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DIRECTIVITY PATTERNS OF ACOUSTIC RADIATION FROM BOWED VIOLINS

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Directivity patterns of acoustic radiation have been measured in the far-field of a violin, excited with an open-frame mechanical bowing machine. Analysis of the directivity patterns confirms that, at frequencies below 600 Hz, the violin radiates omnidirectionally, while above 600 Hz, certain trends are apparent as the patterns become increasingly complex. It is noted that when different strings are excited, the far-field radiation patterns observed at nearly the same frequency are similar, even in higher frequency ranges where modal overlap is high. When the difference in frequency between two directivity patterns exceeds some fraction of a semitone, though, the measured radiation patterns differ significantly. This is demonstrated quantitatively by computing an rms difference around the polar plots between patterns. The sensitivity of the directivity patterns to percent changes in frequency increases with frequency.

To make valid experimental measurements on a violin, the instrument must be excited in a manner which is consistent, reproducible, and not harmful to the instrument (Jansson *et al.* 1986). An open-frame mechanical bowing machine, introduced below, has been constructed for the task. This paper documents measurements of the far-field radiation patterns in two planes around the violin which were performed while using the bowing apparatus. Certain trends with frequency are evident upon closer study of the directivity patterns. These are related to directivity results from previous researchers.

The measured directivity patterns have also been contrasted to each other, first at matching frequencies produced from bowing different strings to assess the effect of the difference in excitation. Variances in the radiated patterns due to

different excitation are expected to be minimal in the lower frequency range, where radiating eigenmodes of the instrument are distinct in frequency and modal overlap is low (Rodgers 1991). Modal overlap becomes greater, though, at frequencies above 1000 Hz; thus, changes in the excitation may influence the weighting of modes in the summed response generated at a particular frequency, and subsequently the sound radiation.

To quantify the similarity between two directivity patterns, the rms difference between the patterns was computed. This quantity has moreover been utilized to study at what percent change in frequency do the directivity patterns begin to alter significantly. After initial studies, it became apparent that this percent change is itself a function of frequency. At lower frequencies, a certain

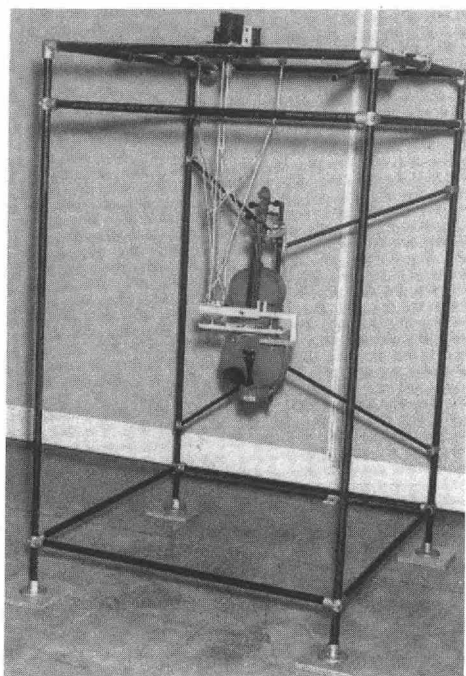
percent change in frequency did not produce a significant variation in the directivity pattern. At higher frequencies, however, that same percent change lead to notable pattern differences. These results corroborate Weinreich's "Directional Tone Color" theory, which states that violin radiation patterns vary rapidly with frequency above 1 kHz, impacting the subjective perception of sound produced by violins (1997).

The Open-Frame Mechanical Bowing Machine

Ideally, the excitation of a violin under a testing situation would simulate real playing conditions. Assorted tests have been made while hand-bowing violins (Saunders 1937, Jansson 1976, Rodgers 1993). However, it is in general difficult to make reproducible measurements with violinists holding and

playing the instrument, as they naturally adapt their playing to what they hear. In the search for acceptable alternatives, researchers have in the past tried various mountings and excitations (Hutchins 1973). Violins have been hung by rubber bands, clamped at edges, fit in Styrofoam casings, and held loosely at the neck and tailpiece with foam. Both transient and steady state excitations of the instrument have been applied. The most common transient excitation employs impact hammers (Marshall 1985, Bork 1993, Bailey and Bissinger 1997). Steady state excitation, on the other hand, often involves electromagnetic excitation of the strings or electromechanical shakers driving the bridge, with input signals which range from single frequencies and swept sine signals to maximum length sequences (MLS) (Morset *et al.* 1998) and those using digitally stored friction curves (Weinreich and Caussé 1986, Müller and Lauterborn 1996).

Figure 1 ■ The open-frame mechanical bowing machine. The violin is held at its neck and tailpiece in the center of the apparatus and driven by a belt sewn with horsehairs on a pulley system.



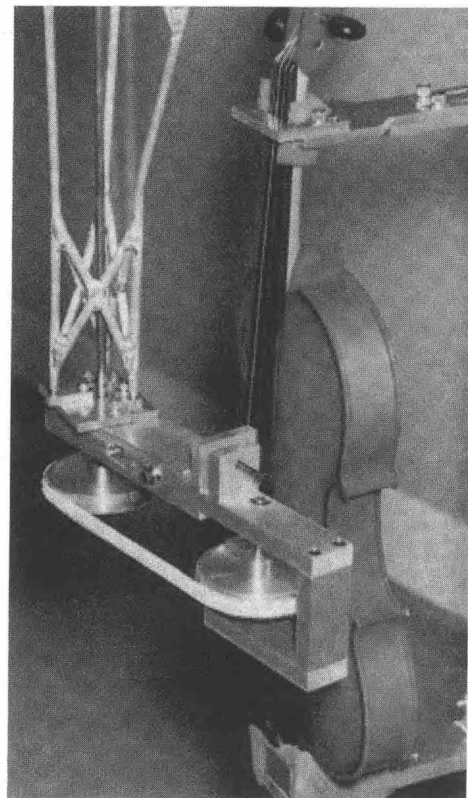
Reciprocal techniques have also been used, during which loudspeakers surrounding the violin stimulate vibrations of the instrument body (Moral and Jansson 1982, Weinreich 1985a, 1985b).

Other scientists have sought a controlled bowing excitation, as some maintain that there is no substitute for a bowing excitation (Rodgers 1993), which produces proper harmonic content and mechanical forces on the string (Hutchins 1973). Raman utilized a bowing machine which moved the violin back and forth across a fixed bow, while Saunders' bowing machine from the 1930s had a number of celluloid disks at the end of an arm which descended upon the instrument (Hutchins 1983). Other bowing machines have employed real bows controlled by computers and motors (Pickering 1991, Schumacher 1994).

Many of the above methods have been slightly problematic, as concerns have been raised about changing boundary conditions, mass loading, accurate driving of the bridge, and the amount of energy input. Jansson *et al.* (1986) compared a group of holding and mechanical excitation schemes, each with pros and cons, not including bowing excitations. They concluded that much of the decision about which method is best to use depends on what the desired output is, such as input admittance, cavity or body mode shapes, or sound radiation.

The desired output in the authors' ongoing investigation is the radiated sound field from a violin, which results directly from the vibration of the instrument. The instrument's vibration is itself a sum of normal modes, and the final sum is affected by the method of excitation. This fact has been documented by a number of researchers, including Weinreich through the differences in radiativity from two excitation directions (1985a), and Kondo *et al.* whose results showed different holographic shapes for the top and back plates of a violin when different angles of string excitation at the bridge were implemented (1980).

Figure 2 ■ A close-up view of the driven pulley system. The arm rotates so that different violin strings may be excited. A screw on top of the pulley assembly increases the separation between the driving and idle pulleys, thereby increasing the belt tension.



Consequently, to generate most accurately the sum of modes which result from real playing situations, a bowing excitation was deemed necessary.

A new open-frame mechanical bowing machine was designed and constructed for the authors' violin studies. The apparatus mounts the violin in the center of an open frame where it is loosely held at the neck with a foam pad and stands on a foam pad at the base of the tailpiece, the same locations at which a human player may hold it (Figure 1). The open frame, which measures three feet by three feet by five feet (0.9144 m x 0.9144 m x 1.524 m), is constructed from 1/2 inch black iron pipe shaft tubing, connected by prefabricated fittings.

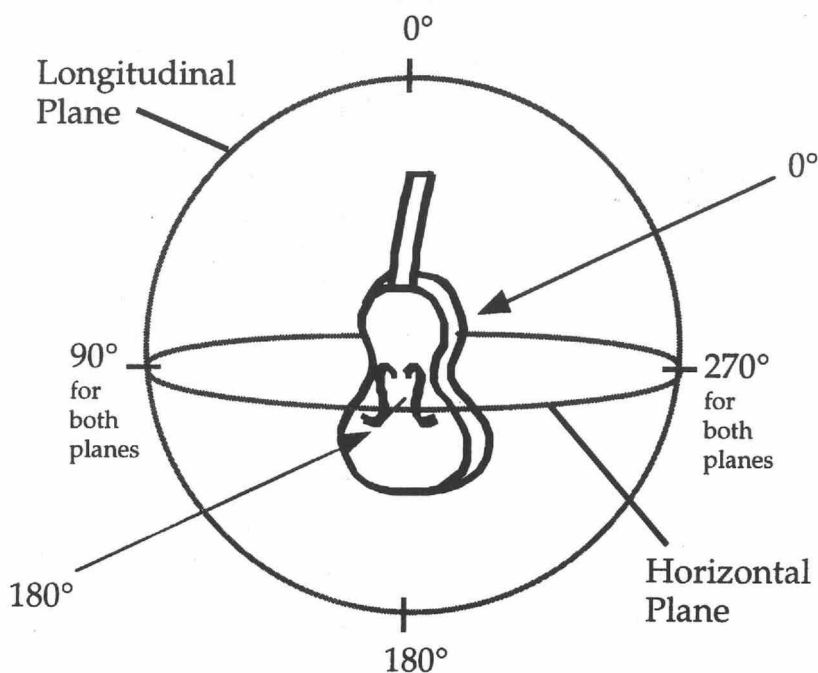
Meanwhile, a driven pulley system, hanging down from the top of the apparatus frame, runs a belt which is hand-woven with horsehairs, creating a continuous bowing excitation. The belt has a polyester carcass with a polyurethane covering, and dimensions of 600 mm x 10 mm x 1 mm. Approximately 30 horse hairs are handsewn evenly across the 10 mm width, with the ends of the hairs glued to the interior side of the belt. The entry and exit points of the hairs are staggered along the belt's length. A rubber-based glue is used so that the bond does not become brittle and break when maneuvering around the pulley. The number of hairs is much less than the 200 ordinarily found on a bow; however, the sound produced is consistent with that from a normally bowed violin.

The open-frame mechanical bowing apparatus allows for adjustment of various bowing parameters. The pulley mechanism may be rotated to attack different violin strings (Figure 2). On top of the pulley assembly is a screw which alters the belt tension by increasing the separation between the driving and idle pulleys. The bowing or belt velocity is increased or decreased by varying the DC voltage input to the motor which drives the pulley system. One can adjust the bow distance from the bridge, as well, by moving the violin fixture up and down. Meanwhile, the force of the belt against the violin string changes as counterweights are moved forward or backward on the moment arms atop the apparatus. This force may be calculated by balancing the moments around the pivot point.

Power Spectra Measurements

A series of power spectra and directivity measurements were conducted while using the open-frame mechanical bowing machine. Data were taken for excitation on each of the four open strings in two different planes: one parallel to the plane of the bridge at the height of the excitation area (approximately 2.5 cm above the bridge), henceforth referred to as the horizontal plane;

Figure 3 ■ The two planes along which power spectra measurements were made: the horizontal plane, parallel to the plane of the bridge; and the longitudinal plane, parallel to the plane of the strings. Each plane is marked with degree angles matching the directivity plots presented in this paper.



and one parallel to the plane of the strings again at the height of the excitation area, referred to as the longitudinal plane (Figure 3). The instrument used was a machine-made full-size Scherl and Roth student violin (base model #R270E4), rented from a local music store.

The bowing apparatus with violin was situated on a wire mesh which runs across the mid-section of an anechoic chamber. The chamber has dimensions of roughly 25 ft (7.6 m) on each side and a cutoff frequency below 100 Hz. A 1/2 inch omnidirectional B&K microphone was mounted on a boom which circled the center of the violin in the far-field, approximately 2 m away. This distance ensured that the measurement was in the far-field for frequencies down to 196 Hz, or the fundamental frequency of the open G string (Bies and Hansen 1988). The boom, which was computer-controlled, moved the microphone in 5° increments around the violin, measuring an average

narrowband magnitude spectrum from 100 to 5000 Hz at each location; no phase information was obtained. For the measurements in the longitudinal plane, the bowing apparatus was placed on its side so that the boom circled the violin on a plane parallel to the strings.

Measurements were conducted with the bowing speed and force set at typical values of 0.35 m/s and approximately 1 N (Askenfelt 1989). Representative spectra are shown in Figures 4 and 5. Figures 4(a-d) depict sample spectra measured at 180° in the horizontal plane, directly in front of the violin's top plate; Figures 5(a-d) were produced at the 180° position in the longitudinal plane, out from the tailpiece. The results demonstrate that the excitation was steady, producing power spectra which have strong narrow peaks at the fundamental frequency of the string and harmonics which are 30-50 dB above the background noise level.

Figure 4 ■ The power spectra obtained from bowing the a) open G string, b) open D string, c) open A string, and d) open E string, as measured in the horizontal plane at 180°, directly in front of the violin.

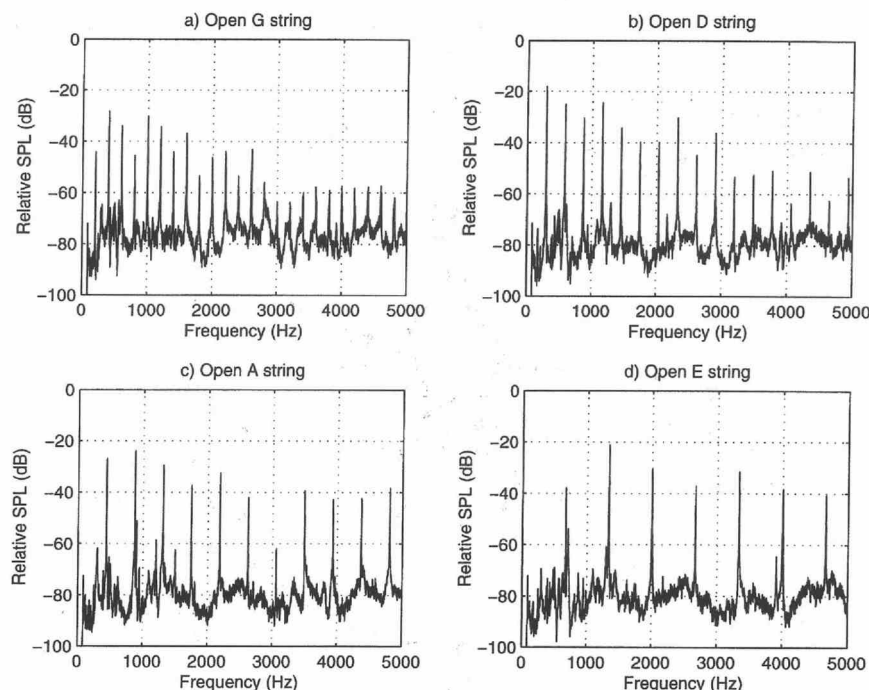
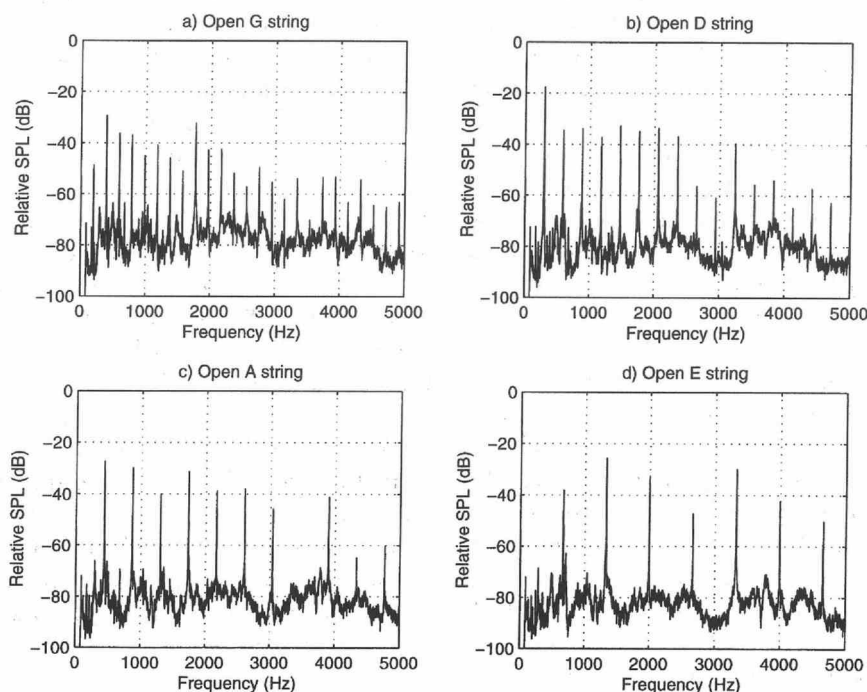


Figure 5 ■ The power spectra obtained from bowing the a) open G string, b) open D string, c) open A string, and d) open E string, as measured in the longitudinal plane at 180°, directly out from the tailpiece.



Directivity Patterns of the Bowed Violin

From the power spectra measurements, directivity patterns were formed specifically at each fundamental and harmonic frequency in the 100 to 5000 Hz measurement band. The directivity patterns for the lowest four partials from bowing the open D and A strings are shown in Figures 6(a-b) for the horizontal plane and Figures 7(a-b) for the longitudinal plane. Although the directivity patterns of every harmonic up to 5000 Hz have been studied, they are not all presented here.

Upon closer study, one finds that there are certain trends in the directivity patterns in the horizontal plane. At frequencies less than 600 Hz, the radiated patterns are omnidirectional. Above that frequency range, between 800 and 1000 Hz, the directivity patterns begin to exhibit a cardioid-like pattern, with a maximum oriented out from the top plate of the violin (Figure 8a). At even higher frequencies, ranging from 1100 to 1600 Hz, the patterns resemble a skewed doublet pattern, with nulls that spin clockwise as the frequency increases (Figure 8b). Above 1700 Hz, the patterns become increasingly irregular, some with deep nulls forming full lobes, and others with shallow nulls (Figure 8c). These directivity plots are asymmetric, and the largest lobes are not necessarily directed perpendicular to the top plate.

The measurements made in the longitudinal plane also demonstrate some interesting trends. Again under about 600 Hz, the patterns look omnidirectional. Between 600 and 800 Hz, however, the directivity resembles an ellipsoidal or egg shape, with the maxima oriented towards the upper bassbar and lower soundpost quadrants (Figure 9a). From 800 to 900 Hz, the violin produces a cardioid radiation pattern, with a maximum directed towards 0° (Figure 9b). These forms change in the range of 950 to 1400 Hz to become a doublet directivity pattern. In the lower part of this range, the maxima seem to lie in the same quad-

Figure 6 ■ The directivity patterns of the lowest four partials from bowing a) the open D string and b) the open A string, as measured in the horizontal plane. The labeled frequencies are rounded to the nearest whole number. The patterns are normalized to the highest of the measured levels for each string.

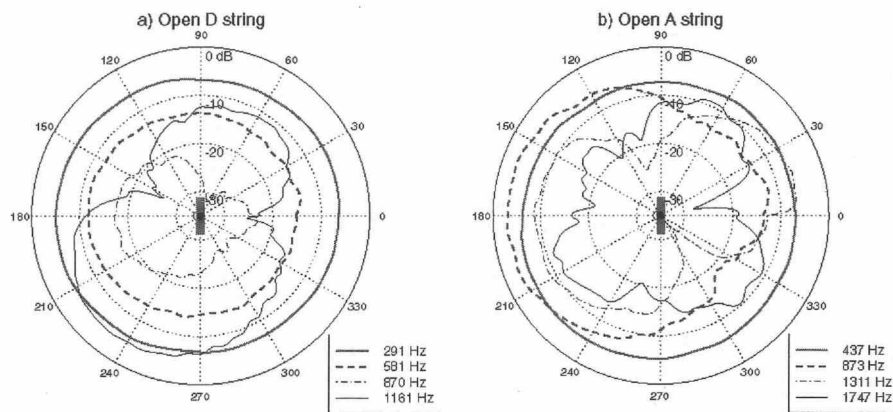
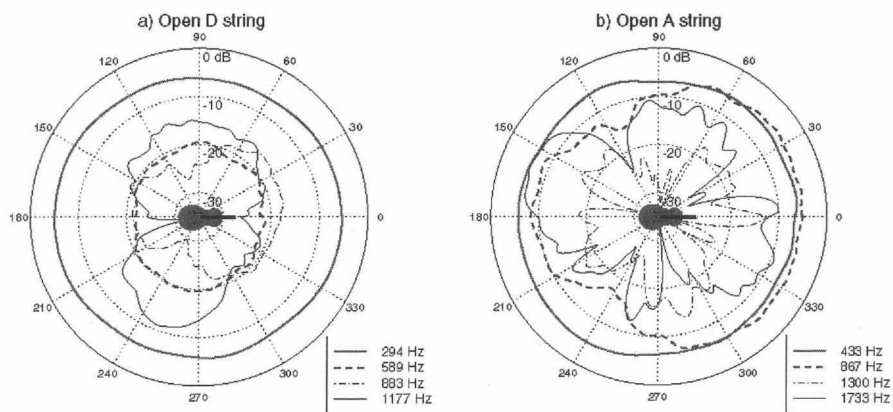


Figure 7 ■ The directivity patterns of the lowest four partials from bowing a) the open D string and b) the open A string, as measured in the longitudinal plane. The labeled frequencies are rounded to the nearest whole number. The patterns are normalized to the highest of the measured levels for each string.



rants as those of the ellipsoid, while nulls are oriented close to 0° and 180° (Figure 9c). Above 1400 Hz, the patterns become increasingly complicated with many bumps, nulls, and lobes, although nulls near 70° and 295° were common for some of the patterns (Figure 9d).

Other published work, despite using different violins, different mountings and different excitations, have produced similar patterns which match the trends shown here (Meinel 1937, Meyer 1972,

Cremer 1984, Bissinger 1995). Looking more closely at Meyer's seminal work (1972), which shows histograms depicting the principal directions of radiation for a number of violins, one finds patterns that follow the trends outlined here. In the horizontal plane, Meyer's results reveal mostly omnidirectional behavior below 600 Hz, a cardioid pattern at 1000 Hz, and a skewed doublet pattern at 1500 Hz. Frequencies above 2000 Hz exhibit many lobes and com-

plexity. The other plane on which Meyer made measurements is similar to the longitudinal plane used in this paper, but with the violin in an inclined position as it is when played. Nevertheless, his results and the trends noted here are similar. Below 500 Hz, patterns are omnidirectional, while the ellipsoidal or egg shape dominates from 550-700 Hz with peaks in the lower soundpost and upper bassbar quadrants. Then at 800 and 1000 Hz, a cardioid develops. Further similarities exist at 1250 Hz with the skewed doublet pattern; above this frequency, the plots become more complex.

Bissinger (1995) has also measured far-field directivity patterns in the horizontal plane. His results are omnidirectional below 600 Hz, radiate symmetrically outward from the top plate around 800 Hz, and show a skewed doublet pattern at 1158 Hz. Comparing Bissinger's longitudinal plane results to these trends also shows some correlation. There is roughly an omnidirectional behavior at low frequencies, which becomes ellipsoidal and doublet-like up to 1 kHz. At 1038 Hz, the doublet pattern clearly emerges with the peak lobes oriented in the lower soundpost and upper bassbar quadrants.

Consequently there is evidence that the trends denoted in this paper may apply to violins globally and are not unique to the violin tested or method used here.

Changes in Directivity Patterns with Frequency

From the large number of directivity patterns accumulated in this study at frequencies between 196 Hz and 5000 Hz, it is possible to learn more about how directivity patterns of violins change with frequency. Weinreich has formulated the idea of "Directional Tone Color" which is based on violin radiation patterns above 1000 Hz changing drastically with frequency (1997). He compared the radiativity at two locations around the violin, measured using a reciprocal technique, and concluded that patterns

Figure 8 ■ Directivity trends in the horizontal plane: a) cardioid-like patterns emerge between 800 and 1000 Hz; b) skewed doublet patterns appear between 1100 and 1600 Hz; c) patterns become increasingly complicated above 1700 Hz.

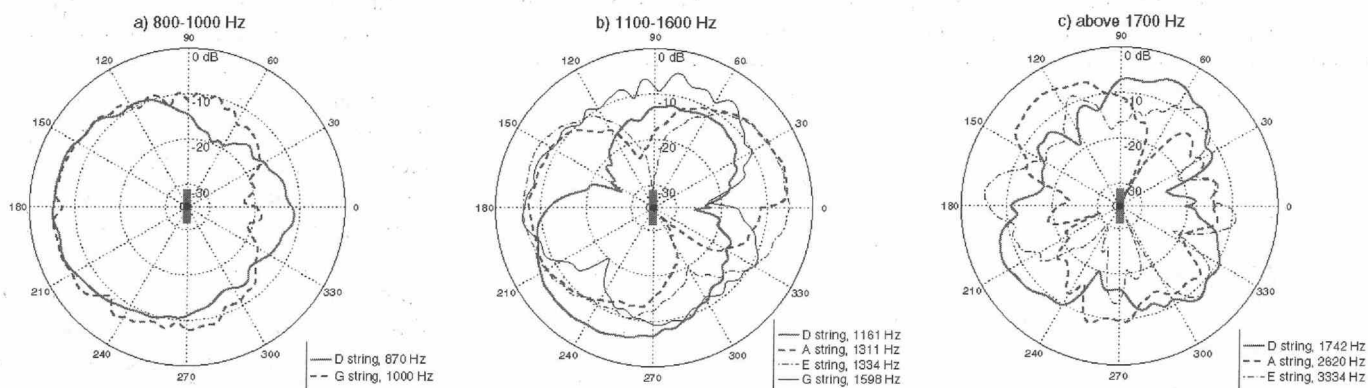
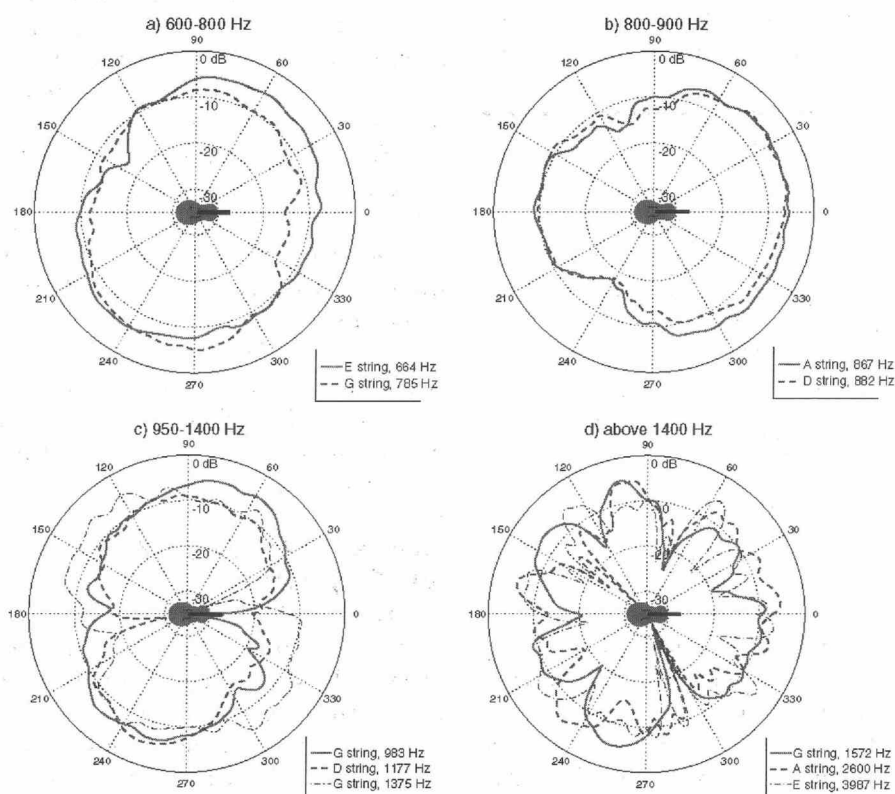


Figure 9 ■ Directivity trends in the longitudinal plane: a) ellipsoidal or egg shape patterns emerge between 600 and 800 Hz; b) cardioid patterns appear between 800 and 900 Hz; c) doublet patterns develop between 950 and 1400 Hz; d) patterns become increasingly complicated above 1400 Hz.



differ between frequencies that are even a semitone apart. From the current database of planar radiation patterns, the degree of similarity between patterns at different frequencies can be compared

quantitatively also.

Since the relative and not absolute levels are of interest for comparison, the directivity patterns were first normalized to each other. The average sound pres-

sure levels around two measured patterns were calculated, and the data set with the lower average value was multiplied by a normalization factor so that its average sound pressure level matches the other's. The rms difference between the normalized patterns was then calculated as follows:

$$\Delta_{rms} = \left\{ \frac{1}{n} \sum_n |L_{1n} - L_{2n}|^2 \right\}^{\frac{1}{2}}$$

where n is the number of measurement points around the directivity pattern ($n=72$ in this paper), and L_{1n} and L_{2n} are the relative sound pressure levels of the two patterns at the n th measurement location, after normalization. This quantity is essentially an average dB difference taken around the polar plots.

Before examining changes in directivity patterns with frequency, consideration was given to differences between directivity patterns at nearly equal frequencies when different strings were bowed. Harmonics with nearly equal frequencies were selected from excitation of different strings (Figures 10(a-c)). (The harmonic frequency values are very close to one another but not precisely the same, because each string was tuned by ear.) As one can see from the examples in Figure 10, the differences in the patterns caused by bowing different strings appear to be small, with calculated rms

Figure 10 ■ Comparison of directivity patterns at similar frequencies, obtained from bowing different strings: a) 1177 Hz from bowing D string and 1178 Hz from bowing G string in longitudinal plane; b) 1742 Hz from bowing D string and 1747 Hz from bowing A string in horizontal plane; and c) 2648 Hz from bowing D string and 2658 Hz from bowing E string in longitudinal plane. The rms difference, Δ_{rms} for each case is provided.

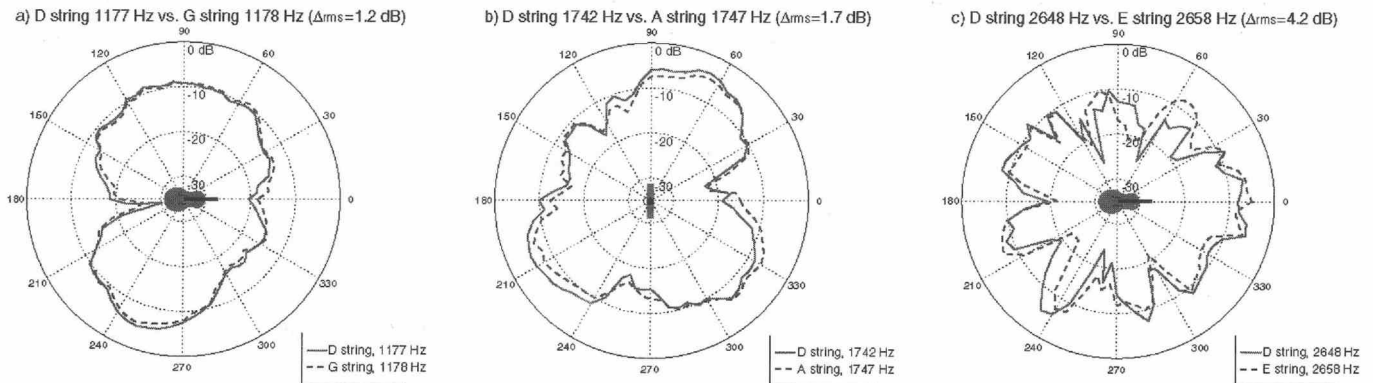
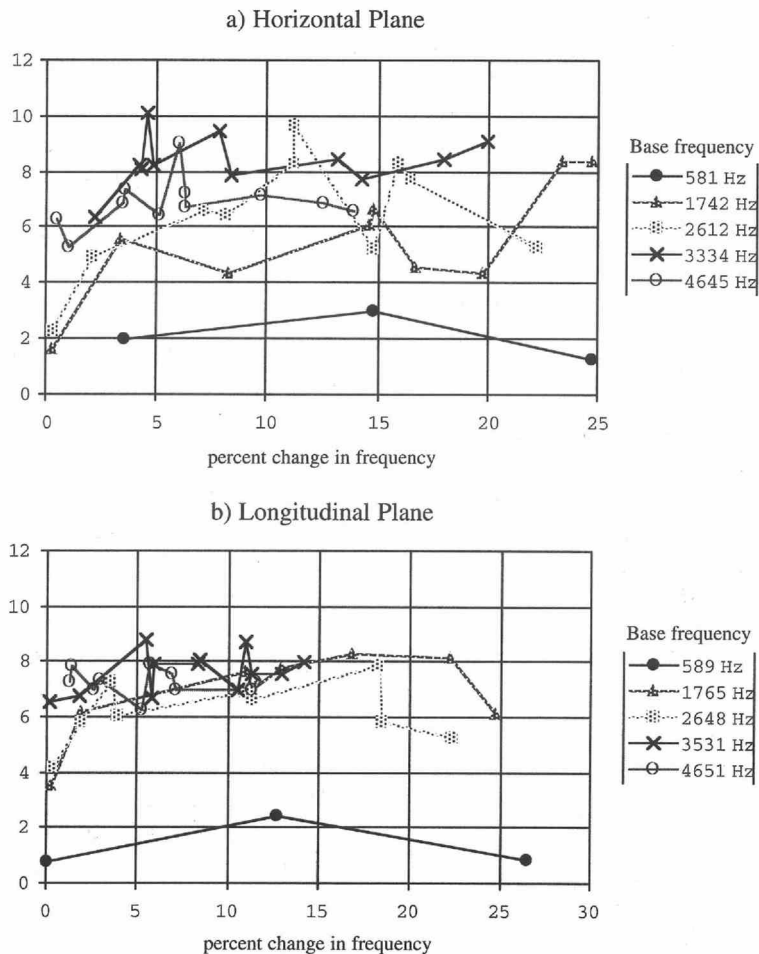


Figure 11 ■ The rms difference between directivity patterns as a function of the percent change in frequency for the base frequency shown, in the a) horizontal plane and b) longitudinal plane.

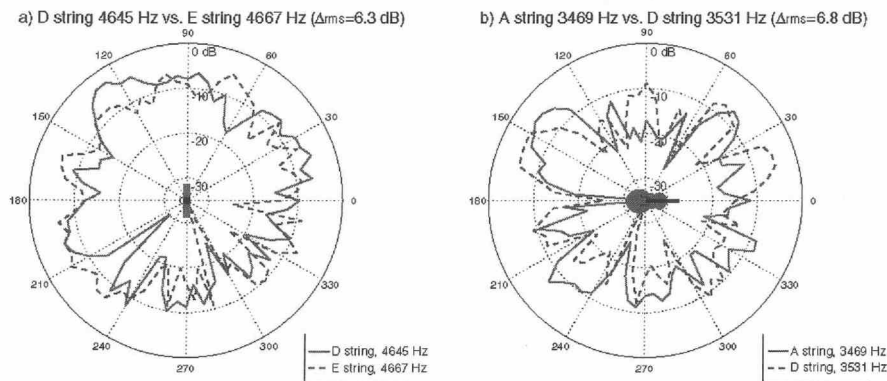


differences around 4 dB or less. Consequently, even in higher frequency ranges where modal overlap is high, the far-field radiation response due to a sum of eigenmodes is barely altered.

Finding that the string excitation has minimal effect on the far-field patterns, the rms difference was then calculated for a wide variety of frequency combinations on the horizontal and the longitudinal planes, to determine how large of a change in frequency is required before significant changes in radiation patterns emerge. After preliminary study, though, it was apparent that the frequency change that produced a significant alteration in the radiation pattern is itself a function of frequency. Computing the rms difference as a function of the percent change in frequency for different base frequencies produced results as shown in Figures 11(a-b) for the horizontal and longitudinal planes. These demonstrate that, at base frequencies below 800 Hz, the deviation remains consistently low, under 3 dB. This physically stands to reason, since below 800 Hz, the violin patterns are essentially omnidirectional with no lobes or strong nulls. With base frequencies above 800 Hz, one finds that the deviation can increase up to values of 10 dB.

One further notes that as the base frequency becomes higher, the range of

Figure 12 ■ Comparisons of directivity patterns at high frequencies which are less than a semitone apart: a) 4645 Hz from bowing D string and 4667 Hz from bowing E string in horizontal plane; and b) 3469 Hz from bowing A string and 3531 Hz from bowing D string in longitudinal plane. The rms difference, Δ_{rms} for each case is provided.



frequency difference percentage in which the rms difference remains under 4 dB becomes smaller. A small frequency change is more likely to result in significantly different patterns at higher frequencies; the patterns are much more complicated at these higher frequencies so that small shifts in frequency have more drastic effects on the radiation patterns. Even a change in frequency less than a semitone apart, which corresponds to approximately a 6% change of frequency, leads to significant differences between radiation patterns above 1000 Hz. Examples of such comparisons are shown in Figures 12(a-b).

Conclusions

Power spectra and directivity patterns have been acquired in two planes around a violin, while using a recently constructed open-frame mechanical bowing machine. The far-field directivity patterns have omnidirectional characteristics at frequencies less than 600 Hz, then show distinct trends up to 1600 Hz in the horizontal plane and up to 1400 Hz in the longitudinal plane, becoming more highly varied and directional above that. Directivity patterns published by others follow these trends, indicating that they may be generally applicable to

all violins.

Comparisons between the patterns obtained in this study show similarities between the directivity patterns at harmonics of different strings which are located at nearly the same frequency, demonstrating that the location of the bowed excitation has small effect on the far-field radiation. Computed rms differences between radiation patterns furthermore illustrate that the changes in patterns are small with frequencies below 800 Hz. However, for frequencies above 800 Hz, radiation patterns alter significantly for changes in frequency as small as a semitone, corroborating Weinreich's "Directional Tone Color" phenomenon. It has also been noted that the sensitivity of the radiation patterns to percent changes in frequencies increases with frequency. ■ CASJ

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