.[8]

In the A sector, a homogeneous good is produced under perfect competition and constant returns to scale. One unit of L is required for each unit of the agricultural good. The A good is also chosen as the numéraire. This implies that in the short-run equilibrium, where p_A denotes the agricultural price and w the unitary wage, we have $p_A = w = 1$.

In the M sector, N varieties of a differentiated good are produced under monopolistic competition and increasing returns. For each identical firm, the technology involves a fixed component, one entrepreneur, and a variable component, η units of unskilled labor for each unit of the differentiated variety.

No economies of scope are allowed. Given consumers' preference for variety (see below) and increasing returns, each firm will always produce a variety different from those produced by the other firms. Moreover, since one entrepreneur is required for each manufacturing firm, the total number of firms/varieties is always equal to the total number of entrepreneurs, $E = N$. The number of regional varieties produced in region r is $n_r = \lambda_r N = \lambda_r E$, where λ_r is the share of entrepreneurs located in region r and $r = 1, 2, 3$.

The regions are separated by trade costs. These depend on natural geography, national borders, language, and cultural differences, and other factors including those that could be directly affected by the

E.U. cohesion policy. We assume that the trade distance between the regions is not necessarily the same. Specifically, the distance between regions 1 and 2 is smaller than that between regions 1 and 3, which in turn is smaller than that between regions 2 and 3. Assuming that the cost of trading the industrial good is independent of the direction of trade, $T_{rs} = T_{sr}$, r, $s = 1, 2, 3$, we can write: $T_{12} < T_{13} < T_{23}$. Moreover, we assume $T_{rr} = 0$ for $r = 1, 2, 3$.

In order to facilitate the comparison with the empirical analysis, we define T as the trade costs between regions 2 and 3 (the two more distant regions), $T_{23} = T$. Moreover, we define the distance between 1 and 2 and between 1 and 3 respectively as $T_{12} = T - \epsilon_1$ and $T_{13} = T - \epsilon_2$. By reducing T, we are considering "closer" regions facing lower trading costs, ϵ_1 and ϵ_2 represent the relative distance between the three regions, with $0 < \epsilon_2 < \epsilon_1 < T$.[9]

Concerning consumption choices, preferences are quasilinear (see Ottaviano et al., 2002). The utility function is composed of a quadratic part defining the preferences across the N varieties of the M good and a linear component for the consumption of the A good:

$$U=\alpha\sum_{i=1}^{N} c_i-\left(\frac{\beta-\delta}{2}\right)\sum_{i=1}^{N} c_i^2-\frac{\delta}{2}\left(\sum_{i=1}^{N} c_i\right)^2+C_A \tag{1}$$

where c_i is the consumption of variety i and C_A the consumption of the agricultural good. In this expression, $\alpha > 0$ represents the intensity of preferences for the manufactured varieties; δ represents the degree of substitutability across those varieties; and $\beta - \delta > 0$ represents the taste for variety.

The representative consumer's (unskilled worker or entrepreneur) budget constraint is:

$$\sum_{i=1}^{N} p_i c_i+C_A=y+\overline{C_A} \tag{2}$$

where $\overline{C_A}$ is the individual endowment of the agricultural good, sufficiently large to allow for positive consumption in equilibrium; p_i is the price of variety i inclusive of trade costs; and y is the consumer's income.

Solving the utility maximization problem involving (1) and the constraint (2), we obtain the following first order conditions for $i = 1, 2, \ldots, N$:

$$\frac{\partial U}{\partial c_i}=\alpha-(\beta-\delta)c_i-\delta\sum_{j=1}^{N} c_j-p_i=0$$

from which

$$p_i=\alpha-(\beta-\delta)c_i-\delta\sum_{j=1}^{N} c_j.$$

Solving for c_i the system, we obtain the linear demand function:

$$c_i=\max\,[0, a-(b+cN)p_i+cP]$$

where $P=\sum_{j=1}^{N} p_j$ is the price index,

$$a=\frac{\alpha}{(N-1)\delta+\beta},\ b=\frac{1}{(N-1)\delta+\beta},\ c=\frac{\delta}{(\beta-\delta)[(N-1)\delta+\beta]}$$

where $c_i \geq 0$ for $p_i \leq \tilde{p}=\frac{a+cP}{b+cN}$, and $\tilde{p}$ represents the reservation price below which demand is positive.

Focusing on a specific region, let's say region $s(=1, 2, 3)$, after dropping the subscript i, under the assumption of symmetric firm behavior, the consumer's demand originating from regions s for a good produced in region $r(=1, 2, 3)$ is:

$$c_{rs}=\max\left[0, a-(b+cN)p_{rs}+cP_s\right] \tag{3}$$

where c_{rs} is the demand of a consumer living in region s for a good produced in region r; p_{rs} is the price of a good produced in region r and consumed in region s;

$$P_s=\sum_{k=1}^{3} n_k p_{ks}=\sum_{k=1}^{3} \lambda_k E p_{ks} \tag{4}$$

is the price index in region s; and, as before, $c_{rs}\geq 0$ for $p_{rs}\leq \tilde{p}_s=\frac{a+cP_s}{b+cN}$, $\tilde{p}_s$ representing the reservation price of a consumer living in region $s(=1, 2, 3)$ above which the demand originating from region s for a variety produced in r is zero.

Considering that $L_1=L_2=L_3=\frac{L}{3}$, the operating profit of a representative firm in region $r(=1, 2, 3)$ is

$$\pi_r=\sum_{s=1}^{3}(p_{rs}-\eta-T_{rs})q_{rs}\left(\frac{L}{3}+\lambda_s E\right).$$

Introducing the short-run equilibrium condition according to which in each segmented market $s=1, 2, 3$ demand equals supply, $c_{rs}=q_{rs}$, from profit maximization, considering further that $N=E$ and that firms take the price index as given, the first order conditions for r, $s=1, 2, 3$ are:

$$\frac{\partial \pi_r}{\partial p_{rs}}=\left[a+(\eta+T_{rs})(b+cE)+cP_s-2p_{rs}(b+cE)\right]\left(\frac{L}{3}+\lambda_s E\right)=0.$$

Taking into account trade costs and letting $\tilde{p}_r=\frac{a+cP_r}{b+cN}>\eta$ (for $r=1, 2, 3$), to allow for positive production in the local market, profit maximizing prices correspond to

$$p_{rr}=\frac{a+cP_r+\eta(b+cE)}{2(b+cE)}=\frac{\tilde{p}_r}{2}+\frac{\eta}{2} \tag{5}$$

which is the price that firms quote in the market in which they are located; and to

$$p_{rs}=\begin{cases}\dfrac{a+cP_s+(\eta+T_{rs})(b+cE)}{2(b+cE)}=\dfrac{\tilde{p}_s}{2}+\dfrac{\eta}{2}+\dfrac{T_{rs}}{2} & \text{if} \quad T_{rs}<\tilde{p}_s-\eta \\ \tilde{p}_s & \text{if} \quad T_{rs}\geq \tilde{p}_s-\eta\end{cases} \tag{6}$$

which is the price that a firm located in region r quotes in region s, with $r\neq s$.

Using the demand and price functions, we can write:

$$q_{rr}=(b+cE)(p_{rr}-\eta) \tag{7}$$

which is the quantity sold by an r firm in the local market; and

$$q_{rs}=\begin{cases}(b+cE)(p_{rs}-\eta-T_{rs}) & \text{if} \quad T_{rs}<\tilde{p}_s-\eta \\ 0 & \text{if} \quad T_{rs}\geq \tilde{p}_s-\eta\end{cases} \tag{8}$$

which is the quantity that a firm located in region r sells in region s, with s, $r=1, 2, 3$ and $r\neq s$.

According to expressions (6) and (8), if a firm located in r quotes in market s a price larger than the reservation price $\tilde{p}_s$ of consumers living in s, the export from region r to region s is zero. The boundary condition for trade as reported in these expressions is crucial for the following analysis when determining all the possible patterns of trade between the regions.

The indirect utility for r is given by

$$V_r = S_r + y + \overline{C_A}$$

where S_r corresponds to the consumer's surplus:

$$S_r = \frac{a^2 E}{2b} + \frac{b + cE}{2} \sum_{s=1}^{3} \lambda_s p_{sr}^2 E - aP_r - \frac{c}{2} P_r^2.$$

The indirect utility is crucial in determining the long-run evolution of the economy; in particular, the entrepreneurial migration process. In this paper, we focus on the short run, fixing the distribution of the industrial economic activity, given by the regional shares λ_r.

3.2 | Trade costs and trade network structures (triads)

3.2.1 | General conditions

Here we make explicit the conditions of trade between the three regions and study how a reduction of T could affect the occurrence of the different trade network structures known, in the language of network analysis, as triads.

Considering the three regions, 1, 2, and 3, the creation of a trade link from one of them, labeled r, to a second one, labeled s, depends on trade costs and competition in the local market originating both from local and foreign firms. The latter is affected by the existence (or absence) of a previously existing trade link from the third region k to region s, with r, s, $k = 1, 2, 3$ and $r \neq s \neq k$. If such a link is absent, r-firms (i.e. those firms located in region r) only face competition from the local s-firms; instead, if it is present (and region s is already an importer from k) r-firms face competition also from k-firms exporting to region s.

In general (see above expressions 6 and 8), the condition for trade (resp. no trade) from r to s corresponds to:

$$T_{rs} < (\geq) \tilde{p}_s - \eta = 2(p_{ss} - \eta).$$

When no previous link from region r to s exists, $T_{rs} \geq 2(p_{ss} - \eta)$, and trade costs are too high for a link from k to s as well, $T_{sk} \geq 2(p_{ss} - \eta)$, the local price fixed by s-firms (obtained from Equations (4), (5), and (6) is only (negatively) affected by competition from local firms, depending on λ_s:[10]

$$p_{ss} = \frac{a + \eta(b + c\lambda_s E)}{2b + c\lambda_s E}. \tag{9}$$

Thus, owing to the competition effect $\frac{\partial p_{ss}}{\partial \lambda_s} < 0$, which implies that increasing competition in the local market leads to a smaller price set in the local market and lower profitability, making that market less permeable to imports.

When trade costs become sufficiently low so that $T_{rs} < 2(p_{ss} - \eta)$, and a trade link from r to s is formed, but a link from k to s is still missing, $T_{sk} \geq 2(p_{ss} - \eta)$, the local price is also (negatively) affected by competition from r-firms and (positively) by costs of trading goods from r to s:

$$p_{ss}=\frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \tag{10}$$

where $\frac{\partial p_{ss}}{\partial \lambda_s}<0$, $\frac{\partial p_{ss}}{\partial \lambda_r}<0$ for $T_{rs}<\frac{2(a-\eta b)}{2b+c\lambda_s E}$ and $\frac{\partial p_{ss}}{\partial T_{rs}}>0$. That is, increasing competition, both from local s-firms and r-firms, has a negative effect on the local price. Again this makes the local market less permeable to imports owing to the negative impact on profitability.[11]

If either Equation (9) or (10) holds, the condition for the presence (absence) of one-way trade from r to s is:

$$T_{rs}<(\geq)\frac{2(a-\eta b)}{2b+c\lambda_s E}. \tag{11}$$

These conditions can be alternatively expressed as

$$\lambda_s<(\geq)\frac{2(a-\eta b-bT_{rs})}{cET_{rs}}=\lambda^*. \tag{12}$$

These inequalities clearly show that trade from r to s could only take place by reducing sufficiently trade costs for a given level of local competition, determined by the share λ_s, to a lower λ_s corresponding a larger threshold for T_{rs}.

When a trade link from region k to region s exists, $T_{sk}<2(p_{ss}-\eta)$, and trade costs between r and s are high, $T_{rs}\geq 2(p_{ss}-\eta)$, the price fixed locally by s-firms is

$$p_{ss}=\frac{a+\eta[b+c(\lambda_s+\lambda_k)E]+\frac{T_{sk}}{2}c\lambda_k E}{2b+c(\lambda_s+\lambda_k)E}. \tag{13}$$

Finally, when $T_{sk}<2(p_{ss}-\eta)$ and $T_{rs}<2(p_{ss}-\eta)$ and both r-firms and k-firms access the market in s, the local price fixed by s firms is affected by a weighted average of the costs of trading goods from r to s and from k to s, where the weights are given by the number of goods imported from r and k:

$$p_{ss}=\frac{a+\eta(b+cE)+\left(\frac{T_{rs}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_k\right)cE}{2b+cE}. \tag{14}$$

If either Equation (13) or (14) holds, the condition for the presence (absence) of one-way trade from r to s becomes:

$$T_{rs}<(\geq)\frac{2(a-\eta b)+cE\lambda_k T_{sk}}{2b+c(\lambda_s+\lambda_k)E} \tag{15}$$

that, for our purposes, can be alternatively expressed as:

$$\lambda_s<(\geq)\frac{2(a-\eta b-bT_{rs})}{cET_{sk}}+\frac{(1-\lambda_r)(T_{sk}-T_{rs})}{T_{sk}}=\lambda^{**}. \tag{16}$$

If we substitute into this expression the condition $\lambda_r=1-\lambda_s-\lambda_k$ and rearrange, we have that:

$$\lambda_s<(\geq)\frac{2(a-\eta b-bT_{rs})}{cET_{rs}}+\frac{T_{sk}-T_{rs}}{T_{rs}}\lambda_k.$$

This expression shows that, when a link from k to s exists and $T_{rs}<T_{sk}$, the condition for trade from r to s is less restrictive (resp. the condition for no trade more restrictive) than in the absence of such a

link. We conclude that, for a given level of competition in the local market (determined by λ_s and λ_k): (i) given T_{sk}, sufficiently small trade costs between r and s allow for trade from r to s; (ii) a trade link from r to s is more likely the larger the difference between T_{sk} and T_{rs}; finally, (iii) since the same reasoning applies to the symmetric case of trade from s to r (facing a possible trade link from k to r), the two closer regions engage more easily in mutual trade and are less open to trade with the third region.

Notice the interesting result, following from the choice of the quasilinear utility function in (1), that regional incomes do not directly affect price and quantity determination and therefore the formation of trade links. However, they could indirectly affect link formation via the long-run distribution of the manufacturing activity across the economy, that is, via the regional shares of entrepreneurs λ_r, λ_s, and λ_k, as shown, for example, in (11) and (15).

It is possible to show that, when Equation (9) holds, a sufficient condition for no trade from r to s is $\lambda^* \leq 0$; and, when Equation (13) holds, a sufficient condition for no trade from r to s is $\lambda^{**} \leq 0$. Moreover, a sufficient condition for trade when Equation (10) or Equation (14) holds is $\lambda^* > 1$. Analogous conditions can be derived considering the link from s to k.[12]

In what follows, we use conditions (12) and (16) to sort out the most likely trade pattern configurations in our three-region economy.

Given our assumption on trade costs not all configuration are possible. Indeed the following proposition can be used to discard some of them:

Proposition 1 *Let trade costs from* r *to* s *be smaller than (or equal to) those from* s *to* k, *if a trade link from* k *to* s *exists then a trade link from* r *to* s *must exist as well.*

Let us suppose there is a link from k to s but no link from r to s, moreover $T_{sk} \geq T_{rs}$. From condition (12), after a suitable relabeling, the following inequality must hold

$$\lambda_s < \frac{2(a-\eta b-bT_{sk})}{cET_{sk}}$$

and from condition (16) the following inequality must hold:

$$\lambda_s \geq \frac{2(a-\eta b-bT_{rs})}{cET_{rs}} + \frac{T_{sk}-T_{rs}}{T_{rs}}\lambda_k.$$

However, this is not possible, since these two inequalities imply

$$\frac{2(a-\eta b+c\lambda_k ET_{sk})(T_{rs}-T_{sk})}{cET_{rs}T_{sk}} > 0,$$

which is not verified for $T_{sk} \geq T_{rs}$. Therefore, if a link from k to s exists and $T_{sk} \geq T_{rs}$, a link from r to s must exist as well. *Q.E.D.*

From Proposition 1, a configuration that includes a link from k to s, but does not allow for a link from r to s, cannot occur.

Conjecture 1 *Let* $T_{rk} > T_{rs}$, *if a specific configuration (triad) characterized by a link from* r *to* k *(but not by a link from* r *to* s*) is possible, then another configuration (triad) of the same type that allows for a link from* r *to* s *(but not for a link from* r *to* k*) must be possible as well.*

We verify this conjecture only for the case of a single-edge configuration (triad) and for a specific example. A single-edge configuration is characterized by the existence of a unidirectional trade link

from a region to another and no other link. In our example, we consider two single-edge configurations the first involving a one-way link from 1 to 2 and no other link; and the second involving a one-way link from 1 to 3 and no other link. We show that if the second configuration is possible (meaning that there exists a set of values of λ_1, λ_2, and λ_3, which allows it), because the distance from 1 to 3 is larger than the distance from 1 to 2, then the first must be possible as well (meaning that a set of values of λ_1, λ_2, and λ_3, different from the previous and bigger exists that allows this other configuration). Starting from the conditions for trade or no trade when another trade link is absent (12) or present (16), we proceed as follows.

Combinations of λ_1 and λ_2, which allow for a single-edge configuration involving a link from 1 to 3, must satisfy the following conditions:

$$\lambda_1 \geq \tilde{\lambda}\,(\text{no trade from 2 to 1}),\ \lambda_2 \geq \tilde{\lambda}\,(\text{no trade from 1 to 2})$$
$$\lambda_1 \geq \bar{\lambda}\,(\text{no trade from 3 to 1}),\ \lambda_3 < \bar{\lambda}\,(\text{trade from 1 to 3})$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,(\text{no trade from 3 to 2}),\ \lambda_3 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{13}}{T_{23}}\lambda_1\,(\text{no trade from 2 to 3}).$$

Combinations of λ_1 and λ_2, which allow for a single-edge configuration involving a link from 1 to 2, must satisfy the following conditions:

$$\lambda_1 \geq \tilde{\lambda}\,(\text{no trade from 2 to 1}),\ \lambda_2 < \tilde{\lambda}\,(\text{trade from 1 to 2})$$
$$\lambda_1 \geq \bar{\lambda}\,(\text{no trade from 3 to 1}),\ \lambda_3 \geq \bar{\lambda}\,(\text{no trade from 1 to 3})$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\ (\text{no trade from 3 to 2}),\ \lambda_3 \geq \bar{\bar{\lambda}}\,(\text{no trade from 2 to 3})$$

where: $\tilde{\lambda} = \frac{2(a-\eta b-bT_{12})}{cET_{12}}$, $\bar{\lambda} = \frac{2(a-\eta b-bT_{13})}{cET_{13}}$, and $\bar{\bar{\lambda}} = \frac{2(a-\eta b-bT_{23})}{cET_{23}}$. Owing to our assumption on trade costs $T_{12} < T_{13} < T_{23}$, it follows $\bar{\bar{\lambda}} < \bar{\lambda} < \tilde{\lambda}$.

Through comparison of the two groups of conditions, we verify that (1) the first condition of the first group and the first condition of the second group are the same; (2) since $\bar{\lambda} < \tilde{\lambda}$, the second condition of the first group is more restrictive than the fourth condition of the second group; (3) the third condition of the first group and the third condition of the second group are the same; (4) since $\bar{\lambda} < \tilde{\lambda}$, the fourth condition of the first group is more restrictive than the second condition of the second group; (5) the fifth condition of the first group is equivalent to the sixth condition of the second group; and (6) since $T_{13} > T_{12}$ the sixth condition of the first group is more restrictive than the fifth condition of the second group. Thus, for the chosen example, our conjecture is confirmed.

From Conjecture 1, once we allow for different trade costs between the regions, some of the configurations of a type are more likely than others of the same type.

3.2.2 | Analysis of the specific network configurations (triads)

In what follows, we discuss how the likelihood of the trade network configurations (or triads)—defined as the set of values of λ_1 and λ_2 that satisfy the respective trade or no trade conditions—is affected by trade costs. The analysis is carried out with the help of Figures 2(a) and (b). Both figures have been plotted for $a=b=c=\frac{1}{3}$, $\eta = 0$, $E = 10$, $\epsilon_1 = 0.1$ and $\epsilon_1 = 0.2$. Moreover, we have set $T = 0.6$ in Figure 2(a) and $T = 0.5$ in Figure 2(b). In these figures, the combinations of λ_1 and λ_2 (after taking into account that $\lambda_3 = 1-\lambda_1-\lambda_2$) that allow for a specific network configuration or triad are represented by areas with different colors; each color marked by the conventional code of the corresponding triad. A dotted line corresponds to the case (12), that is, there is not a link from k to s affecting the existence of

a link from r to s; and a solid line to the case (16), that is, there is a a link from k to s affecting the existence of a link from r to s. Moreover, a line is red, blue or green when regions 1 and 2, regions 1 and 3, or regions 2 and 3, are involved, respectively. We compare the likelihood of each configuration for the two different values of T (0.6 and 0.5). In concluding, we will also concisely report results from simulations, not presented in this paper, which consider more values of T. We now turn to our study of each triad; note that, for expository purposes, the conditions for their occurrence were moved to the Appendix:

(1) For sufficiently high trade costs no trade link is formed between any of the three regions. The case of full autarky corresponds to an empty-graph configuration. The likelihood of this configuration, which is obviously unique, decreases by reducing trade costs (i.e., by reducing T). Indeed, this can be easily verified by comparing Figure 2(a) with 2(b). In Figure 2(a), all the combinations of λ_1 and λ_2 corresponding to the empty graph are represented by the yellow area (marked by 003). This area disappears when T is reduced from 0.6 to 0.5 as showed in Figure 2(b).

(2) As mentioned above, a single-edge configuration is characterized by a one directional link from region r to region s and no other links. According to Proposition 1, given our assumption on trade costs, the only possible single-edge configurations are: one-way trade from region 1 to region 2; one-way trade from region 2 to region 1; and one-way trade from region 1 to region 3.

Owing to our assumption on trade costs $T_{12} < T_{13} < T_{23}$ (from which it follows $\bar{\bar{\lambda}} < \bar{\lambda} < \tilde{\lambda}$), the configuration of one-way trade from 1 to 2 is at least as likely as the configuration from 2 to 1 (and its is more likely when all conditions are binding); and the configuration from 2 to 1 is more likely than the configuration from 1 to 3 (as discussed in Conjecture 1. This is verified by looking at Figure 2(a), where the light green areas (marked by 012) represent all the combinations of λ_1 and λ_2, which allows for a single-edge configuration (the area close to the horizontal side the one from 1 to 2; the area close to the vertical side the one from 2 to 1; and the area close to the hypotenuse the one from 1 to 3). The comparison between Figures 2(a) and 2 (b) shows that the single-edge configuration becomes less likely by reducing trade costs from 0.6 to 0.5.

(3) A mutual edge configuration involves bilateral trade between regions r and s and no other link. From Proposition 1, a two-way trade link involving only two regions can only be formed between regions 1 and 2.

In Figures 2 (a) and 2(b) all the possible combinations of λ_1 and λ_2 corresponding to a mutual edge configuration are colored in pink (marked by 102). The comparison between Figures 2(a) and 2(b) shows that a mutual edge becomes more likely by reducing trade costs from 0.6 to 0.5.

(4) An Out-star configuration is characterized by unilateral trade from r to s and from r to k and no other trade link. This type of configuration is represented in Figures 2(a) and 2(b) by the light gray colored area on the right bottom corner (marked by 021D). As shown by the comparison of these figures the likelihood of this type of configuration could decrease with T.

(5) Allowing only for unilateral trade from region s to region r and from k to r gives the In-star configuration. All configurations of this type are possible. From Conjecture 1, it follows that some configurations of this type are more likely than others:[13] Indeed, as confirmed by Figures 2(a) and 2(b), where all the combinations of λ_1 and λ_2 that corresponds to an In-star configuration are colored in lilac (marked by 021U), one-way trade from 2 to 1 and from 3 to 1 (represented by the area close to the vertical axis) is more likely than one-way trade from 3 to 1 and from 3 to 2 (represented by the area close to the hypotenuse); and one-way trade from 3 to 1 and from 3 to 2 is more likely than one-way trade from 3 to 1 and from 3 to 2 (represented by the area close to the

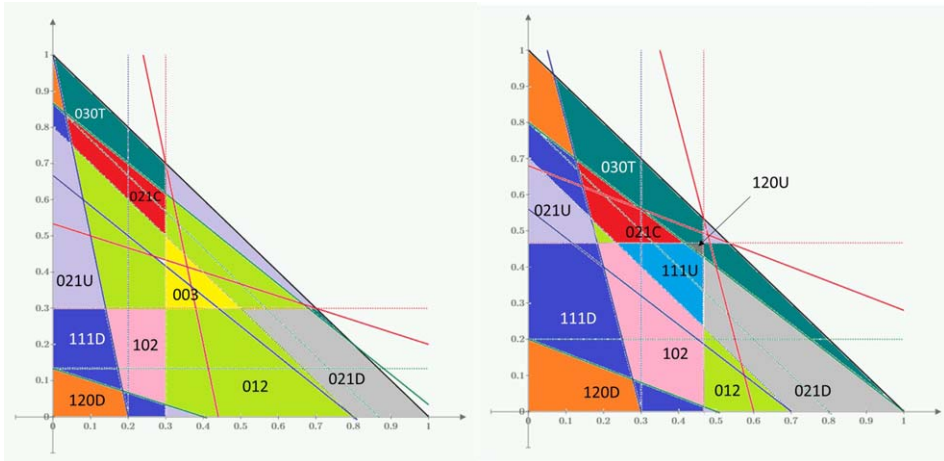

FIGURE 2 Analysis of the network configurations (triads): (a) $T = 0.6$; (b) $T = 0.5$

Note. These figures represent all possible combinations of λ_1 and λ_2 corresponding to specific trade network configurations or triads. These combinations are colored differently: each color marked on the basis of the conventional code for triads. The coloring of the areas within the triangle plotted in the λ_1 and λ_2 space is based on the conditions in (12) and (16) with the help of the boundary lines involved in those conditions and taking into account that $\lambda_3 = 1 - \lambda_1 - \lambda_2$. Dotted lines correspond to (12) when the presence of the third region is irrelevant for existence of a link from a region to a second; and solid lines to (16) when the presence of a third region impacts on trade links involving the first two. We have chosen the following parameters values: $a = b = c = 1/3$, $\eta = 0$, $E = 10$, $\epsilon_1 = 0.1$, $\epsilon_1 = 0.2$, and $T = 0.6$ in Figure 2(a), 1 and $T = 0.5$ in Figure 2(b)

vertical axis—absent in Figure 2(b)). The comparison between Figures 2(a) and 2(b) shows that this type of configuration becomes less likely by reducing sufficiently trade costs.

(6) A Line configuration is characterized by only two links: one from region s to region r and the second from region r to region k. The only possible configuration of this type is one-way trade from 2 to 1 and from 1 to 3. Looking at Figures 2(a) and 2(b), where all the possible combinations of $\lambda_1$and λ_2 corresponding to a Line configuration are represented by the red area (marked by 021C), the likelihood of this configuration is substantially similar for the values of T chosen to plot these figures.

(7) A Mutual edge + In configuration is characterized by bilateral trade between r and s and unilateral trade from k to r and no other link. From Proposition 1, the possible configurations of this type are three: two-way trade between 1 and 2 and one-way trade from 3 to 1; two-way trade between 1 and 2 and one-way trade from 3 to 2; and two-way trade between 1 and 3 and one-way trade from 2 to 1.

From Conjecture 1, the first configuration of this type is more likely than the second one;[14] whereas the third configuration could be more or less likely than the other two. This is confirmed looking at Figures 2(a) and 2(b), where the configuration Mutual edge + In is represented by the areas colored in dark blue (marked by 111D): the combinations of λ_1 and λ_2 corresponding to the first and second configurations are represented by areas close to the bottom left corner, sharing a common point and the third configuration is represented by the area near to the top left corner. Comparing Figures 2(a) and 2(b), the likelihood of this configuration increases by reducing T from 0.6 to 0.5.

(8) A Mutual edge + Out configuration is characterized by bilateral trade between r and s and unilateral trade from r to k. From Proposition 1, only one configuration of this type is possible: two-way trade between 1 and 2 and one-way trade from 1 to 3. Given our setting of parameters, a Mutual edge + Out

configuration only appears in Figure 2(b), where all combinations of λ_1 and λ_2 corresponding to this type of configuration are represented by the light blue area (marked by 111U).

(9) Three links arranged from *s* to *r*, from *k* to *r*, and from *s* to *k* correspond to a Transitive configuration. From Proposition 1, only three configurations of this type are possible: one-way trade from 1 to 2, from 1 to 3, and from 3 to 2; one-way trade from 1 to 2, from 1 to 3, and from 2 to 3; and one-way trade from 1 to 3, from 2 to 1, and from 2 to 3.

Looking at Figures 2(a) and 2(b)—where Transitive configurations correspond to the area colored in dark green (marked by 030T)—we infer that the configuration one-way trade from 1 to 2, from 1 to 3, and from 3 to 2 is less likely than the other two since, given the chosen parameter it does not appear (but it does by reducing sufficiently T_{23} or ϵ_2). Comparing these figures, the likelihood of a Transitive configuration increases by reducing T from 0.6 to 0.5.

(10) Three links arranged from *r* to *s*, from *s* to *k*, and from *k* to *r* correspond to a Cycle configuration. From Proposition 1, a combination of λ_1 and λ_2 does not exist that allows for this type of configuration.

(11) A Mutual star configuration is characterized by bilateral trade between *r* and *s* and between *r* and *k*. From Proposition 1, the only possible configuration of this type is two-way trade between 1 and 2, and 1 and 3. This configuration is not represented in Figures 2(a) and 2(b), since it only occurs for values of $T < 0.5$.

(12) A Mutual edge + double In configuration is characterized by bilateral trade between *s* and *k* and unilateral trade from *r* to *s* and from *r* to *k*. All configurations are possible: one-way trade from 1 to 2 and from 1 to 3, and bilateral trade between 2 and 3; one-way trade from 2 to 1 and from 2 to 3, and bilateral trade between 1 and 3; and one-way trade from 3 to 1 and from 3 to 2, and bilateral trade between 1 and 2.

From Conjecture 1,[15] the configuration one-way trade from 3 to 1 and from 3 to 2, and bilateral trade between 1 and 2 is more likely then the other two. Moreover, the configuration one-way trade from 2 to 1 and from 2 to 3 and bilateral trade between 1 and 3 is more likely than the configuration one-way trade from 1 to 2 and from 1 to 3, and bilateral trade between 2 and 3. This is confirmed looking at Figures 2(a) and 2(b), where the configuration Mutual edge + double In is represented by the area colored in orange (marked by 120D). Comparing Figures 2(a) and 2 (b), the likelihood of this configuration increases by reducing T from 0.6 to 0.5.

(13) A Mutual edge + double Out configuration is characterized by bilateral trade between *s* and *k* and unilateral trade from *s* to *r* and from *k* to *r*. The only configuration possible is two-way trade between 2 and 3, and one-way trade from 2 to 1 and from 1 to 3. Given our parameter setting, a Mutual edge- + Out configuration only appears in Figure 2(b), where all combinations of λ_1 and λ_2 corresponding to this type of configuration are represented by the dark green area (marked by 120U).

(14) A Mutual edge + Cycle configuration is characterized by bilateral trade between *s* and *k*, and unilateral trade from *s* to *r* and from *r* to *k*. From Proposition 1, only one configuration is possible: two-way trade between 1 and 2, and one-way trade from 1 to 3 and from 3 to 2. This configuration occurs only for values of $T < 0.5$.

(15) An Almost Complete graph configuration is characterized by bilateral trade between *s* and *k* and between *r* and *k*, and unilateral trade from *s* to *r*. From Proposition 1, the only possible configurations of this type are: two-way trade between 2 and 3 and between 1 and 2, and one-way trade from 1 to 3; two-way trade between 1 and 3 and between 1 and 2, and one-way trade from 3 to 2; and two-way trade between 1 and 3 and between 1 and 2, and one-way trade from 2 to 3. From Conjecture 1, the third configuration of this type is more likely than the first one.[16] Also this configuration only occurs for values of $T < 0.5$.

(16) A Complete graph configuration is characterized by bilateral trade involving all three regions and it is unique. This configuration occurs only for values of $T < 0.5$.

As mentioned above, we have also carried out simulations, not presented here, considering values of T from 1.2 (corresponding to the sufficient condition of no trade $\tilde{\lambda}=0$) to 0.21 (corresponding to a distance between 1 and 2 that is sufficiently close to zero). By reducing trade costs from $T = 1.2$ to $T = 0.21$, we have detected clear patterns for the different types of triad: the likelihood of the empty-graph configuration decreases by reducing trade costs (until it becomes zero); the likelihood of the complete graph configuration always increases by reducing trade cost; finally, the likelihood of all other configurations, except the cycle, which can never occur, first increases then decreases by reducing T. Each configuration reaches a peak with this approximate order: (1) Single edge; (2) In-star; (3) Line; (4) Mutual edge, Out-star and Transitive; (5) Mutual edge + In; (6) Mutual edge + Out; (7) Mutual edge + Double In; (8) Mutual edge + Double Out and Mutual edge + Cycle; (9) Mutual star; and (10) almost Complete graph.

Finally, we observe that by reducing T all the configurations involving bilateral trade increase. This is clearly shown through the comparison of Figure 2(a) and 2(b) where all these configurations are included in the squares at the bottom left corner, with sides of length $\tilde{\lambda}=0.3$ (in Figure 2a) and $\tilde{\lambda}=0.467$ (in Figure 2b).

4 | DISCUSSION AND CONCLUSION

4.1 | Discussion: The empirical results in the light of the theoretical analysis

In this section, the theoretical analysis is used to shed some light on the empirical results looking at the impact of trade costs on network structures. To this end, we compare the findings from the triad census analysis applied to the European regional trade network with the theoretical results in correspondence of different values of trade costs. Reassuringly, we find some notable associations.

First, the outcome of the triad census in correspondence of the 2nd and 5th deciles (subnetworks $\mathcal{N}_2$ and $\mathcal{N}_5$, respectively) as reported in Table 1 shows that the relative frequency of simple trade patterns increases for higher values of the trade costs; whereas that of mutual and more connected structures is reduced.[17] On the whole, these findings are confirmed by the results of the theoretical model as reported in Figures 2(a) and 2(b). Indeed, those figures suggest a transition from simple, less connected to more connected trade structures when reducing the trade costs. For most of the structures (with the exception of empty and complete graphs, for which this relationship is respectively always increasing and decreasing), this relationship is not linear with more connected structures emerging at lower values of T. However, details differ: in the empirical analysis, even at small distances the proportion of simpler structures is still quite substantial; and that of more connected ones is not too large.

Second, considering the two extreme deciles, the 1st and the 10th (subnetworks $\mathcal{N}_1$ and $\mathcal{N}_{10}$, respectively), we can infer the following: for the 1st decile, we note that by reducing further trade costs also the frequency of configurations of intermediate connectedness decreases by decreasing distance.[18] Moreover, given the strong increase in frequency of the Mutual Edge configuration and the increase of that of the complete graph configuration, overall the structures involving only bilateral trade increases. This is in line with the theoretical results. However, for the 10th decile, we observe an increase in percentage terms of the frequency of structures characterized by trade inflow.[19] This observation is not in line with the theoretical results.

4.2 | Conclusion and further research lines

While on the whole empirical findings correspond to theoretical results, some stunning differences prevail and ask for further research. This concerns, in particular, the persistence at small distances of a large number of simple structures.

Various explanations can be taken into account. The first is related to trade costs: the objective of the model is to explain how the presence of trade links in a three-region economy is determined. We showed that the likelihood of a network structure depends on trade costs T. In the model T is homogeneous and has a very broad meaning encompassing various impediments to trade. Turning to the empirics, the triad census is considering the overall European regional network, which is probably not homogeneously connected. The presence of a large number of empty graph and mutual edge configurations hints to a network composed of highly connected regions and much less connected ones. Moreover, the concept of trade costs in the empirical analysis especially that used to produce Table 1 is not equivalent to the one underlying the theoretical part; it only serves as proxy of pure transport costs and does not capture the broad meaning of T in the theoretic model. Therefore, it is well possible that a reduction in transport costs corresponds to a smaller and nonhomogeneous reduction in T. In order to add another dimension to distance, we compare the triad census for regions belonging to the same country and to different countries. Indeed, the empirical analysis presented in Tables A1 and A2 in the Appendix shows a "border effect." Disregarding the empty graphs,[20] mutual edges appear to be relatively more frequent when regions of the same country are involved; and relatively less frequent when regions of different countries are involved (note that a similar intuition is grasped simply by comparing the absolute number of mutual edge configurations for the two groups of regions). This again confirms the theoretical results according to which reducing trade costs favors bilateral links between closer regions.

Another issue that may contribute to explain differences between the empirical and the theoretical analysis is the following. A model, which analyzes just three regions in isolation, neglects that triplets of real world trade networks will be different with respect to the connections to regions outside the triplet. In particular, regions that are not well connected to each other will presumably (owing to the effect of competition) have more connections to other regions. These connections to other regions make triads with few connections more likely (than for triplets that do not have any outside connection).

Finally, notice that in the theoretical framework we have chosen, there is not a market size effect. However, local demand could play an important role in the real world, explaining, for example, the emergence of "In-star" structures in correspondence of the last decile. Note that, even though the NUTS classification actually homogenizes slightly the size of regions (with respect to population); to there is still a considerable variation in regional population size leaving ample scope for a home market effect.

To sum up: we have developed a simple NEG model that allows us to analyze the emergence of trade structures depending upon trade costs and we have elaborated on how the results correspond to an empirical analysis of the European regional trade network. Given the simple structure of the model, various extensions have been pointed out to increase the explanatory power. These directions suggest that more empirical regularities of the trade network can be explained from a NEG perspective. However, they are left to further research.

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NOTES

[1] It is worth noticing that the methodology used to construct the data is a parameter-free approach and therefore deviates from earlier methods based on the gravity model that suffer from analytical inconsistencies (Thissen et al., 2013).

[2] The countries included in the dataset are Austria, Belgium, Bulgaria, Cyprus, the Czech Republic, Germany, Denmark, Estonia, Spain, Finland, France, Greece, Croatia, Hungary, Ireland, Italy, Lithuania, Luxembourg, Latvia, Malta, the Netherlands, Poland, Portugal, Romania, Sweden, Slovenia, Slovak Republic, and United Kingdom.

[3] The new version of the data includes Croatia regions, Baltic states (Estonia, Latvia, and Lithuania), two more Portuguese regions. Moreover, Branderburg is considered as a single region, while in the previous version it was split in two subregions (North-East and South-West).

[4] Since there are three nodes in a triad, and each node can be connected to two other nodes, this gives rise to six possible links. Since each link can be present or absent, there are $2^6 = 64$ possible realizations of the triads. Excluding isomorphic cases (e.g., if $\mathscr{V}_s$, $\mathscr{V}_r$, and $\mathscr{V}_k$ are not linked, $\mathscr{T}_{srk}$, $\mathscr{T}_{rks}$, and $\mathscr{T}_{ksr}$ are isomorphic), we remain with 16 isomorphic classes for 64 different triad states.

[5] See Basile et al. (2016) on the choice of the threshold. The threshold has been compared with possible alternatives: a less stringent threshold starts to modify the structure of connection for values below 10 million; while a more stringent threshold leaves the structure unchanged as far it reaches a level of $w > 500$ million.

[6] The subnetwork including links with $w > 25$ is weakly connected, that is not every vertex r is reachable from every s by a directed walk. In probability terms, taking two regions s and r at random, the chance of observing a link between them is now 34 percent.

[7] Bilateral distances are measured in terms of great circle distance between the centroids of the regions. The left-edge cutoffs (in km) are: 384.64, 587.66, 768.56, 943.15, 1,123.67, 1,320.81, 1,539.63, 1,800.79, 2,203.10, and 5,179.90.

[8] We present here a three-region new economic geography linear model that extends the two-region model developed by Ottaviano et al. (2002). Our analysis, which is limited to the short run, also extends other previous contributions, which consider a three-region economy and assume a quasilinear utility function (see Ago et al., 2006; Behrens, 2011). These analyses, however, consider a less general geographical structure by assuming that the regions are equally spaced along a line (Ago et al., 2006) or that two regions in a trade bloc are at the same distance from a third outside region. Similarly to Basile et al. (2016), we let the distance between all regions differ. In this paper, we explicitly number the regions and specify their distance. Moreover, we study how trade costs impact on the distribution of triadic configurations.

[9] Notice that the definition of trade distance used here is more general than that of geographical distance that we used in the empirical part.

[10] Owing to market segmentation exports do not affect pricing in the local market. Therefore, there are no effects of a link from r to s on the price fixed in r.

[11] Profitability may also decrease owing to the higher trade costs impacting on the price set by local and outside firms in the local market.

[12] We proceed as follows. When Equation (9) applies, sufficient conditions for no trade from r to s and from s to k are:

$$T_{rs} \geq \frac{a}{b} - \eta = T_{M1} \text{ and } T_{sk} \geq \frac{a}{b} - \eta = T_{M1}.$$

When Equation (10) applies, sufficient conditions for trade from r to s and no trade from s to k are:

$$T_{rs} < \frac{2(a-\eta b)}{2b+cE} = T_{m1} \text{ and } T_{sk} \geq \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c\lambda_s E} = T_{M2}$$

with $T_{M2} < T_{M1}$ since $T_{rs} < T_{m1}$.
When Equation (13) applies, sufficient conditions for no trade from r to s and trade from s to k are:

$$T_{rs} \geq \frac{2(a-\eta b)+cE\lambda_k T_{sk}}{2b+c\lambda_k E} = T_{M3} \text{ and } T_{sk} < \frac{2(a-\eta b)}{2b+cE} = T_{m1}$$

with $T_{M3} < T_{M1}$ since $T_{sk} < T_{m1}$.
Finally, when Equation (14) applies, sufficient conditions for trade from r to s and from s to k are:

$$T_{rs} < T_{m1} \text{ and } T_{sk} < T_{m1}.$$

Condition $T_{rs} < T_{m1}$ has been obtained as follows: consider the term $\frac{2(a-\eta b)+cE\lambda_k T_{sk}}{2b+c(\lambda_s+\lambda_k)E}$ in (15), defined as the ratio A/B, where $A=2(a-\eta b)+cE\lambda_k T_{sk}$ and $B=2b+c(\lambda_s+\lambda_k)E$.
$\min(A)=2(a-\eta b)$ is obtained in correspondence of $\lambda_k = 0$, whereas $\max(B)=2b+cE$ is obtained when $\lambda_s + \lambda_k = 1$. Given that both A and B are positive and different from zero, it follows $\frac{\min(A)}{\max(B)} = \min\left(\frac{A}{B}\right)$, which can only occur at $\lambda_k = 0$. These conditions extend to the case of a three-region economy those derived by Ottaviano et al. (2002) for the case of two regions.
It is easy to check that condition $T_{rs} \geq T_{M1}$ is equivalent to $\lambda^* \leq 0$; condition $T_{rs} < T_{m1}$ to $\lambda^* > 1$ and $\lambda^{**} > 1$ (letting $\lambda_k = 0$); and condition $T_{rs} \geq T_{M3}$ to $\lambda^{**} \leq 0$.

[13] More specifically, according to Conjecture 1, the link 3 to 1 in the first In-star configuration should prevail over the link 3 to 2 in the second configuration; and the link 1 to 2 in the second configuration should prevail over the link 1 to 3 in the second configuration.

[14] Here, the link 3 to 1 in the first Mutual edge + In configuration prevails over the link 3 to 2 in the second configuration.

[15] Comparing the configuration 3 to 1, 3 to 2, and 1 and 2, and the configuration 2 to 1, 2 to 3, and 1 and 3, the link 1 to 2 prevails over the link 1 to 3. Comparing this latter configuration with the configuration 1 to 2, 1 to 3, and 2 and 3, the link 3 to 1 prevails over the link 3 to 2. It also follows that the configuration 3 to 1, 3 to 2, and 1 and 2 is more likely than the configuration 1 to 2, 1 to 3, and 2 and 3 (indeed consider that here the link 2 to 1 prevails over the link 2 to 3).

[16] Indeed, the link 3 to 1 in the configuration 1 and 3, 1 and 2, and 3 to 2 prevails over the link 3 to 2 in the configuration 2 and 3, 1 and 2, and 1 to 3.

[17] Concerning simple structures, these are: Empty graph, Single edge, In-star, Line, Transitive, and Cycle; with the exception of Out-star. Notice that as suggested by the theoretical model and confirmed by the empirical analysis no (or almost no) cycle configuration should occur. Concerning more complex structures, these are: Mutual edge, Mutual edge + out, Mutual star, Mutual edge + double In, Mutual edge + Double out, almost complete and complete graph; with the exception of Mutual edge + In, that increases only in absolute terms.

[18] These are: Out-star, Mutual edge + In, Mutual edge + Out, Mutual star; with the exceptions of Mutual edge and Transitive.

[19] These are: Single edge, In-star, Line, Mutual edge + In, Transitive, Mutual edge + double In.

[20] Notice that also at the country scale trade networks appears to be nonhomogeneous (this is revealed by the large number of empty graphs).

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APPENDIX

A | Features of the E.U. interregional trade network

This Appendix provides some information on the main features of the E.U. interregional trade network emerging from the PBL dataset. To give an account of the heterogeneity of the data points, trade flow values go from a minimum of 0.000010400 million of exports from Algarve (PT15) to the Åland

islands (FI20), to a maximum of 11185.284 million of exports from Vzhodna Slovenija (SI01) to Zahodna Slovenija (SI02), and the Gini coefficient of the frequency distribution of trade flows is 0.75, showing very high inequality among values.

Figure A1 represents the backbone structure of regional trade flows in Europe in 2010. It is obtained following the procedure proposed by De Benedictis, Nenci, Santoni, Tajoli, and Vicarelli (2014) and applied by Zhou, Wu, and Xu (2016) to international trade flows. A complete graph of 268 region nodes and 71,556 edges is too dense to be visualized in a meaningful way to obtain interesting information on the structure of the network. Therefore, just for visualization purposes, an information-shrinking procedure has to be applied. One way to sparsify the network is to fix the number of outgoing links for every region. The way we proceeded is through the ordering of trade flows for every region, keeping only the two most important export ties, in terms of trade value, for every region, and discarding the rest. As an example, Vienna (Wien, AT13) is exporting 3.15 percent of its total E.U. export to Oberösterreich (AT31) and 2.96 percent to Oberbayern (DE21) in Germany. These are

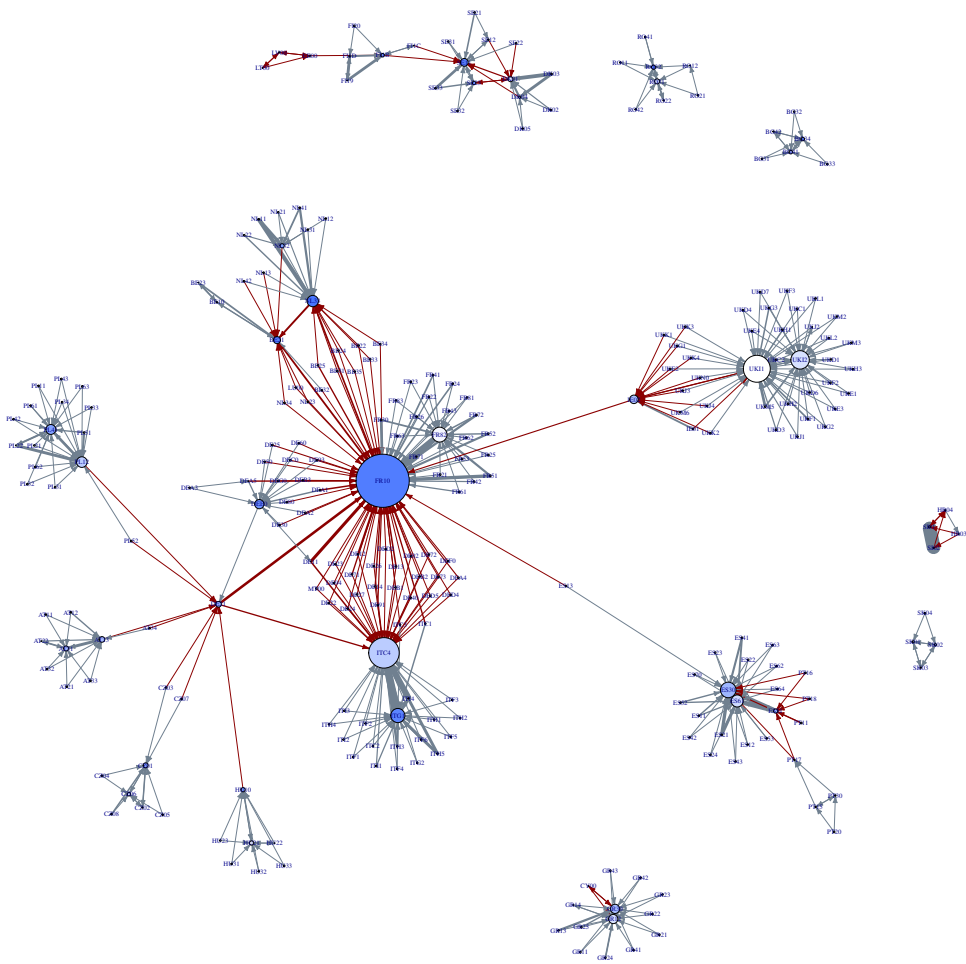

FIGURE A1 The E.U. regional trade network—top two export regions

Note. Our elaboration on bilateral trade flows between E.U. regions. Nodes are E.U. regions, labeled by their NUTS-2 code. Outdegree is fixed to 2 for every region. The size of the node is proportional to the indegree and the shades of blue coloring of the node are proportional to the GDP of the region in 2010. Edges are colored in gray if they link regions of the same country and in dark red if they connect regions of different countries. The thickness of the edge is proportional to the value of exports

Vienna's two major exporting markets at the regional level in E.U. Only those two corresponding edges are therefore selected in the case of Vienna. The same applies to all E.U. regions and the network dimension is reduced to 268×2.

The nodes of the network in Figure A1 are E.U. regions, labeled by their NUTS-2 code. For every region the number of outgoing links (outdegree) is fixed to two by construction, while the number of incoming links (indegree) depends on the number of regional trade partners that include the region as a major exporting market. Vienna has an indegree of eight, and the distribution of indegrees has a range that goes from 0 (201 regions are never included as a major export market) to 75, in the case of Île-de-France (FR10). The size of the node is proportional to the indegree of the region and the shades of blue coloring the node goes from dark blue to white in a proportional way to the GDP of the region in 2010. Edges are colored in gray if they link regions of the same country and in dark red if they connect regions of different countries.

From Figure A1 a star-like structure clearly emerges. At the core of the network are French, German, and Italian regions (and their regional partners in Belgium, Luxembourg, the Netherlands, and Malta); the U.K. is connected to the main component of the network through its link with the South-East of Ireland (IE02); Germany, with Oberbayern (DE21), bridges the core of Europe with Austria, Poland, Hungary, and the Czech Republic. All other E.U. countries are isolates: Sweden, Finland, Denmark, and the Baltic Republics form an isolated subnetwork, as Greece and Cyprus, Spain, and Portugal (with Madeira, the Azores islands and the Algarve further isolated), and Croatia and Slovenia. Slovakia, Romania, and Bulgaria are fully national subnetworks.

The large majority of the edges (73%) link regions belonging to the same country and core regions tend (with notable exceptions) to be the richest regions in Europe in terms of GDP. Île-de-France, Inner London (UKI1), and Madrid (ES30) are the key players in the network (Ballester, Calvó-Armengol, & Zenou, 2006; Borgatti, 2006; An, 2015). They share a common characteristic: each of them is a fundamental node in the domestic subnetwork and bridges intranational and international regional trade flows. This role is crucial in keeping the network as united as possible. Without them the level of fragmentation will increase notably and the number of subnetworks will increase substantially.

B | Domestic and international trade

TABLE A1 Triadic census conditional on distance: Domestic trade

	MAN code	Figure	Class	1st decile <384km	2nd decile	5th decile	9th decile	10th decile >2,207km
1	003		Empty graph	2,363,487	943,608	1,036	84	0
2	012		Single edge	33,132	12,246	79	0	0
3	102		Mutual edge	352,495	78,709	186	0	0
4	021D		Out-star	129	23	0	0	0
5	021U		In-star	213	200	6	0	0
6	021C		Line	0	1	0	0	0
7	111D		Mutual edge + In	225	95	4	0	0
8	111U		Mutual edge + Out	703	319	0	0	0
9	030T		Transitive	9	0	0	0	0
10	030C		Cycle	0	0	0	0	0
11	201		Mutual-star	4,157	2,834	19	36	0
12	120D		Mutual edge + double In	1	0	0	0	0
13	120U		Mutual edge + double Out	666	25	0	0	0
14	120C		Mutual edge + Cycle	0	0	0	0	0
15	210		Almost complete graph	725	18	0	0	0
16	300		Complete graph	7,578	142	0	0	0
			Number of nodes	256	185	21	10	2
			Number of edges	3,189	1,004	29	18	2
			Number of edges before trimming	7,156	7,154	7,156	7,154	7,156
			Number of zero-trade flows	3,967	6,150	7,127	7,136	7,154
			Edge—example (top)	SI01→ SI02	ITC4→ ITI4	ITC4→ ITG1	ES21→ ES70	ES51→ ES70

TABLE A2 Triadic census conditional on distance: International trade

	MAN code	Figure	Class	1st decile <384km	2nd decile	5th decile	9th decile	10th decile >2,207km
1	003		Empty graph	1,200,303	1,728,942	2,075,924	1,256,612	353,704
2	012		Single edge	53,496	112,926	141,070	61,833	21,307
3	102		Mutual edge	196,096	270,450	150,778	26,156	4,655
4	021D		Out-star	681	2,112	1,777	474	233
5	021U		In-star	424	1,801	2,337	4,042	2,279
6	021C		Line	233	537	714	408	157
7	111D		Mutual edge + In	2,313	6,812	4,940	1,982	516
8	111U		Mutual edge + Out	3,184	8,389	4,219	911	267
9	030T		Transitive	42	38	15	3	19
10	030C		Cycle	0	0	0	0	0
11	201		Mutual-star	16,986	29,573	9,351	969	156
12	120D		Mutual edge + double In	104	70	22	2	4
13	120U		Mutual edge + double Out	156	120	31	5	8
14	120C		Mutual edge + Cycle	48	44	13	0	0
15	210		Almost complete graph	1,243	451	142	3	1
16	300		Complete graph	2,947	675	111	0	0
			Number of nodes	208	236	224	202	133
			Number of edges	2,709	3,564	2,144	683	298
			Number of edges before trimming	7,156	7,154	7,156	7,154	7,156
			Number of zero-trade flows	4,447	3,590	5,012	6,471	6,858
			Edge—example (top)	NL33→ BE21	AT11→ DE11	AT12→ BE25	AT21→ ES61	BE21→ CY00

C | Boundary conditions

In this Appendix, we present the boundary conditions that have to be satisfied for all the allowed network configurations or triads. Each inequality refers to a specific link according to the following order: the first to

the link from 2 to 1; the second to the link from 1 to 2; the third to the link from 3 to 1; the fourth to link from 1 to 3; the fifth to the link from 3 to 2; and the sixth to the link from 2 to 3. Moreover, recalling that $\lambda_3 = 1 - \lambda_1 - \lambda_2$, a strict inequality sign corresponds to trade and a weak inequality sign to no trade.

C1 | Empty graph

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1$$

where

$$\tilde{\lambda} = \frac{2(a-\eta b-bT_{12})}{cET_{12}}$$
$$\bar{\lambda} = \frac{2(a-\eta b-bT_{13})}{cET_{13}}$$
$$\bar{\bar{\lambda}} = \frac{2(a-\eta b-bT_{23})}{cET_{23}}.$$

C2 | Single edge

One-way trade from 1 to 2:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

One-way trade from 2 to 1:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

One-way trade from 1 to 3:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C3 | Mutual edge

Two-way trade between 1 and 2:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

C4 | Out-star

One-way trade from 1 to 2 and from 1 to 3:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C5 | In-star

One-way trade from 2 to 1 and from 3 to 1:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

One-way trade from 1 to 2 and 3 to 2:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

One-way trade from 1 to 3 and 2 to 3:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C6 | Line

One-way trade from 2 to 1 and from 1 to 3:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\frac{T_{13}}{T_{23}}\lambda_1.$$

C7 | Mutual edge + In

Two-way trade between 1 and 2, and one-way trade from 3 to 1:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

Two-way trade between 1 and 2, and one-way trade from 3 to 2:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

Two-way trade between 1 and 3, and one-way trade from 2 to 1:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\frac{T_{13}}{T_{23}}\lambda_1.$$

C8 | Mutual edge + Out

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C9 | Transitive

One-way trade from 1 to 2, from 1 to 3, and from 3 to 2:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

One-way trade from 1 to 2, from 1 to 3, and from 2 to 3:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

One-way trade from 1 to 3, from 2 to 1, and from 2 to 3:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C10 | Cycle

None.

C11 | Mutual edge + double In

One-way trade from 1 to 2 and from 1 to 3, and bilateral trade between 2 and 3:

$$\lambda_1 \geq \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda}\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

One-way trade from 2 to 1 and from 2 to 3, and bilateral trade between 1 and 3:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 \geq \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 \geq \bar{\bar{\lambda}}\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

One-way trade from 3 to 1 and from 3 to 2, and bilateral trade between 1 and 2 occurs when:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}}-\lambda_1.$$

C12 | Mutual edge + double Out

Two-way trade between 2 and 3, and one-way trade from 2 to 1 and from 1 to 3:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 \geq \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 \leq (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 < \bar{\bar{\lambda}}\,,\ \lambda_2 > 1-\bar{\bar{\lambda}}-\lambda_1.$$

C13 | Mutual star

Two-way trade between 1 and 2 and 1 and 3:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C14 | Mutual edge + Cycle

Two-way trade between 1 and 2, and one-way trade from 1 to 3 and from 3 to 2:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C15 | Almost complete graph

Two-way trade between 2 and 3 and 1 and 2, and one-way trade from 1 to 3:

$$\lambda_1 < \tilde{\lambda}\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 \geq \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

Two-way trade between 1 and 3 and 1 and 2, and one-way trade from 3 to 2:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > 1-\bar{\lambda}-\lambda_1$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 \leq 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

Two-way trade between 1 and 3 and 1 and 2, and one-way trade from 2 to 3:

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \tilde{\lambda}$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 \geq \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$

C16 | Complete graph

$$\lambda_1 < \tilde{\lambda}\frac{T_{12}}{T_{13}} + \frac{T_{13}-T_{12}}{T_{13}}(1-\lambda_2)\,,\ \lambda_2 < \frac{T_{12}}{T_{23}}\tilde{\lambda} + \frac{T_{23}-T_{12}}{T_{23}}(1-\lambda_1)$$
$$\lambda_1 < \bar{\lambda} - \frac{T_{13}-T_{12}}{T_{13}}\lambda_2\,,\ \lambda_2 > (1-\bar{\lambda}-\lambda_1)\frac{T_{13}}{T_{23}}$$
$$\lambda_2 < \bar{\bar{\lambda}} - \frac{T_{23}-T_{12}}{T_{23}}\lambda_1\,,\ \lambda_2 > 1-\bar{\bar{\lambda}} - \frac{T_{13}}{T_{23}}\lambda_1.$$