

October, 2009

economics becomes science

Lester G Telser

Economics Becomes a Science: 1900-1999

L. G. Telser

Economics became a science in the 20th century thanks to major developments in economic theory and analysis of economic data, called econometrics. These developments could flourish because a new helper came on scene, the digital computer. Now it was feasible to employ a new mathematical technique capable of solving large scale problems in business and economics: linear programming. It became possible to make and test elaborate models of various aspects of the economy using numerical data: econometrics. It is no exaggeration to assert that economics took its place among the hard sciences in the 20th century.

Although many contributed to these achievements, a few deserve recognition for these major advances. Among them one stands out – John von Neumann. It is my belief that he together with David Hume are the only two geniuses so far who have contributed to the science of economics. Economics is not the only beneficiary of v. Neumann's genius. His contributions to the design of digital computers and automatons are legendary (Goldstine, 1972, parts 2 & 3).

John von Neumann proved the minimax theorem in an article published in 1928. He created the mathematical foundations of economic theory in a lecture delivered at Princeton in German in 1932, subsequently published in 1937. In 1944 von Neumann and his co-author, Oskar Morgenstern, published their great work, *The Theory of Games and Economic Behavior*. It set economics on a new course. Rigorous models of a competitive economy such as by Debreu (1959) and linear programming can be traced to v. Neumann's 1932 lecture. Linear programming by itself would be useless without the simplex algorithm to solve LP problems. George Dantzig (1949), the inventor of this algorithm, tells his story of how linear programming began in (1997, pp. xxi-xxxvi) and v. Neumann's role in its development in (1997, pp. xxvi-vii).

The simplex algorithm is one of the most remarkable achievements in mathematical computing. Although it can be stumped by deliberately contrived mathematical problems, it rarely fails in practice, even for very large problems. Nor is this all. It can also solve some problems that demand solutions in integers although this was no part of its original intention. The assignment problem is an example. Equations in integers, called Diophantine equations, are hard to solve. Indeed, Hilbert's Problem 10 asks whether there is a computing algorithm that can determine in a finite number of steps whether a polynomial Diophantine equation with integer coefficients can be solved in integers. In 1970, Yuri Matijasevich proved this was not possible (Davis, 1982, Appendix 2 and Gray, 2000, pp. 226-32). Because integer programming problems are particularly important in many economic applications, this result means we must seek methods that will work for suitable classes of problems, but no single method can succeed for all problems.

Before 1928, economic models had to ignore crucial aspects of the economy owing to the absence of tools capable of handling them. No economic model could say whether an equilibrium could exist, what would be produced, what would be free and what would command a positive price. Tom Sawyer, whitewashing a fence, had more to say on what would be free than did any economic theory of his time or long after. All that economists could do who spoke of a general equilibrium was count the number of equations and variables and claim this sufficed. This is nonsense.

The minimax theorem is the keystone of most modern economic models. The theory of demand is a familiar example of this. It starts with a consumer assumed to maximize utility subject to a budget constraint. This is the Primal Problem. It is equivalent to finding the least outlay needed to attain a given level of utility. This is the Dual Problem. The relation between the Primal and the Dual Problems depends on the Minimax Theorem suitably generalized.

Statistical tools developed in the 19th century could measure the direction and strength of associations among variables but none is suited to the peculiar nature of economic data. The received statistical methods stood on safest ground only for analyzing the results of experiments, but not for studying relations among variables an observer cannot control, the typical situation for economic data. Economists and statisticians at the Cowles Commission for Research in Economics at the University of Chicago were among the first to be aware of the special difficulties posed by economic data. They contrived complicated ways in an effort to overcome these difficulties but to little avail. Henri Theil's work was brought to the attention of the econometricians at Cowles in 1952 and the scales fell from their eyes. I was present at the seminar when Roy Radner explained Theil's two-stage least squares. It was a revelation. Theil's methods led the way to measure and test sophisticated models of the whole economy. It was especially useful for models based on the general theory of Keynes. (See Theil (1958, chap. 6) for a description of his method.)

Important applications of game theory to economics soon followed publication of *The Theory of Games*. Models of a competitive economy could establish in detail with rigor the relation between the number of traders and the properties of an equilibrium in pure exchange (Scarf, 1962). The crucial role of constant returns to scale in the neo classical models of production and consumption became manifest. Until the advent of game theory economics resembled the drunk at night looking for his lost key by the lamp post where the light was better although he knew he had lost his key elsewhere.

Of more importance to economics is the development of core theory derived from game theory. Shubik (1959) was among the first to publicize the importance of core theory to economic theory. Bondareva (1963) made feasible the step from theory to practice. She showed that linear programming could readily determine the status of the core. (Telser, 1997, pp. 104-5 explains the relations between the original First Core and her core I call the Bond Core.)

Core theory furnishes explicit models combining cooperation and competition by introducing coalitions. The security value of a coalition describes the most a coalition can get under the worst conditions. This security value for a coalition depends on v. Neumann's Minimax Theorem. Coalitions do two things. They compete for members. Keeping in mind that individuals can join any coalition that wants them, competition for members among coalitions determines members' returns. Members cooperate within coalitions and thereby contribute to the value of coalitions. These contributions place limits on what coalitions are willing to offer potential members. Core theory can describe the conditions conducive to a stable, efficient outcome. In the absence of these conditions the core is empty. Core theory supplies economists with the tools to study a variety of problems beyond the reach of the older theories, problems such as vote trading, division of estates, division of the assets of a bankrupt, returns to share holders of a corporation, why there is limited liability, pricing computer programs, electricity rates, dispersion of prices in an organized market and much more.

Game theory led to a second major development in economics, noncooperative games by John F. Nash, Jr. (1950 and 1951). Although Cournot (1838), the first true mathematical economist, introduced to economics models of two or more competing firms and showed how the results depend on the number of firms, a general model involving mixed strategies for n firms had to wait more than a century. In Cournot's simplest model there are two firms who decide independently how much to sell. Because consumers know their products are identical, can costlessly obtain the prices each firm quotes and because the firms decide in advance how much to sell at whatever price it can fetch, there is an implication of inefficiency. Any noncooperative equilibrium using mixed strategies is inefficient (Telser, 2007, IV.4.2). Although only a deterministic noncooperative equilibrium can be efficient, Cournot's model shows that even a deterministic noncooperative equilibrium yields an implication of inefficiency. Nonetheless an important application of noncooperative theory to economics proper

must not elude mention. The neo classical competitive equilibrium with constant returns to scale can be cast as a noncooperative game. It is the leading example of an efficient noncooperative equilibrium. Indeed, in Cournot's model if vendors could sell as much of their mineral water as they please at a constant unit price, then there would be an implication of an efficient, deterministic noncooperative equilibrium. However, this would beg the question of who or what determines this constant unit price, a key problem addressed by core theory with its deeper analysis of competition.

A major drawback of noncooperative models is that often there are many equilibria with little guidance from the theory on how to choose among them. Economic models aimed at predicting what happens with a small number of rivals face the problem of describing how expectations of what rivals will do affect their strategy. The problem is exacerbated if there are many equilibria, a common occurrence in these models.

Because game theory introduces mixed strategies whereby players choose according to optimal probabilities they assign to their available alternatives, uncertainty is intrinsic to the outcomes. Hence von Neumann and Morgenstern had to show how this affects the strategy of the players. They were led to propose cardinal utility, a form wholly at odds with the ordinal version accepted by economists. Cardinal utility turned out to be a mixed blessing. Claiming a game is zero-sum in cardinal utility is dubious. Hence it weakens the case for the application of the minimax theorem in two-person game and even more in n-person games (Telser, 2007, sec. 1.2.7). The economic drama is still in the first act.

References

Bondareva, O.N. 1963. Some Applications of Linear Programming Methods to the Theory of Cooperative Games (in Russian) Problemy Kibernetika . 10:119-39.

- Cournot, Augustin. 1960 [1838]. *Researches into the Mathematical Theory of Wealth*.
Trans. Nathaniel Bacon from 1838 French edition. New York: Kelley.
- Dantzig, George B. 1951 [1949]. *Maximization of a Linear Function of Variables
Subject to Linear Inequalities*. In ed. Tjalling C. Koopmans. *Activity Analysis of
Production and Allocation*, Cowles Commission Monograph No. 13. *Proceedings of
Linear Programming Conference*, July-October, 1949, New York: Wiley.
- Dantzig, George B. and Mukund N. Thapa. 1997. *Linear Programming 1: Introduction*.
New York, Berlin, Heidelberg: Springer-Verlag.
- Davis, Martin. 1982. *Computability and Unsolvability*. New York: Dover.
- Debreu, Gerard. 1959. *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*.
Cowles Commission Monograph No 17.
- Goldstine, Herman H. 1972. *The Computer: From Pascal to von Neumann*. Princeton:
Princeton University Press.
- Gray, Jeremy J. 2000. *The Hilbert Challenge*. Oxford: Oxford University Press.
- Koopmans, Tjalling C., ed. 1950. *Statistical Inference in Dynamic Economic Models*.
Cowles Commission Monograph 10. New York: Wiley.
- Nash, John F., Jr. 1950. *Equilibrium Points in n-Person Games*. *Proceedings National
Academy of Sciences*. 36: 48-9.
- _____. 1951. *Non-Cooperative Games*. *Annals of Mathematics Journal*. 54: 286-95.
- Neumann, John von. 1928. *Zur Theorie der Gesellschaftsspiele*. *Mathematische Annalen*.
100:295-320.
- _____. 1937. *Über ein Ökonomisches Gleichungssystem und eine Verallgemeinerung des
Brouwer'schen Fixpunktsatzes*. *Ergebnisse eines Mathematischen Kolloquiums*. Ed
K. Menger. Wien.
- Neumann, John von and Oskar Morgenstern. 1944. *The Theory of Games and Economic
Behavior*. Princeton: Princeton University Press.

- Scarf, Herbert. 1962. An Analysis of Markets with a Large Number of Participants. Recent Advances in Game Theory, pp. 127-55. Princeton, N. J. : Princeton University Conference (private printing for members of the conference).
- Shubik, Martin. 1959. Edgeworth Market Games. Contributions to the Theory of Games, ed. A. W. Tucker and R. D. Luce. Vol. 4. Princeton, N. J. : Princeton University Press.
- Telser, Lester G. 1972. Competition, Collusion and Game Theory. Chicago: Aldine-Atherton.
- _____. 1978. Economic Theory and the Core. Chicago: University of Chicago Press.
- _____. 1987. A Theory of Efficient Cooperation and Competition. Cambridge, U.K.: Cambridge University Press.
- _____. 1997. Joint Ventures of Labor & Capital. Ann Arbor, MI.: University of Michigan Press.
- _____. 2007. The Core Theory in Economics: Problems and Solutions. London: Routledge.
- Theil, Henri. 1958. Economic Forecasts and Policy. Amsterdam: North-Holland Publishing Co.