Where Prices Come From

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Introduction
Computers and game theory entered the civilian economy after 1944, when von Neumann and Morgenstern published their Theory of Games and Economic Behavior. The coincidence of the birth of these two innovations is no accident. John von Neumann and Alan Turing brought them to life during World War II von Neumann for the Manhattan Project and Turing at Bletchley Park for code breaking. Economics is most useful when it evolves in order to solve new problems arising from new developments in the economy. Core theory from cooperative game theory is a leading asset in economics because it helps solve the new economic problems created by digital computers and their offspring.

To assert there are cooperative games seems contradictory, if not fallacious. Because the terms ‘non cooperative games’ have become often used in game theory as being more informative than bare ‘game theory’, it is necessary to use contrasting terms for economic applications of game theory such as to markets. Everybody who actually trades is a winner in a market with unrestricted trade. The buyer may reject an offer, the seller may reject a bid. They trade because each believes it is beneficial to him. Both are winners. Voluntary trade has no losers. Yet a market allows, indeed, encourages rivalry among potential buyers and potential sellers. This rivalry explains why outcomes in a market make everybody a winner. Given this is understood and that the terms ‘non cooperative games’ are entrenched, the appellation, cooperative games, superficially contradictory, has the advantage of provoking thought.

Core Theory Applied to Software Producers
Consider a group of firms such that the cost function of each firm has only one component, an avoidable cost. It is zero when the firm is idle and is a positive constant at any output rate up to its capacity. There are no variable costs. Including fixed costs would complicate but not illuminate my analysis.

If avoidable cost increases less rapidly than capacity, then an efficient regime uses one firm to serve the whole market. One may claim this is a compelling
reason for price controls. However, if avoidable cost increases more rapidly than capacity and there is a capacity that minimizes avoidable cost per user, then the efficient regime is a collection of firms each with this optimal capacity. However, now a difficulty appears. Say all active or potential suppliers offer the identical commodity. Since variable cost is zero, marginal cost is also zero. Market size determines the efficient supply response. The efficient regime allows at most one active firm operating apart from the optimal size. An empty core results. It now becomes necessary to prevent ruinous price cutting. (Telser, 1978, chap. 3, Viner Industries)

Consider software, a commodity available to anybody connected to the internet. The cost of designing software increases with its capabilities. Different customers want different capabilities. At the same price the most commonly used capabilities attract the most customers. Also, software may incur a variable cost depending on how many copies are sold because some customers may seek help or need repairs. Let us agree to ignore this. Let a software firm set a price based on how many potential users it anticipates. Given the total cost of making its software, the more sales the producer anticipates, the lower the price for its software.

Arrange n competing firms by size of capacity from the smallest to the biggest. The bigger is the capacity of the firm, the bigger its cost. A firm incurs avoidable cost if and only if it is active. Avoidable cost does not depend on realized commodity sales but does depend on estimates of sales.

1) \( c_i = \text{total avoidable cost of firm } i, \ c_i < c_{i+1} \)
2) \( k_i = \text{capacity of firm } i, \ k_i < k_{i+1} \)
3) \( q = \text{quantity, } p = \text{unit price,} \)
4) \( r = \text{price, } q = \text{revenue.} \)

The lowest price that can cover the total cost of firm i is \( p_i = c_i / k_i \). This is the price at the capacity of firm i.
The highest price that can cover the total cost of firm i is \( u_i = c_i / k_{i-1} \).
This assumes the least output of firm i capable of covering its cost is at the capacity of firm \( i-1, k_{i-1} < k_i \), a smaller firm.
The break even price for firm i depends on \( q_i \), its break even rate of sales, \( v_i = c_i / q_i \).
The break even rate of sales, \( q_i \), satisfies \( k_{i-1} < q_i < k_i \). The break even price, \( v_i \), obeys \( p_{i-1} > v_i > p_i \).
Break even sales mark the boundary between profit and loss. Sales below the break even level incur loss, sales above this level, get profit.

This model ignores several complications. A commodity can satisfy several different requirements but only some requirements can be satisfied by every commodity. How well differs among them. Users with special requirements may buy several commodities. In a market with all firms capable of satisfying the same requirements, efficiency dictates at most one active firm would have unused capacity. Not all potential suppliers are active.

The Story Begins

The diagram illustrates my analysis. There are three firms described by two parameters.

<table>
<thead>
<tr>
<th>firm</th>
<th>cost</th>
<th>capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>7</td>
</tr>
</tbody>
</table>

The step function shows total cost for firms 1, 2, and 3 the smallest to the biggest. The diagram shows three linear revenue functions. The first revenue function is labelled 15/7. It shows the break even price and quantity for firm 2. It also shows the highest price that yields enough revenue to cover the total cost of firm 3. The second revenue function, 10/7, shows the lowest price at which firm 2 could cover its total cost and the break even price for firm 3. The
third revenue function 15/13 shows the lowest price at which firm 3 could cover its cost. At this price, firm 3 operates at capacity. Firm 2 operates at capacity at price 10/7. For prices in the range 10/7 to 15/7, some demand is unfilled. Although firm 2 operates at capacity, its output is too small to cover the cost of firm 3, a bigger firm. Firm 3 is idle at prices in this range.

It may seem this model assumes a perfectly inelastic demand. This is not so. The linear revenue function has a smaller slope and intersects the avoidable cost at higher rate of sales. Since more is sold, the lower the price per unit, demand is not inelastic.

The von Neumann growth model does not assume firms maximize their profits. It makes two assumptions. First, no active firm incurs a loss. Second, excess supply cannot be sold at a positive price. The unexpected implication of the von Neumann growth model is that the maximum growth rate of the economy equals the minimum real interest rate.

Admittedly, the v. Neumann growth model makes assumptions about technology not all in accord with reality. It assumes inputs and outputs are linear functions of each other. It defines an mXn matrix A of input coefficients and an mXn matrix B of output coefficients. There are no fixed or avoidable costs. There are no households or consumer commodities. No output can command a positive price unless it is an input in a process. A commodity that fails this condition will not be produced. The growth model of von Neumann relies on more inputs to increase outputs. Solow shows this can explain at most half the measured growth rate. To say the other half comes from innovation suggests little more than where we should look.

Conclusions

A software firm sets its price based on its estimate of sales to cover its total cost. This ignores some realistic aspects such as differences among the capabilities of the commodities to satisfy requirements. These could enter the analysis by spelling out these requirements and their relations. Granting these shortcomings of my model should not deny the moral. Profits and losses depend on how well a firm can estimate sales for its commodities defined by the cost of making products with requirements that satisfy buyers.

Without variable cost output can increase without more inputs. Employment included in input does not increase. A deeper lesson is below the surface in this
model. Revenue, not cost, goes up for a successful software firm. This explains why inputs in the software industry do not match its growing revenue. Do the losses of the less successful and unsuccessful software firms offset the profits of the winners? Is the innovation lottery more than fair, fair or unfair?

References

Procedures

cost := ListStepPlot[{{0, 7}, {1, 10}, {7, 15}},
    Joined → False, PlotStyle → {Red, Thick},
    AxesLabel → {"QUANTITY", "COST"}, AxesOrigin → {0, 0},
    DataRange → {0, 14}]

sales[p_] := Plot[p x, {x, 0, 13}, PlotStyle → {Blue},
    AxesLabel → {"QUANTITY", "COST"}]

stuff :=
    Graphics[{{Text[10 / 7, {13.5, 20.1}],
      Text[15 / 7, {13.5, 26.8}], Text[15 / 13, {13.5, 14.8}]}}]

Show[cost, sales[10 / 7], sales[15 / 7], sales[15 / 13],
    stuff, PlotRange → All]