The Wealth of Nations Updated

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Introduction

Few concepts are more important for valid economics than cost. Anything with no alternative use is costless because it forgoes nothing to use it. Ricardo uses land to illustrate this point. To readers familiar with the discovery of the New World this example is not far fetched but it would give pause to a reader having in mind a long settled country. Presumably, Ricardo must mean the cost of land is measured by the best alternative to its current use. Land would be costless if and only if it had no use apart from producing agricultural commodities, all taken to be identical. Of course, this is too simple. If land could be used to produce different agricultural commodities that would compete for the given land, then this would impose a positive cost on land. Even were this not so and there were only a single agricultural commodity, wheat [corn] in classical economics, this would not rule out use of complementary resources that do have alternative opportunities. As a first approximation to take this into account for a better definition of cost, Ricardo needs a measure of the input with the most alternative uses. He chose labor. Of course, Ricardo knows that labor is not the only input even though he sometimes treats capital as stored up labor. Yet it would be hard to find another input as pervasive as labor. His hypothetical economy has three factors of production, land, labor and capital. Labor receives a fixed wage. Land owners receive rent (see Appendix). Capitalists receive profit. For more than three factors it is easier for me to use algebra. My interpretation of his model assumes the objective of the economy is the production of the most food from its given resources.

Classical Cost

There are m commodities, the outputs, and n factors of production, the inputs. The outputs are the coordinates of the m-vector X. All units of
commodity i are identical. The inputs are the coordinates of the n-vector Y. All units of factor j are identical. Inputs and outputs are flows. No stocks are in the model of the economy.

The inequalities in (1) describe demand conditions. They assert that if the output of commodity i exceeds the minimal quantity, \( b_i \), then the shadow price of commodity i, \( \lambda_i \), is 0. On the other hand, if the shadow price is positive, then the output of commodity i equals the required level.

\[
(1) \quad x_i \geq b_i > 0, \quad \lambda_i \geq 0, \quad (x_i - b_i) \lambda_i = 0 \quad \text{and} \quad i = 2, 3, ..., m.
\]

At least one of the m-1 commodities must be subject to a minimal required level as we shall see.

The inequalities in (2) describe the upper bounds on the available resources. If less than the available amount of resource j is used so that \( y_j < c_j \), then the shadow price of resource j, \( \mu_j \), is 0. Now because more of this resource is available than is used, it commands no positive shadow price. However, if the shadow price of resource j is positive, then the available quantity of this resource is fully utilized.

\[
(2) \quad y_j \leq c_j, \quad \mu_j \geq 0 \quad \text{and} \quad (c_j - y_j) \mu_j = 0.
\]

The inequalities in (3) describe the technology of the economy. They show the most that can be produced of the m commodities using the n resources available. It takes \( a_{ij} \) units of input j to produce one unit of commodity i. The mxn matrix \( A = [a_{ij}] \geq 0 \). The quantity acquired cannot exceed the quantity produced. If the quantity of commodity i acquired is below the amount produced by the n inputs, then its shadow price, \( \pi_i \), is 0. On the other hand if the shadow price of commodity i is positive, then the quantity acquired equals the quantity produced.

\[
(3) \quad 0 \leq x \leq Ay, \quad \pi \geq 0 \quad \text{and} \quad \pi_i \left( \sum_{j=1}^{n} a_{ij} y_j - x_i \right) = 0.
\]

The output of food is \( x_1 \). The greater the food output, the less the outputs of the other m-1 commodities. Therefore, the outputs of all these m-1 commodities are costs for the food output. Now we are ready to study the equilibrium of the model of the economy as a solution to
the following primal and dual problems.

The primal problem is

\[ \text{(4)} \quad \text{Maximize } x_1 \text{ with respect to nonnegative } x \text{ and } y \text{ subject to (1)-(3)}. \]

Its Lagrangian is

\[ \mathcal{L}(x,y) = x_1 + \sum_{i=2}^{m} \lambda_i (x_i - b_i) + \pi (A \cdot Y - X) + (c - y) \mu, \quad \pi = \{\pi_i; \lambda_i = \mu; j = 1, 2, ..., n\}. \]

If the primal has a solution, then it must satisfy

\[ \text{(6)} \quad 1 - \pi_1 \leq 0, \quad x_1 \geq 0 \quad \text{and} \quad x_1 (1 - \pi_1) = 0. \]

Therefore, if the output of food is positive, then the price of food is 1. If the price of food were less than 1, then the output of food would be zero, a theoretical curiosum.

\[ \text{(7)} \quad \lambda_i - \pi_i \leq 0, \quad x_i \geq 0 \quad \text{and} \quad x_i (\lambda_i - \pi_i) = 0, \quad i = 2, ..., m. \]

The inequalities in (7) are classical. For any commodity i that is actually produced, the shadow price of the quantity acquired equals the shadow price of the quantity supplied. However, if the shadow price of the quantity acquired is below the shadow price of the quantity supplied, then it would not be produced. Nothing is produced at a loss.

\[ \text{(8)} \quad \pi A - \mu \leq 0, \quad y \geq 0 \quad \text{and} \quad (\pi A - \mu)x = 0. \]

The inequalities in (8) are no less classical than those in (7). If the shadow revenue from the use of any factor j is below its shadow value, then this factor is idle, \( y_j = 0 \). The factor price for any fully utilized factor equals its shadow revenue. The equality in (8) says that no actively produced commodity yields a positive profit. It just covers its cost of production.

The dual problem minimizes \( c \mu - \sum_{i=2}^{m} \lambda_i b_i \) with respect to \( \lambda, \mu \) and \( \pi \) subject to (6), (7) and (8). A solution of the dual must satisfy the primal constraints, (1)-(3).

First, we must see whether this model of the economy is feasible. It follows from (1) and (3) that

\[ \text{(9)} \quad x_i \geq b_i > 0 \quad \text{requires} \quad \sum_{j=1}^{n} a_{ij} y_j \geq x_i. \]

It is an implication of (2), (3) and (9) that

\[ \text{(10)} \quad 0 \leq y_j \leq c_j \quad \text{requires} \quad \sum_{j=1}^{n} a_{ij} c_j \geq \sum_{j=1}^{n} a_{ij} y_j \geq x_i \geq b_i. \]
Therefore, the problems are solvable if and only if the demand requirements given by (1) and the resources available given by (2) can be satisfied by the input-output technology described by input-output coefficients of the mxn matrix A.

\[ \sum_{j=1}^{n} a_{ij} c_j \geq b_i \text{ for all } i \text{ and } j. \]

If the inequalities in (11) are feasible, then the duality theorem of linear programming applies. It says the primal objective that maximizes the output of food equals the dual objective that minimizes the net cost of doing so.

\[ \text{Max } x_1 = \text{Min } (c \mu - \lambda b). \]

Equation (12) may seem strange because it relates a physical term on the left to a monetary term on the right. We must clarify this.

Inequality (6) applies to the shadow price and actual output of food. From (6), \( x_1 > 0 \) implies \( \pi_1 = 1 \) and \( \pi_i < 1 \) implies \( x_1 = 0 \). Therefore, all shadow prices of outputs and inputs are relative to the shadow price of food that equals 1. The integer 1 is implicit in front of \( x_1 \). It multiplies the quantity of food \( x_1 \) in (12). Hence equation (12) is in real terms, not monetary terms.

A sensible model of the economy needs at least one demand constraint. The proof is by contradiction. Suppose all the b’s were zero. This would remove all the demand terms from the Lagrangian in (5). Consequently, the \( \lambda \)'s in (7) would vanish. There would be positive outputs of commodities 2 to m only if they were free. The economy would focus on the maximum output of food. Outputs of any other commodity would be ancillary. This is absurd. Hence a sensible model of the economy must include a positive output of at least one commodity besides food.

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Old Knowledge is Free but New Knowledge Is Not Free

We can see why land is free in classical economics. It is because land has no use outside agriculture. The question is whether there is anything in a modern economy that is costless in the classical meaning. Consider the accumulated knowledge of mathematics. It has many related parts
according to present understanding of this ancient subject. Among these is trigonometry useful in surveying. It is also useful in navigation. Its use in navigation neither diminishes nor even affects the trigonometry available for use in surveying and conversely. For this reason the knowledge of trigonometry is costless. It is obviously also true, however, that anyone contemplating a career that uses trigonometry can pursue either of these occupations. The cost resides in the person who has these alternatives not in the stock of trigonometric knowledge, a huge stock indeed. In the present state of technology a human input can expand the stock of mathematical knowledge. This opens room for debate about the best directions to seek advances of mathematical knowledge. Not only do these incur costs, but they also raise issues that classical economics is ill equipped to handle. To whom should society endow property rights on new knowledge costly to find or create?

Existing knowledge is free but improving and enlarging it is costly. It uses resources that have other uses. There is some parallel to capital goods but the nature of knowledge poses an inherent difficulty. Many can benefit from what is known without reducing the amount or interfering with the use by others. New knowledge typically promises little if any immediate yield. Sometimes one can trace a long and tangled path to practical ends. The prodigious growth of scientific knowledge seems barely amenable if at all to an explanation based on business motives.

A work force of constant size can maintain a constant food output based on their current understanding of this technology. The economy could obtain more food output even with a constant population if some members of the work force move out of food production into research provided they are sufficiently talented to raise the stock of knowledge by more than enough to compensate for the immediate sacrifice of their contribution to food production. This is not the situation I study in the following model. It focuses on the consequences of a rising population, the Malthus problem.
Definition of variables in my model

\[ k[t] = \text{stock of knowledge at time } t \]
\[ l[t] = \text{number of workers at time } t \]
\[ f[l[t]] = \text{number of workers in food production at time } t \]
\[ k[l[t]] = \text{number of workers increasing knowledge at time } t \]
\[ f[t] = \text{food production at time } t \]

Description of my model

The total work force at time \( t \) has two parts, the work force in food production and the work force in research. By writing the total work force as the sum of these two the model assumes the workers in these two activities are perfect substitutes for each other.

(1) \[ l[t] = f[l[t]] + k[l[t]]. \]

The food output is a concave function of the number of food workers and the stock of knowledge as follows.

(2) \[ f[t] = f[l[t]] k[l[t]] \]

This is the simplest version capable of accommodating the hypothesis that a positive human input is necessary for a positive food output. It rules out the possibility of a total robotic takeover of food production. However, this formulation also assumes that no food production can occur without some knowledge. The next equation describes the knowledge sector. Knowledge increases in proportion to the existing stock of knowledge and in proportion to the work force in the new knowledge sector. However, the stock of knowledge does not increase at all if no workers do research. Equation (3) describes the production function for the research sector.

(3) \[ \partial_t \log(k[t]) = \begin{cases} \beta \partial_t \log k[l[t]] & \text{if } k[l[t]] > 0 \text{ and } \beta > 0 \\ 0 & \text{if } k[l[t]] = 0 \end{cases} \]

The food output at time \( t \) is a maximum if the entire work force is assigned to food production and none is assigned to research. In this case the stock of knowledge enters the food production function as a constant \( k \).

(4) \[ f[t] = l[t]^\lambda k^\gamma. \]

Equation (4) shows that even in this case with the constant existing
stock of knowledge, the food output is higher not only thanks to this knowledge but also by virtue of more workers in food production.

A better way to see how research affects food production begins with the production function in equation (2). Take logs and derivatives to obtain

(5) \[ \frac{\partial_t \log f[t]}{f[t]} = \lambda \frac{\partial_t \log l[t]}{l[t]} + \gamma \frac{\partial_t \log k[t]}{k[t]} \]

If all workers are assigned to food production and none to produce knowledge so \( kl[t]=0 \), then equation (3) becomes

(6) \[ \frac{\partial_t \log k[t]}{k[t]} = 0. \]

Together with equation (5) it follows that

(7) \[ \frac{\partial_t \log f[t]}{f[t]} = \lambda \frac{\partial_t \log l[t]}{l[t]} < \frac{\partial_t \log k[t]}{k[t]} \]

because \( \lambda<1 \). This completes the proof of the following Proposition 1. If all the workers are assigned to food production and none to knowledge production, then food output per capita will decrease over time.

To keep food per capita constant over time in this model it is not enough that some workers are assigned to the task of increasing knowledge. They must also be very efficient in a sense we now derive. This analysis begins with equation (5). Replace the term involving the change in the stock of capital with the first expression from equation (3). This gives

(8) \[ \frac{\partial_t \log f[t]}{f[t]} = \lambda \frac{\partial_t \log f[l]}{l[t]} + \gamma \beta \frac{\partial_t \log k[l]}{k[t]} \]

In terms of per capita food production equation (8) becomes

(9) \[ \frac{\partial_t \log f[t]/l[t]}{l[t]} = \lambda \frac{\partial_t \log f[l]}{l[t]} + \gamma \beta \frac{\partial_t \log k[l]}{k[t]} - \frac{\partial_t \log l[t]}{l[t]} \]

It would not be illuminating to stop here and simply assert that to maintain constant per capita food requires the parameters and variables to satisfy equation (10) that assumes \( \frac{\partial_t \log f[t]/l[t]}{l[t]} = 0 \).

(10) \[ \lambda \frac{\partial_t \log f[l]}{l[t]} + \gamma \beta \frac{\partial_t \log k[l]}{k[t]} - \frac{\partial_t \log l[t]}{l[t]} = 0 \]

A more interesting hypothesis asks a more pertinent question. Suppose the ratio of the food production work force to the new knowledge production work force remains constant over time. Under these conditions what must be true of the parameters to maintain a constant output of food per capita over time. The hypothesis says

(11) \[ \frac{\partial_t \log (f[l]/l[t])}{l[t]} = \frac{\partial_t \log (k[l]/l[t])}{l[t]} = 0. \]
Equation (11) and some algebra simplifies equation (10) to

\[(\lambda + \gamma \beta - 1) \partial_t \log l[t] = 0.\]

If the labor force is growing so that \(\partial_t \log l[t] > 0\), then (12) requires

\[(\lambda + \gamma \beta - 1) = 0 \rightarrow \gamma \beta = 1 - \lambda \geq \gamma.\]

Divide the latter expression in (13) through by \(\gamma\) and obtain

\[\beta \geq 1.\]

The next proposition summarizes this result.

Proposition 2. Assume a growing work force and that the ratio of workers in the two sectors, food production and knowledge production stays constant over time. The output of food per capita can also remain constant over time if there are nondecreasing returns in the knowledge production sector.

Before continuing the analysis of the role of knowledge in food production, it is important to note that a constant ratio over time between workers in the two sectors of this model of the economy says nothing about the size of this ratio. It suffices that some workers are in the new knowledge producing sector.

Proposition 2 has a corollary. It says that if the output of food per worker stays constant over time and there are nondecreasing returns in the knowledge producing sector, then the stock of knowledge must be increasing relative to food production. In other words, the stock of knowledge per worker in the food production sector must be increasing. The proof starts with equation (15).

\[\partial_t \log \frac{f[t]}{k[t]} = \left(\frac{1}{\beta} + \gamma - 1\right) \partial_t \log k[t].\]

By equation (13) it follows that \(\left(\frac{1}{\beta} + \gamma - 1\right) < 0\). Consequently, an increasing stock of knowledge implies that \(\partial_t \log \frac{f[t]}{k[t]} < 0\). This proves the Corollary. If the ratio between food production workers and knowledge production workers remains constant over time and the output of food per capita remains constant over time, then knowledge per unit of food output must be rising over time.
Conclusions

Measuring the cost of new knowledge is straightforward. Measuring the effects of new knowledge is exceedingly difficult. It is the rare case that can trace this cost directly to a commodity which can be sold. Support for the production of new knowledge must come outside the private profit seeking sector. Patrons for new knowledge presumably have incentives and motives other than those that drive the private sector. Economic models that meet success in explaining the private sector fail to explain what happens in the new knowledge sector. Here we observe public support and private philanthropy, new terrain for economics.

Appendix: On Rent

Rent is a complicated concept. It is the return to owners of a factor available in a fixed amount that cannot be increased and has only a single use. The return to land would be rent if there were only one crop that could be grown on the given amount of land, none of the land is idle and it is impossible to increase the amount of cultivable land. A more useful definition allows the given factor to have alternative uses. In this case the rent to the owner of the factor in fixed supply is the current return minus the return for the best alternative use. This makes rent a nonnegative surplus to the owner of the factor. On this understanding it does not harm the validity of the analysis and simplifies it to assume the amount of the factor is fixed and that the factor has no alternative use. Therefore, the return to the factor is wholly driven by the demand for it.