Optimal ordering and pricing strategies in the presence of a B2B spot market

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Abstract: In the current paper, we examine the effect of a B2B spot market on the strategic behavior and the performance of a reseller who continues to use the traditional channel while participating in a B2B spot market. We analyze the case in which a risk-neutral reseller faces an additive or multiplicative demand function and identify sufficient conditions under which the optimal order quantity and retail price exist and are unique. We then analytically examine the case in which a risk-averse reseller participates in a fully liquid spot market. We also study numerically how varying liquidity, spot price volatility, demand variability, and correlation coefficient affect a firm’s strategies and performance. We find that demand variability significantly affects both pricing and ordering strategies, whereas the spot price volatility has less influence on pricing decisions. Our results also show that for a risk-averse reseller to charge a lower retail price when the spot market liquidity increases is desirable. We further show that a B2B spot market cannot always improve a reseller’s utility. These findings shed light on how resellers can adjust their procurement and pricing strategies to align with the new business environment created by the emergence of B2B spot markets, as well as have obvious implications for the development of a B2B spot market.

Keywords: Supply chain management; Procurement and pricing strategy; B2B spot market; Market liquidity; Risk

1. Introduction

Many organizations predicted that B2B spot markets (online spot markets or e-marketplaces) would have a grand future (e.g., Gartner Group 2004 and e-Marketer 2002). However, the development of B2B spot markets has not been smooth. Expected by many to be the next major innovation in business, many B2B spot markets sprang up virtually overnight between 1999 and 2000, and a tremendous amount of capital was poured in (Kaplan and Sawhney 2000). However, the frenzied
development came to a halt by the end of 2000 (Grey et al. 2005). During these gloomy years, many B2B spot markets failed or were merged (Brunn et al. 2002). The boom and burst of the dot com during that period created skepticism about the future of B2B spot markets. However, many did not lose hope, and the development continues until today. In recent years, hundreds of B2B spot markets have opened or reopened on the Internet. Many commodity products, such as commodity metal, chemical products, semiconductors, plastics, and agricultural products, are traded over B2B spot markets. In China alone, over 100 B2B spot markets have been established since 2000, many of which have achieved reasonable success. BOCE (www.boce.cn, Tianjin), a leading B2B spot market for crude oil, coke, and rebar, has achieved an average daily trading volume of more than 100,000 metric tons on rebar.

Resellers are major participants of B2B spot markets for commodity products. B2B spot markets can be a double-edged sword to resellers. On the one hand, resellers can offload their excessive inventories and eliminate stockout costs through B2B spot markets. They can earn additional revenue by speculating (purchasing more through contracts and then selling) in B2B spot markets. On the other hand, trading in B2B spot markets also exposes resellers to price volatility. Therefore, to participate in a B2B spot market, a reseller needs to tailor his/her pricing and procurement strategies to the new business environment. However, for many resellers, there is a lack of understanding of this evolving business environment, which may limit the participation of resellers in the B2B spot market. Clearly, how a B2B spot market serves or is perceived to serve the reseller needs is crucial to its success.

Another critical concern for resellers is the lack of liquidity in the B2B spot market. As many B2B spot markets are still in their early development stage, most of them cannot provide buyers or sellers with perfect matches. In such business environment, resellers cannot effectively offload their excessive inventories or procure the shortfall in the spot market. In addition, the speculation behavior may also be affected by the imperfectness of the spot market access. Therefore, resellers should develop different pricing and procurement strategies for different market liquidities, but it cannot be done properly without a clear understanding of the effect of the spot market liquidity.
Motivated by the concerns elaborated above, we consider a single-period inventory model in which a reseller participating in a B2B spot market faces an uncertain price-sensitive demand. The reseller makes a procurement-quantity decision and a selling-price (retail price) decision simultaneously before spot trading with the objective of maximizing his/her expected utility. We consider two types of resellers, that is, the risk-neutral reseller and the risk-averse reseller, and investigate how the B2B spot market affects the reseller’s strategies by focusing on such characteristics as price sensitivity, demand uncertainty, and price volatility.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the basic model. Section 4 presents the strategies for a risk-neutral reseller. In Section 5, we examine the strategies for a risk-averse reseller. We first analytically investigate optimal strategies with a fully liquid B2B spot market and an additive demand function, and then numerically investigate the effects of the B2B spot market on the reseller’s strategies. Finally in Section 6, we give the concluding remarks and present some issues for future research.

2. Literature Review

The popularity and development of B2B spot markets have stimulated extensive research in the last 10 years. Detailed reviews of research on B2B spot markets from a supply chain perspective can be found in Swaminathan and Tayur (2003), Grieger (2003), Eng (2004), Grey et al. (2005), and Haksöz and Seshadri (2007). Here, we only review the works closely related to this paper.

The emergence of B2B spot markets has provided manufacturers and resellers with a new business avenue while forcing them to adjust their strategies to align with the new business environment. Thus, a key question is how the procurement and pricing strategies of a reseller will be affected by the presence of a B2B spot market. Motivated by HP’s Internet-based exchange TradingHub.com, Lee and Whang (2002) use a two-period newsvendor model to explore the effect of a secondary market, which can readjust resellers’ inventories. They show that although the sales volume of the manufacturer may increase or decrease, the secondary market can always improve the supply chain performance. Peleg et al. (2002) compare three procurement strategies using the long-term
relationship-based contract, the short-term spot market, and the combination of both. They find that the optimal strategy depends on the market characteristics. Etzion and Pinker (2008) study the asymmetric competition between two types of suppliers in a B2B spot market environment; one uses the combination of forward contracts and spot market, whereas the other utilizes only the spot market. They find that the supplier with forward contracts can benefit from the spot market more than the supplier without using forward contracts. Wu et al. (2002) examine the capacity reservation option contract strategy for a capital-intensive product. The buyer purchases a certain amount of option contracts from the seller. On exercising day, both the buyer and the seller can sell excess capacities or buy additional quantities in a spot market. Buyers’s optimal reservation quantity depends on both the reservation cost and execution cost, whereas the seller’s optimal strategy is to set the execution cost as low as possible. Spinler and Huchzermeier (2006) extend Wu et al.’s (2002) results to the case in which the buyer’s demand function is state dependent. They show that the combination of the option contract and the spot market is a Pareto improvement compared with other market structures. Seifert et al. (2004) analyze the benefits of an online spot market from the supply chain operational perspective. In their model, the buyer can procure through forward contract or the spot market. In spot trading, the commodity can be both bought and sold through the spot market. The optimal procurement strategies are analyzed based on different market situations. Their result shows that a significant profit can be achieved through the readjustment of the order quantity. Martínez-de-Albéniz and Simchi-Levi (2005) study a risk-averse manufacturer’s portfolio procurement strategy option contract and having access to a spot market. The analysis of Serel et al. (2001) and Inderfurth and Kelle (2011) of the capacity reservation in the presence of a spot market is also an interesting source of information. The model presented in the current paper differs from the above mentioned studies in that it incorporates optimal pricing and ordering decisions in the presence of price-dependent stochastic demands in the resell channel and an exogenous random price in the spot market. The reseller’s risk attitude is also considered. These factors enable us to study from different angles the effects of a B2B spot market on the performance and strategic behaviors of a reseller.
Our work relates to the literature on joint ordering and pricing decisions. A recent work on this area is by Petruzzi and Data (1999), who study a newsvendor problem in which the reseller has to make the stocking and pricing decision simultaneously. They analyze the effect of the nature of the stochastic demand function on the pricing and stocking decisions. Elmaghraby and Keskinocak (2003) and Yano and Gilbert (2003) examine the various related issues and provide comprehensive reviews. Recently, Bernstein and Federgruen (2005), Ray et al. (2005), and Song et al. (2008), among others, have studied the joint decision problem in a price-dependent stochastic setting. Studies also address the combined pricing and ordering strategy of a risk-averse newsvendor. For example, Agrawal and Seshadri (2000) model risk aversion with the general utility function, Chen et al. (2007) extend Agrawal and Seshadri’s (2000) model to a multi-period setting, and Chen et al. (2009) use CVaR as risk criteria. Our focus is different. We examine the effect of B2B spot markets with a volatile price on the strategies and performance of a reseller.

3. Basic Model Description

We consider a one-period model in which the decisions and events form a three-stage process, as shown in Figure 1. Before the selling season, i.e., at $T_0$, knowing the procurement price $w$ and facing a random price-sensitive demand $D$, the reseller decides the quantity $q$ to order through forward contract and the retail price $p_r$ for the resell channel simultaneously. During the selling season $T_1$, knowing the realized demand and spot price, the reseller can sell his/her excessive inventory or buy some extra products through the B2B spot market. After the selling season, i.e., at $T_2$, the spot market is closed, and the unsold surplus is salvaged at price $s$. To avoid trivial outcomes, we assume that $s < w$. A newsvendor type model is formulated to study the procurement and pricing decision of a reseller who faces uncertain future demand and a spot market with an uncertain exogenous spot price. We assume that the reseller cannot reorder from the manufacturer during the selling season. The reseller, such as catalog resellers or resellers with planned promotional campaigns, cannot change the price during the whole selling season. These kinds of decisions under uncertainty and risk are common in reality, and they have been studied extensively.
Owing to the large number of participants in the spot market, individual participants cannot manipulate the market. Therefore, the spot price $p_e$ of the product in the spot market is assumed an exogenous stochastic variable, with mean $\mu_e$ and variance $\sigma^2_e$, and all the participants in the spot market are price-takers. As most B2B spot markets are still at their development stage, not many spot markets have achieved full liquidity, and as such a reseller’s order may not be completely executed. In the current paper, we assume that when an order is placed in the spot market, it will be executed with a probability of $m$, which measures the liquidity of the spot market (Kleindorfer and Wu 2003).

The reseller’s profit function can then be expressed as follows:

$$
\pi_r = p_r D + p_e m(q-D)^+ + s(1-m)(q-D)^+ - p_e m(D-q)^+ - wq - p_r (1-m)(D-q)^+. \quad (1)
$$

The first term on the right-hand side (RHS) represents the revenue from the resell channel if the demand is met in full. The second term is the revenue obtained by selling the surplus in the spot market. The third is the salvage value of the unsold surplus. We assume that $s$ is lower than the realized spot price, so that the reseller will first try to salvage his/her inventory in the spot market, as reflected by the second and third terms on the RHS \(^1\). The fourth is the cost for procurement shortage from the spot market, the fifth is the procurement cost through contract, and the last represents the value of the unmet demand in the resell channel. In summary, to maintain customer goodwill and prevent the erosion of the customer base in the resell channel, a reseller usually goes the extra mile to meet customer demands, and he/she usually maintains a stable reselling price. In other words, the reseller takes the risk of price volatility in the spot market and an uncertain demand in the resell channel in return for a higher profit margin in the resell channel.

\(^1\) A more general form for the second and third terms is $I(p_e - s)[p_e m + s(1-m)](q-D)^+ + I(s - p_e) s(q-D)^+$, and $I(\cdot)$ is the indicator function that takes the value of 1 if its argument is positive or 0 otherwise.
We consider two types of demand functions: the additive demand function and the multiplicative demand function. Demand is defined as $D = y(p_r) + \varepsilon$ in the additive case and $D = y(p_r)\varepsilon$ in the multiplicative case, where $y(p_r)$ is a deterministic and decreasing function of the selling price $p_r$, and $\varepsilon$ is a random factor with mean $\mu_d$ and standard deviation $\sigma_d$. Furthermore, we take the form of $y(p_r) = \theta - ap_r$ and $y(p_r) = \theta p_r^{-a}$, where $\theta > 0$ is the market base and $a > 0$ measures the price sensitivity (Petruzzi and Dada 1999). Usually, a fixed setup/initial participation cost is required to trade in a B2B spot market, and there are also some risk-associated costs; thus, most small-volume end-users do not participate in the B2B spot trading. In our example, although some large-volume end-users participate in the BOCE, over 95% of the users of BOCE are steel distributors\textsuperscript{2}. Therefore, we assume that the spot price $p_e$ does not directly affect end-user demand $D$ in the retail channel.

For each demand function, we consider the decisions of two types of resellers, the risk-neutral reseller and the risk-averse reseller. In the risk-neutral case, the reseller makes decisions under the expected value criterion. He/she intends to maximize his/her expected profit $E[\pi_r]$ by setting the retail price and deciding the quantity to order from the manufacturer. In the risk-averse case, the reseller intends to maximize his/her mean–variance utility. Thus, in this case, the reseller’s decision problem is

$$\max_{p_r, q \geq 0} U_r = E[\pi_r] - kVar[\pi_r],$$

where $k > 0$ represents the reseller’s risk attitude. Mean–variance utility is first proposed in the seminal work of Markowitz (1959), and it has been widely adopted by some operations management studies, such as Chen and Federgruen (2000), Ding et al. (2007), and Buzacott et al. (2011). To study the effect of online spot market, Seifert et al. (2004), Martínez-de-Albéniz and Simchi-Levi (2006), and Dong and Liu (2007) also use mean–variance to measure risk exposure. An alternative approach to model risk aversion is by the expected utility framework. Van Mieghem (2003) provides an extensive discussion of utility theory and mean–variance analysis.

4. Strategies of a Risk-neutral Reseller

In this section, we consider the case in which the reseller is risk-neutral ($k = 0$). Let $F(\cdot)$ and $f(\cdot)$ represent CDF and PDF of $\varepsilon$, respectively, and assume that this distribution has support on $[A, B]$ with $0 \leq A < B$. Let $r(x) = f(x) / [1 - F(x)]$ be the hazard rate of the distribution $F(\cdot)$.

Following Petruzzi and Dada (1999), we denote $z \equiv q - y(p_r)$ in the additive demand case. The profit function can be rewritten as

$$
\pi_r(z, p_r) = \begin{cases} 
  p_r[y(p_r) + \varepsilon] - w[y(p_r) + z] + [(1 - m)s + mp_e](z - \varepsilon), & \varepsilon \leq z, \\
  p_r[y(p_r) + z] - w[y(p_r) + z] - m(p_e - p_r)(\varepsilon - z), & \varepsilon > z.
\end{cases}
$$

Define $\Lambda(z) = \int_A^z (z - x)f(x)dx$ and $\Theta(z) = \int_z^B (x - z)f(x)dx$. If $\varepsilon$ is independent of $p_e$, the expected profit can be written as follows:

$$
E[\pi_r(z, p_r)] = \Psi(p_r) - L(z, p_r),
$$

where $\Psi(p_r) \equiv (p_r - w)[y(p_r) + \mu_d]$, and

$$
L(z, p_r) \equiv [w - s(1 - m) - m\mu_e]\Lambda(z) + [p_r + m(\mu_e - p_r) - w]\Theta(z).
$$

Similar to Petruzzi and Dada (1999), we can verify that $E[\pi_r(z, p_r)]$ is concave in $z$ for a given $p_r$. We can then follow a sequential procedure to find the optimal solution. We first find the optimal retail price $p_r^*(z)$ given $z$, and then find $z^*$ to maximize $E[\pi_r(z, p_r^*(z))]$.

**Theorem 1** For the additive demand, if $\varepsilon$ is independent of $p_e$, the following properties hold:

1. For a fixed $z$, the unique optimal retail price is given by

$$
p_r^*(z) = \frac{\theta + \mu_d + aw}{2a} - \frac{(1 - m)\Theta(z)}{2a}.
$$

2. If $F(\cdot)$ is a distribution function whose hazard rate satisfies $2r(z)^2 + dr(z)/dz > 0$ for $A \leq z \leq B$, and the market liquidity satisfies $m < (w - s)/(\mu_e - s)$, then $z^*$ is the largest $z$ in the region $[A, B]$ that satisfies $dE[\pi_r(z, p_r(z))] / dz = 0$.

3. If the conditions for (2) are met, and

$$
2ma\mu_e - (1 + m)aw + (1 - m)\theta + (1 - m)^2A + m(1 - m)\mu_d > 0,
$$

then $z^*$ is uniquely determined by $dE[\pi_r(z, p_r(z))] / dz = 0$. 

For convenience, we relegate all the proofs in this paper to the appendix. The condition \( m < (w - s)/(\mu_e - s) \) can be rewritten as \( m(\mu_e - w) < (1 - m)(w - s) \). Note that \( w - s \) is the loss per unit because of the inefficiency of the spot market, and \( \mu_e - w \) is the earning per unit from speculation. If the expected gain from the speculation is less than the expected loss due to the inefficiency of the spot market, the reseller will order a finite quantity by contract. Otherwise, the reseller should either procure an infinite quantity through contract if \( w < E[p_e] \) or order zero if \( w > E[p_e] \), which is similar to Milner and Kouvelis’s (2007) result.

**Proposition 1** With additive demand, if the conditions in Theorem 1(2) are met and \( w < \mu_e \), both \( z^* \) and \( p^*_r \) are increasing in \( m \).

In the additive demand case, the effect of \( m \) on the optimal order quantity is a tradeoff between the two opposing effects: the optimal order quantity increases in \( z^* \) and decreases in \( p^*_r \).

To understand further how the market liquidity and demand uncertainty affect the reseller’s strategies and performance, we use the parameter values \( a = 1.5 \), \( w = 10 \), and \( s = 6 \), and assume \((\theta + \varepsilon) \sim N(100, 20^2)\), \( p_e \sim N(11, 2^2) \) and \( \rho = 0.1 \) (the correlation coefficient between \( \varepsilon \) and \( p_e \)). We focus on analyzing three scenarios with different liquidities: partially liquid with \( m = 0.4 \) and \( m = 0.7 \), and no liquidity with \( m = 0 \). Hence, the reseller does not participate in the spot market.

As shown in Figure 2, the optimal retail price still increases in \( m \), although the spot price and the demand are correlated. The optimal order quantity increases in \( m \) for different demand uncertainties. With a higher liquidity spot market, the reseller can unload his/her excessive inventory for a higher value, and so he will order more through contract. As \( \sigma_d \) increases, the reseller orders more to protect the demand uncertainty and sets a lower retail price for all three scenarios.

In the multiplicative demand case, we denote \( z \equiv q/y(p_r) \). The profit function can be rewritten as

\[
\pi_r(z, p_r) = \begin{cases} 
    p_r y(p_r) \varepsilon - w y(p_r) z + [(1 - m)s + mp_e] y(p_r) (z - \varepsilon), & \varepsilon \leq z, \\
    p_r y(p_r) z - w y(p_r) z - m(\mu_e - p_r) y(p_r) (\varepsilon - z), & \varepsilon > z.
\end{cases}
\]
Analogous to the additive case, if $\varepsilon$ is independent of $p_c$ the expected profit can be written as follows:

$$E[\pi_r(z, p_r)] = \Psi(p_r) - L(z, p_r), \quad (5)$$

where $\Psi(p_r) \equiv (p_r - w)y(p_r)\mu_d$ and

$$L(z, p_r) \equiv y(p_r)[(w - s(1 - m) - m\mu_c)\Lambda(z) + (p_r + m(\mu_c - p_r) - w)\Theta(z)].$$

**Theorem 2** For multiplicative demand, if $\varepsilon$ is independent of $p_c$, the following properties hold:

1. For a fixed $z$, the unique optimal retail price is given by

$$p_r^*(z) = \frac{aw}{a - 1} + \frac{a}{a - 1} \left[\frac{w - (1 - m)s - m\mu_c}{\mu_d - (1 - m)\Theta(z)}\right], \quad (6)$$
(2) If $F(\cdot)$ is a distribution function satisfying $2r(z)^2 + dr(z)/dz > 0$ for $A \leq z \leq B$, and $m < (w - s)/(\mu_e - s)$ and $a \geq 2$, then $z^*$ is the largest $z$ in the region $[A, B]$ that satisfies $dE[\pi_r(z, p_r(z))]/dz = 0$.

(3) If the conditions for (2) are met, and $ma(\mu_e - w)\mu_d + [m\mu_d + (1 - m)A](w - m\mu_e) > 0$, then $z^*$ is uniquely determined by $dE[\pi_r(z, p_r(z))]/dz = 0$.

In the multiplicative demand case, we use the parameter values $\theta = 2000$, $a = 3$, $w = 10$, and $s = 6$, and assume $\varepsilon \sim N(100, 20^2)$, $p_e \sim N(11, 2^2)$, and $\rho = 0.1$. As shown in Figure 2, the optimal retail price slightly decreases in $m$, and the order quantity increases in $m$. We also observe that the reseller will set a higher retail price for higher $\sigma_d$. However, the order quantity tends to increase in $\sigma_d$ when the liquidity is relatively high and decreases in $\sigma_d$ when the liquidity is relatively low.

From the proofs of Theorem 1 and 2, we also find that the spot market uncertainty $\sigma_e$ does not affect the reseller’s strategies if $\varepsilon$ is independent of $p_e$. If $\varepsilon$ is correlated with $p_e$, our numerical study shows that the effects of $\sigma_e$ and $\rho$ on the reseller’s strategies and performance are insignificant.

5. Strategies of a Risk-averse Reseller

In this section, we first analyze the scenario in which the spot market achieves full liquidity. We then present some numerical examples to illustrate how the related parameters affect the optimal strategies of a risk-averse reseller facing both additive and multiplicative demands.
5.1. Strategies under a Fully Liquid Spot Market

Suppose full liquidity is achieved in the spot market, i.e., \( m = 1 \). We can simplify (1) into

\[
\pi_r = p_e D + p_e (q - D)^+ - wq - p_e (D - q)^+ \\
= p_r D - p_e (D - q) - wq.
\]

(7)

Usually, price movements in the spot market reflect the overall demand trend. Therefore, when the spot price \( p_e \) is high, demand in the resell channel is usually strong. For example, the price of hot-rolled sheets in a spot market in December 2008 was about 3000 RMB per metric ton, down from the 5000 RMB per metric ton in June of that year because of the sluggish demand in China caused by the on-going financial crisis. Therefore, assuming a positive correlation between the demand in the resell channel and the spot price is reasonable (Seifert et al. 2004). Accordingly, we assume further that \( \varepsilon \) and \( p_e \) together satisfy a bivariate normal distribution with correlation coefficient \( 0 \leq \rho < 1 \), i.e., \( (\varepsilon, p_e) \sim \mathcal{BN}[\mu_d, \mu_e, \sigma^2_d, \sigma^2_e] \). This distribution helps the tractability of the model, and it has been commonly used in the literature (Chod and Rudi 2005).

We first consider the additive demand function in the resell channel, i.e., \( D = \theta - a p_r + \varepsilon \). The expected value and the variance of the reseller’s profit can be expressed, respectively, as follows (see Appendix A for details):

\[
E[\pi_r] = -ap_r^2 + (a \mu_e + \theta + \mu_d)p_r + (\mu_e - w)q - \rho \sigma_d \sigma_e - (\theta + \mu_d)\mu_e,
\]

(8)

\[
\text{Var}[\pi_r] = (1 - \rho^2)\sigma^2_d (p_r - \mu_r)^2 + \sigma^2_e + \alpha^2 + 4\alpha \beta \mu_e + 4\beta^2 \mu_e^2 + 2\beta^2 \sigma_e^2,
\]

(9)

where \( \alpha = (a \sigma_e + \rho \sigma_d)p_r + q \sigma_e - (\theta + \mu_d)\sigma_e + \rho \mu_e \sigma_d \) and \( \beta = -\rho \sigma_d \). Clearly, the expected profit is a linear function of the order quantity \( q \). A risk-neutral reseller would order an infinite quantity through contract to speculate in the spot market if \( \mu_e > w \) or order zero if \( \mu_e < w \). Such strategy is unreasonable in practice. Similar to Seifert et al.’s (2004) argument, we only consider the situation with \( k > 0 \), i.e., the case of risk-averse resellers.

To solve the optimization problem, we need the property given in the following lemma:
Lemma 1 For the additive demand, if $k > 0$ and $\sigma_e > 0$, the utility function $U_r(p_r, q)$ is strictly joint concave in $p_r$ and $q$.

We have the following theorem to describe the optimal strategy:

Theorem 3 For the additive demand, the optimal retail price and the optimal order quantity for the risk-averse reseller are unique. They are given as follows:

\begin{align*}
p^*_r &= \mu_e + \frac{\theta + \mu_d - a\mu_e}{2\sigma(k)} - \lambda(\mu_e - w), \\
q^* &= \theta + \mu_d - ap^*_r + \frac{\mu_e - w}{2k\sigma^2_e} + \frac{\rho\sigma_d}{\sigma_e}(\mu_e - p^*_r),
\end{align*}

where $\sigma(k) \equiv a + k\sigma^2_d(1 - \rho^2)$ and $\lambda \equiv (a + \rho\sigma_d/\sigma_e)/[2\sigma(k)]$.

The optimal utility for the reseller is now given by

\begin{align*}
U^*_r = \frac{\sigma_e(\theta + \mu_d - 2a\mu_e + aw) - \rho\sigma_d(\mu_e - w)}{4\sigma^2_e\sigma(k)} + \frac{(\mu_e - w)^2}{4k\sigma^2_e} - \rho\sigma_d\sigma_e \\
+ (\mu_e - w)(\theta + \mu_d - a\mu_e) - k\sigma^2_e\sigma^2_d(1 + \rho^2).
\end{align*}

From (10), we observe that the reseller’s pricing strategy is different when the reseller participates in a fully liquid spot market. Instead of using the wholesale price as a starting point, the expected spot price is used. Starting from $\mu_e$, the reseller marks the price up by a margin determined by the risk-adjusted expected potential demand in the resell channel. The parameter $\lambda$ is positive, and it represents the retail price sensitivity with respect to the wholesale price. If $\mu_e > w$, the reseller benefits from the price premium $\mu_e - w$, which allows him/her to lower the retail price proportionally. Conversely, when $\mu_e < w$, the reseller will set a higher retail price to compensate for a higher procurement cost through contract.

Equation (11) shows that the optimal order quantity is also strongly influenced by the spot market. The optimal order quantity is composed of three parts, reflecting a rational procurement strategy. The first part is the expected demand in the resell channel. The second part is the strategic quantity. If $\mu_e > w$, the reseller should order more to speculate in the spot market. However,
when $\mu_e < w$, the reseller should order less and expect to meet a part of his/her demand from the spot procurement. Clearly, this term decreases in $k$ and $\sigma_e$. For a more risk-averse reseller, there is less incentive to get involved in speculation (procurement) in the spot market. As price volatility increases, speculating (procuring) in the spot market becomes less attractive to the reseller. The third part reflects the adjustment function of the spot market. When $p_r^* < \mu_e$, it is positive, indicating that the reseller can order more from the manufacturer to hedge against demand uncertainty, as selling this quantity in the spot market may still gain utility if it is not consumed by the demand in the resell channel. When $p_r^* > \mu_e$, it is negative, indicating that the reseller will order less from the manufacturer. The reseller can protect himself/herself from the demand uncertainty by buying from the spot market in case of a shortage in the resell channel.

When $\rho = 0$, (10), (11) and (12) become less complex, and we have the following result.

**Proposition 2** For the additive demand and $\rho = 0$, the following properties hold:

1. The optimal retail price is unaffected by $\sigma_e$; the optimal order quantity decreases in $\sigma_e$ if $\mu_e > w$, and increases in $\sigma_e$ if $\mu_e < w$; and the optimal utility always decreases in $\sigma_e$.

2. If $\theta + \mu_d - 2a\mu_e + aw > 0$, the optimal retail price decreases in $\sigma_d$, and the optimal order quantity increases in $\sigma_d$; if $\theta + \mu_d - 2a\mu_e + aw < 0$, the optimal retail price increases in $\sigma_d$, and the optimal order quantity decreases in $\sigma_d$; the optimal utility always decreases in $\sigma_d$.

Let

$$T = \frac{\sigma_e(\theta + \mu_d - 2a\mu_e + aw)}{\rho \sigma_d}. \quad (13)$$

**Proposition 3** For the additive demand and $\rho \neq 0$, there exists a price premium threshold $T$ such that when $\mu_e - w < T$, the optimal retail price decreases in $k$ and becomes greater than the expected spot price, and the optimal order quantity increases in $k$ if $\mu_e < w$. When $\mu_e - w > T$, the optimal retail price increases in $k$ and becomes smaller than the expected spot price, and the optimal order quantity decreases in $k$ if $\mu_e > w$. 
If the value of $\sigma_e$ is not very small, the condition $\mu_e - w < T$ will be satisfied. In these cases, the spot market is not a good channel for buying or selling the product. Then, a more risk-averse reseller will decrease his/her retail price to stimulate demand in the resell channel. By extensive numerical experiments, we can show that $\mu_e - w < T$ is always true except under extreme parameter settings (e.g., a very small price volatility). In practice, price volatilities in most spot markets in China are moderate. For example, the maximum price change of steel or crude is less than 25% in a month.

**Proposition 4** For the additive demand, if $\rho \neq 0$, the following hold:

1. The optimal retail price increases in $\sigma_e$ if $\mu_e > w$ and decreases in $\sigma_e$ if $\mu_e < w$.

2. There exists an expected market base threshold $B_1$ that if $\theta + \mu_d > B_1$, the optimal order quantity increases in $\sigma_e$; otherwise, the optimal order quantity decreases in $\sigma_e$.

3. There exists an expected market base threshold $B_2$ that if $\theta + \mu_d > B_2$, the optimal utility increases in $\sigma_e$; otherwise, the optimal utility decreases in $\sigma_e$.

Our numerical experiment in Section 5.2 shows that the effect of price volatility in the spot market on the retail price is not very strong. The effect of price volatility on the order quantity is strong when $\sigma_e$ is small and is insignificant when $\sigma_e$ is large.

Next, we investigate the multiplicative demand case, i.e., $D = \theta p_r^{-\alpha}$. Similar to the additive demand, the expected value and the variance of the reseller’s profit can be expressed, respectively, as follows:

$$E[\pi_r] = p_r y(p_r) \mu_d - y(p_r) (\mu_d \mu_e + \rho \sigma_d \sigma_e) + (\mu_e - w) q, \quad (14)$$

$$\text{Var}[\pi_r] = y(p_r)^2 (1 - \rho^2) \sigma_d^2 [(\mu_e - \mu_r)^2 + \sigma_e^2] + \alpha^2 + 4 \alpha \beta \mu_e + 4 \beta^2 \mu_e^2 + 2 \beta^2 \sigma_e^2, \quad (15)$$

where $y(p_r) = \theta p_r^{-\alpha}$, $\alpha = \sigma_e q - y(p_r) \mu_d \sigma_e + y(p_r) \rho \sigma_d (p_r + \mu_e)$ and $\beta = -\rho \sigma_d y(p_r)$.

**Lemma 2** For the multiplicative demand, if $k > 0, \sigma_e > 0$, and given the retail price $p_r$, the utility function $U_r(p_r, q)$ is strictly concave in $q$, and

$$q^* = \mu_d \theta p_r^{-\alpha} + \frac{\mu_e - w}{2k \sigma_e^2} + \frac{\rho \sigma_d}{\sigma_e} \theta p_r^{-\alpha} (\mu_e - p_r). \quad (16)$$
Comparing (16) with (11), in the multiplicative case, the optimal order quantity is also composed of three parts: the expected demand in the resell channel, the speculation (procurement) quantity, and the adjustment quantity. However, we cannot obtain the closed-form solution for the optimal retail price. In the next subsection, we provide some numerical examples to examine how the demand uncertainty, spot price volatility, and correlation coefficient affect the optimal retail price.

5.2. Comparative Statics

In this subsection, we study the effect of the spot market on the strategies and performance of a risk-averse reseller numerically. We also compare the differences as a result of additive and multiplicative demands. Throughout this section, we use the general profit function (1) and the same parameter values as in Section 4, except with $k = 0.01$.

5.2.1. Effect of Demand Variability. As shown in Figure 3, the optimal retail price decreases in the demand variability in the additive case but increases in the demand variability in the multiplicative case. We also observe that the optimal retail price decreases in the market liquidity in both the additive case and multiplicative case. A reseller participating in a fully liquid spot market will sell at the lowest price in the resell channel.

When $\sigma_d$ increases, a reseller participating in a fully liquid spot market will increase (decrease) his/her order quantity in the additive (multiplicative) case. In the additive case, the reason is the sharply reduced retail price stimulating higher demand in the resell channel. However, in the
multiplicative case, a higher order quantity implies a higher risk than that in the additive case so that the reseller has to reduce his/her order quantity to control risk. With a lower liquidity, the optimal order quantity decreases in $\sigma_d$ in both cases.

Figure 3 also shows that the reseller’s utility decreases as the demand variability increases in both additive and multiplicative cases. We also find that the utility is more sensitive to market liquidity as demand variability increases.

5.2.2. Effect of Price Volatility. Figure 4 shows that price volatility has a weaker influence on the retail price in both the additive and multiplicative cases. The reason is that the main purpose of setting a retail price is to optimize the utility of the resell channel, and the price volatility has
less effect on it.

In Figure 4, price volatility has a significant effect on the order quantity in scenarios with a fully liquid spot market in both the additive and multiplicative cases, whereas it has a smaller effect on the order quantity in other scenarios. The reason is that, as $\sigma_e$ increases, the spot market becomes less attractive for the reseller to trade, and then he/she will reduce his/her speculative activity in a fully liquid spot market. When market liquidity is lower (in our example, the liquidity threshold for speculating is about 0.8), the reseller is not involved in any speculative activity in the spot market; thus, price volatility has less effect on the order quantity.

We also find that the reseller cannot always benefit from spot trading in the additive case. When $\sigma_e$ is small, the reseller participating in a fully liquid spot market can achieve a significantly higher
utility than those participating in a lower liquid spot market in the additive case. The reason is that higher liquidity facilitates the reseller to sell his/her surplus, which in turn stimulates the speculative activity and improves his/her utility. As $\sigma_e$ increases, the spot market becomes less attractive because of the risk caused by spot trading.

5.2.3. Effect of Correlation Coefficient. Figure 5 shows that the optimal retail price slightly increases as $\rho$ increases in both the additive and multiplicative cases. When market liquidity increases, the effect of correlation coefficient on the optimal order quantity becomes more significant. In scenarios with higher values of liquidity, the optimal order quantity is reduced sharply as $\rho$ increases. With an increase in $\rho$, the incentive for ordering more excess inventory to sell through
the spot markets decreases. That is, when excess inventory is high, there is less demand in the resell channel, and the spot price is more likely to be low, and vice versa. Furthermore, in the additive case, the utility is convex in $\rho$ with a fully liquid spot market, and $\rho$ increases when market liquidity is lower. However, in the multiplicative case, the utility decreases in these three scenarios.

5.2.4. Summary. Table 1 summarizes the sensitivity analysis on both risk-neutral and risk-averse cases. The retail price is less sensitive to spot price volatility and the correlation coefficient in all four cases. However, the risk-averse reseller will significantly reduce the retail price when demand uncertainty increases in the additive demand case. In sum, to set an appropriate retail price, a reseller should pay more attention to demand uncertainty and his/her own risk attitude.
When demand uncertainty is large, the reseller should set a lower retail price if he/her faces an additive demand, and set a higher price if he/she faces a multiplicative demand. When the liquidity in the spot market is higher, a reseller should set a lower retail price if he/she faces a multiplicative demand. However, facing an additive demand, a risk-averse reseller should set a lower retail price, whereas a risk-neutral reseller should set a higher price.

The optimal order quantity is more sensitive to demand uncertainty and the correlation coefficient. The optimal order quantity may increase or decrease as demand uncertainty increases depending on different demand functions and risk attitudes. The correlation coefficient has a significant negative effect on the optimal order quantity if a risk-averse reseller participates in the spot market with a higher liquidity. Moreover, price volatility significantly affects the optimal order quantity when market liquidity is high enough for the reseller to speculate in the spot market, as a spot market with higher liquidity can better facilitate the reseller’s buying and selling in the spot market, motivating the reseller to become more involved in speculative activities. However, when market liquidity is low, the effect of price volatility on the optimal order quantity is insignificant.

Higher demand uncertainty always hurts the reseller in all four cases. However, a higher liquid spot market does not always improve the reseller’s utility. In the additive case, only when price volatility is small can participating in a spot market with a higher liquid improve the reseller’s utility.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary of Numerical Illustration</th>
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<tr>
<td></td>
<td>Additive</td>
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<tr>
<td></td>
<td>Risk Neutral</td>
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<tr>
<td>( p^*_R )</td>
<td>( q^* )</td>
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<tr>
<td>( m )</td>
<td>+</td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>-</td>
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<td>( \sigma_e )</td>
<td>I</td>
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<td>( \rho )</td>
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</table>

Note: +, −, U, I represent increase, decrease, unclear (increase or decrease), and insignificant, respectively.
6. Conclusion

The advent of B2B spot markets has greatly changed the traditional supply chain structure, and many resellers have started to conduct business in spot markets, thus adapting themselves to and taking advantage of this newly–emerging business environment. In this paper, we analyze how the spot market affects the reseller’s decisions and performance. We study two types of resellers: the risk-neutral reseller and the risk-averse reseller. For both types, we consider the additive demand function and the multiplicative demand function, respectively.

We first analyze the cases in which the risk-neutral reseller faces an additive or multiplicative demand function and obtain the sufficient conditions under which the optimal strategies exist and are unique. Then, we examine analytically the special case in which a risk-averse reseller participates in a fully liquid market with additive demand and obtain closed-form solutions to the optimal pricing and stocking policy. Our study shows that instead of using the wholesale price as a starting point, the expected spot price is used. Starting from the expected spot price, the reseller marks the retail price up by a margin determined by the risk-adjusted expected potential demand in the resell channel. To determine the optimal ordering quantity, a reseller should consider the speculation function and the adjustment function of the spot market.

Our numerical study shows that to set an appropriate retail price, a reseller should pay close attention to demand uncertainty and his/her own risk attitude. To decide on the optimal order quantity, the reseller should be careful of the demand uncertainty, spot price volatility, market liquidity, and risk attitude. Our numerical study also shows that participating in spot markets will not always benefit resellers. When price volatility is high, the spot market is less attractive to risk-averse resellers. Intuitively, a spot market provides a good channel for a reseller to deal with his/her excessive inventory or shortage, and the reseller can also profit through speculation. These advantages will benefit the reseller. However, the increased risk taken by the reseller in a spot market may also offset part or all of the advantages. This finding has an important management implication for the development of B2B spot markets. For example, there are thousands of resellers (traders) in the Chinese steel industry. However, only a very small fraction of them has participated
in spot markets. Avoiding exposure to risk in the spot market may have been a major factor preventing more resellers from participating in spot markets. As most of the steel resellers in China are small or medium sized, they have little experience in dealing with such risk. One way to reduce risk is by hedging, which includes operational hedging and financial hedging (Van Mieghem 2003, 2007). The former involves operational strategies such as postponement decisions and inventory policies, and the latter involves forward and option contracts.

Note that some factors are not considered in our model and analysis, such as asymmetric information and reseller competition. Including these factors in a model should be an interesting topic for future research. The model presented in this paper assumes one ordering opportunity by the reseller. Repeated ordering from manufacturers is a prominent characteristic in some industries. Allowing multiple ordering opportunities by the reseller may also be worth studying in the future.

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References


Appendix

A. Computing the Mean and Variance in the Additive Case

Let $d = \mu_d + \varepsilon$. The profit for the reseller when he/she participates in a fully liquid spot market with an additive demand can be rewritten as

$$\pi_r = p_r D + p_r(q - D)^+ - wq - p_r(D - q)^+ = (ap_r + q)p_r - d(p_e - p_r) - wq - ap_r^2.$$ 

The expected profit can be computed as

$$E[\pi_r] = E[(ap_r + q)p_r - d(p_e - p_r)] - ap_r^2 - wq = (ap_r + q - \theta - \mu_d)\mu_e - \rho \sigma_d \sigma_e + (\theta + \mu_d)p_r - ap_r^2 - wq = -ap_r^2 + (a\mu_e + \theta + \mu_d)p_r + (\mu_e - w)q - \rho \sigma_d \sigma_e - (\theta + \mu_d)\mu_e.$$

The variance of profit can be computed as follows:

$$\text{Var}[\pi_r] = \text{Var}[p_r d + (ap_r + q)p_r - dp_r] = E\{\text{Var}[p_r d + (ap_r + q)p_r - dp_r|p_e]\} + \text{Var}\{E[p_r d + (ap_r + q)p_r - dp_r|p_e]\} = E\{(p_r - p_e)^2\text{Var}[d|p_e]\} + \text{Var}\{(ap_r + q)p_e + (p_r - p_e)E[d|p_e]\} = E[(p_r - p_e)^2]\text{Var}[d|p_e] + \text{Var}\{(ap_r + q)p_e + (p_r - p_e)[\theta + \mu_d + \frac{\sigma_d}{\sigma_e}(p_e - \mu_e)]\} = (1 - \rho^2)\sigma_e^2[(p_e - \mu_e)^2 + \sigma_e^2] + \alpha^2 + 4\alpha \beta \mu_e + 4\beta^2 \mu_e^2 + 2\beta^2 \sigma_e^2,$$

where $\alpha = (a\sigma_e + \rho \sigma_d)p_r + q\sigma_e - (\theta + \mu_d)\sigma_e + \rho \mu_e \sigma_d$ and $\beta = -\rho \sigma_d$.

B. Proofs

Proof of Theorem 1. The proof of the first part is similar to that in Petruzzi and Dada (1999), and we thus omit the details.

$$\frac{dE[\pi_r(z,p_e(z))]}{dz} = m\mu_e + (1 - m)s - w + (1 - m)\left[\frac{\theta + aw + \mu_d}{2a} - \frac{(1 - m)\Theta(z)}{2a} - s\right][1 - F(z)].$$

Let $R(z) \equiv dE[\pi_r(z,p_e(z))]/dz$, then

$$\frac{dR(z)}{dz} = -(1 - m) f(z) \left[\frac{\theta + aw + \mu_d}{2a} - (1 - m)\Theta(z) - \frac{(1 - m)[1 - F(z)]}{r(z)}\right].$$
and,

\[
d^2 R(z) = \left[ \frac{dR(z)/dz}{f(z)} \right] \frac{df(z)}{dz} - \frac{(1-m)^2 f(z)}{2a} \left[ 1 - F(z) + \frac{f(z)}{r(z)} + \frac{1-F(z)dr(z)/dz}{r(z)^2} \right].
\]

Then, we obtain

\[
\left. \frac{d^2 R(z)}{dz^2} \right|_{dR(z)/dz=0} = -\frac{(1-m)^2 f(z)[1-F(z)]}{2ar(z)^2} \left[ 2r(z)^2 + \frac{dr(z)}{dz} \right].
\]

Under the condition \(2r(z)^2 + dr(z)/dz > 0\), \(R(z)\) is either monotone or unimodal, which implies that \(R(z) \equiv dE[\pi_r(z, p_r(z))]/dz\) has at most two roots. If \(m < (w-s)/(\mu_e - s)\), \(R(B) < 0\). Therefore, if \(R(z)\) has only one root, it indicates a change in sign for \(R(z)\) from positive to negative. Thus, it corresponds to a local maximum of \(E[\pi_r(z, p_r(z))]\). If it has two roots, the larger of the two values corresponds to a local maximum, and the small corresponds to a local minimum of \(E[\pi_r(z, p_r(z))]\). In either case, \(E[\pi_r(z, p_r(z))]\) has only one local maximum, identified either as the unique value of \(z\) that satisfies \(R(z) = dE[\pi_r(z, p_r(z))]/dz = 0\) or as the larger of two values of \(z\) that satisfies \(R(z) = 0\). As \(E[\pi_r(z, p_r(z))]\) is unimodal if \(R(z)\) has only one root, a sufficient condition for unimodality of \(E[\pi_r(z, p_r(z))]\) is \(R(A) > 0\) or, equivalently, \(2aR(A) > 0\), where \(2aR(A) = 2am\mu_e - (1 + m)aw + (1-m)\theta + (1-m)^2 A + m(1-m)\mu_d\). We obtain the condition in part (3).

This completes the proof. \(\square\)

**Proof of Proposition 1.** We still use the definition in the proof of Theorem 1. By the implicit function rule, \(dz^*/dm = L(z^*)/(\partial R(z^*)/\partial z^*)\), where \(L(z^*) = -\partial R(z^*)/\partial m\). Then, we have

\[
L(z^*) = s - \mu_e - (1-m)[1-F(z^*)] = \frac{\Theta(z^*)}{2a} + [1-F(z^*)]\left[ \frac{\theta + aw + \mu_d}{2a} - s - \frac{(1-m)\Theta(z^*)}{2a} \right].
\]

As

\[
[1-F(z^*)][\frac{\theta + aw + \mu_d}{2a} - s - \frac{(1-m)\Theta(z^*)}{2a}] = \frac{w - (1-m)s - m\mu_e}{1-m},
\]

so we have that

\[
L(z^*) = \frac{\mu_e - w}{1-m} - \frac{(1-m)\Theta(z^*)[1-F(z^*)]}{2a}.
\]

If \(w < \mu_e\), \(L(z^*) < 0\). We also have \(\partial R(z^*)/\partial z^* < 0\), as \(R(B) < 0\), and \(z^*\) is the largest \(z\) in the region \([A, B]\) that satisfies \(dE[\pi_r(z, p_r(z))]/dz = 0\), thus, \(dz^*/dm > 0\). We complete the first part of the proof.

Next, we have \(\frac{dp^*_r}{dm} = \frac{dp^*_r}{dx} + \frac{dp^*_r}{dz} \frac{dz^*/dm}{dz^*/dm}\). We can verify that \(\frac{dp^*_r}{dm} = \frac{\Theta(z^*)}{2a} > 0\), and \(\frac{dp^*_r}{dx} = \frac{(1-m)[1-F(z^*)]}{2a} > 0\). In conjunction with \(dz^*/dm > 0\), we obtain \(dp^*_r/dm > 0\). \(\square\)
Proof of Theorem 2. A proof of this theorem can be carried out by following a procedure similar to Theorem 1 and Petruzzi and Dada’s (1999). Thus, we omit the details. □

Proof of Lemma 1. The first-order derivatives of $U_r(p_r, q)$ are as follows:

$$\frac{\partial U_r}{\partial p_r} = -2ap_r + a\mu_e + \theta + \mu_d - 2k[(1 - \rho^2)\sigma^2_d(p_r - \mu_e) + (a\sigma_e + \rho\sigma_d)(\alpha + 2\beta\mu_e)],$$
$$\frac{\partial U_r}{\partial q} = \mu_e - w - 2k\sigma_e(\alpha + 2\beta\mu_e).$$

The second-order derivatives of $U_r(p_r, q)$ are given by

$$\frac{\partial^2 U_r}{\partial p_r^2} = -2a - 2k[(1 - \rho^2)\sigma^2_d + (a\sigma_e + \rho\sigma_d)^2],$$
$$\frac{\partial^2 U_r}{\partial q^2} = -2k\sigma_e^2,$$
$$\frac{\partial^2 U_r}{\partial p_r \partial q} = -2k\sigma_e(a\sigma_e + \rho\sigma_d).$$

Let $H$ denote the Hessian matrix of the reseller’s utility function $U_r(p_r, q)$. For $k_r > 0$ and $\sigma_e > 0$, we have

$$H_{11} = \frac{\partial^2 U_r}{\partial p_r^2} < 0,$$
$$\det(H) = 4ak\sigma_e^2 + 4k^2\sigma_d^2\sigma_e^2(1 - \rho^2) > 0.$$

Therefore, the Hessian matrix is negatively definite, and the utility function $U_r(p_r, q)$ is strictly joint concave in $p_r$ and $q$. This completes the proof. □

Proof of Theorem 3. By Lemma 1, a unique optimal retail price $p^*_r$ and a unique optimal order quantity $q^*$ exist. Applying the first–order necessary and sufficient conditions, we obtain (10) and (11), respectively. □

Proof of Proposition 4. We first define two thresholds:

$$B_1 = 3a\mu_e - 2aw + \frac{2(a + k\sigma_d^2)(\mu_e - w)}{k\rho\sigma_d\sigma_e},$$
and

$$B_2 = 2ap_e - aw + \frac{(a + k\sigma_d^2)(\mu_e - w)}{k\rho\sigma_d\sigma_e} + \frac{2\sigma(k)\sigma_d^2}{\mu_e - w} + \frac{4k\sigma(k)\sigma_d^3\sigma_d(1 + \rho^2)}{\rho(\mu_e - w)}.$$

Then, using the first derivative of which, we can obtain the results. □