May, 2015

How Low Interest Rates Impede Recovery

Lester G Telser, *University of Chicago*

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1. Introduction

A super market and a bank in the money market treat their customers differently. The super market stands ready to sell to any customer any amount of what is on hand at a given price without regard to the credit standing of the customer. The credit standing of a potential borrower has paramount concern to a bank. Whether a bank accommodates a potential borrower depends on its assessment of a potential borrower's chance of default. Whether the bank offers a loan, its size, if it does make an offer, and the interest rate it will ask all depends on its assessment of default risk.

There is another special aspect of banking. Banks belong to a clearing house. Members of the Federal Reserve System have their reserves on deposit at a Federal Reserve Bank. They settle differences in net liabilities among each other by over night credits and loans at the Federal Funds Rates. Borrowers pay and lenders receive interest at these rates. A bank that lends more to its customers may incur an increase in its net liabilities and must borrow in the over night market paying the Federal Funds Rate on its new debt. A loan by a bank takes the form of a deposit in the bank to the credit of the borrower. The borrower may write checks in favor of persons with accounts at other banks. This raises net liabilities of the lending bank. Consequently, the clearing house creates relations among banks unlike any other non financial industry.

2. Basic Identities for Bank i

Bank i: assets a_i = loans l_i plus reserves r_i.

(1) $a_i = l_i + r_i$, i = 1, 2, … , n.

Bank i loans yield interest rate ρ . Bank i pays interest rate δ on deposits. Assume the loan interest rate is bigger than the interest rate

it pays on deposits so that $\rho > \delta$. A bank loan to a borrower takes the form of a deposit in the bank from which the borrower can draw funds by writing checks. A loan enters as an asset and the corresponding deposit enters as a liability of the bank. The liabilities of a bank are its deposits. Deposits, d_i, equal loans plus other deposits from non borrowers, s_i.

(2) $d_i = l_i + s_i$.

These deposits are means of payment. Some may include them in their definition of the money supply.

The liabilities of bank, d_i, equals its assets, a_i.

(3) $d_i = a_i$.

Equation (3) says that the means of payment equals assets. The ratio of reserves to deposits is θ.

(4) $\theta = r_i / d_i = r_i / a_i \leq 1$.

This ratio, θ, depends on the probability that withdrawals, w_i, do not exceed its reserves, r_i, equals β.

(5) $\Pr\{w_i \leq r_i\} = \beta$.

It follows from (1) and (2) that

(6) $r_i = s_i$.

Since $l_i = a_i - r_i$ and $r_i = \theta a_i$,

(7) $l_i = (1 - \theta) a_i$

It is plausible to regard loans that default as random draws from a distribution function, F(x), such that

(8) $\Pr\{\text{bad loans} \leq x\} = F(x) = \int_o^x f(\xi)\, d\xi$,

and F(x) is a non decreasing function of x. If the bank makes loans l_i then, according to (8), the fraction of loans that default would be $F(l_i)$. Let $\alpha(l_i)$ denote this fraction so that $\alpha(l) = F(l_i)$. The expected loss on bad loans, those that default is shown in equation (9).

(9) Expected loss on bad loans $= \int_o^l \xi\, f(\xi)\, d\xi$,

keeping in mind that bad loans cannot exceed total loans. A conservative estimate, an upper bound, of this loss is $\alpha(l_i)\ l_i$.

The profit to bank i, π_i, equals the revenue from its loans, minus its payments to non borrowing depositors minus its loss on bad loans.

(10) $\pi_i = \rho\, l_i - \delta\, s_i - \alpha\, l_i = \rho(1-\theta)\, a_i - \delta\, \theta\, a_i - \alpha\,(1-\theta)\, a_i$.

Equation (10) assumes the bank pays no interest on the deposits of its loans to those who borrow from it. Simplify (10), divide by a_i and obtain the profit rate in (11).

(11) $\pi_i / a_i = (\rho - \alpha)(1-\theta) - \delta\, \theta$.

If $\pi_i = 0$, then equation (11) implies

(12) $\rho - \alpha = \delta\, \theta/(1-\theta) \rightarrow -\delta$ as $\theta \rightarrow 1$.

Perhaps the result in (12) explains bank hostility to 100 % reserves.

3. When Can Loans Increase

Now consider the effects of more bank's loans.

(1) $\Delta\, a_i = \Delta\, l_i + \Delta\, r_i = \Delta\, d_i = \Delta\, l_i + \Delta\, s_i - \Delta\, t_i$.

Equation (1) has a new term, Δt_i. It shows the loss of deposits in bank i as a result of checks written on the borrower's deposit account that went to those who have deposits in other banks. It follows that $\Delta\, t_i \leq \Delta\, l_i$ and, normally, there is strict inequality. Simplify equation (1) and assume s_i does not change. Hence $\Delta\, s_i = 0$ and

(2) $\Delta\, r_i = -\Delta\, t_i$.

This shows that more loans by bank i reduces its reserves by the amount shown in equation (2). The next step analyzes the effect of more loans on profits. To this end rewrite π_i from equation (8) above as follows:

(3) $\pi_i = [(\rho - \alpha)(1-\theta) - \delta\, \theta]\, a_i$.

Consequently,

(4) $\Delta\pi_i = [(\rho - \alpha)(1-\theta) - \delta\, \theta]\, \Delta a_i + a_i\, \Delta[(\rho - \alpha)(1-\theta) - \delta\, \theta]$.

The most important case to consider starts from an initial position at which the net revenue of the bank is zero. Equation (5) assumes this.

(5) $(\rho\text{-}\alpha)\,(1\text{-}\theta)\text{-}\delta\, \theta = 0$.

Equation (5) simplifies equation (4) so that $\Delta\, \pi_i > 0$ if and only if $\Delta[(\rho\text{-}\alpha)\,(1\text{-}\theta)\text{-}\delta\, \theta]>0$. This leads to the central question. When would it be true that

(6) $\Delta[(\rho\text{-}\alpha)\,(1\text{-}\theta)\text{-}\delta\, \theta]>0$?

Part of the answer is on hand. Because the increase in loans reduces reserves, $\Delta\, r_i = -\Delta t_i \leq 0$ and $\Delta\, a_i = \Delta\, l_i - \Delta\, t_i > 0$, it follows that the reserve ratio θ must have gone down. If this were all that happens, then,

according to (6), profits would rise, assuming ρ - α does not change. But this assumption is not plausible. To lend more, the bank needs more borrowers. But more borrowers means more loans likely to default so that α would rise. Figure 1 shows the probability of bad loans, as a function of loans. This graph reflects the fact that more loans on mortgages went to riskier prospects so the fraction of bad loans among the total went up. Not only did this happen in the last decade but also it happened from 1928 to 1929.

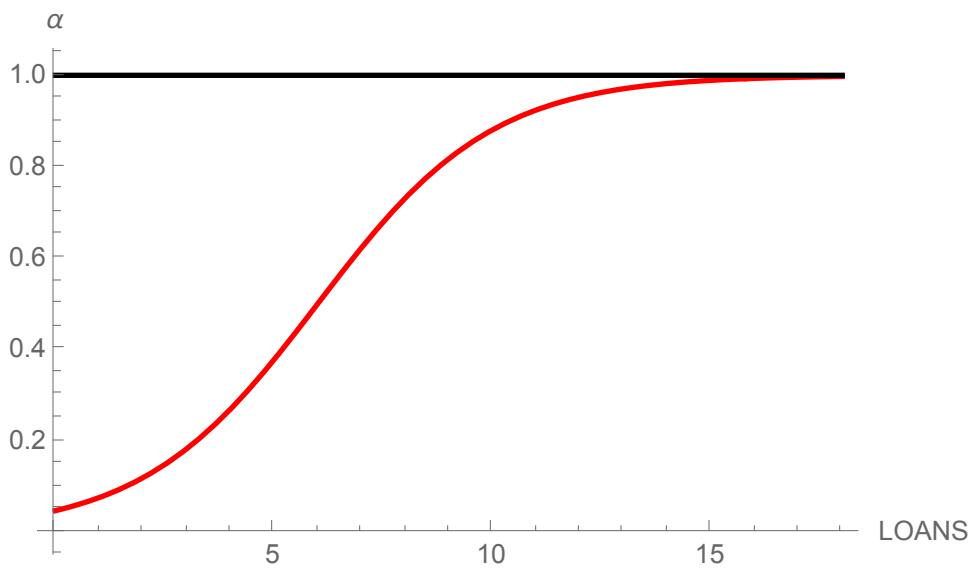

Figure 1

To compensate for greater risk, the bank asks a higher interest on its new loans. This raises ρ to offset the bigger α. Indeed the most favorable case for more loans in view of the lower reserve ratio is for ρ and α to rise equally keeping their difference the same. Now there is a determinate result, profits are bigger on the new loans but risks from a lower reserve ratio are also higher. Summarizing, it takes a higher interest rate on loans to encourage more bank loans at the cost of higher risk. An important conclusion follows. If interest rates cannot rise, loans will not increase.

4. What If All Banks Move Together

Assuming all banks move together simplifies study of basic determinants of loans because it may ignore the drain on an individual bank's reserves

due to a loss of deposits to other banks.. The capital letters refer to all banks and correspond to the lower case letters that refer to individual banks. Total reserves are default free and are proportional to total assets according to (3). A fraction α of all loans default. Equations (1) - (6) are counterparts for all banks to the similar equations for one bank.

(1) $A = L + R$,

(2) $D = L + S$.

(3) $R = \theta A$,

(4) $L = (1 - \theta) A$.

(5) $\pi = \rho L - \delta S - \alpha L$

(6) $\pi = [(\rho - \alpha)(1-\theta) - \theta \delta] A$

Let Γ represent the rate of return on total assets.

(7) $\Gamma = [(\rho - \alpha)(1-\theta) - \theta \delta]$.

(8) $\pi = \Gamma A$.

(9) $\Delta\pi = \Gamma \Delta A + A \Delta\Gamma$.

If $\Gamma = 0$, then $\Delta\pi = A \Delta\Gamma$. To assume $\Gamma = 0$ simply means that revenue covers cost. Equation (9) assumes all the variables are deterministic. It ignores how random variables, notably bad loans, affect profits.

To see what lies behind the reserve ratio needs a closer look at the loss from bad loans. To this end It is necessary to distinguish between the ex ante variables that are deterministic and the ex post variables that are random, especially bad loans. Let L_o denote ex ante loans and L denote ex post loans. Let W denote the bad loans. Bad loans reduce bank assets similar to the effect of withdrawals. The loss on bad loans cannot exceed the total ex ante loans, $W \leq L_o$.

(10) $\Pr\{W \leq R\} = \alpha(R)$.

This is the same as β in (2.5).

(11) $\Pr\{R \leq W \leq L_o\} = \alpha(L_o) - \alpha(R)$.

is the probability that bad loans exceed reserves. Nor is this all. It is an implication of equation (11) that the change in reserves necessary to keep the probability of loss unchanged as loans increase depends on the size of ex ante loans. For the cdf in Figure 2, the required change is biggest in the middle range of L and is smallest at both the lower and upper ends of the cdf. An increase in loans causes a more than proportionate increase in bad loans. To cover these losses requires a

more than proportionate increase in the interest rate. A lid on interest rates discourages new loans because such loans entail more risk.

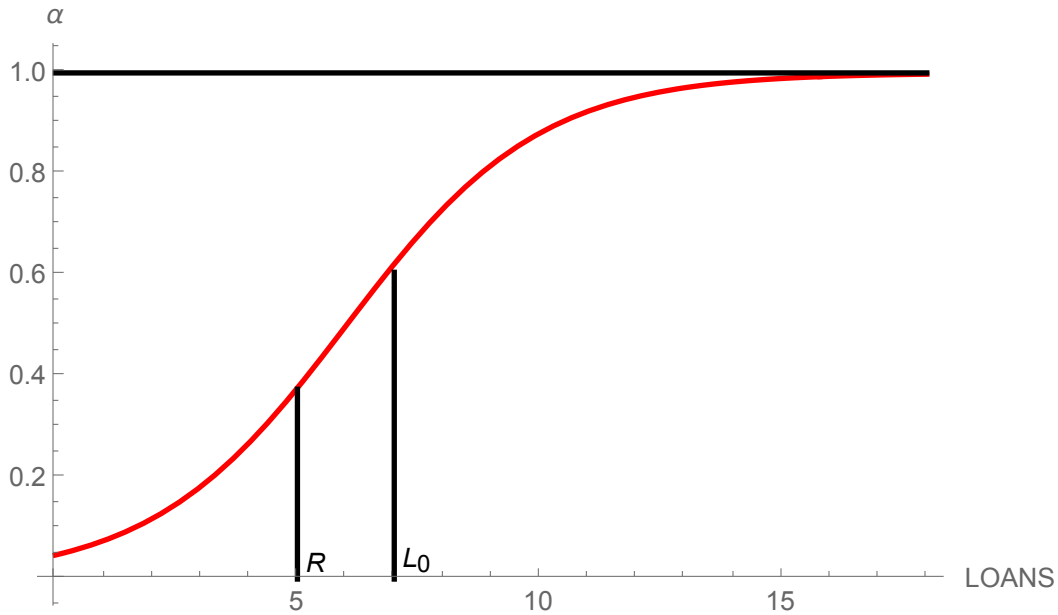

Figure 2

References

Telser, L. G. 1955. Safety First and Hedging. Review of Economic Studies. XXIII (1- 16)

Post Script

The post script handles some tricky material on the relation between bad loans and total loans. Let w denote bad loans and l denote total loans. Necessarily $0 \le w \le l$ because bad loans cannot exceed the total. Suppose w/l were an increasing function of total loans, say,

(1) $\frac{w}{l} = l^{\beta}$, $\beta > 0$.

(2) $w = l^{\beta+1}$.

Since w cannot exceed l, substitute w = l in (2) and obtain $l = l^{\beta+1}$. This says that $l(l^{\beta} - 1) = 0$. If $l \neq 0$, then $l^{\beta} = 1$ and $l = 1$. Therefore, equation (1) can hold only for $0 < w/l \le 1$. For example, suppose $f(\alpha) = \alpha^2(3 - 2\alpha)$ and $0 \le \alpha \le 1$. Write $w = f(\alpha)\, l$. Although w is now an increasing function of l as required, $0 < f(\alpha) \le 1$, w equals l only at $\alpha = 1$.

Hence this hypothesis about the ratio does not lead to desired results.

We now need the cdf for bad loans, F[l]. As is reasonable for all $w \leq l_o$, $F[w] < F[l_o] < 1$. It is never true that all loans default for sure as long as loan size is finite. The probability that loss from bad loans will not exceed reserves is $F[r_o]$. The probability that loss from bad loans will exceed reserves is $F[l_o] - F[r_o]$. This probability goes up if loans go up and reserves do not go up. Also, depending on the shape of the cdf F[l], more loans may require a more than proportional increase in reserves to maintain a constant probability of the bank's survival. Bank failure means being unable to meet its obligations because it has run out of reserves.